Introduction to Robotics

[ME_639]

Assignment-2

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Assignment-2

(1) <u>To Show:</u> Columns of the rotation matrix R_0^1 are orthogonal.

→ <u>Solution:</u>

$$R_0^1 = \begin{bmatrix} i_1 \cdot i_0 & j_1 \cdot i_0 & k_1 \cdot i_0 \\ i_1 \cdot j_0 & j_1 \cdot j_0 & k_1 \cdot j_0 \\ i_1 \cdot k_0 & j_1 \cdot k_0 & k_1 \cdot k_0 \end{bmatrix} \quad - (1)$$

Inverse of the transformation R_0^1 can be written as;

$$R_1^0 = \begin{bmatrix} i_0 \cdot i_1 & j_0 \cdot i_1 & k_0 \cdot i_1 \\ i_0 \cdot j_1 & j_0 \cdot j_1 & k_0 \cdot j_1 \\ i_0 \cdot k_1 & j_0 \cdot k_1 & k_0 \cdot k_1 \end{bmatrix} \quad - (2)$$

Also; $R_1^0 = (R_0^1)^{-1} = (R_0^1)^T$ — (3)

$(R_0^1)(R_1^0) = I$ — (4)

$$\begin{bmatrix} i_1 \cdot i_0 & j_1 \cdot i_0 & k_1 \cdot i_0 \\ i_1 \cdot j_0 & j_1 \cdot j_0 & k_1 \cdot j_0 \\ i_1 \cdot k_0 & j_1 \cdot k_0 & k_1 \cdot k_0 \end{bmatrix} \begin{bmatrix} i_0 \cdot i_1 & j_0 \cdot i_1 & k_0 \cdot i_1 \\ i_0 \cdot j_1 & j_0 \cdot j_1 & k_0 \cdot j_1 \\ i_0 \cdot k_1 & j_0 \cdot k_1 & k_0 \cdot k_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \cdot i_0 \\ i_1 \cdot j_0 \\ i_1 \cdot k_0 \end{bmatrix} \cdot \begin{bmatrix} i_0 \cdot i_1 \\ i_0 \cdot j_1 \\ i_0 \cdot k_1 \end{bmatrix} = 0$$

— Continued on Next Page

So, the column vectors of the orthogonal matrix R_0^1 are of unit length and column vectors are mutually orthogonal.

(2) $\det(R_0^1) = 1$

<u>Solution:</u>

We restrict ourselves to right-hand coordinate system.

By definition;

$$(R_0^1)^T = (R_0^1)^{-1} = R_1^0$$

or $(R_0^1)(R_0^1)^T = I$

$$\det(R_0^1)(R_0^1)^T = \det(I)$$

$$\det(R_0^1)\det(R_0^1)^T = 1$$

$$\left[\because \det(A) = \det(A^T) \quad A \to R_0^1\right]$$

$\therefore \det(R_0^1)^2 = 1$

$$\det(R_0^1)^2 - 1 = 0$$

$$(\det(R_0^1) - 1)(\det(R_0^1) + 1) = 0$$

$\det(R_0^1) - 1 = 0$ or $\det(R_0^1) + 1 = 0$

$\det(R_0^1) = 1$ or $\det(R_0^1) = -1$

$\because$ we refer to right hand coordinate system

$\therefore \boxed{\det(R_0^1) = 1}$

Hence, Proved

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(3) Reviewed

(4) Reviewed

(5) Show :- $RS(a)R^T = S(Ra)$

Solution :-

For any $R \in SO(3)$ and $b \in \mathbb{R}^3$, it follows that ;

