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M. Tech [Mechanical]

Task-1

Singular Configuration :-

Singular Configurations are robot Configurations where a mani-pulator loses one or more of its degrees of freedom and therefore can't move in the corresponding directions. Singularities represent Configurations from which certain directions of motion may be unattainable.

Finding Singular Configuration :-

Singular Configurations are those at which the Jacobian is rank-deficient. We can find conditions for singularity by solving det[J] = 0.

The last row of J being all ones means that no matter the Configuration, you can generate some angular velocity. This implies that the Conditions we get from solving det(J) = 0 will be the Conditions for linear velocity singu-larities.

→ Yes, we can detect if a particular Configuration is close to a Singular Configuration by finding the determinant of the Manipulator Jacobian Matrix. All Joints angle sets which result in zero or near zero determinant are at or near Singular Configurations.
Singularity instantaneously causes the loss of one or more DoF. In the neighbourhood of Singularities, a finite Cartesian velocity command requires joints velocities approaching infinity.

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Task-4

Verification Using Code :-

(1) Stanford Manipulator;

Assuming numerical values for joint parameters;

$d_1 = 0$; $d_2 = 2$; $d_3 = 2$

$a_1 = 0$; $a_2 = 0$; $a_3 = 0$

$\alpha_1 = -90$; $\alpha_2 = +90$; $\alpha_3 = 0$

$\theta_1 = 45$, $\theta_2 = 90$, $\theta_3 = 0$

$$\therefore \text{Jacobian Matrix} = \begin{bmatrix} -1.41 & -1.41 & 7.071 \\ 1.41 & 1.41 & 7.071 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- The values in the Jacobian matrix matches with the values of matrix we derived earlier.

Similarly;

(2) SCARA Manipulator;

Assuming numerical values for the joints parameters;

$d_1 = 0$; $d_2 = 0$; $d_3 = 1$

$a_1 = 1$; $a_2 = 2$; $a_3 = 0$

$\alpha_1 = 0$; $\alpha_2 = 180$; $\alpha_3 = 0$

$\theta_1 = 90$; $\theta_2 = -45$ $\theta_3 = 0$

we get';

Jacobian Matrix :-

$$\begin{bmatrix} -2.414 & -1.414 & 8.65 \\ 1.414 & 1.414 & -8.65 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

The Values given by the Code are same as obtained by the derived Jacobian Matrix.

Hence, Verified.

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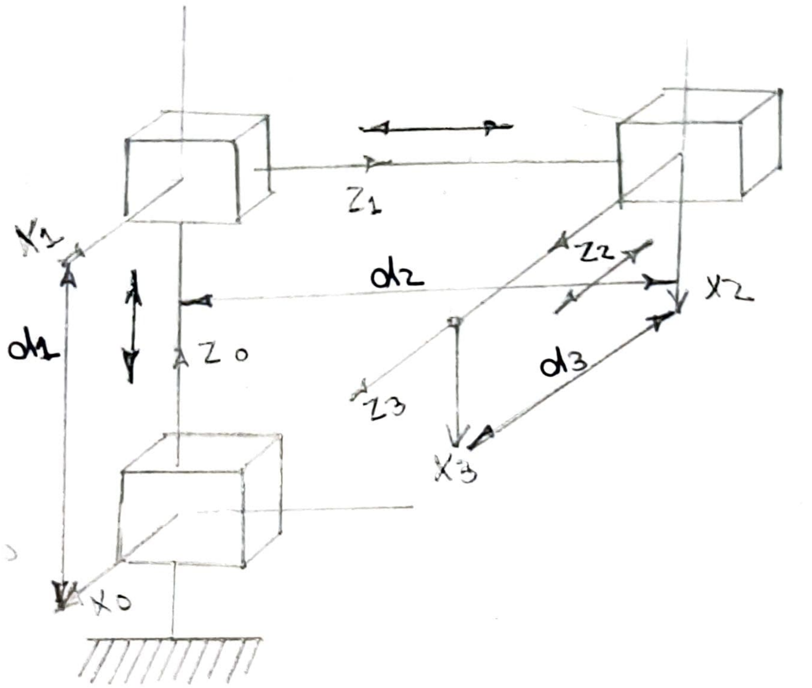
Task-5

