Course Project

Trajectory tracking of Four-Wheel Drive

Shubham Balasaheb Wakchaure

131701026

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Submitted to

Dr. Santhakumar Mohan

Indian Institute of Technology Palakkad

Introduction

Autonomous mobile robots are finding widespread application is many areas like mining, space exploration and in-service industry. The Four-wheel Mobile Robot FWD is one such robot that has gained wide popularity due to its simplicity and ease of control. The four-wheel mobile robot consists of four non steerable wheels. By adjusting the power applied to motors, the robot can be operated to go forward, rotate in place or perform movement on any arbitrary curve in plane

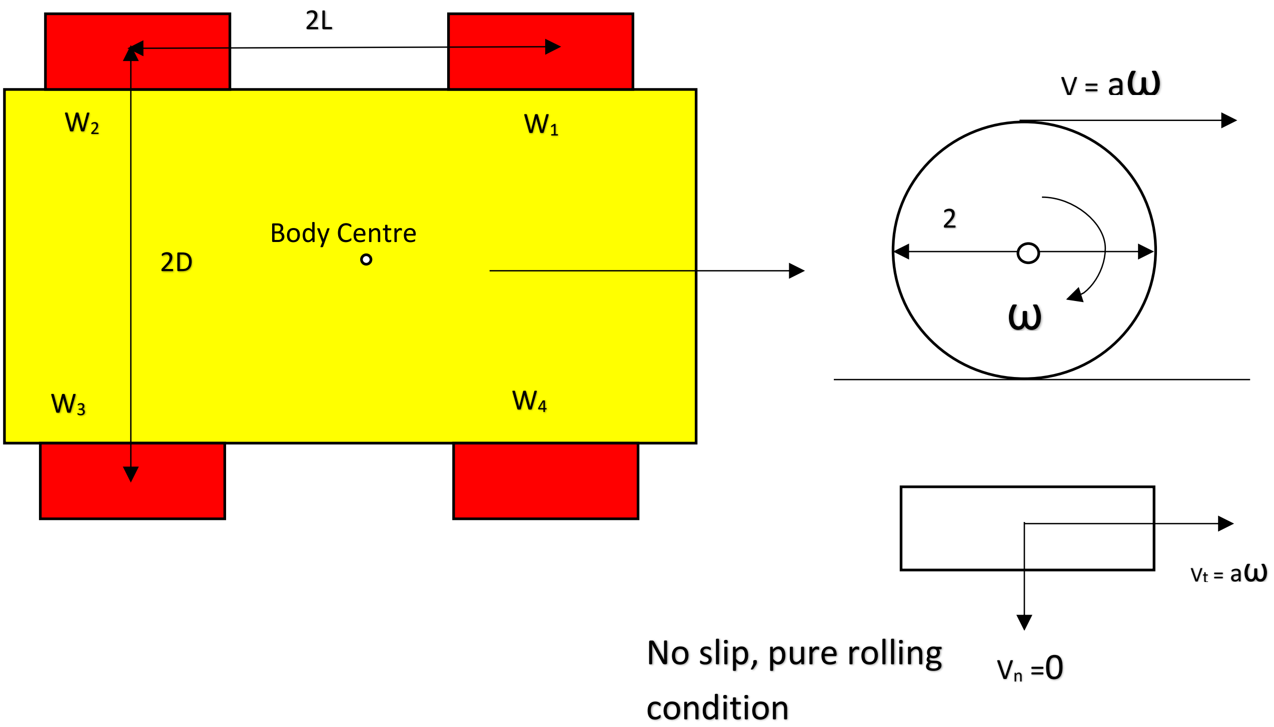
This project discusses the trajectory tracking of four wheeled mobile robot. Both forward and inverse kinematic model are described over here. Along with MATLAB based animation of trajectory tracking of different shaped curve. Simple P controller is used for controlling the velocities of the robot.

Modelling of the four-wheel drive

Four-wheel drive consists of the four non steerable wheel powered by DC motor independently which is powered by battery. By varying the voltage applied to each motor robot can be made to follow certain trajectories.