$$S(a)P = a \times P$$

and, $$R(a \times b) = Ra \times Rb$$

$$\therefore \; RS(a)R^T b = R(a \times R^T b)$$

$$= \cancel{R^T}(Ra) \times (RR^T b)$$

$$= (Ra) \times b \qquad [\because R \text{ is orthogonal}]$$

$$= S(Ra)b$$

$$\therefore \; \boxed{RS(a)R^T = S(Ra)}$$

Hence, Proved

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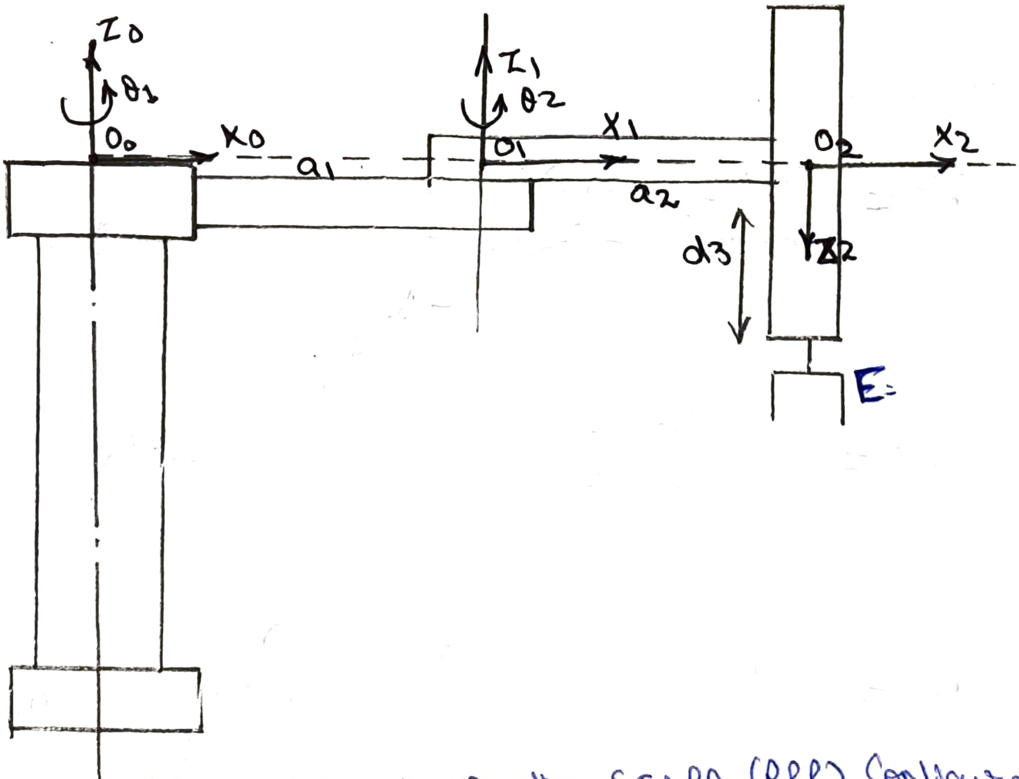
(6) RPP SCARA Configuration



The line diagram (above) of the SCARA (RPP) Configuration shows the various Coordinate frames.

Using the figure ;

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -(1)$$

$$d_0^1 = \begin{bmatrix} a_1\cos\theta_1 \\ a_1\sin\theta_1 \\ 0 \end{bmatrix} \quad -(2)$$

$$\therefore \quad H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \ H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -③$$

Similarly;

$$R_1^2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ \sin\theta_2 & -\cos\theta_2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad -④$$

$$d_1^2 = \begin{bmatrix} a_2\cos\theta_2 \\ a_2\sin\theta_2 \\ 0 \end{bmatrix} \quad -⑤$$

$$H_1^2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & -\cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -⑥$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -⑦$$

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix} \quad -⑧$$

$$\therefore \ H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -⑨$$

Now;

$$P_0 = H_0^1 \times H_1^2 \times H_2^3\ P_3$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}$$

On substituting the values from 3, 6 & 9, we get;

$\therefore$ P_0;

$$\boxed{P_0 = \begin{bmatrix} a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) \\ a_1 \sin\theta_1 + a_2 \sin(\theta_1+\theta_2) \\ -d_3 \\ 1 \end{bmatrix}}$$

