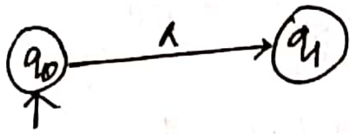
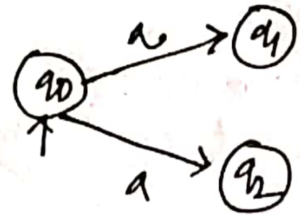
Non- Deterministic FA : (NDFA/NFA)



$\delta(q_0, a) = q_1$



$\delta(q_0, a) = \{q_1, q_2\}$

Deterministic ND

Non-Deterministic : Example LIFT, celling FAN. Human being.

* Can we want a m/c to be non- derterministic?

No
why? because if we are makeing a m/c then we are expecting the m/c to respond as we command.
for example if we are in a lift and we press button 4 then we ~~12~~ want that lift to go to 4th floor not in 3rd or any other floor.

* Every h/w & s/w is made of with deterministic m/c.

* Deterministic m/c is very difficult to implement but implementation is possible.

Why we are using NDFA/NFA?

because,
① It is easy to implement theoritically.
② It is simple to design.
③ NDFA/NFA $\longrightarrow$ DFA $\longrightarrow$ MDFA

Def^n of NFA :

NFA = $\{Q, \Sigma, q_0, F, \delta\}$

$Q \rightarrow$ Set of non empty finite set

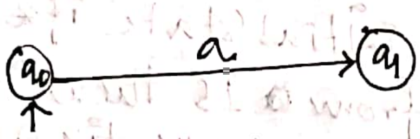
$\Sigma \rightarrow$ Non-empty set of finite state.

$q_0 \rightarrow$ only initial state

$F \rightarrow$ set of final state $[0 \leq |F| \leq |Q|]$

$\delta \rightarrow \delta : Q \times \Sigma \longrightarrow 2^Q$

PowerSet is a all possible subset of a set



$P\{q_0, q_1\} \rightarrow \Phi, \{q_0\}, \{q_1\}, \{q_0, q_1\}$

$Q \rightarrow \{q_0, q_1\}$; $\Sigma = \{a\}$; $q_0 \rightarrow \{q_0\}$; $F \rightarrow \{\Phi\}$

At state q_0 if I give a then NS may be q_0

" " q_0 " I " a " " " " ~~q_0~~ q_1

" " q_0 " I " a " " " " q_0 & q_1 both

" " q_0 " I " a " " " " Φ

Deterministic

① For one char say 'a' ∃ only one edge from PS to NS.

② No concept of dead state

③ TF: $\delta : Q \times \Sigma \rightarrow Q$

④ Φ Set never generates

Non-Deterministic

① For one char say 'a' ∃ more than one edge or no edge from PS to NS.

② concept of dead state is not present.

③ TF: $\delta : Q \times \Sigma \rightarrow 2^Q$

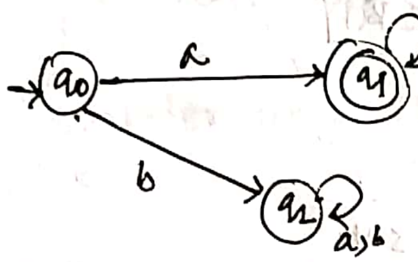
④ Φ Set may generate.

Scanned with CamScanner

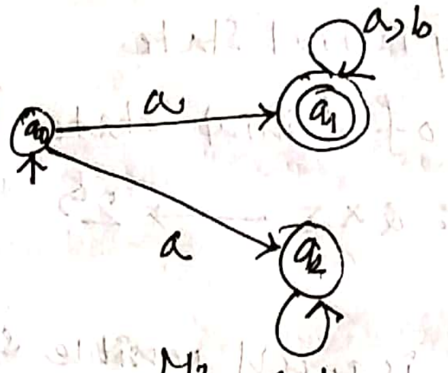
Acceptance by NFA:

A string 'w' is said to be accepted by a NFA if $\exists$ at least one transition path on which we start at initial state & ends at FS.

$$\delta^*(q_0, w) = F$$



M_1



M_2

After giving a string from initial state if many times do not reach the FS from IS then it is okay. But, if we reach the FS at one time than we can say that the string is accepted by the NFA.

Ex: 'ba' the string is not accepted by M_1

because,

At q_0 if I give b then we have to go to q_2

" q_2 " I " a " " " " q_2

$\therefore$ Ultimate state is q_2 hence the string is not accepted.

