Regular Languages

Regular Expressions:

a way of representing RL

expression of strings & operators like

i) $*$ (kleen cloosure) $[a^*]$

ii) $+$ (positive closure) $[a^+]$

iii) $\cdot$ (concatenation) $[a.b]$

iv) $+$ (union) $[a+b]$

A L is said to be regular if $\exists$ an expression to represent it.

$$\begin{array}{ccc} & RG & \\ & \downarrow & \\ & RL & \\ \swarrow & & \searrow \\ FA & & RE \end{array}$$

$a+b =$ choice. $a|b$

$a^* = \varepsilon, a, aa, aaa, \ldots\ldots$

$a^+ = a, aa, aaa, \ldots\ldots$

Just like BODMAS in equation we need to solve RE

RE is said to be valid iff it can be derived from the primitive RE by a finite no. of application of the rule. $r^*, r^+, r_1, r_2, r_1+r_2$

if Σ is a given alphabet then ϕ, ϵ/λ, $a \in \Sigma$ are primitive RE.

FIND L FROM RE:

1. $r = \phi$, $L(r) = \{\ \} / \phi$
2. $r = \epsilon$, $L(r) = \{\epsilon\}$
3. $r = a$, $L(r) = \{a\}$
4. $r = a+b$, $L(r) = \{a, b\}$
5. $r = a \cdot b$, $L(r) = \{ab\}$
6. $r = a+b+c$, $L(r) = \{a, b, c\}$
7. $r = (ab+a) \cdot b$, $L(r) = \{abb, ab\}$
8. $r = a^+$, $L(r) = \{a, aa, aaa, \ldots\}$
9. $r = a^*$, $L(r) = \{\varepsilon, a, aa, aaa, \ldots\}$
10. $r = (a+ba)(b+a)$, $L(r) = \{ab, bab, baa\}$
11. $r = (a+\epsilon)(b+\phi)$, $L(r) = \{ab, b\}$
12. $r = (a+b)^2$, $L(r) = \underset{(a+b)(a+b)}{=} \{aa, ab, ba, bb\}$
13. $r = (a+b)^*$, $L(r) = \{\epsilon, (a+b), (aa, ab, ba, bb), \ldots\}$
14. $r = (a+b)^*$, $L(r) = \{\epsilon, (a, b), \{aa, ab, ba, bb), \ldots\}$
15. $r = \underbrace{(a+b)^*(a+b)}_{=(a+b)^+}$, $L(r) = \{a, b, aa, bb, \ldots\}$
16. $r = a^* \cdot a^*$, $L(r) = \{\varepsilon, a, a^2, a^3, \ldots\} = a^*$
17. $r = (ab)^*$, $L(r) = \{\varepsilon, ab, abab, \ldots\}$

FORMULA & OPERATIONS IN RE :

1. $r^+ \cup r^* = r^*$
2. $r^+ \cap r^* = r^+$
3. $r^* . r^+ = r^+$
4. $(r^*)^* = r^*$
5. $(r^*)^+ = r^*$
6. $(r^+)^* = r^*$
7. $((r^*)^+)^* . r^+ = r^+$
8. $(a+b)^* = (a^*+b)^*$
9. $(a+b)^* = (a+b^*)^*$
10. $(a+b)^* = (a^*+b^*)^*$
11. $(a+b)^* \neq (a.b)^*$
12. $(a+b)^* \neq (a^*.b)^*$
13. $(a+b)^* = (a.b^*)^*$
14. $(a+b)^* = (a^* b^*)^*$

:1

$r_1 = a^*$

$r_2 = a^* + (aa)^*$

a) $L(r_1) \subseteq L(r_2)$

b) $L(r_2) \supseteq L(r_2)$

c) $L(r_1) = L(r_2)$

d) $L(r_1) \neq L(r_2)$

$a^* = \{a^0, a^1, a^2, a^3, \ldots, a^\infty\}$

$= \{\epsilon, a^1, a^2, a^3, \ldots, a^\infty\}$

$(aa)^* = \{\epsilon, a^2, a^4, a^6, \ldots\}$

$r_1 = a^* = (aa)^* + a^* = r_2$

$\therefore r_1 \cong r_2$

Two different RE generates same RL. or multiple RE generates same RL ~~or viceversa~~. But one RE cannot generates multiple RL. One RE can generates one RL.

When we can say 2 RE are equal?

When they represents the same RL.