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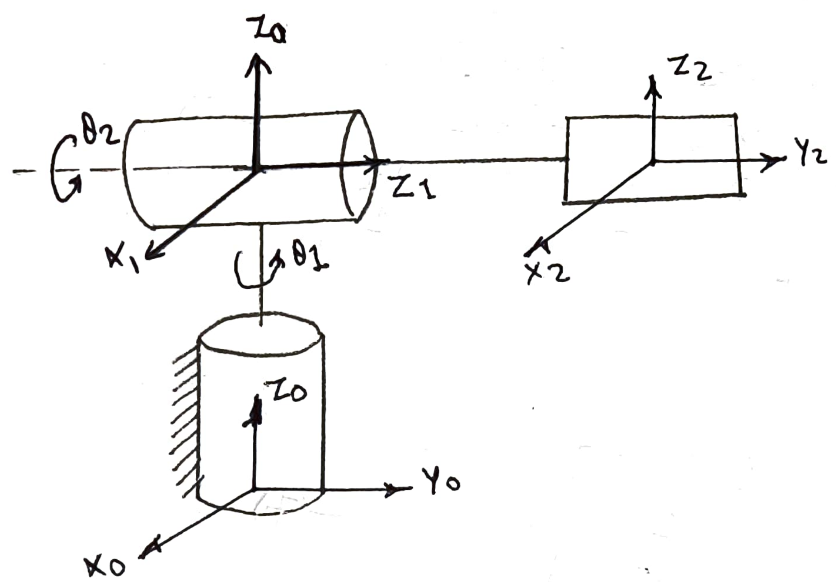
(8) Stanford Type RRP Manipulator :-



The diagram of Stanford type RRP Manipulator is shown above;

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & -1 & 0 \end{bmatrix} \quad -(1)$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad -(2)$$

$$\therefore H_0^1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -(3)$$

$$R_1^2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ \sin\theta_2 & 0 & -\cos\theta_2 \\ 0 & 1 & 0 \end{bmatrix} \quad -(4)$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ d_2 \end{bmatrix} \quad -(5)$$

$$\therefore H_1^2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -(6)$$

Similarly;

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad -(7)$$

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix} \quad -(8)$$

$$\therefore H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -(9)$$

$$\therefore P_0 = H_0^1 \times H_1^2 \times H_2^3 \, P_1$$

On substituting, we get;

$$\boxed{\therefore P_0 = \begin{bmatrix} \cos\theta_1 \sin\theta_2 d_3 - \sin\theta_1 d_2 \\ \sin\theta_1 \sin\theta_2 d_3 + \cos\theta_1 d_2 \\ (\cos\theta_2) d_3 \\ 1 \end{bmatrix}}$$

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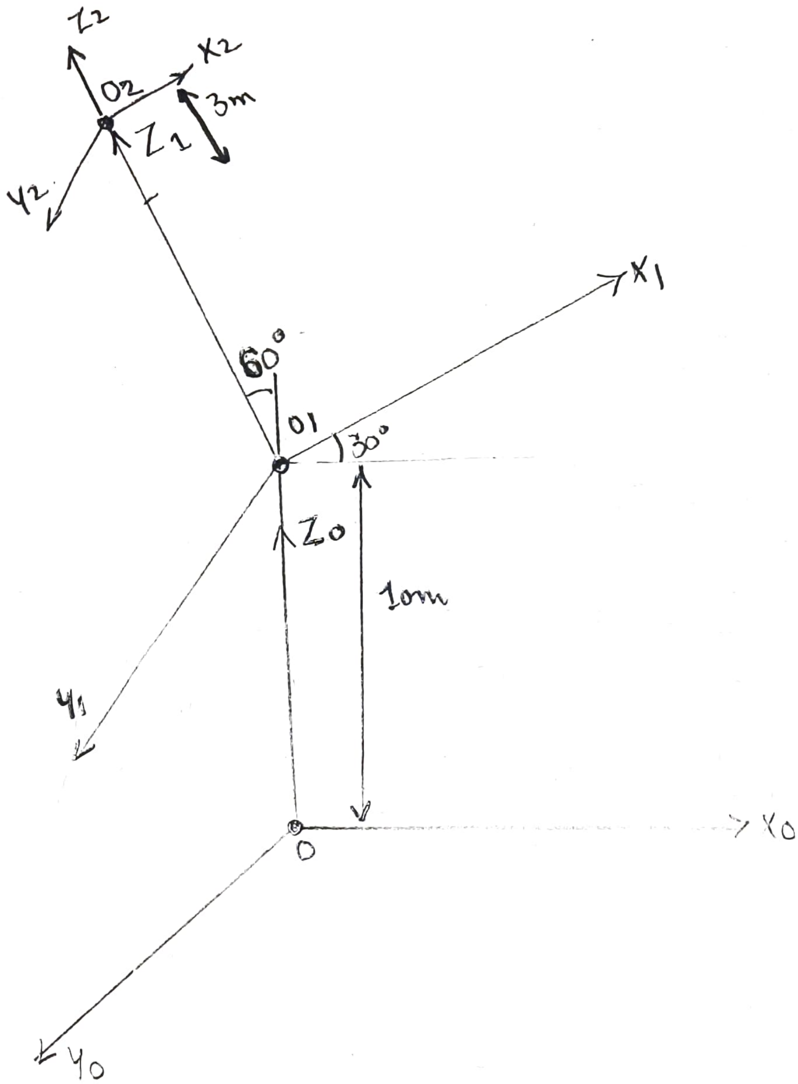
(9) To find :- Position vector of the obstacle w.r.t base Coordinate frame.



