Introduction to Robotics

[ME_639]

Mini-Project

[Task_0]

Submitted by:

Shubham Singh

21250025

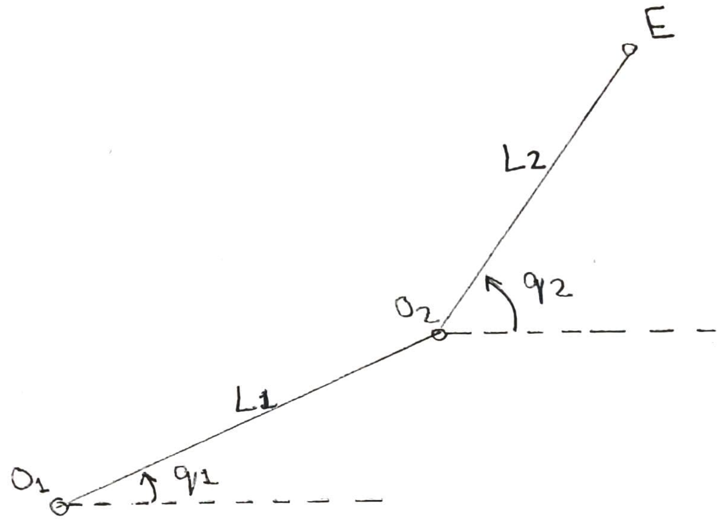
M.Tech (Mechanical)

Date of Submission: 16/08/2021

Shubham Singh, 21250025

⇒ Task-0

Consider the line diagram of a 2R manipulator as shown in the figure below;



⇒ Equation-1 (Forward Kinematics)

Let O_1 be the origin of the 2-R manipulator (elbow manipulator)

For link-1;

$l_1 \rightarrow$ length of the link-1

$m_1 \rightarrow$ mass of link-1

$I_1 \rightarrow$ Inertia of the link-1

For link-2;

$l_2 \rightarrow$ length of the link-2

$m_2 \rightarrow$ mass of the link-2

$I_2 \rightarrow$ Inertia of the link-2

At O_2;

$$O_{2x} = l_1 \cos(q_1) \quad [1-a]$$

$$O_{2y} = l_1 \sin(q_1) \quad [1-b]$$

Similarly,

At End effector (E);

$$E_x = O_{2x} + l_2 \cos(q_1+q_2)$$

Using eq (1-a), we get;

$$\rightarrow E_x = l_1 \cos(q_1) + l_2 \cos[q_1+q_2] \quad [1-c]$$

$$E_y = O_{2y} + l_2 \sin[q_1+q_2]$$

Using eq(1-b), we get;

$$\rightarrow E_y = l_1 \sin(q_1) + l_2 \sin[q_1+q_2] \quad [1-d]$$

Let's call the coordinates of the end effector as X & Y;

$\therefore$

$$\boxed{\begin{aligned} E_x = X &= l_1 \cos(q_1) + l_2 \cos[q_1+q_2] \\ E_y = Y &= l_1 \sin(q_1) + l_2 \sin[q_1+q_2] \end{aligned}} \rightarrow \text{Equation}-1$$

Equation-1 gives the forward kinematics of the end effector (E) of the 2-R manipulator.

$\Rightarrow$ Equation-2 [Velocity of End Effector]

Differentiating Equation-1; we get;

$$\dot{X} = -l_1 \sin q_1 \cdot \dot{q}_1 - l_2 \sin[q_1+q_2] \cdot [\dot{q}_1+\dot{q}_2]$$

$$\dot{Y} = l_1 \cos q_1 \cdot \dot{q}_1 + l_2 \cos[q_2+q_1] \cdot [\dot{q}_1+\dot{q}_2]$$