Given :- 3 link Cartesian Manipulator

To Derive :- Forward Kinematic equations using DH-Conventions



DH Parameters Table for 3-link Cartesian Manipulator

Link	a_i	α_i	d_i	θ_i
1	0	-90°	d_1	0
2	0	90°	d_2	90°
3	0	0	d_3	-90°

$$A_1 = \begin{bmatrix} C\theta & -S\theta C\alpha & S\theta S\alpha & a_1 C\theta \\ S\theta & C\theta C\alpha & -C\theta S\alpha & a_1 S\theta \\ 0 & S\alpha & C\alpha & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Substituting the values from the DH Table, we get;

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad -(1)$$

Similarly;

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad -(2)$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad -(3)$$

The forward kinematics is given by;

$$T_0^3 = A_1 A_2 A_3$$

on solving; we get

$$T_0^3 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The above matrix gives the forward kinematics of the 3-link Cartesian Manipulator.

<u>Verification using the Code of Task-3;</u>

Using the Numerical Value of d_1, d_2, d_3 as;

$d_1 = 1$

$d_2 = 2$

$d_3 = 3$

Using the parameters of the DH Table as input in the code, we get;

$$T_0^3 = \begin{bmatrix} 0 & 0 & 1 & 3 \\ -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Position of end effector in X-direction :- 3.0

Position of end effector in Y-direction :- 2.0

Position of end effector in Z-direction :- 1.0

T_0^3 given by the Code matches with that of desired Value. Also, the Value of end effector position of Code matches with the desired Values. Hence, Verified.

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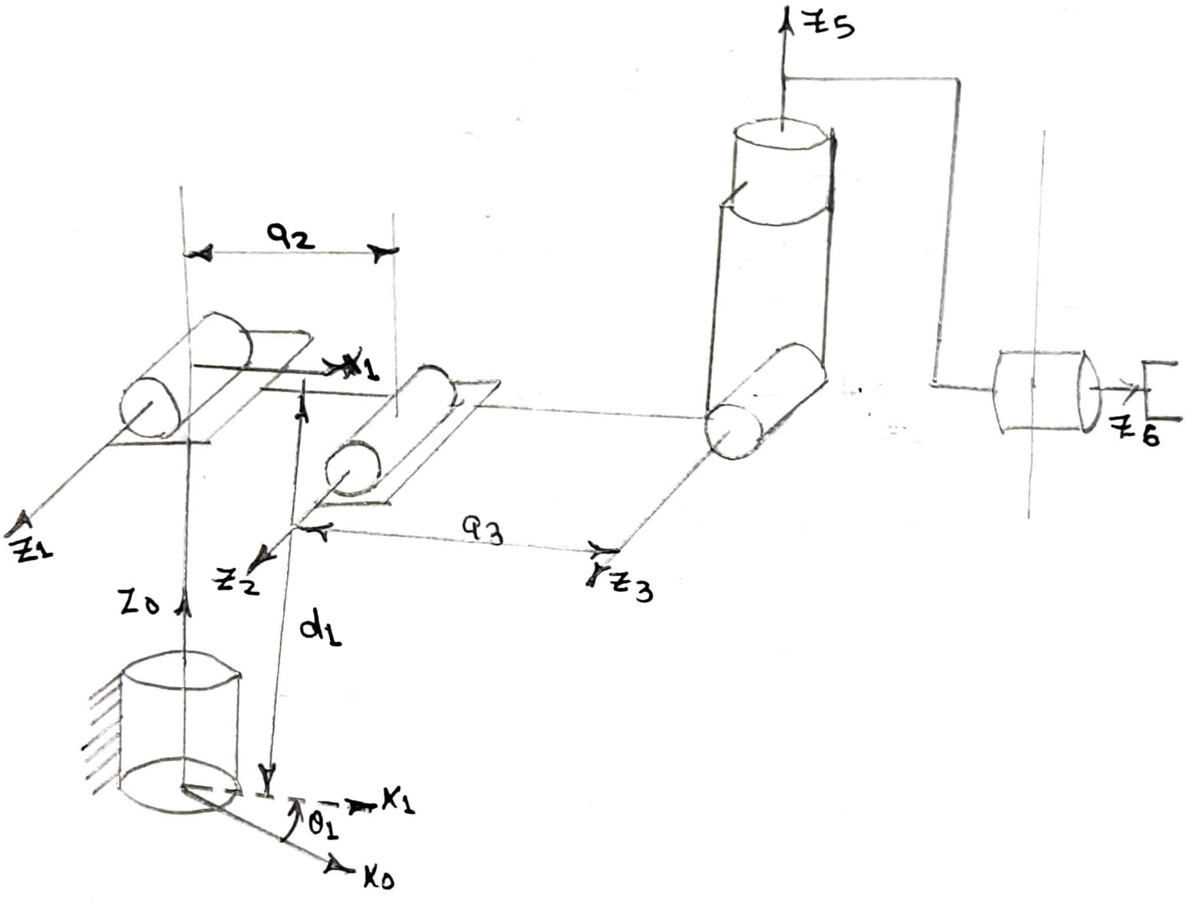
Task - 6