$\therefore L(r_1) = L(r_2)$

Q:2

$r = \epsilon^*$; $r = \epsilon^+$; $r = \phi^*$; $r = \phi^+$

$r = \epsilon^*$; $L(r) = \{\epsilon^0, \epsilon^1, \epsilon^2, \ldots\} = \{\epsilon, \epsilon, \epsilon, \ldots\}$

$= \{\epsilon\}$

$r = \epsilon^+$

$= \{\epsilon^1, \epsilon^2, \epsilon^3, \ldots \epsilon^\infty\}$

$= \{\epsilon, \epsilon, \epsilon, \ldots, \epsilon\}$

$= \{\epsilon\}$

$r = \phi^*$

$= \{\phi^0, \phi^1, \phi^2 \ldots \infty\}$

$= \{\epsilon, \phi, \phi, \ldots \infty\}$

$= \{\epsilon\}$

$r = \phi^+$

$= \{\phi^1, \phi^2, \phi^3, \ldots \infty\}$

$= \{\ \}$

$= \phi$

$A = \{a, b\}$

$A^0 = \{\epsilon\}$

$A^1 = \{a, b\}$

$A^2 = \{aa, ab, ba, bb\}$

$A^3 = \{aaa, \ldots, bbb\}$

$\vdots$

$A^k = \{w \mid |w| = k\}$

DESIGN A RE

① Starts with 'ab' $\Sigma = \{a, b\}$

$$ab\underbrace{(a+b)^*}_{\text{Any combination of a \& b or } \epsilon}$$

② Starts with 'bba'

$$bba\,(a+b)^*$$

③ ends with 'ab'

$$(a+b)^* \cdot ab.$$

④ ends with 'abb'

$$\underbrace{(a+b)^*}_{\text{Anything}} \cdot ab$$

⑤ Contains a substring 'aab'

$$(a+b)^* aab\, (a+b)^*$$

⑥ Starts and ends with 'a'

$$a + a\,(a+b)^*\, a$$

⑦ Starts and ends with same symbol.

$$a + b + a\,(a+b)^* a + b\,(a+b)^* b$$

⑧ Starts and ends with different symbol.

$$a\,(a+b)^* b + b\,(a+b)^* a$$

⑨ $|w| = 3$

$$(a+b)\,(a+b)\,(a+b)$$

⑩ $|w| \geq 3$

$$(a+b)^3\,(a+b)^*$$

⑪ $|w| \leq 3$

$$\cancel{(a+b)^3} \qquad \epsilon + (a+b) + (a+b)^2 + (a+b)^3$$

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⑫ $|w|_a = 2$

$b^* a b^* a b^*$

$|w| = 2$ means all ~~strings~~ strings of length 2 generated by the symbols present in Σ.

$|w|_a = 2$ means all strings of length 2 generated with only 'a'.

⑬ $|w|_a \geq 2$

$(a+b)^* a (a+b)^* a (a+b)^*$

⑭ $|w|_a \leq 2$

$b^* (a+\varepsilon) b^* (a+\varepsilon) b^*$ ~~or~~ $b^* + b^* a b^* + b^* a b^* a b^*$

⑮ 3rd symbol from left end is b.

$(a+b)^2 b (a+b)^*$

⑯ 28th symbol from right end is 'a'

$(a+b)^* a (a+b)^{27}$

⑰ $|w| = 0 \pmod 3$

$\frac{\text{length of the string}}{3} = 0$ ↑ reminde

$[(a+b)^3]^*$ 0, 3, 6, 9, ...

⑱ $|w| = 2 \pmod 3$ 2, 5, 8, 11, 14, ...

$(a+b)^2 [(a+b)^3]^*$

⑲ $|w|_b = 0 \pmod 3$

$a^* +$ ~~$[(b)^3]^*$~~ $(a^* b a^* b a^*)^*$ 0, 3, 6, 9, ...