Therefore,

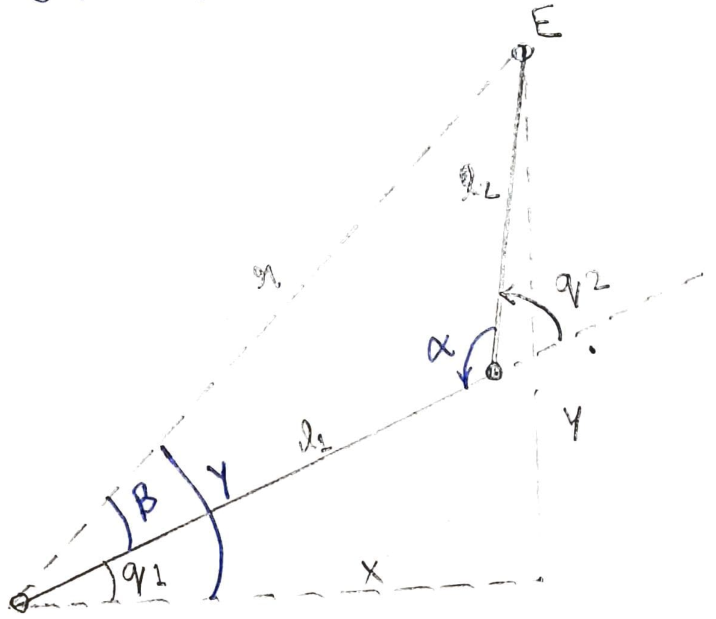
Equation of End-Effector Velocity can be written as:

$$\boxed{\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin[q_1+q_2] \\ l_1 \cos q_1 & l_2 \cos[q_1+q_2] \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}}$$

↳ Equation-2

⇒ Equation-3 [Inverse Kinematics]

The distance from the end effector to the robot base (shoulder-Joint) is r and can be written in terms of X & Y using Pythagoras theorem;



$$r^2 = x^2 + y^2 \qquad [3\text{-}a]$$

Using Law of Cosines, we get;

$$r^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos(\alpha) \qquad [3\text{-}b]$$

$$\because \alpha = \pi - q_2 \qquad [3\text{-}c]$$

$$x^2+y^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos(\alpha)$$

$$x^2+y^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos[\pi - q_2] \qquad [3\text{-}d]$$

$$2 l_1 l_2 \cos[\pi - q_2] = l_1^2 + l_2^2 - [x^2+y^2]$$

$$\cos[\pi - q_2] = \frac{l_1^2 + l_2^2 - [x^2+y^2]}{2 l_1 l_2} \qquad [3\text{-}e]$$

$$\because \cos[\pi - q_2] = -\cos(q_2)$$

$$\therefore \cos q_2 = \frac{x^2+y^2 - [l_1^2 + l_2^2]}{2 l_1 l_2}$$

$$\therefore \boxed{q_2 = \cos^{-1}\left[\frac{x^2+y^2 - [l_1^2+l_2^2]}{2 l_1 l_2}\right]} \qquad [3\text{-}f]$$

Also, $$\beta = \tan^{-1}\left(\frac{l_2 \sin(q_2)}{l_1 + l_2 \cos(q_2)}\right) \qquad [3\text{-}g]$$

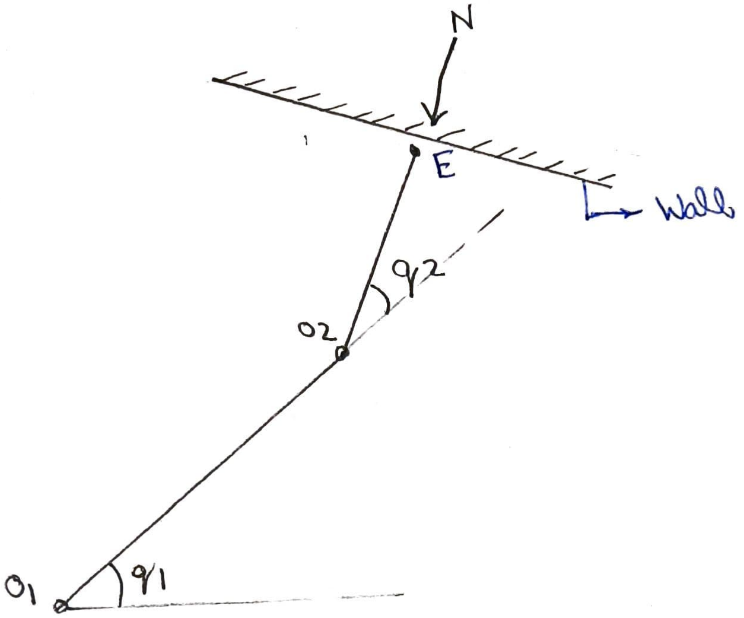
$$\gamma = q_1 + \beta \qquad [3\text{-}h]$$