Attach a spherical wrist to the three link articulated manipulator

To derive :- Forward kinematics equations for this manipulator



DH Parameters Table :-

Link	a_i	α_i	d_i	θ_i
1	0	90°	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	-90°	0	θ_4
5	0	0	0	θ_5
6	0	0	d_6	θ_6

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T_0^6 is given by;

$$T_0^6 = A_1 A_2 A_3 A_4 A_5$$

$$\therefore T_0^6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$r_{11} = C_1[C_5C_6C_{234} - S_6S_{234}] - S_1S_5S_6$

$r_{12} = -C_1[C_5S_6C_{234} + C_6S_{234}] + S_1S_5S_6$

$r_{13} = C_1S_5C_{234} + S_1C_5$

$d_x = a_2C_1C_2 + a_3C_1C_{23} + d_6[C_1S_5C_{234} + S_1C_5]$

$r_{21} = C_1S_5S_6 + S_1C_5C_6C_{234} - S_1S_6S_{234}$

$r_{22} = -C_1S_5S_6 - S_1C_5S_6S_{234}$

$r_{23} = -C_1C_5 + S_1S_5C_{234}$

$d_y = a_2S_1C_2 + a_3S_1C_{23} - d_6[C_1C_5 + S_1S_5C_{234}]$

$r_{31} = S_6C_{234} + C_5S_6S_{234}$

$r_{32} = C_6S_{234} - C_5S_6S_{234}$

$r_{33} = S_5S_{234}$

$d_z = a_2S_2 + a_3C_2S_{23} + d_6S_5S_{234}$

on simplifying, we get;

$$T_0^6 = \begin{bmatrix} -C_6S_5 & S_5S_6 & C_5 & (d_3 + d_6S_5) \\ (-C_4C_5C_6 + S_4S_6) & (C_4C_5S_6 + C_6S_4) & (-C_4S_5) & (d_2 - d_6C_4S_5) \\ (-C_4S_6 - C_5C_6S_4) & (-C_4C_6 + C_5S_4S_6) & (-S_4S_5) & (d_1 - d_6S_4S_5) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Substituting the Values from the DH Parameters Table; we get;

$$T_0^6 = \begin{bmatrix} -C_6S_5 & S_5S_6 & C_5 & d_6S_5 \\ (-C_4C_5C_6+S_4S_6) & (C_4C_5S_6+C_6S_4) & -C_4S_5 & d_6C_4S_5 \\ (-C_4S_6-C_5C_6S_4) & (-C_4C_6+C_5S_4S_6) & -S_4S_5 & d_6S_4S_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The above matrix gives the forward kinematics of a three link articulated manipulator with a spherical wrist.

Verification :-

Let's assume some numerical values in the DH Table;

$a_2 = 1$

$a_3 = 2$

$d_6 = 3$

$\theta_1 = 45$

$\theta_2 = 45$

$\theta_3 = -45$

$\theta_4 = 90$

$\theta_5 = -90$

$\theta_6 = 0$

$\therefore$ we get ;

$$T_0^6 = \begin{bmatrix} 0.707 & 0 & -0.707 & -0.202 \\ -0.707 & 0 & -0.707 & -0.202 \\ 0 & 1 & 0 & 0.707 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End Effector Position; $P_x = -0.202$

$P_y = -0.202$

$P_z = 0.707$

Hence, the values given by code matches with that of derivation.

Hence, Verified

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Task - 7

Three different Configurations for 2R Manipulator :-

(1) Direct Drive 2R Manipulator :-

In direct drive 2R Manipulators; the link is driven directly through motor without any gears / pulleys / belts. These are simple & inexpensive and avoids the need of a mechanism for transferring motion.

→ Advantages :-

(1) Simple & Inexpensive as compared to Indirect / Remotely driven drive.

(2) No or ~~Negligible~~ Negligible loss of power as motor is directly mounted on the link.

(3) These can be used where the weight of the links is not a factor of concern, as motors adds up to the total weight of the links.