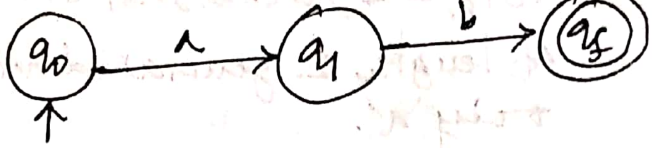
⑳ $|w|_a = 1 \pmod 3$

$(b^* a b^* a b^* a b^*)^* + b^* a b^*$

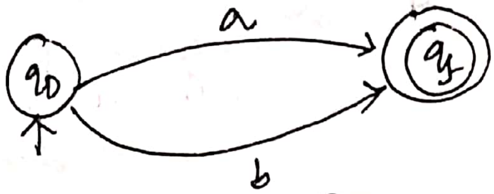
㉑ $|w|_b = 2 \pmod 3$

$(a^* b a^* b)^* + (a^* b a^* b a^* b a^*)^*$

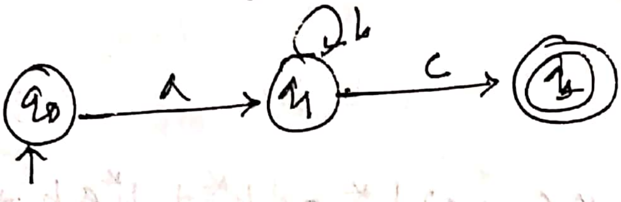
CONVERSION OF FA TO RE :

① 

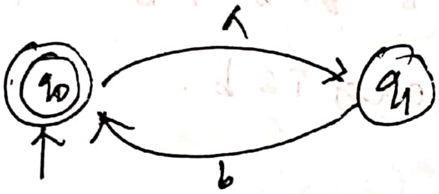
RE: ab

② 

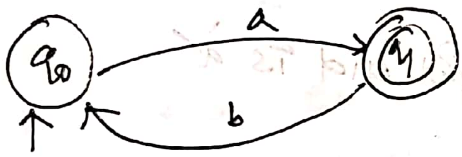
RE: $a+b$

③ 

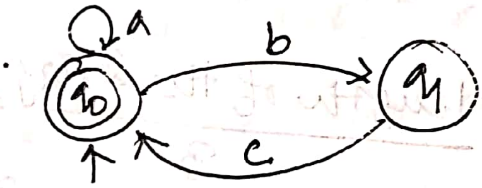
RE: $a \cdot b^* c$

④ 

RE: $(a \cdot b)^*$

⑤ 

RE: $a \cdot (ba)^*$

⑥ 

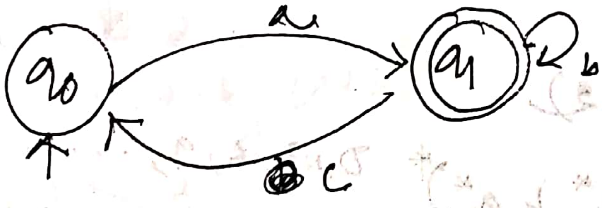
RE: $a^* \cdot (b \cdot c)^*$

$\times \; a^* + (bc)^*$

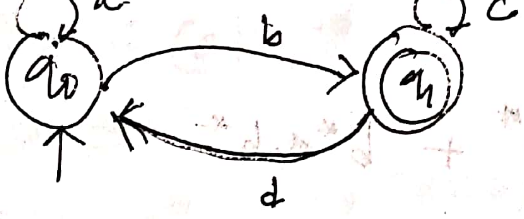
$\cancel{(b.c)^* \, a^* \, (b.c)^*}$

$(a^* + (bc)^*)^*$

$= (a + bc)^*$

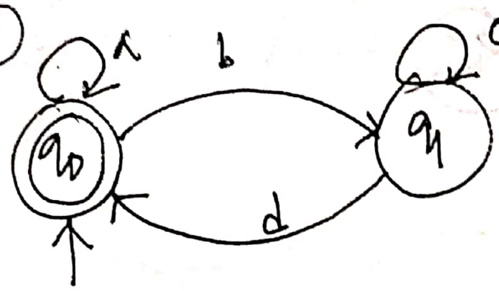
⑦ 

RE: $a(b+ca)^*$

⑧ 

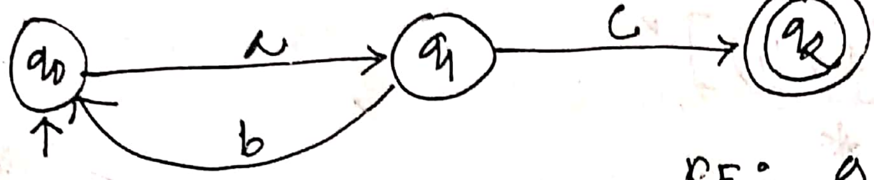
RE: $a^* b (c + d a^* b)^*$

(9)