~~But in~~ And as because M_1 is a DFA it is possible.

But at M_2, which is a NFA,

At q_0 if I give 'a' NS is q_1 i.e. FS

" " " " " " " " q_2 i.e. Non-FS.

So, $\exists$ atleast one transition path on which we start at IS & ends at FS.

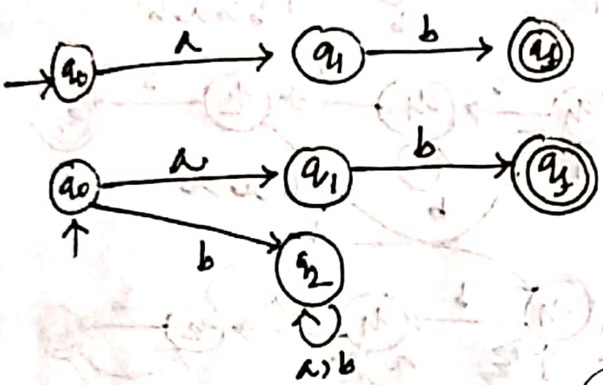
'ba' is not the member of the M_2 So, ∃ no chance of its acceptance.

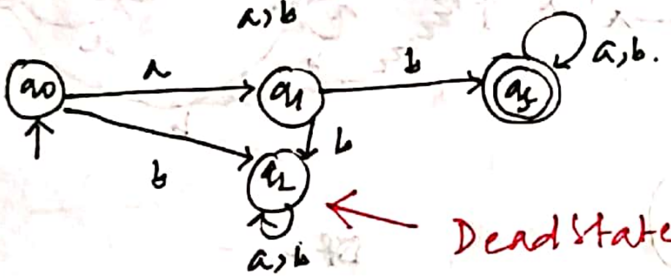
'ab' is the member of M_2 So, ∃ atleast one ~~chance~~ transition that it will be accepted by the NFA/M_2.

Problems:

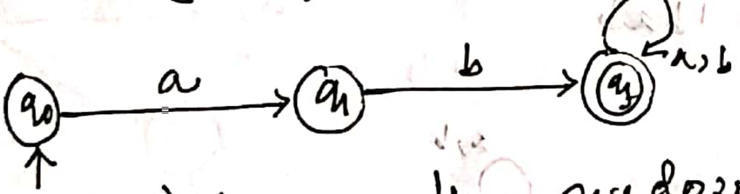
* Design a NFA over an alphabet $\Sigma = \{a, b\}$ s.t. every string accepted must start with,

(i) ab x





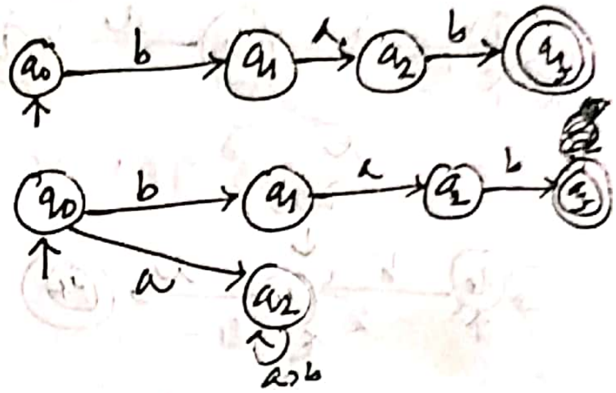
(DFA)

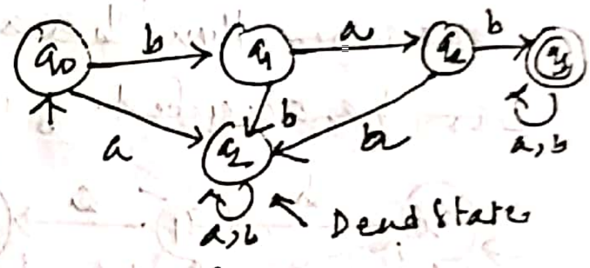


if at q_0 'b' comes then we do not need to respond.

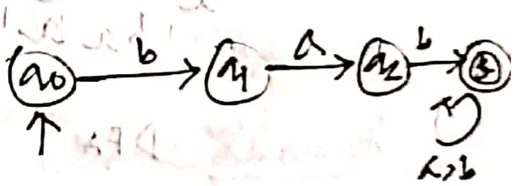
(NFA)

(ii) bab x





(DFA)



(NFA)

Note:

{W}x0 if $|W| = n$.