Kinematic Model

Kinematic model deals with relationship between motion of robot in space with geometric parameters without considering the cause of the motion. For four-wheel drive radius of each wheel is a longitudinal centre distance between wheel is “2l” and lateral distance between wheel centre is “2d”



Generalized wheel model for any robot of 3 degrees of freedom is given by

$$\omega_i = [\frac{1}{a_i} \quad \frac{1}{a_i}\tan \emptyset_i] \quad * \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} * \begin{bmatrix} 1 & 0 & -dy_i \\ 0 & 1 & dx_i \end{bmatrix} * \begin{bmatrix} u \\ v \\ r \end{bmatrix} \text{................ (1)}$$

Where

$\omega_i = angular\ veloctiy\ of\ ith\ wheel$

$\emptyset_i = angle\ of\ rollers\ of\ ith\ wheel$

$a_i = radius\ of\ the\ ith\ wheel$

$\theta_i = angle\ of\ ith\ wheel\ with\ longitudinal\ direction\ of\ robot$

$dy_i = transverse\ distance\ of\ wheel\ centre\ from\ body\ centre\ of\ robot$

$dx_i = longitudinal\ distance\ of\ wheel\ centre\ from\ body\ cetnre\ of\ robot$

u = linear velocity of robot in body frame

v = linear velocity of robot in body frame

r = angular velocity of robot in body frame about its COM

for four-wheel drive

$a_1 = a_2 = a_3 = a_4 = a$

$dx_1 = L\ ;\ dx_2 = -L\ ;\ dx_3 = -L\ ;\ dx_4 = L$

$dy_1 = D\ ;\ dy_2 = D\ ;\ dy_3 = -D\ ;\ dy_4 = -D$

$\theta_1 = 0\ ;\ \theta_2 = 0\ ;\ \theta_3 = 0\ ;\ \theta_4 = 0$

$\emptyset_1 = 0\ ;\ \emptyset_2 = 0\ ;\ \emptyset_3 = 0\ ;\ \emptyset_4 = 0$

By substituting these values in equation 1

$$\omega_1 = \frac{u - dr}{a} \qquad \omega_2 = \frac{u - dr}{a} \qquad \omega_3 = \frac{u + dr}{a} \qquad \omega_4 = \frac{u + dr}{a}$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{a}\begin{bmatrix} 1 & 0 & -d \\ 1 & 0 & -d \\ 1 & 0 & d \\ 1 & 0 & d \end{bmatrix}\begin{bmatrix} u \\ v \\ r \end{bmatrix}$$

As we know that;

$\dot{\eta} = J(\psi)\xi$ .... Forward Kinematic model

Where $\dot{\eta} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}$ $J(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\xi = \begin{bmatrix} u \\ v \\ r \end{bmatrix}$

$\xi = J(\psi)^{-1}\dot{\eta}$ .... Inverse Kinematic model

$\xi = W\omega$

$$\xi = \begin{bmatrix} u \\ v \\ r \end{bmatrix} \quad \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} \quad W^{-1} = \frac{1}{a}\begin{bmatrix} 1 & 0 & -d \\ 1 & 0 & -d \\ 1 & 0 & d \\ 1 & 0 & d \end{bmatrix}$$

Trajectory tracking of FWD

Trajectory tracking problem is given as follows

Given a robot at some pose say (x,y, ψ) and the desired trajectory xd,yd, ψd find a control law for linear and angular velocity of robot such that $\lim_{t\to\infty} |xd(t) - x(t)| = 0$,

$\lim_{t\to\infty} |yd(t) - y(t)| = 0$ and $\lim_{t\to\infty} |\psi d(t) - \psi(t)| = 0$ Simple Proportional controller is used.