(2) Indirect Drive 2R Manipulator :- [Remotely Driven]

In Indirect drive 2R Manipulator; the link is driven by motors mounted at the base. The first joint is turned directly by one of the motors, while the other is turned via a gearing mechanism or a timing belts.

→ Advantages :-

(1) It can be configured to yield almost any speed ratio.

(2) The weight of the motor doesn't account with that of the links, so lighter links.

(3) 5- bar Parallelogram Arrangement :-

It is a 2 DOF mechanism that is constructed from the five links that are connected together in a closed chain. The ground is counted as an addition linkage apart from 4 links (moving).

→ Advantages :-

(1) Unlike a serial manipulator, this configuration has an advantage of having both motors grounded at the base.

(2) As the motor can be quite massive, this significantly decreases the total moment of inertia of the linkage.

(3) Workspace reached by the endpoints is smaller than that of a serial manipulator.

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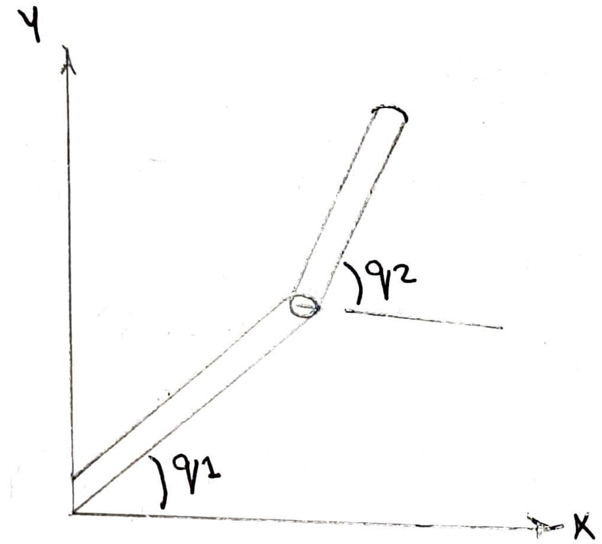
Task-8 Compare the derivation of the dynamic equations of 2R manipulator discussed in class and compare the results with those in the mini project.
Remark on any discrepancies or observations.

Consider a planar elbow manipulator (remotely driven link).



$$V_{c1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 \\ \frac{l_1}{2}\cos q_1 \\ 0 \end{bmatrix} \dot{q}_1 \qquad -(1)$$

$$V_{c2} = \begin{bmatrix} -l_1\sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1\cos q_1 & \frac{l_{c2}}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \qquad -(2)$$

$$\left.\begin{aligned} w_1 &= \dot{q}_1 \hat{k} \\ w_2 &= \dot{q}_2 \hat{k} \end{aligned}\right\} -(3)$$

Hence, the Kinetic energy of the Manipulator;

$$K = \frac{1}{2}\sum_{j=1}^{n} m_j V_{cj}^T V_{cj} + \frac{1}{2}\sum_{j=1}^{n} \omega_j^T I_j \omega_j \qquad -(4)$$

$$\left.\begin{aligned} V_{cj} &= J_{V_{cj}}(q)\dot{q} \\ \omega_j &= R_j^T J_{\omega j}(q)(\dot{q}) \end{aligned}\right\} -(5)$$

Substituting (5) in (4), we get;

$$K = \frac{1}{2}\dot{q}^T \sum_{j=1}^{n} \left[m_j J_{V_{cj}}(q)^T J_{V_{cj}}(q) + J_{\omega j}(q)^T R_j(q)\dot{q}\, R_j(q)^T J_{\omega j}(q) \right]\dot{q}$$

$$= \frac{1}{2}\dot{q}^T D(q)\dot{q} \qquad -(6)$$

$$D(q) = \begin{bmatrix} m_1\frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2}\cos[q_2 - q_1] \\ m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) & m_2\frac{l_2^2}{4} + I_2 \end{bmatrix} \qquad -(7)$$

Computing the Christoffel Symbols given;

$$C_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = -m_2 l_1 \frac{l_2}{2}\sin[q_2 - q_1]$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2}$$

$$= m_2 l_1 \frac{l_2}{2}\sin[q_2 - q_1]$$

$$C_{212} = C_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0 \qquad -(8)$$

Next, the Potential Energy of the manipulator in terms of q_1 & q_2, we get;

$$V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left[l_1 \sin q_1 + \frac{l_2}{2} \sin q_2 \right] \qquad -(9)$$