DFA: $n + 2$

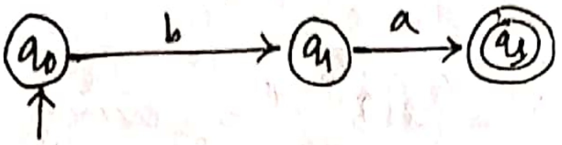
NFA: $n + 1$ (without dead state)

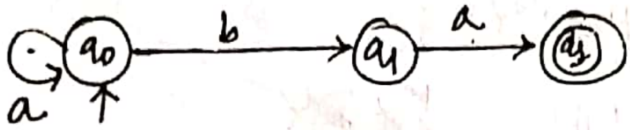
Q:2

Design a NFA over an alphabet $\Sigma = \{a, b\}$ s.t. every String ~~is~~ accepted ~~and~~ ends with,

(i) X.ba

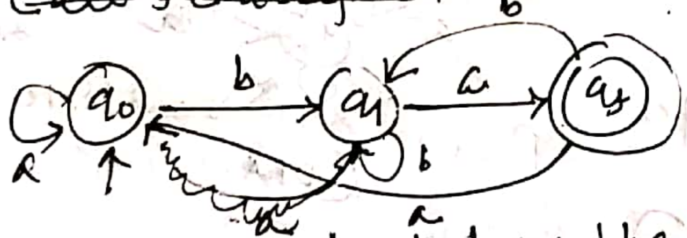
(ii) X.bab





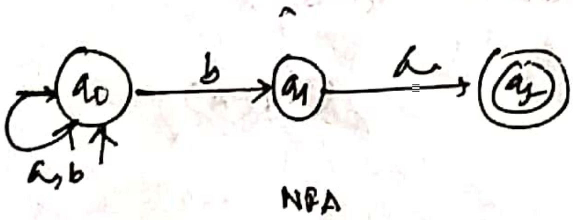
not

~~because then 'baaaa a' Can be accepted~~



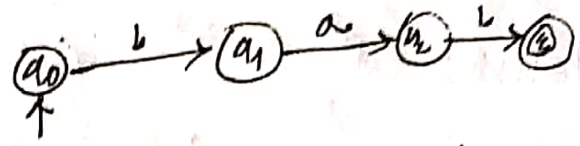
'baa' or 'bab'

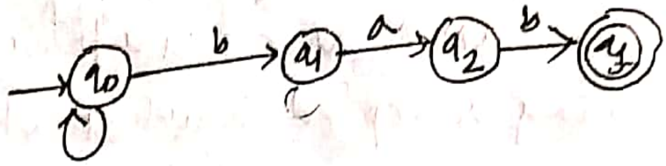
DFA

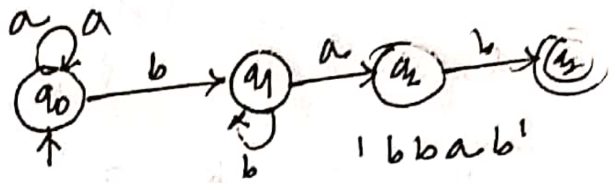


NFA

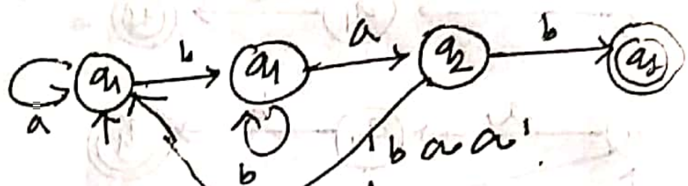
' —— ba'



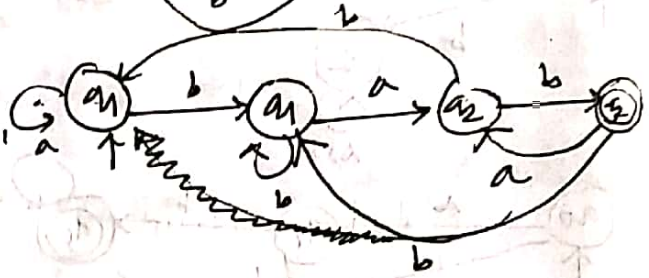




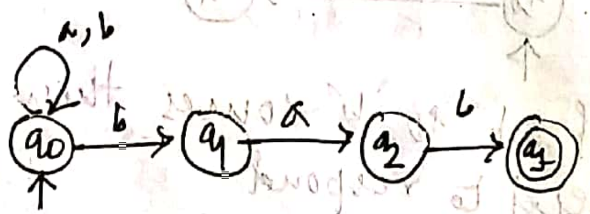
'bbab'