$$q_1 = \gamma - \beta$$

$$\therefore \boxed{q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left[\frac{l_2 \sin(\theta_2)}{l_1 + l_2 \cos(\theta_2)}\right]} \qquad [3\text{-}i]$$

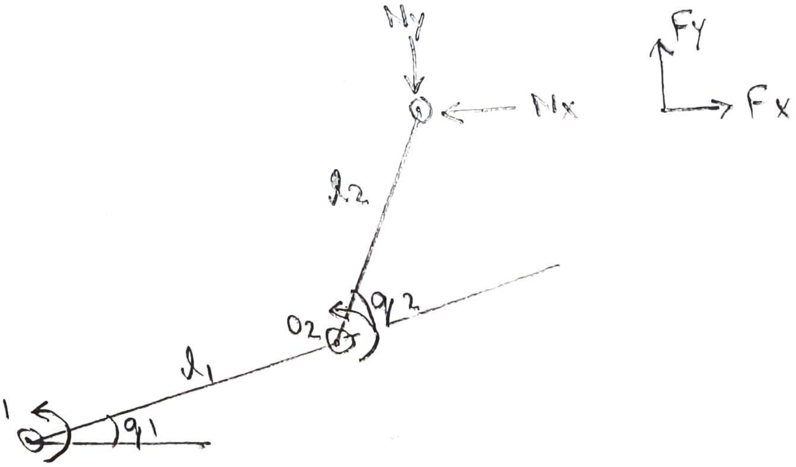
Equation (3-f) & (3-i) combined together Constitute Equation-3 of Inverse Kinematics of the end effector of 2R Manipulator.

⇒ Equation-4 [Force Control]



Consider the line diagram of the 2-R manipulator as shown in the figure above;

FBD :-



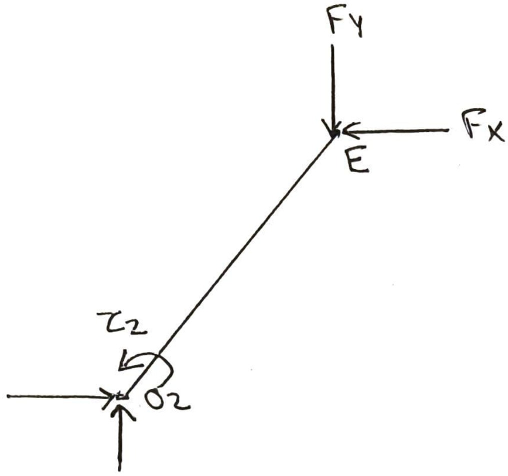
Considering static equilibrium;

$$\Sigma M_{O_1} = 0$$

$$\Sigma M_{O_2} = 0$$

Now, we will analyze each link separately;

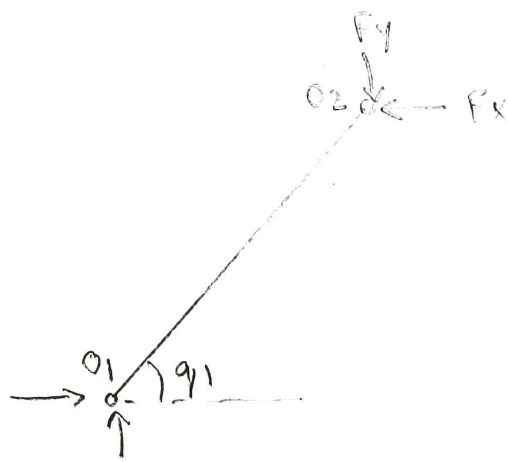
FBD of link-2;



$\Sigma M_{O2} = 0$

$$\boxed{F_y l_2 \cos q_2 - F_x l_2 \sin q_2 = \tau_2} \qquad [4-a]$$

FBD of link-1;



$\Sigma M_{O1} = 0$

$$\boxed{F_y l_1 \cos q_1 - F_x l_1 \sin q_2 = \tau_1} \qquad [4-b]$$

Combining eq-[4-a] and [4-b], we get Equation-4.