Now; $\phi_1 = \frac{\partial V}{\partial q_1}$ & $\phi_2 = \frac{\partial V}{\partial q_2}$

$$\left.\begin{aligned} \therefore\ \phi_1 &= \left[m_1 \frac{l_1}{2} + m_2 l_1 \right] g \cos q_1 \\ \phi_2 &= m_2 \frac{l_2}{2} g \cos q_2 \end{aligned}\right\} -(10)$$

$\therefore$ Dynamic equations are;

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + C_{221}\dot{q}_2^2 + \phi_1 = \tau_1$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + C_{112}\dot{q}_1^2 + \phi_2 = \tau_2$$

Substituting the terms, we get;

$$\left[m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1 \right]\ddot{q}_1 + \left[m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \right]\ddot{q}_2 - \left[m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1) \right]\dot{q}_2^2$$
$$+ \left[m_1 \frac{l_1}{2} + m_2 l_1 \right] g \cos q_1 = \tau_1 \qquad -(11)$$

and,

$$\left[m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \right]\ddot{q}_1 + \left[m_2 \frac{l_2^2}{4} + I_2 \right]\ddot{q}_2 + \left[m_2 l_1 \frac{l_2}{2} \sin[q_2 - q_1] \right]\dot{q}_1^2$$
$$+ m_2 \frac{l_2}{2} g \cos q_2 = \tau_2 \qquad -(12)$$

The equations obtained matches with that of the mini-project. τ_1 & τ_2 gives the dynamic equations of 2R manipulator (planar elbow manipulator) discussed in the class.

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Task-10 Summarize neatly in your own handwriting, the key steps to derive equations of motions, provided $D(q)$ & $V(q)$.

$$K = \frac{1}{2}\sum_{i,j}^{n} d_{ij}(q)\,\dot{q}_i\,\dot{q}_j = \frac{1}{2}\dot{q}^T D(q)\dot{q}$$

Where the $n \times n$ inertia matrix $D(q)$ is symmetric and positive definite for each $q \in \mathbb{R}^n$.

Second, the potential energy $V = V(q)$ is independent of $\dot{q}$.

The Euler-Lagrange equations for such a system can be derived as;

$$L = K - V$$

$$= \frac{1}{2}\sum_{i,j} d_{ij}(q)\,\dot{q}_i\dot{q}_j - V(q)$$

$$\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q)\dot{q}_j$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q)\ddot{q}_j + \sum_j \frac{d}{dt} d_{kj}(q)\dot{q}_j$$

$$= \sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j}\frac{\partial d_{kj}}{\partial q_i}\dot{q}_i\dot{q}_j$$

$$\frac{\partial L}{\partial q_k} = \frac{1}{2}\sum_{i,j}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j - \frac{\partial V}{\partial q_k}$$

Thus;
$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{1}{2}\frac{\partial d_{ij}}{\partial k}\right\}\dot{q}_i\dot{q}_j - \frac{\partial V}{\partial q_k} = \tau_k$$

$$k = 1, \text{——}, n$$

By interchanging the order of summations, we can show that;

$$\sum_{i,j} \left\{ \frac{\partial d_{kj}}{\partial q_i} \right\} \dot{q}_i \dot{q}_j = \frac{1}{2} \sum_{i,j} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} \right\} \dot{q}_i \dot{q}_j$$

Hence,

$$\sum_{i,j} \left\{ \frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2} \frac{\partial d_{ij}}{\partial q_k} \right\} \dot{q}_i \dot{q}_j$$

$$= \sum_{i,j} \frac{1}{2} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\} \dot{q}_i \dot{q}_j$$

The terms,

$$c_{ijk} = \frac{1}{2} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\}$$

c_{ijk} terms are known as Christoffel Symbols.

$$\phi_k = \frac{\partial V}{\partial q_k}$$

Therefore, Euler-Lagrange equation is;

$$\boxed{\sum_j d_{kj}(q) \ddot{q}_j + \sum_{i,j} c_{ijk}(q) \dot{q}_i \dot{q}_j + \phi_k(q) = \tau_k \qquad k = 1, \ldots, n}$$

This can also be written as;

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

where the k,j-th element of the matrix $C(q,\dot{q})$ is defined as;

$$c_{kj} = \sum_{i=1}^{n} c_{ijk}(q) \dot{q}_i$$

$$= \sum_{i=1}^{n} \frac{1}{2} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\} \dot{q}_i$$