'baa'



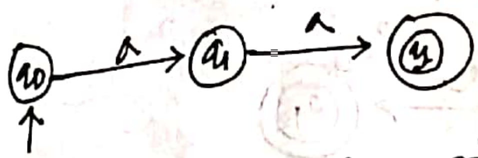
DFA

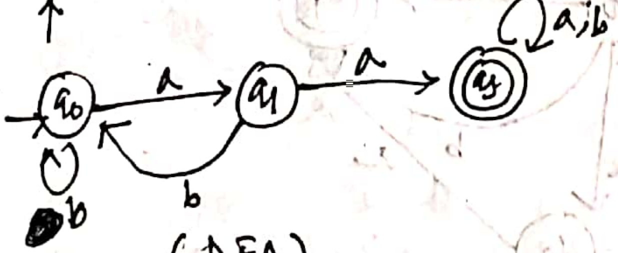


Q: 3

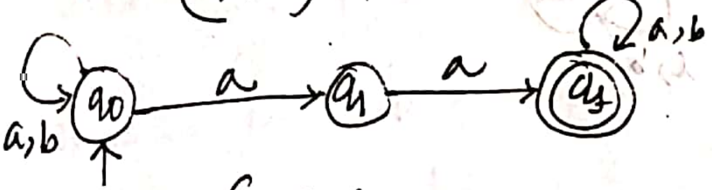
Design a NFA over an alphabet $\Sigma = \{a, b\}$ s.t. every string accepted must contain a substring.

(i) X a a X



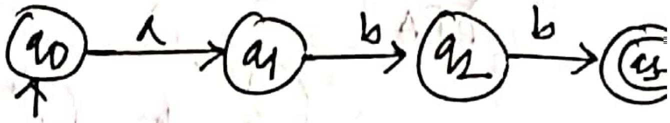


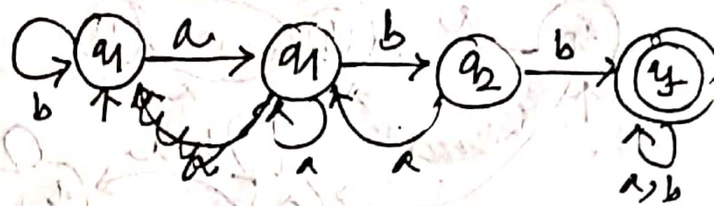
(DFA)



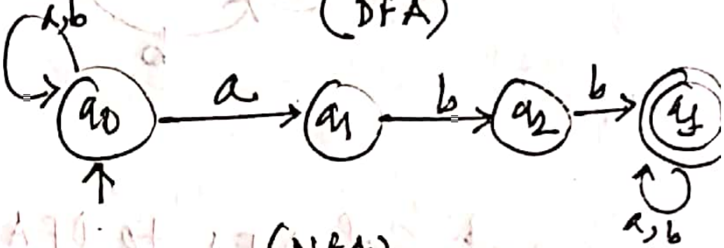
(NFA)

(ii) X a b b X





(DFA)

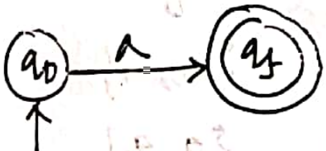


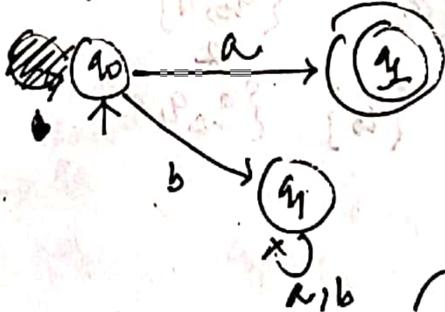
(NFA)

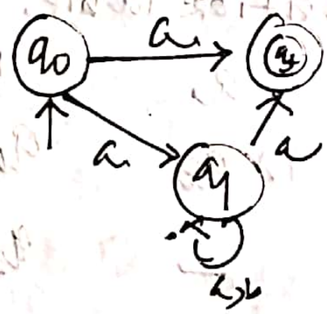
Q: 4

Design a NFA over an alphabet $\Sigma = \{a, b\}$ s.t. every string accept starting ~~with~~ and ending with a