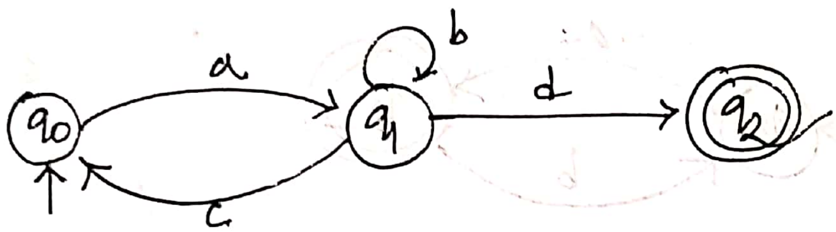
RE: $(a^{*} + bc^{*}d)^{*}$

(10)



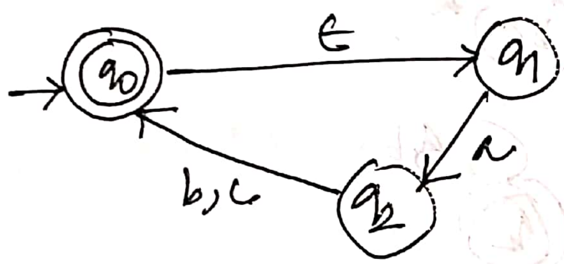
RE: $a(b \cdot a)^{*}c$

(11)



RE: $a(b+ca)^{*}d$

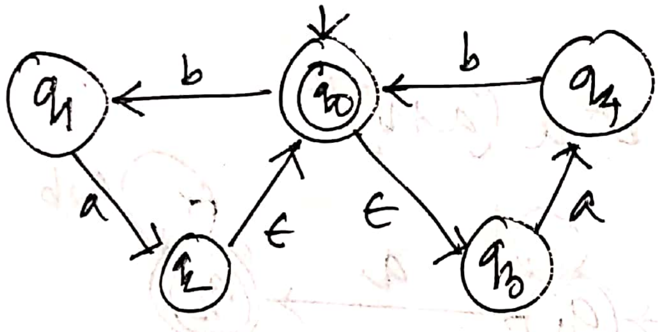
(12)



RE: $[\epsilon \cdot a(b+c)]^{*}$

$[a(b+c)]^{*}$

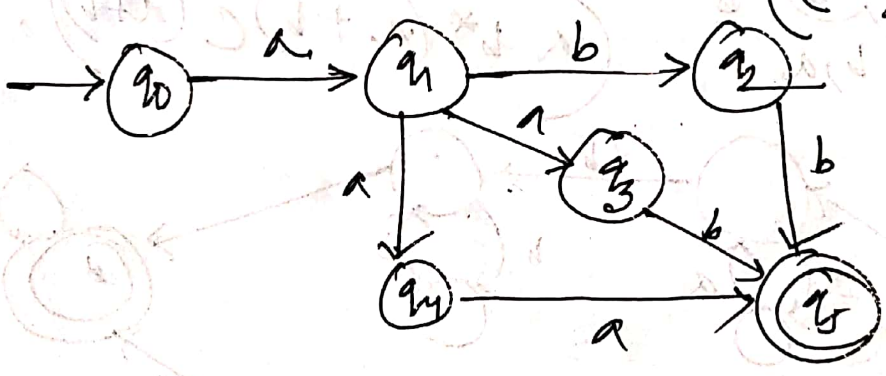
(13)



RE: $((ba\epsilon)^{*} + (\epsilon ab)^{*})^{*}$

$((ba) + (ab))^{*}$

(14)



RE: $a(bb+ab+aa)$

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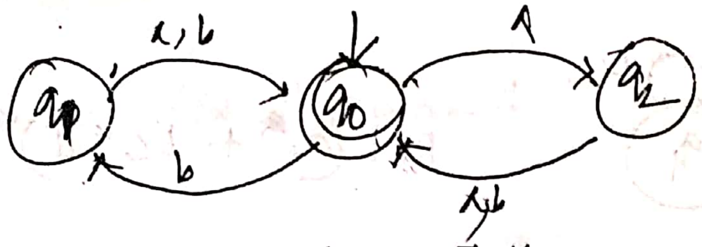


