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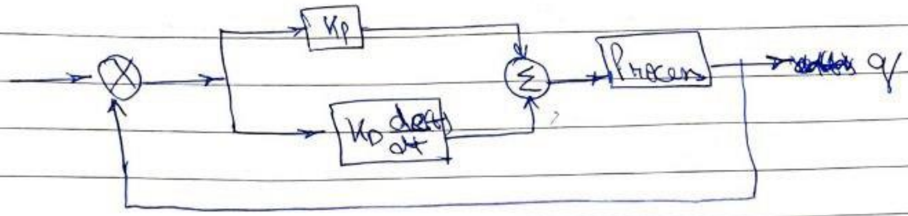
Secret Code:- DEV

Task - 3(a)



$$q = K_p e(t) + K_D \frac{d}{dt} e(t)$$

$$q(s) = K_p E(s) + K_D s E(s)$$

$$q(s) = E(s)\{K_p + sK_D\}$$

$$\frac{q(s)}{E(s)} = K_p\{1 + sT_D\}$$

(b) Motor dynamics:-

$$\frac{L\,di_a}{dt} + Ri_a = V - V_b$$

$$\tau_m = K_1 \phi i_a = K_m i_a$$

$$V_b = K_2 \phi \omega_m = K_b \omega_m = K_b \frac{d\theta_m}{dt}$$

$$K_m = \frac{R\tau_0}{V_R}$$

$$\frac{J_m\, d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} = \tau_m - r\tau$$

$$= K_m i_a - r\tau$$

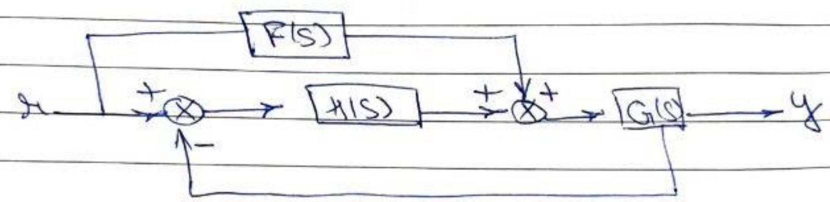
$$[LS+R]\,I_a(S) = V(S) - K_b S\,\theta_m(S)$$

$$[J_m S^2 + B_m S]\,\theta_m(S) = K_t I_a(S) - r\,T_d(S)$$

$$\frac{\theta_m(S)}{V(S)} = \frac{K_m}{S[(LS+R)(J_m S+B_m)+K_b K_m]}$$

$$\frac{\theta_m(S)}{T_d(S)} = \frac{-r[LS+R]}{S[(LS+R)(J_m S+B_m)+K_b K_m]}$$

(3)(c)



$$G(S) = \frac{q(s)}{p(s)}$$

$$H(S) = \frac{c(s)}{d(s)}$$

$$F(s) = \frac{a(s)}{b(s)}$$

$$T(s) = \frac{q(s)c(s)b(s) + a(s)d(s)}{b(s)p(s)d(s) + q(s)c(s)}$$

If $F(S) = \frac{1}{G(S)}$; i.e $a(s) = p(s)$; $b(s) = q(s)$

$q(s) =$

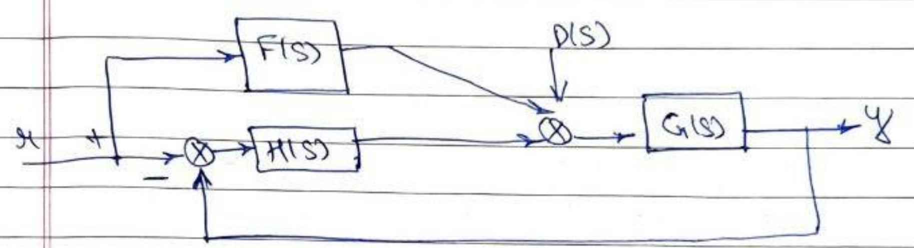
$$q(s)p(s)d(s) + q(s)c(s)Y(s) = q(s)p(s)d(s) + q(s)c(s)R(s)$$

