Course Project

Cylinder Rolling in Trough

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6th Semester Mechanical Engineering

Course - Mechanical Vibration (ME3040)

Submitted to

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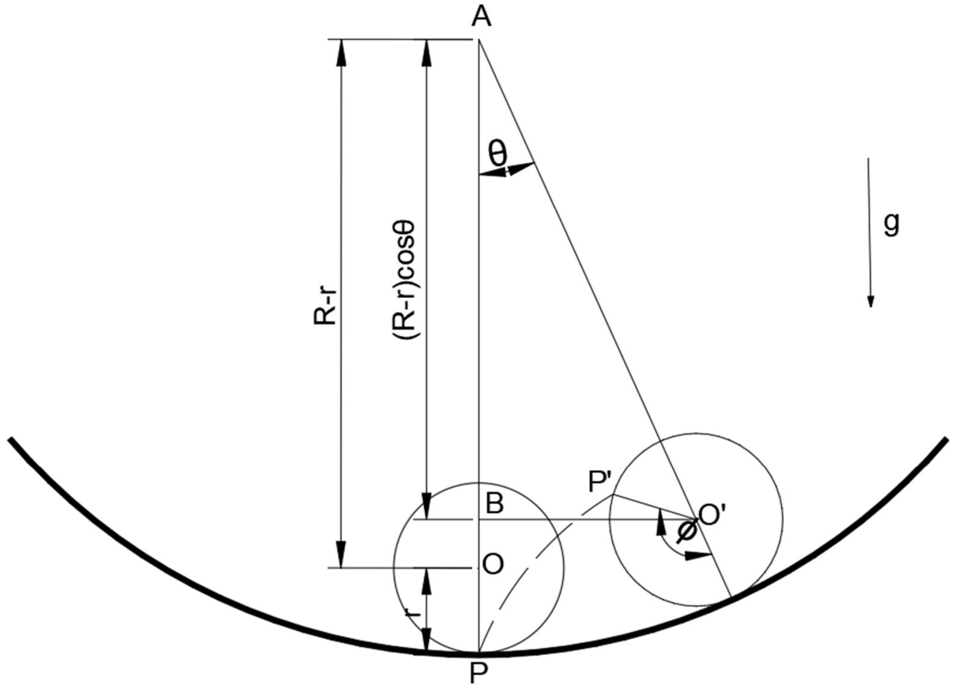
Problem Statement -

Derive the differential equation for the motion of cylinder rolling in the trough as shown in the figure. Plot the animation of the motion of cylinder using MATLAB. Show how potential and kinetic energy changes w.r.t time.



Pure Rolling of the cylinder in a circular trough (larger radius)

Mass of the cylinder is = m

The radius of the smaller cylinder is =AP = r

The radius of trough = OP = R

Moment of Inertia of the cylinder about point O = $J_o = \frac{mr^2}{2}$

When Cylinder is given little disturbance -

Point P move to point P'

Point O move to point O'

Distance covered by the point P = Distance covered by the centre point O

$$R\,\theta = r\,\emptyset$$

$$R\,\dot{\theta} = r\,\dot{\emptyset}$$

$$R\,\ddot{\theta} = r\,\ddot{\emptyset}$$

$$\therefore \qquad \dot{\emptyset} = \frac{R}{r}\,\dot{\theta}$$

Change in height of Centre of cylinder = BO = (R-r)(1-cosθ)

<u>Sign Convention</u> –

θ is measured in a counter-clockwise direction as positive (+ve)

∅ is measured in a clockwise direction as positive (+ve)

∴ Potential energy of Cylinder = mg(R-r) (1-cosθ)

∴ Kinetic Energy of Cylinder = Linear Kinetic energy + Angular kinetic Energy

$$= \frac{1}{2}m\{(R-r)\dot{\theta}\}^2 + \frac{1}{2}J_o(\dot{\emptyset} - \dot{\theta})$$

$$= \frac{m}{2}(R-r)^2\dot{\theta}^2 + \frac{mr^2}{4}\left(\frac{R}{r}\dot{\theta} - \dot{\theta}\right)^2$$

$$= \frac{m(R-r)^2\dot{\theta}^2}{4}(2+1)$$

$$= \frac{3m}{4}(R-r)^2\dot{\theta}^2$$

$$\frac{d}{dt}(KE + PE) = 0$$

$$mg(R-r)sin\theta\frac{d\theta}{dt} + \frac{3m}{4}(R-r)^2 2\dot{\theta}\frac{d\dot{\theta}}{dt} = 0$$

Since $\frac{d\theta}{dt} = \dot{\theta} \neq 0 \Rightarrow gsin\theta + \frac{3}{2}(R-r)\ddot{\theta} = 0$

$$\ddot{\theta} + \frac{2gsin\theta}{3(R-r)} = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 1$$