$$R_0^1 = R_{X\theta} R_{Z\theta}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.7499 & 0.433 & -0.5 \\ 0.433 & 0.25 & 0.866 \end{bmatrix}$$

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$$d_0^1 = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\therefore \quad P_0 = H_0^1 H_1^2 P_1$$

$$= \begin{bmatrix} 0.5 & -0.866 & 0 & 0 \\ 0.7499 & 0.433 & -0.5 & 0 \\ 0.433 & 0.25 & 0.866 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & -0.866 & 0 & 0 \\ 0.7499 & 0.433 & -0.5 & -1.5 \\ 0.433 & 0.25 & 0.866 & 12.598 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad P_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.6 \\ 1 \end{bmatrix}$$

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(10) Different types of Gearboxes :-

(a) Planetary Gearbox :-

It is made up of three types of gears, a sun gear, planet gears and a ring gear. These are used when space and weight are an issue, but a large amount of speed reduction and torque are needed.

(b) Harmonic drive Gearbox :-

It consists of a strainwave gear, consisting of three parts; An outer circular spline, a fixed ring with gear teeth on the inside and a flexible ring.

(c) Worm Gearbox :-

It consists of two parts, worm gear & Worm wheel. The worm can turn the gear but not the other way round.

(d) Spur Gearbox :-

It consists of train of spur gears on parallel shafts. They are mainly used to decrease the speed with high output torque.

(e) Helical Gearbox :-

It consists of train of helical gears mounted on parallel shafts inside the gear box. These are used for quite and smooth operations.

→ other gearbox used in robotics applications include Bevel Gearbox, Hypoid Gearbox, Rack & Pinion Gearbox, Double helical Gearbox etc.

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→ Explanation for use of Gearbox in Drone Applications:-

Gearbox are not used along with motors in Drone Applications. The main consideration while selecting a motor for drones include: Light Weight & High Torque.

BLDC motors are very powerful and generate high torque, because of high torque, there is no need to use a gearbox for the same. Also if a gearbox is used it will increase the weight of the drone, thus decreasing its efficiency and eventually its endurance. Therefore more focus is given on improving the torque characteristics of the BLDC motor, ~~rather~~ than using ~~of~~ a Gearbox.

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(11) Manipulator Jacobian for the RRP SCARA Configuration

→ Refer the figure drawn in problem-7.

→ Joint 1 & 2 are revolute

→ Joint 3 is prismatic

$$J = \begin{bmatrix} Z_0 \times [O_3 - O_0] & Z_1 \times [O_3 - O_1] & Z_2 \\ Z_0 & Z_1 & 0 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} a_1 \cos\theta_1 \\ a_1 \sin\theta_1 \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} a_1 \cos\theta_1 + a_2 \cos\theta_1 \cos\theta_2 \\ a_1 \sin\theta_1 + a_2 \sin\theta_1 \sin\theta_2 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} a_1 \cos\theta_1 + a_2 \cos\theta_1 \cos\theta_2 \\ a_1 \sin\theta_1 + a_2 \sin\theta_1 \sin\theta_2 \\ d_3 \end{bmatrix}$$

$$Z_0 = Z_1 = K$$

$$\& \quad Z_2 = -K$$

$$\cancel{Z_0 \times [O_3 - O_0] = K \begin{bmatrix} a_2 \cos\theta_1 \cos\theta_2 \\ a_2 \sin\theta_1 \sin\theta_2 \\ 0 \end{bmatrix}}$$