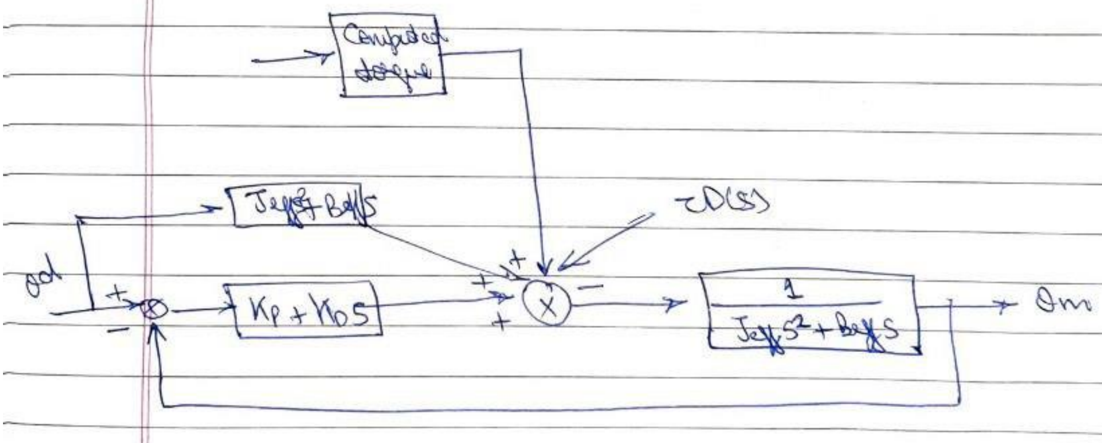
$$E(s) = R(s) - Y(s)$$

$$q(s)\, P(s)\, d(s) + q(s)\, c(s)\, E(s) = 0$$

Tracking ...:-

$$F(s) = \frac{q(s)\, d(s)}{P(s)\, d(s) + q(s)\, c(s)}\, D(s)$$





$$d^d = \sum d_{jk}(q^d)\, \ddot{q}_j^d + \sum C_{ijk}(q^d)\, \dot{q}_i^d \dot{q}_j^d + g_k(q^d)$$

(3)(d) Independent Joint P-D Control can be written as;

$$u = K_P\tilde{q} - K_D\dot{q}$$

$$\tilde{q} = q^d - q$$

$$V = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\tilde{q}^T K_P\tilde{q}$$

$$\dot{V} = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T K_P(\tilde{q})$$

Solving for $M(q)\ddot{q}$ with $g(q) = 0$

$$\dot{V} = \dot{q}^T[u - C(q,\dot{q})\dot{q}] + \frac{1}{2}\dot{q}^T \dot{D}(q)\dot{q} - \dot{q}^T K_P\tilde{q}$$
$$= \dot{q}^T[u - K_P\tilde{q}]$$

Substituting PD Control law for u; we get

$$\dot{V} = -\dot{q}^T K_D\dot{q} \leq 0$$

$$\dot{V} = \dot{q}^T[u - g(q) - K_P\tilde{q}]$$

$$K_P[q^d - q] = g(q)$$

$$u = K_P\tilde{q} - K_D\dot{q} + g(q)$$

Inverse Dynamics:-

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = g(q) = u$$

$$u = f(q,\dot{q},t)$$

$$u = M(q)a_q + C[q,\dot{q}]\dot{q} + g(q)$$

$$\ddot{q} = a_q$$

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$$a_q = -K_0 q - K_1 \dot{q} + r$$