$w = a \; x \; a.$, a

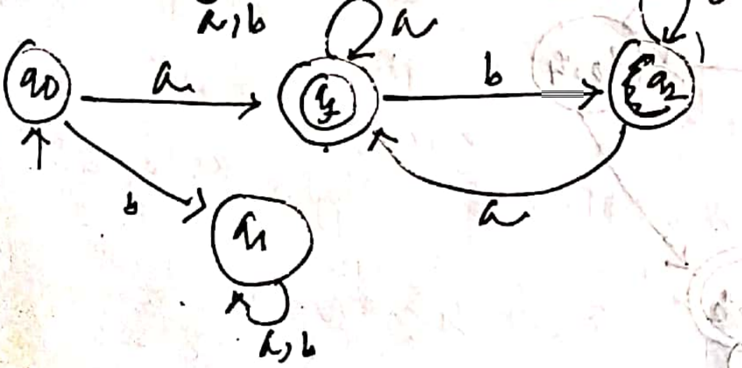






only a ababbb ... a

only a



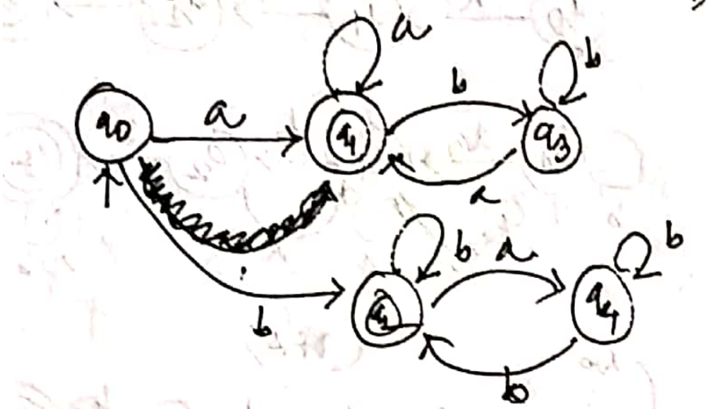
(DFA)

Q:5

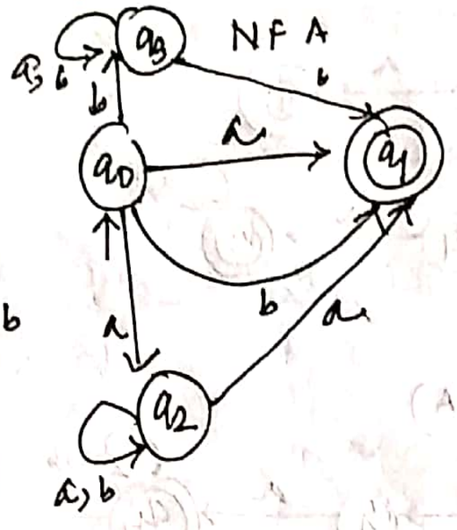
Design a NFA over $\Sigma = \{a, b\}$ s.t. it accepts every string starting & end with same symbol.

$\omega = axa,\ bxb,\ a,\ b$

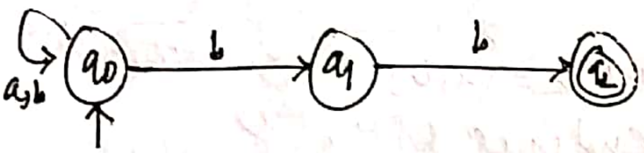
DFA



NFA



Conversion of NFA to DFA:

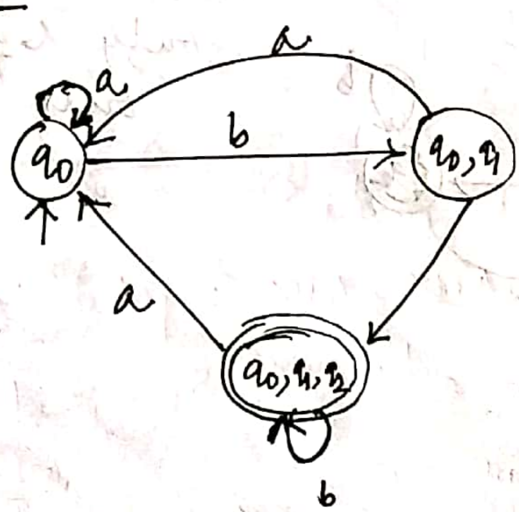


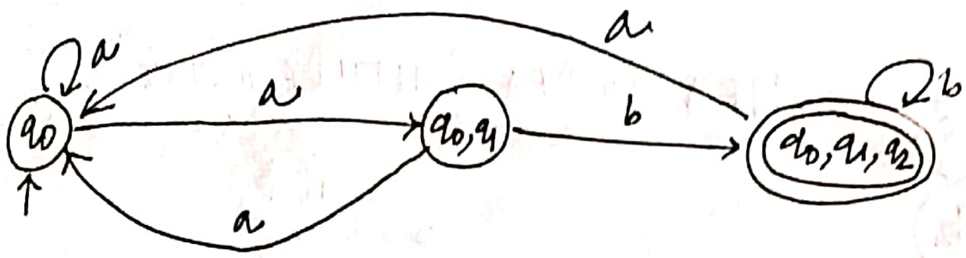
Transition Table (NFA)

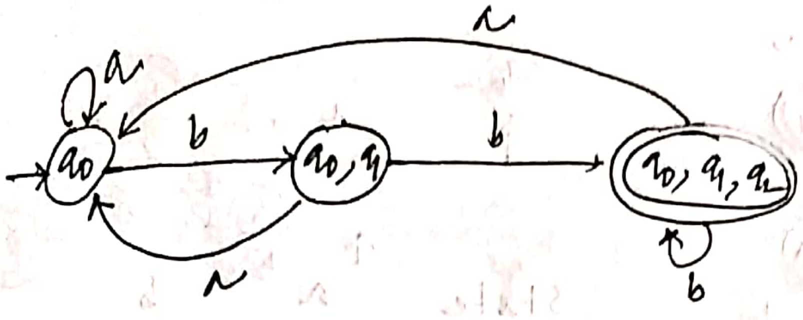
State	a	b
$\rightarrow q_0$	q_0	q_0, q_1
q_1	—	q_2
q_2	—	—

Transition Table (DFA)

State	a	b
$\rightarrow q_0$	q_0	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0\}$	$\{q_0, q_1, q_2\}$







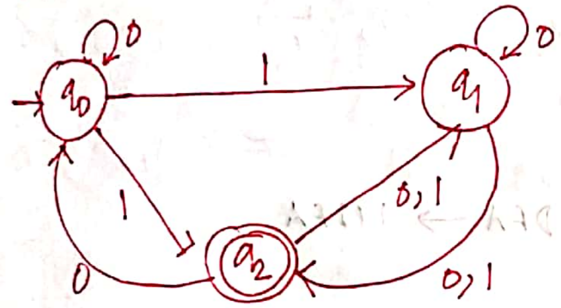
b	q_0	q_0, q_1
b	q_1	q_2
b	q_2	—

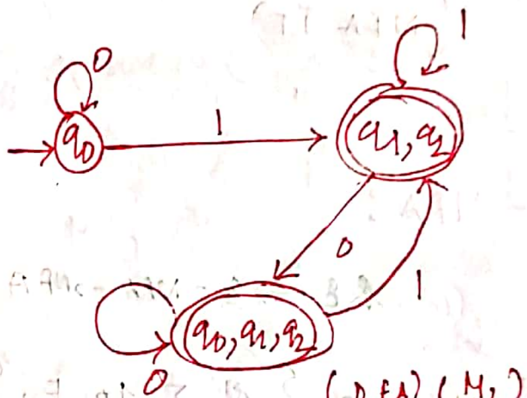
$NFA = \{\Sigma, Q, q_0, F, \delta\}$ $\quad$ $DFA = \{\Sigma, Q', q_0, F', \delta'\}$

Note: $|Q| = m$ (in NFA) & $|Q| = \lambda$ (in DFA) then,

$$1 \le \lambda \le 2^m$$

Problem2:





NFA (M_1) $\quad$ (DFA) (M_2)

