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B.C.A. II Semester Examination, 2024

Paper : V

Discrete Mathematics

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt any **five** questions. **All** questions carry equal marks.

1. (a) Define set, subset and powerset with example and show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

   (b) Define Cartesian product and prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

P.T.O.

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2. (a) Define composition of relation and if $A=\{1, 2, 3\}$, $B==\{p, q, r\}$, $C=\{x, y, z\}$ and let $R=\{(1, p) (1, r) (2, q) (3, q)\}$ and $S=\{(p, y) (q, x), (r, z)\}$ complete ROS.

(b) State mathematical function, exponential and logarithmic function with example.

3. (a) Define group and prove that the fourth roots of unity 1, −1, i, −i form an abelian multiplicative group.

(b) Define cyclic group and permutation group also prove that multiplicative group $\{1, w, w^2\}$ is a cyclic group.

4\. (a) Define Homomorphism and isomorphism of group. Let (G, O) and (G′, *) be two groups and $f: G \to G'$ be a homomorphism then

(i) $f(e) = e'$ where e and e′ are identity element of G and G′ respectively.

(ii) $f(a^{-1}) = \{f(w)\}^{-1} \ \forall \ a \in G$.

(b) Define Ring and field. For the set $I_4 = \{0, 1, 2, 3\}$ show that the modulo 4 system is a field.

5\. (a) Define conjunction and disjunction, also construct the truth table for each compound proposition.

$p \wedge (\sim p \vee q)$ and

$\sim(p \vee q) \vee (\sim p \wedge \sim q)$.

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5\. (a) Define conjunction and disjunction, also construct the truth table for each compound proposition.

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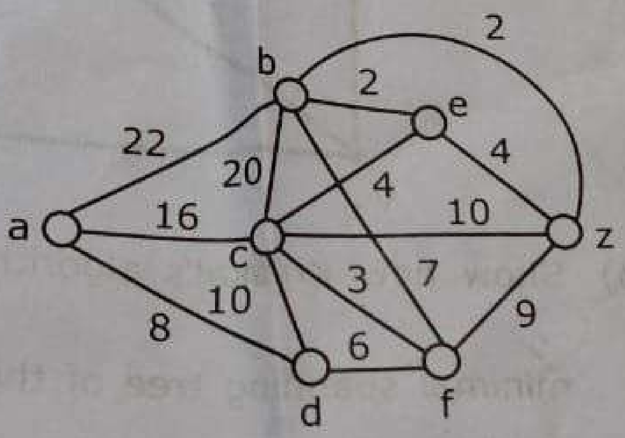
(b) Write short note on Eulerian and Hamiltonian graph.

8. (a) Define Path and Circuits. Determine a shortest path between the vertices a to z as shown below.



(b) State Tree and their properties. A tree has two vertics of degree 2 one vertex of degree 3 and three vertex of degree 4. How many vetices of degree 1 does it have?

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(b) Obtain the disjunctive normal form of the followings : (4)

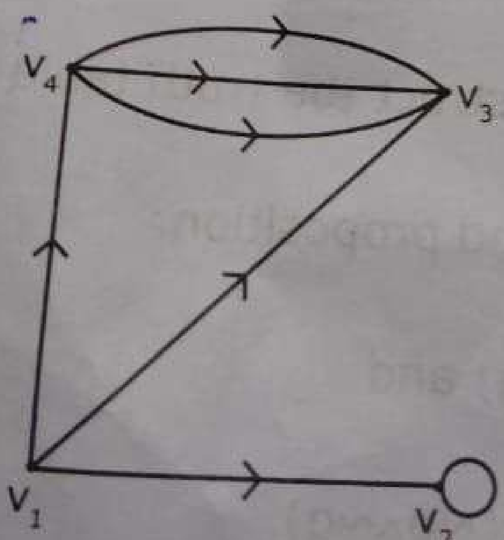
(i) $p \wedge (p \rightarrow q$

(ii) $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r))$.

6. (a) Define simple graph, Multigraph and Pseudograph with example.

(b) State and prove that Handshaking theorem.

7. (a) Find the in degree out degree and of total degree of each vertex of the following graph-



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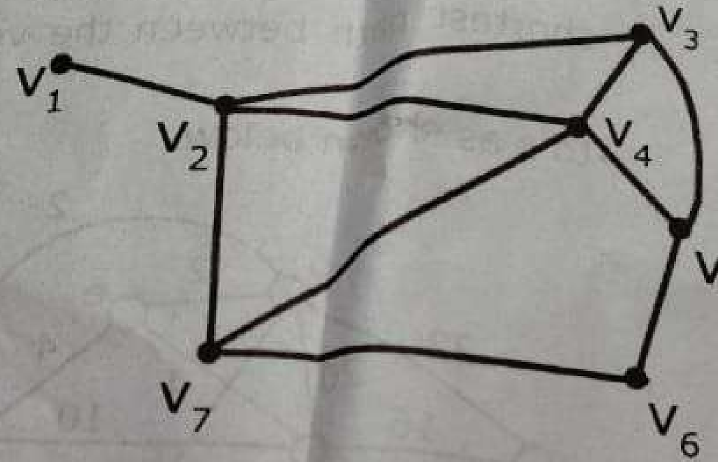
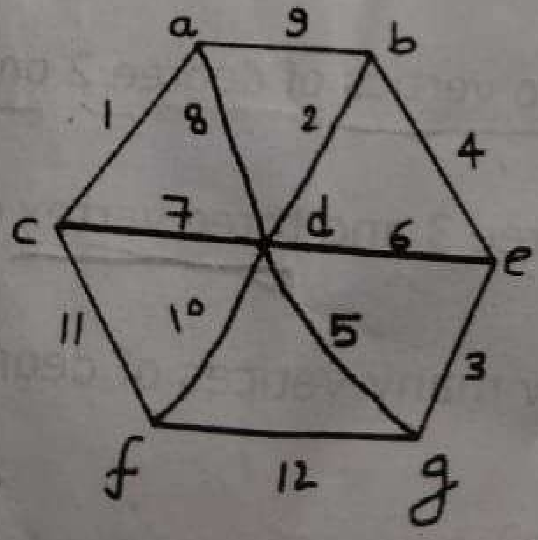
9. (a) Define Fundamental circuits. Find fundamental circuits for the graph shown below :



(b) Show how Kruskal's algorith finds a minimal spanning tree of the path of fig.



10. (a) Write note on Network Flows.

(b) Use Ford Fulkerson algorithm to find the maximum flow for the following network. Find the cut with capacity equal to this maximum flow.