$$\ddot{q} + K_1 \dot{q} + K_0 = r$$

$$r(t) = \ddot{q}^d(t) + K_0 q^d(t) + K_1 \dot{q}^d(t)$$

$$e(t) = q - q^d$$

$$\ddot{e}(t) = -K_1 e(t) - K_0 e(t)$$

$$K_0 = \operatorname{diag}\{\omega_1^2, \ldots, \omega_n^2\}$$

$$K_1 = \operatorname{diag}\{2\omega_1, \ldots, 2\omega_n\}$$

$$\ddot{q} = M^{-1}\{u - C[q, \dot{q}]\dot{q} - g(q)\}$$

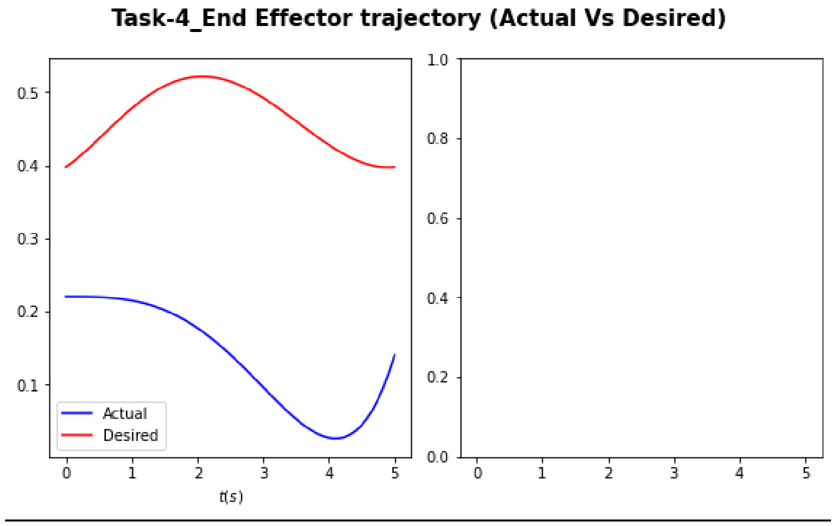
$$\ddot{q}(t) = a_q(t)$$

$$M^{-1}\{u(t) - C(q, \dot{q})\dot{q} - g(q)\} = a_q$$

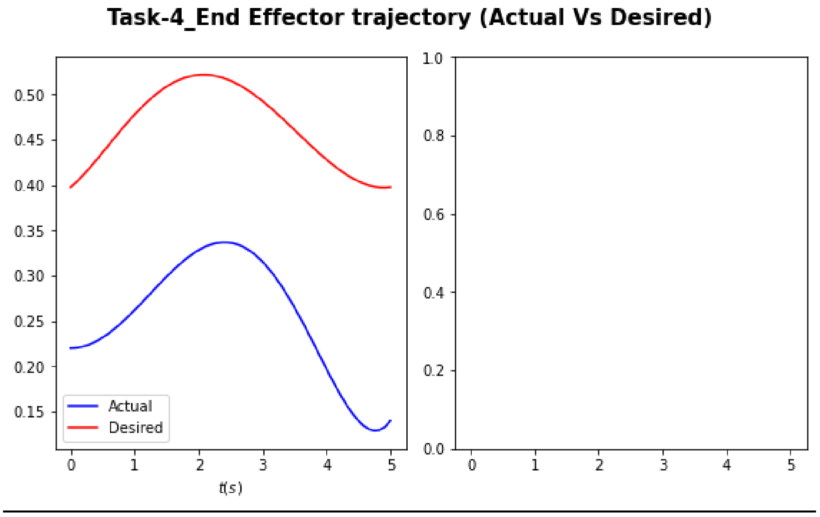
$$u = M(q)a_q + C[q, \dot{q}]\dot{q} + g(q)$$

Task-4 20% error in one of the length of one of the link

- **PD controller with steady state error**



- **For PD control with feedforward control using motor dynamics**



Task -7

Feedforward Control with disturbance cancellation: Since the disturbance was known, feedforward control is an effective means for cancelling the effects of disturbance on the system output. This is advantageous, since in a simple feedback system, the corrective action starts after the effect of disturbance is reflected at the output. On the other hand, in feedforward control the change in disturbance signal is measured and the corrective action takes place immediately. As a result, the speed and performance of the overall system improves.

Multivariable Control: The RRR configuration chosen for the task is a multi-DOF robotic manipulator and is a nonlinear system, However the joint dynamics was assumed to be linear and independent. Hence the linear control scheme developed for the individual joint is applied to each joint individually. Such a control scheme may work reasonably well in many industrial applications but does not perform well compared to a nonlinear control scheme. In particular, one cannot guarantee critical damping or the same settling time for all the joints uniformly at every point in the workspace. Thus, it is also not possible to analyze and predict the exact performance of a PD scheme for this configuration.

Inverse Dynamics Control: Here the controller cancels the non-linearities in the dynamic model of the robot under control. Such control is effectively a two-stage or two-loops control system, where one stage or the inner-loop takes care of making the nonlinear system appearing linear system of unit inertia, and the outer-loop or the second stages takes care of the servo compensation. Such control is effectively a two-stage or two-loops control system, where one stage or the inner-loop takes care of making the nonlinear system appearing linear system of unit inertia, and the outer-loop or the second stages takes care of the servo compensation. In the feedforward control of a robotic system, the nonlinear dynamic model, namely, the equations of motion of the robot manipulator governed by can be treated as disturbance.