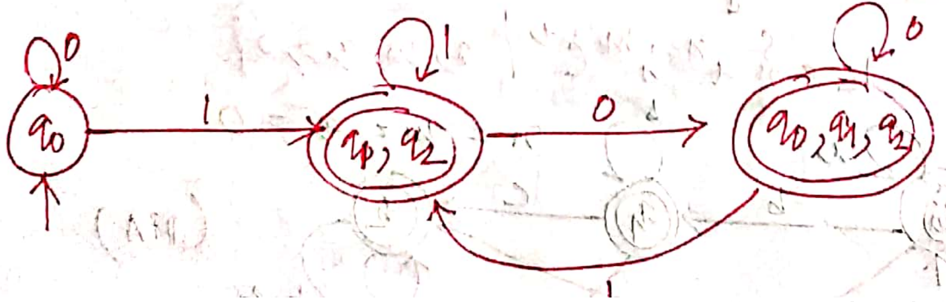
State	0	1
$\rightarrow q_0$	q_0	q_1, q_2
q_1	q_1, q_2	q_2
q_2	q_0, q_1	q_1

(NFA TT)

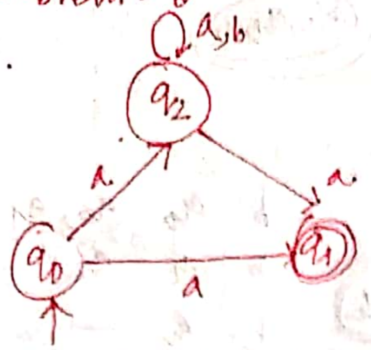
$\longrightarrow$

(DFA → TT)

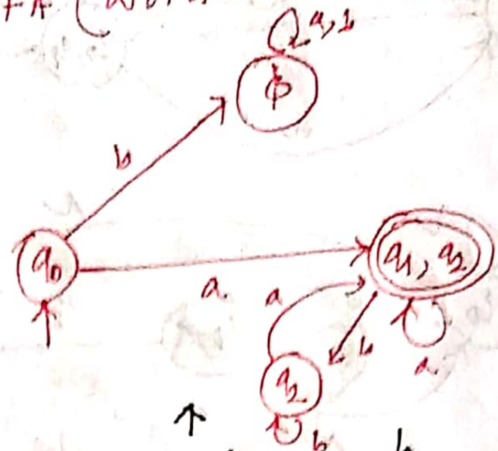
State	0	1
$\rightarrow q_0$	q_0	$\{q_1, q_2\}$
$\{q_1, q_2\}$ FS	$\{q_1, q_2, q_0\}$ ⇓ $\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_0, q_1, q_2\}$ FS	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$



Problem: 3 NFA to DFA (WITH DEAD STATE)



M_1 (NFA)



State	a	b
q_0	q_1, q_2	—
q_1	—	—
q_2	q_1, q_2	q_2

(NFA TT)

State	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	$\{\phi\}$
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$\{\phi\}$	$\{\phi\}$	$\{\phi\}$
$\{q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$

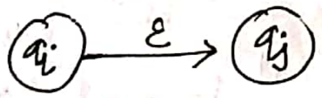
(DFA TT)

ε-NFA:

RE → ε-NFA → NFA → DFA → MDFA

$\varepsilon\text{-NFA} = \{Q, \Sigma, q_0, F, \delta\}$

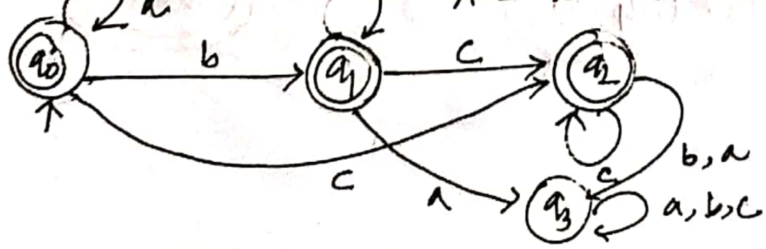
- DFA → $\delta: Q \times \Sigma \rightarrow Q$
- NFA → $\delta: Q \times \Sigma \rightarrow 2^Q$
- ε-NFA → $\delta: Q \times (\varepsilon \cup \Sigma) \rightarrow 2^Q$ $|\varepsilon| = 0$



L generated by ε-NFA, NFA, DFA, MDFA is same.