$$[a(a+b) + b(a+b)]^*$$

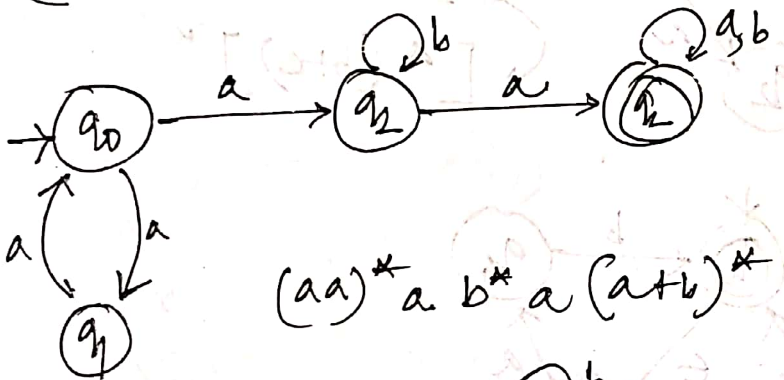
$$= [(a+b)(a+b)]^*$$

$$= ((a+b)^2)^*$$

16

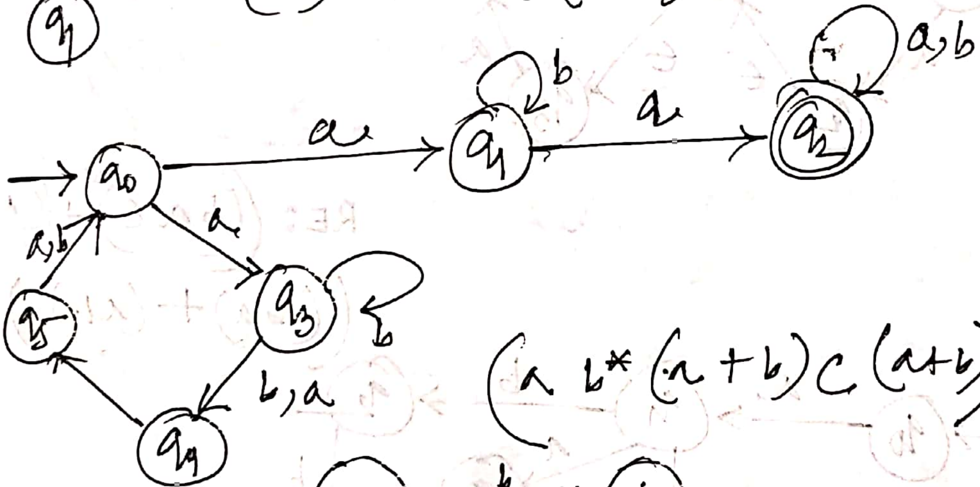
$\cancel{(b^*.a + ab)^*}$ $\quad b^*aa(ba)^*$