Equation – 5; [Lagrange's Equation]

Lagrange's equation is given by;

L = Kinetic Energy [K] – Potential Energy [V]

$\therefore$ $L = K - V$

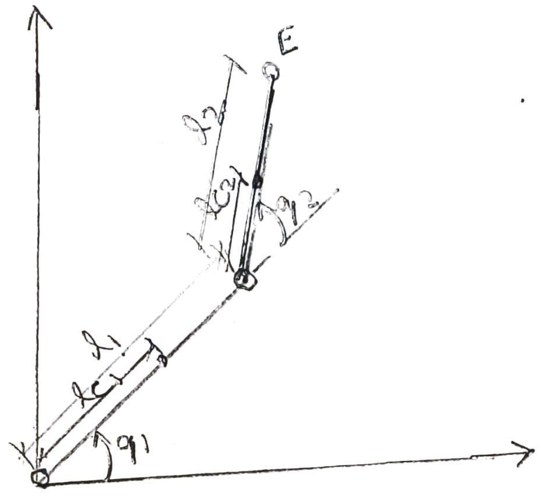
Also;

$$\boxed{\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right] - \left[\frac{\partial L}{\partial q_i}\right] = Q_i'}$$

Q_i' is generalised forces derived using principle of Virtual work.

Equation – 6; [Dynamic's Equation]

Consider a 2-R manipulator as shown in figure below;



Kinetic energy term;

$$K = \frac{1}{2} m_1 V_{C1}^T V_{C1} + \frac{1}{2} m_2 V_{C2}^T V_{C2} = \frac{1}{2} \dot{q} \left[m_1 J_{Vc_1}^T J_{Vc_1} + m_2 J_{Vc_2}^T J_{Vc_2}\right] \dot{q}$$

Angular velocity terms;

$\omega_1 = \dot{q}_1 K$, $\quad \omega_2 = [\dot{q}_1 + \dot{q}_2] K$

Rotational energy term;

$$\frac{1}{2}\dot{q}^T \left\{ I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + I_2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\} \dot{q}$$

Now, Inertia Matrix;

$$D(q) = m_1 J_{vc1}^T J_{vc1} + m_2 J_{vc2}^T J_{vc2} + \begin{bmatrix} I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

Carrying out the multiplications of the above equations and using the standard trigonometric identities;

$$d_{11} = m_1 l_{c1}^2 + m_2 [l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} \cos q_2] + I_1 + I_2$$

$$d_{12} = d_{21} = m_2 [l_{c2}^2 + l_1 l_{c2} \cos q_2] + I_2$$

$$d_{22} = m_2 l_{c2}^2 + I_2$$

$$C_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -m_2 l_1 l_{c2} \sin q_2 = h$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = h$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -h$$

$$C_{122} = C_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

For each link, potential energy is just its mass multiplied by the gravitational acceleration and the height of its centre of mass.

Thus,

$$V_1 = m_1 g l_{c1} \sin q_1$$

$$V_2 = m_2 g [l_1 \sin q_1 + l_{c2} \sin[q_1 + q_2]]$$

$$V = V_1 + V_2$$
$$= [m_1 l_{c1} + m_2 l_1] g \sin q_1 + m_2 l_{c2} g \sin[q_1 + q_2]$$

Now,

$$\phi_1 = \frac{\partial V}{\partial q_1} = [m_1 l_{c1} + m_2 l_1] g \cos q_1 + m_2 l_{c2} g \cos[q_1 + q_2]$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 l_{c2} \cos[q_1 + q_2]$$

$\therefore$

$$\boxed{\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + \phi_1 &= \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 &= \tau_2 \end{aligned}}$$

→ Equation-6