& $Z_1 [O_3 - O_1] = K \begin{bmatrix} a_2 \cos\theta_1 \cos\theta_2 \\ a_2 \sin\theta_1 \sin\theta_2 \\ 0 \end{bmatrix}$

$Z_0 [O_3 - O_0] = K \begin{bmatrix} a_1 \cos\theta_1 + a_2 \cos\theta_1 \cos\theta_2 \\ a_1 \sin\theta_1 + a_2 \sin\theta_1 \sin\theta_2 \\ 0 \end{bmatrix}$

Therefore, the Jacobian Matrix can be written as;

$$\boxed{J = \begin{bmatrix} -a_1 \sin\theta_1 - a_2 \sin[\theta_1 + \theta_2] & -a_2 \sin\theta_1 & 0 \\ a_1 \cos\theta_1 + a_2 \cos\theta_1 & a_2 \cos\theta_1 & 0 \\ 0 & 0 & \cancel{0} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & -1 \end{bmatrix}}$$

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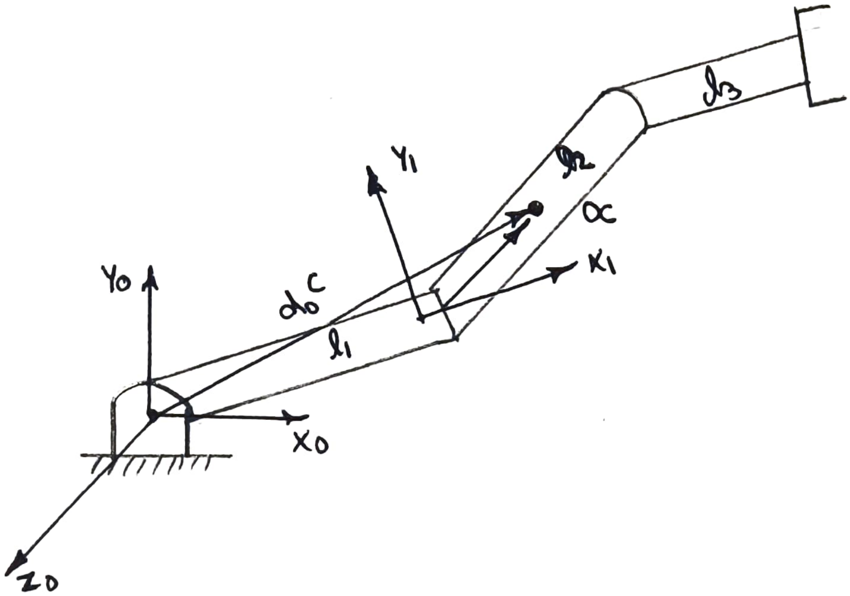
(13) Manipulator Jacobian for the RRR Configuration

→ Since the robot is planar like the elbow manipulator.



Consider the three-link planar manipulator as shown in the figure above;

$$\begin{bmatrix} V \\ w \end{bmatrix} = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix} \dot{q}$$

where, V is the linear velocity of the center of link-2
w is the angular velocity of the center of link-2

Columns of the Jacobian are determined using;

$$J_1 = Z_0 \times [O_c - O_0]$$

$$J_2 = Z_1 \times [O_c - O_1]$$

$$J_3 = 0$$

Since the velocity of the second link is unaffected by motion of link-3.

Therefore, we can write the Jacobian as;

$$J = \begin{bmatrix} -l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3) & -l_2 S_{12} + l_3 S_{123} & -l_3 S_{123} \\ l_1 C_1 + l_2 C_{12} + l_3 C_{123} & l_2 C_{12} + l_3 C_{123} & l_3 C_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Task-7, 8, 12 & 14 $\longrightarrow$ Python Code submitted along with this PDF on Github.