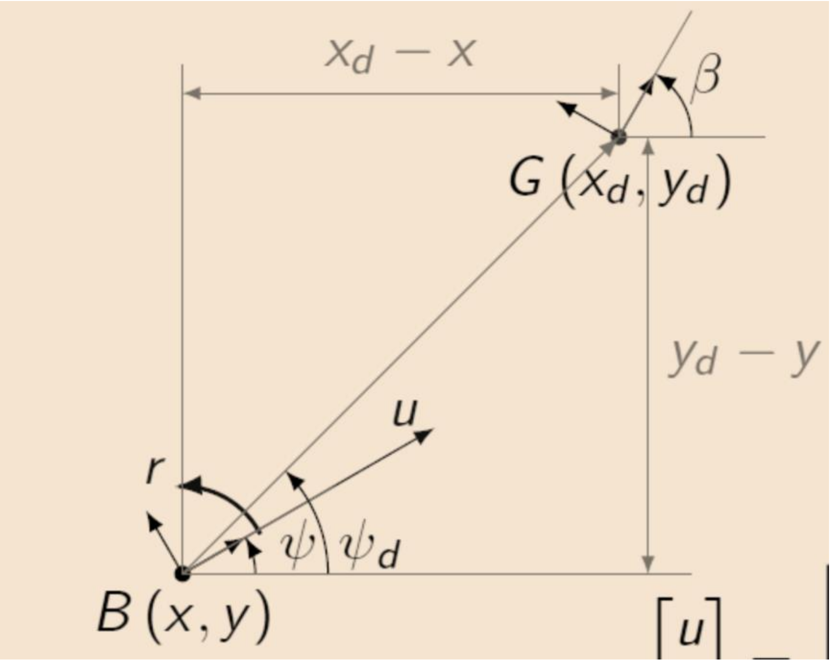


Image courtesy: Prof. Santhakumar Sir Slides

$$\psi_d = \tan^{-1} \frac{y_d - y}{x_d - x}$$

$$\begin{bmatrix} u \\ v \\ r \end{bmatrix} = J(\psi)^{-1} \left(\begin{bmatrix} \dot{x_d} \\ \dot{y_d} \\ \dot{\psi_d} \end{bmatrix} + K \begin{bmatrix} x_d - x \\ y_d - y \\ \psi_d - \psi \end{bmatrix} \right)$$ where K = proportional gain