For small θ , $sin\theta \approx \theta$

$$\ddot{\theta} + \frac{2g\theta}{3(R-r)} = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 2$$

$$\omega_n = \sqrt{\frac{2g\theta}{3(R-r)}}$$

Solving the governing differential equation for following initial condition

$$\ddot{\theta} + \frac{2g\theta}{3(R-r)} = 0$$

$@\, t = 0 \quad \theta = \theta_0$ ………angular displacement

$@\, t = 0 \quad \dot{\theta} = \dot{\theta}_0$ ………angular velocity

$\theta = Asin(\omega t + \varphi)$ (φ and $\emptyset$ are different, φ is phase angle and $\emptyset$ is an angular position of a point on cylinder periphery)

$$\dot{\theta} = A\omega\cos(\omega t + \varphi)$$

$$\ddot{\theta} = -A\omega^2\sin(\omega t + \varphi)$$

Substituting the initial condition

$\theta_0 = Asin(\omega * 0 + \varphi) = Asi\,n(\varphi)$ …………3

$\dot{\theta}_0 = A\omega\cos(\omega * 0 + \varphi) = A\omega\cos(\varphi)$ ……4

Dividing the equation 3 by 4

$$\frac{\theta_0}{\dot{\theta}_0} = \frac{\tan\varphi}{\omega}$$

$$\boxed{\therefore\ \varphi = \tan^{-1}\frac{\theta_0\omega}{\dot{\theta}_0}}$$

Substituting the above φ into the equation no 3

$$\boxed{\therefore A = \frac{\theta_0}{si\,n(\varphi)}}$$

$\theta = \frac{\theta_0}{si\,n(\varphi)} sin(\omega t + \tan^{-1}\frac{\theta_0\omega}{\dot{\theta}_0})$ ………Angular Displacement

$\dot{\theta} = \frac{\theta_0}{si\,n(\varphi)} \omega\cos(\omega t + \tan^{-1}\frac{\theta_0\omega}{\dot{\theta}_0})$ ……. Angular Velocity

$\ddot{\theta} = -\frac{2g}{3(R-r)}$ ……Angular Acceleration

Similarly, for the equations of motion for the point on the periphery (i.e. Point P) of the cylinder.

$\ddot{\theta} = \frac{r}{R}\ddot{\theta}$ $\theta = \frac{r}{R}\emptyset$ substituting in the original Solution