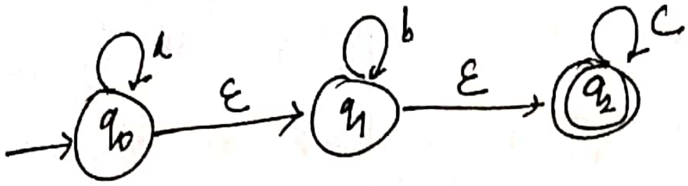
Prob 1: $L = \{a^n b^m c^q \mid n, m, q \geq 0\}$

$n = m = q = 0$



(DFA)

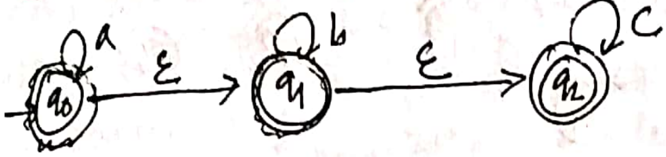
Scanned with CamScanner



(ε−NFA)

$a^* b^* c^*$

ε−NFA to NFA :



(কোনো state থেকে কোনো Symbol পড়ে বা ε transition এর মাধ্যমে কোন কোন state -এ যাওয়া যায়)

State	a	b	c
$\rightarrow q_0$	q_0, q_1, q_2	q_1, q_2	q_2
$\rightarrow q_1$	ϕ	q_1, q_2	q_2
$\rightarrow q_2$	ϕ	ϕ	q_2

Q	ε−Closure
q_0	$\{q_0, q_1, q_2\}$
q_1	$\{q_1, q_2\}$
q_2	$\{q_2\}$

$$\delta'(q_i, x) = \epsilon\text{-closure}[\delta[\epsilon\text{-closure}(q_i), x]]$$

$$\delta'(q_0, a) = \{q_0, q_1, q_2\}, a$$

$$q_0;$$

$$\{q_0, q_1, q_2\}$$

$$\delta'(q_0, b) = [\{q_0, q_1, q_2\}, b]$$

$$\varepsilon\ \text{closure}[q_1]$$

$$q_2$$

$$\delta'(q_0, c) = \varepsilon\text{-c}\ [(q_0, q_1, q_2), c]$$

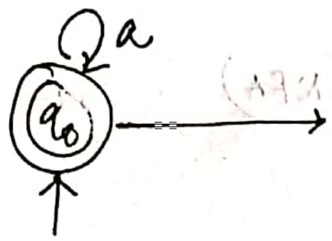
$$= \varepsilon\text{-c}\ [q_2]$$

$$= q_2$$

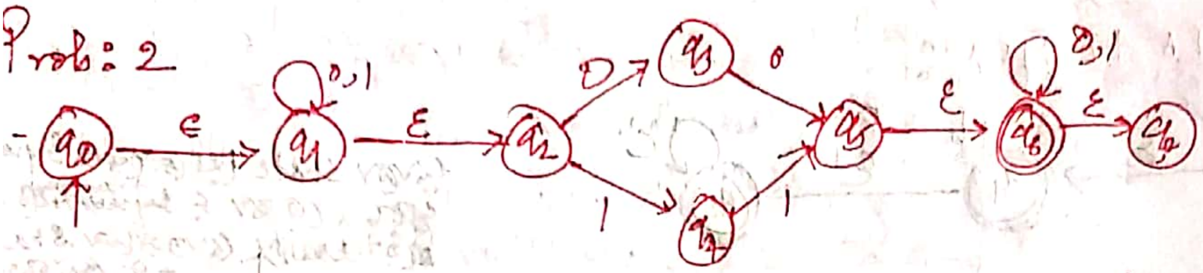
$$\delta'(q_1, a) = \varepsilon\text{-c}\ [(q_1, q_2), a]$$

$$= \varepsilon\text{-c}\ [\phi]$$

$$= \phi$$



Prob: 2



Q	ε closure
q_0	$\{q_0, q_1, q_2\}$
q_1	$\{q_1, q_2\}$
q_2	$\{q_2\}$
q_3	$\{q_3\}$
q_4	q_4
✓ q_5	q_5, q_6, q_7
✓ q_6	q_6, q_7
q_7	q_7

State	0	1
q_0	q_1, q_2, q_3	q_1, q_2, q_4
q_1	q_1, q_2, q_3	q_1, q_2, q_4
q_2	q_3	q_4
q_3	q_5, q_6, q_7	ϕ
q_4	ϕ ~~q_5, q_6, q_7~~	q_5, q_6, q_7
q_5	q_6, q_7	q_6, q_7
q_6	q_6, q_7	q_6, q_7
q_7	ϕ	ϕ

ε – ~~NFA~~ $\hat{c}$: $[\delta[\varepsilon\text{-}c(q_i), x]]$

$q_4, 1$ $\qquad$ $q_4, 0$

$q_4, 1$ $\qquad$ ϕ

q_5, q_6, q_7 $\qquad$ ϕ

$q_5, q_6, q_7, 0$