For animation purpose

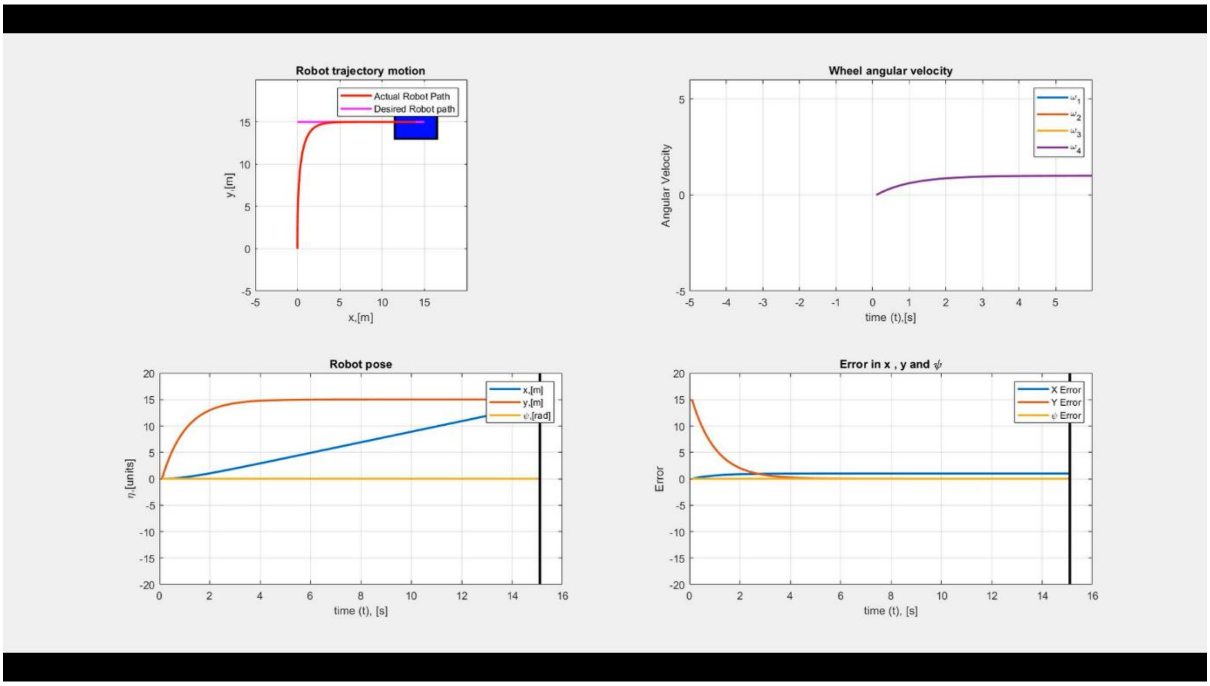
d = 2.5 units

l = 1.5 units

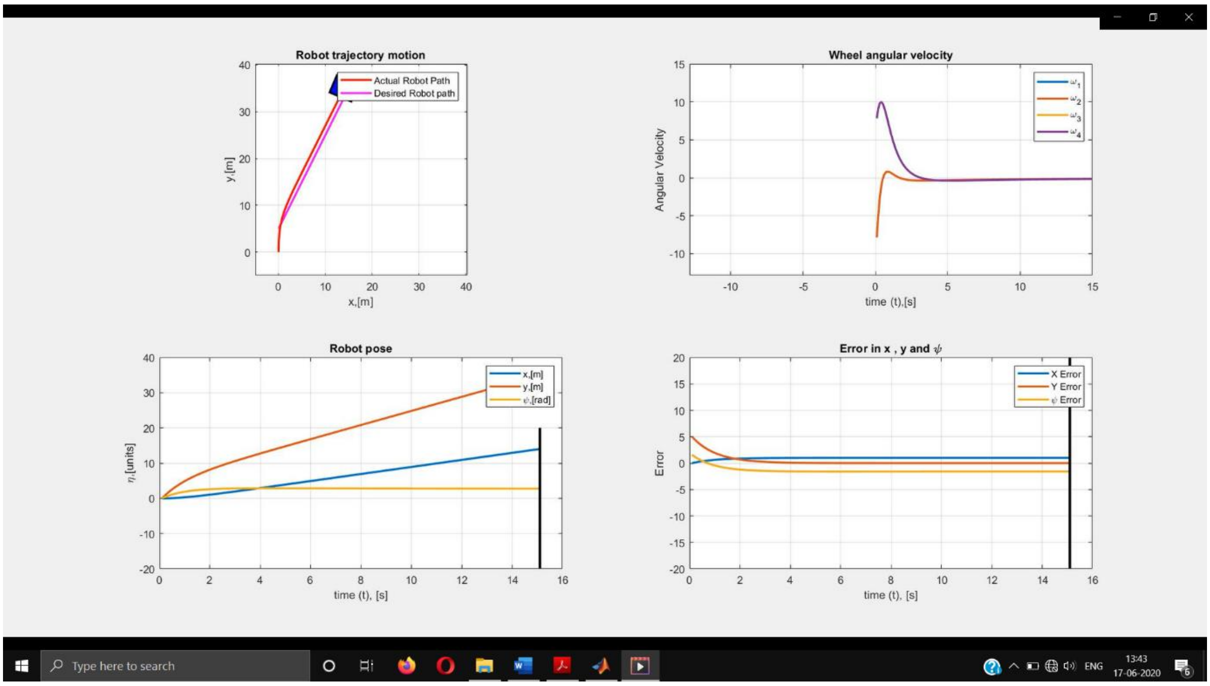
a = 1 units

Animation result

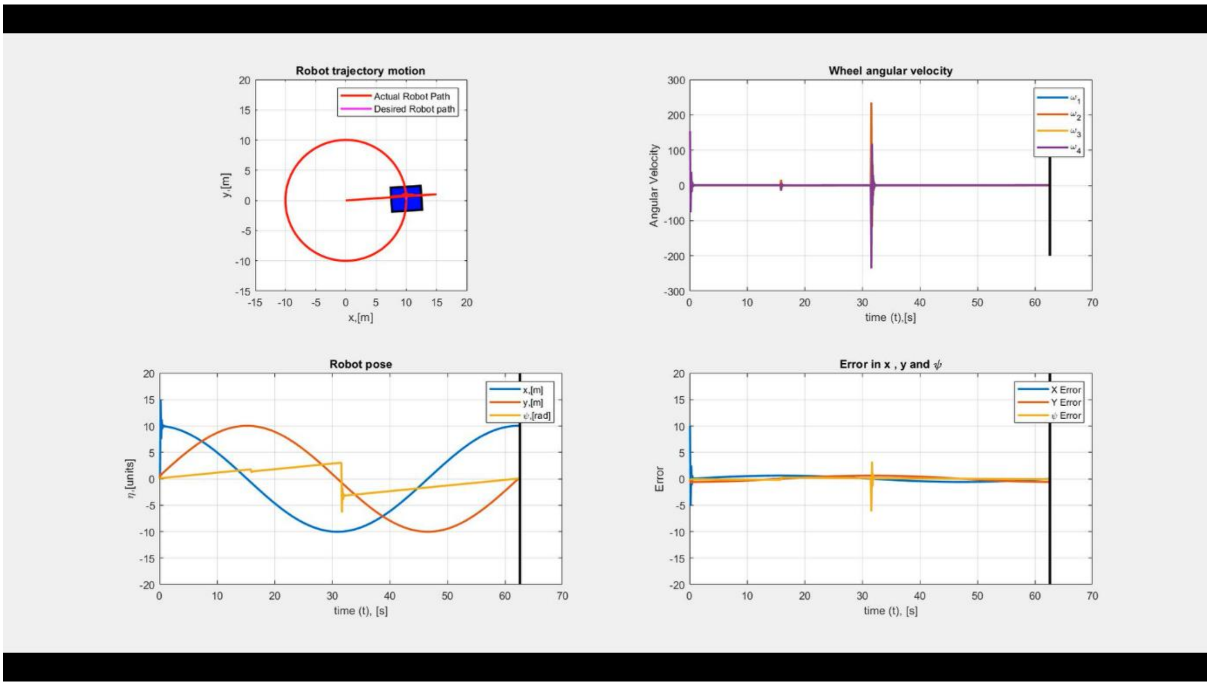
1) Straight line trajectory y = 15
   Proportional gain = 1
   Results are



