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# Answer

1. A)

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ |
|---|---|---|---|---|---|---|
| 0.20 | 1.6596 | | | | | |
| | | 0.0102 | | | | |
| 0.22 | 1.6698 | | | | | |
| | | | 0.0004 | | | |
| | | 0.0106 | | | | |
| | | | | -0.0002 | | |
| 0.24 | 1.6804 | | | | | |
| | | | 0.0002 | | 0.0004 | |
| | | 0.0108 | | | | |
| | | | | +0.0002 | | |
| 0.26 | 1.6912 | | | | | -0.007 |
| | | | 0.0004 | | | |
| | | | | | -0.0003 | |
| | | 0.0112 | | | | |
| | | | | -0.0001 | | |
| 0.28 | 1.7024 | | | | | |
| | | | 0.0003 | | | |
| | | 0.0115 | | | | |
| 0.30 | 1.7139 | | | | | |

For $f(0.23)$,

let

$x_0 = 0.22$ , $x = 0.23$, $h = 0.02$

$$\therefore u = \frac{x - x_0}{h} = \frac{0.23 - 0.22}{0.02} = 0.5$$

According to Newton's forward interpolation Formula,

$$f(0.23) \approx f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{3!}\cdot\Delta^3 f(x_0) + \frac{u(u-1)(u-2)(u-3)}{4!} \times \Delta^4 f(x_0) + \ldots\ldots$$

$$\approx 1.6698 + 0.5 \times 0.0106 + \frac{0.5(0.5-1)}{2} \times 0.0002 + \frac{0.5(0.5-1)(0.5-2)}{6} \times 0.0002 + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{24} \times (-0.0003)$$

$\approx 1.6751$ (Upto 4 decimal places)

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3.b) $y' = x + y + xy$ ; $y(0) = 1$, $h = 0.25$, $y(0.5) = ?$

Forth Order R-K Method,

$k_1 = hf(x_0, y_0) = (0.25)\, f(0,1) = 0.25$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.25 f(0.125, 1.125) = 0.3477$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.25 f(0.125 ; 1.1738) = 0.3614$

$k_4 = hf(x_0 + h + y_0 + k_3) = 0.25 f(0.25, 1.3614)$

$\Rightarrow k_4 = 0.4879$

$\therefore y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$\doteq y_0 + \frac{1}{6}[0.25 + 2(0.3477) + 2(0.3614) + (0.4879)]$

$y_1 = 1.3593$

$\therefore \boxed{y(0.25) = 1.3593}$

Now, take $(x_1, y_1)$ in place of $(x_0, y_0)$;

$k_1 = hf(x_1, y_1) = (0.25) f(0.25, 1.3593)$

$= 0.25 \times 1.9492$

$= 0.4873$

$k_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2})$.

$= (0.25) f(0.375, 1.603) = 0.6448$

$k_3 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2})$

$= (0.25) f(0.375, 1.6817) \doteq 0.6718$

$k_4 = hf(x_1 + h, y_1 + k_3)$.

$= (0.25) f(0.5, 2.0312) = 0.8867$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$\Rightarrow y_2 = 1.3593 + \frac{1}{6}(0.4873 + 2(0.6448) + 2(0.6718) + (0.8867))$.

$\Rightarrow y_2 = 2.0272$

$$\boxed{y(0.5) = 2.0272}$$

4.b)

| $(x, y)$ | $x^2$ | $xy$ |
|---|---|---|
| $(-2,-1)$ | 4 | 2 |
| $(1,1)$ | 1 | 1 |
| $(3,2)$ | 9 | 6 |

$\Sigma x = -2+1+3 = 2$

$\Sigma y = -1+1+2 = 2$

$\Sigma x^2 = 4+1+9 = 14$

$\Sigma xy = 2+1+6 = 6$

$N = 3$

$$m = \frac{N\,\Sigma(xy) - \Sigma x\,\Sigma y}{N\,\Sigma(x^2) - (\Sigma x)^2}$$

$$= \frac{3.6 - 2.2}{3.14 - 2^2} = \frac{7}{19}$$

$$b = \frac{\Sigma y - m\,\Sigma x}{N} = \frac{2 - \frac{7}{19}.2}{3}$$

$$= \frac{24}{57}.$$

So, the line is,

$$57y = 33x + 24$$

5 B) Test wheather the population mean m is less than 40 or not

The test null hypotheses,

$$H_0 : \mu = 40$$

against alternative hypothesis

$$H_1 : \mu < 40$$

We have sample mean $\bar{x} = 38$;

The population standard deviation $\sigma = 5.8$

The sample size $n = 64$

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6 . B) $n = 300$ (large population)

$\bar{x} = ~~16~~ 16$

$\sigma = 5.2$

$\mu = 16.8$

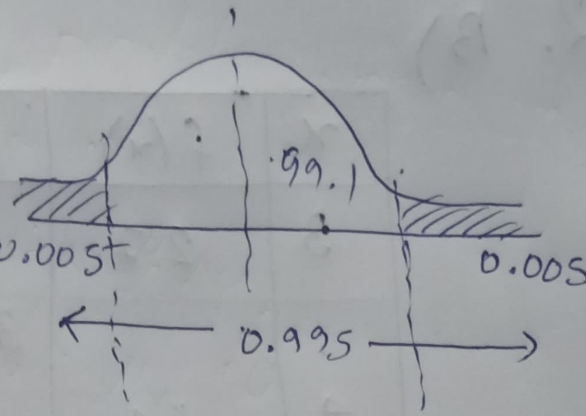
Significance level $= 0.01$, critical region $= 0.01$

Case - I

$\mu_0 = \mu = 16.8$

$\mu_1 = \mu \neq 16.8$ (both sided test)

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{x}}$$




$-z_{\alpha/2} = -2.575$

$z_{\alpha/2} = 2.575$

Critical region $= (-\alpha, -2.575) \cup (2.575, \alpha)$

$\bar{x} = 16$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{16 - 16.8}{\frac{5.2}{\sqrt{300}}}$$

$$= -2.6648$$

Since z lines in the critical region, thus it is not likely as it has confidence level of only 0.01

2. B)

| $x$ | $f(x)$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 3 | 27 | | | |
| | | 37 | | |
| 4 | 64 | | 24 | |
| | | 61 | | 6 |
| 5 | 125 | | 30 | |
| | | 91 | | 6 |
| 6 | 216 | | 36 | |
| | | 127 | | 6 |
| 7 | 343 | | 42 | |
| | | 168 | | |
| 8 | 512 | | | |

For $f(3.5)$ ; $x_0 = 3$

$h = 1$,

$\therefore s = \frac{x - x_0}{h} = \frac{3.5-3}{1} = 0.5$

By Newtoon's forward Interpolation Formula,

$f(3.5) = 27 + 0.5 \times 37 + \frac{0.5(0.5-1)}{2} \times 24 + \frac{0.5(0.5-1)(0.5-2)}{6} \times 6$

$\Rightarrow \boxed{f(3.5) = 42.875}$