$\emptyset = \frac{R}{r}\theta$

$\dot{\emptyset} = \frac{R}{r}\dot{\theta}$

$\ddot{\emptyset} = \frac{R}{r}\dddot{\theta}$

In the following example, Initial condition are

$@\, t = 0$ $\theta = 0°$ ……...angular displacement

$@\, t = 0$ $\dot{\theta} = 0.1\, rad/sec$ ……...angular velocity

M = 5 kg

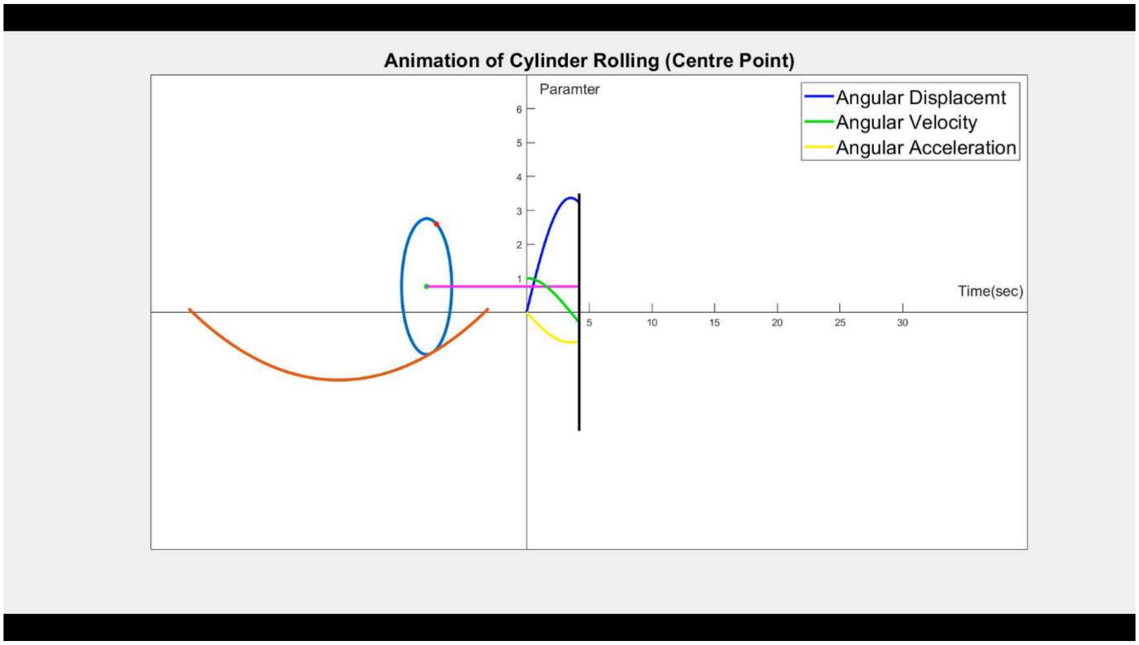
R = 35 m

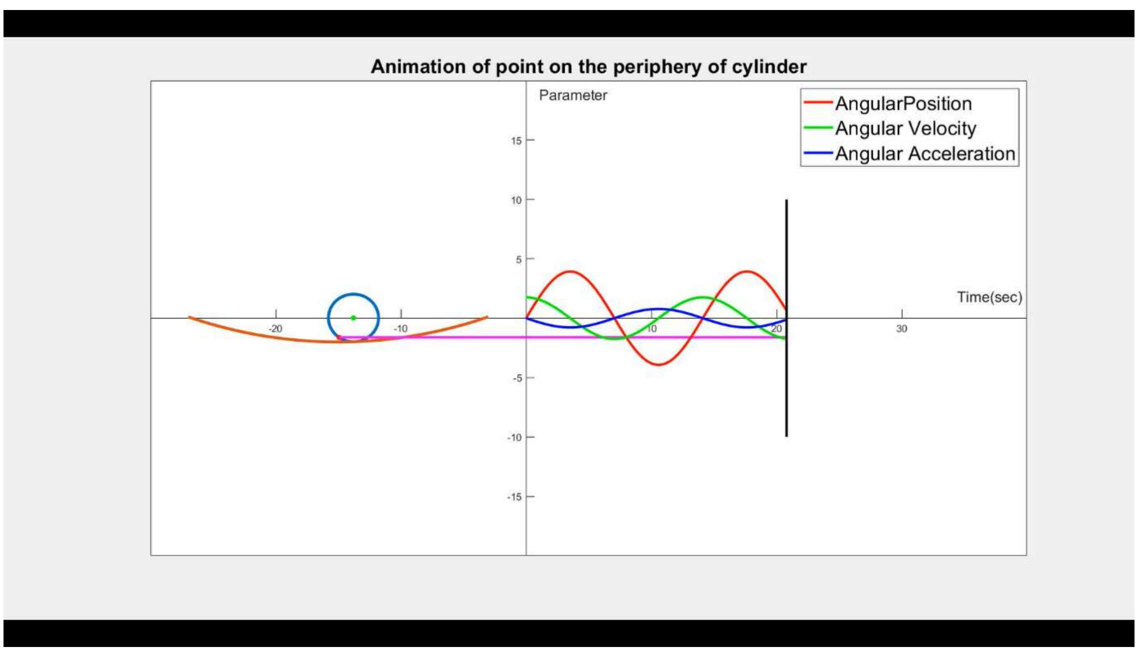
r = 2 m

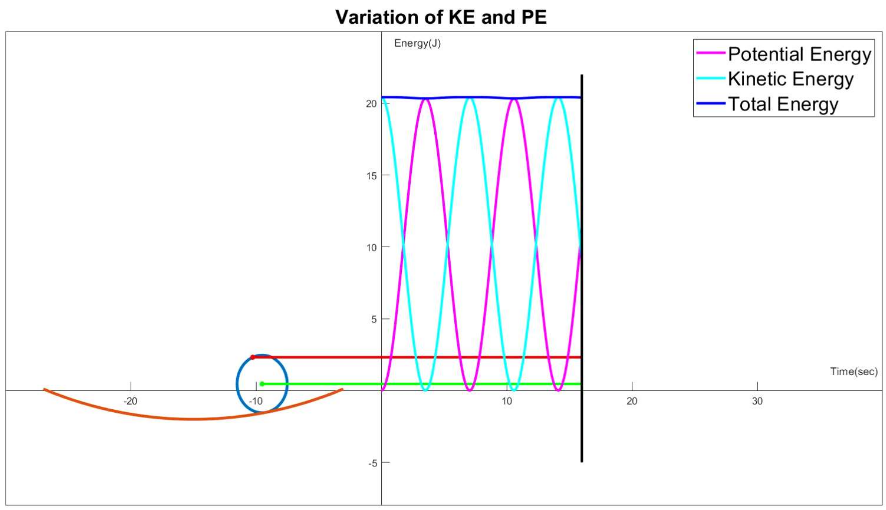
g = 9.81 m/s^2

Solution to the given problem is given by the following plots.

Note: All the plots are amplified for better visualization and understanding.







The line for total energy seems to be varying is due to rounding off of the value of total energy by MATLAB.

References:

- K V N Surendra sir notes