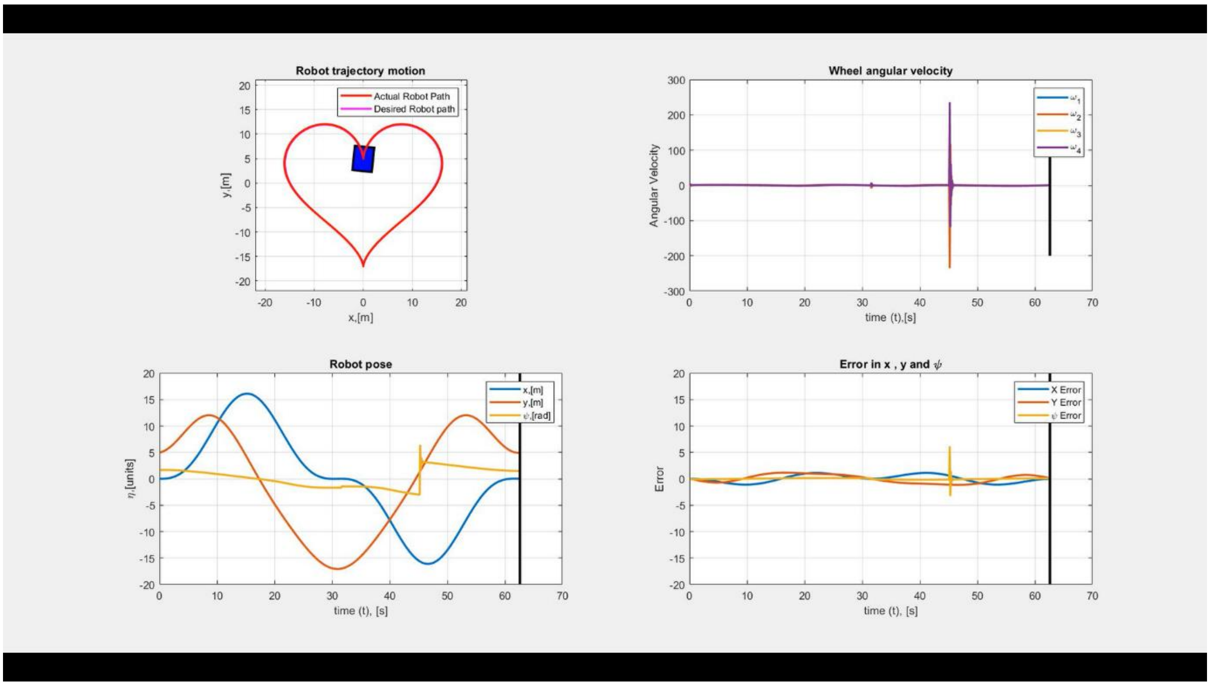
2) Trajectory is straight line y =2x+5
   Proportional gain = 1
   Result are



3) Trajectory is of circle with equation $x^2 + y^2 = 100$
Proportional gain = 15
Results are



4) Trajectory is of heart shape
Proportional gain = 15
$x = 16(sint)^3$
$y = 13\cos(t) - 5\cos(2t) - 2\cos(3t) - \cos(4t)$



Summary
Kinematic model of four-wheel drive is discussed. Kinematic control of four-wheel drive is implemented using proportional controller. Control scheme is good enough for basic tracking problems. Different trajectories are testing MATLAB animation. As we increase the proportional gain, we get more accurate results.