17



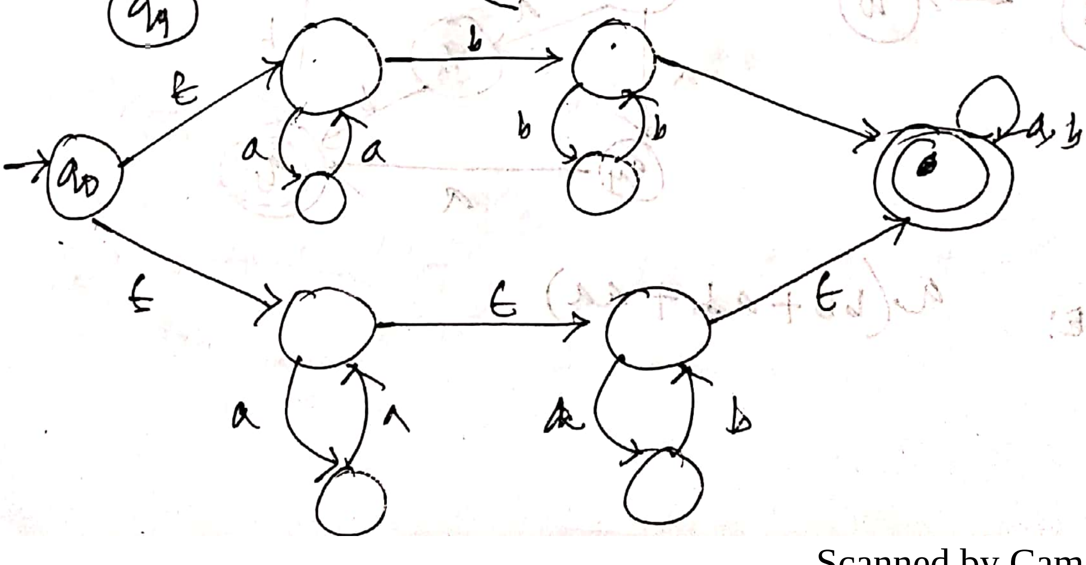
$$(aa)^* a\, b^* a (a+b)^*$$

18

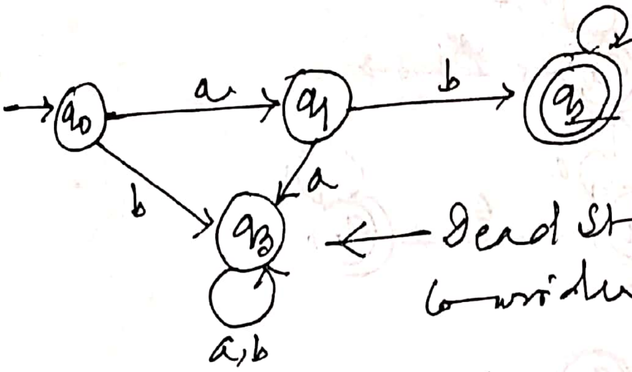


$$(a\, b^* (a+b) c (a+b))^* a\, b^* a (a+b)^*$$

19

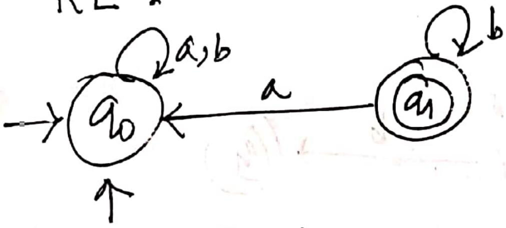


RE: $(\epsilon (aa)^* b (bb)^* \epsilon (a+b)^*) + (\epsilon (aa)^* \epsilon (ab)^* \epsilon (a+b)^*)$

(20) 

Dead state should not be consider.

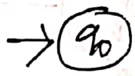
RE: $ab(a+b)^*$

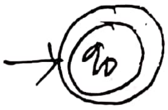
(21) 

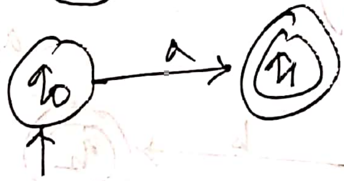
RE: ϕ

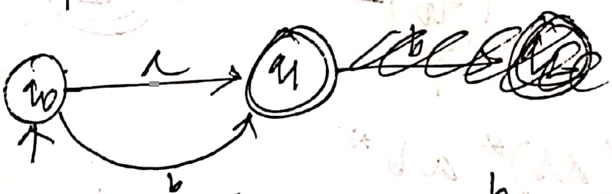
Unreachable state.

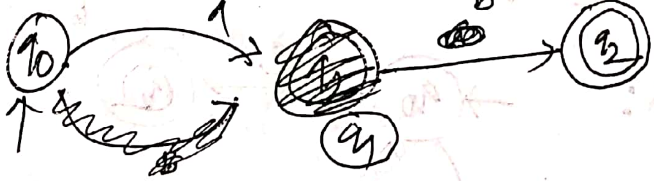
CONVERSION OF RE TO FA:

(1) ϕ FA: 

(2) ϵ FA: 

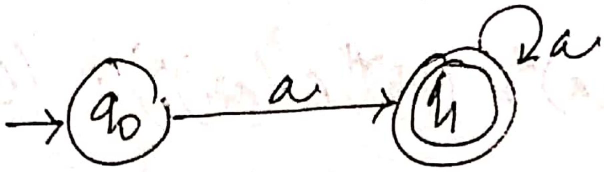
(3) a FA: 

(4) $a+b$ FA: 

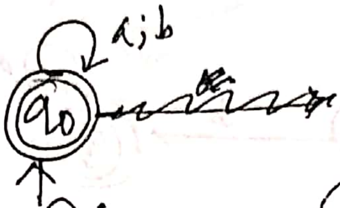
(5) $a.b$ FA: 

(6) a^* FA: 

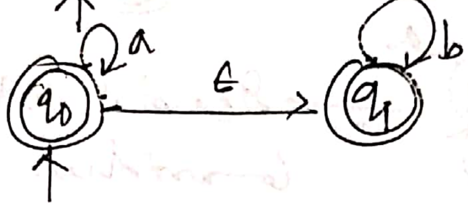
(7) a^+ FA:



(8) $(a+b)^*$ FA:

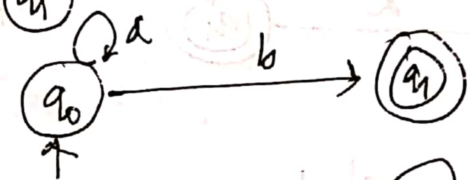


(9) $(a^* b^*)$ FA:

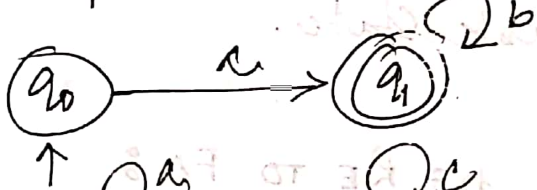


(10) $(ab)^*$ FA:

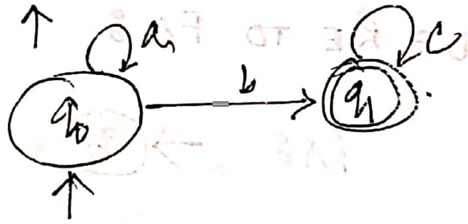
(11) $a^* b$ FA:



(12) ab^* FA:

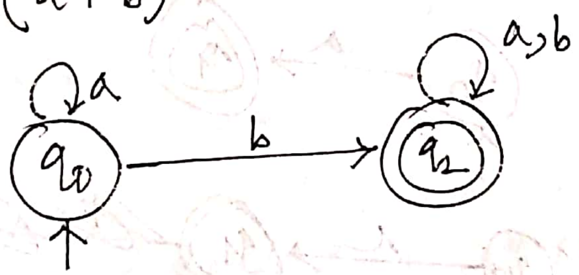


(13) $a^* b c^*$ FA:



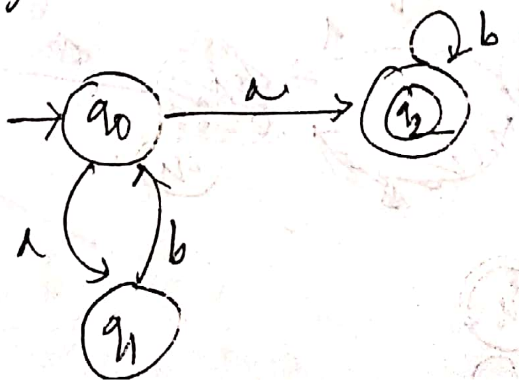
(14) $a^* b (a+b)^*$

FA:



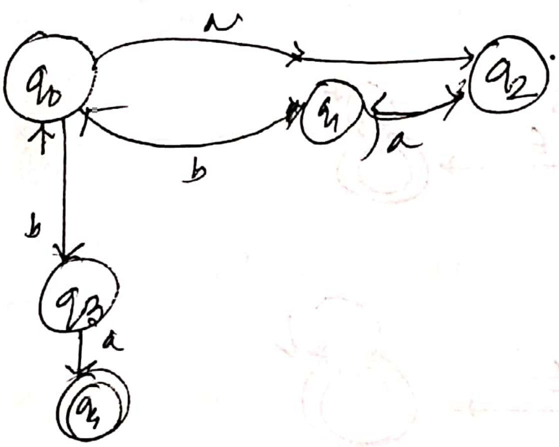
(15) $(ab)^* ab^*$

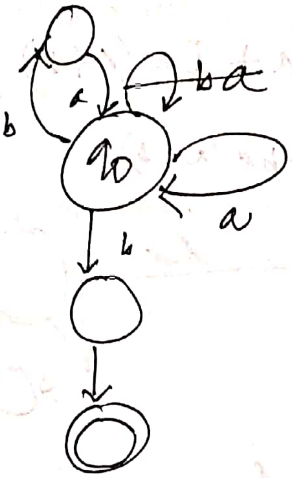
FA:



16. $(a+ba)^* ba$

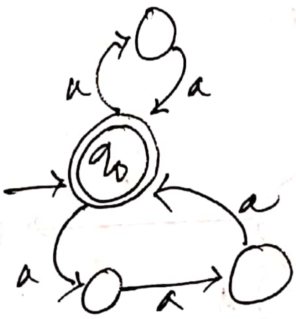
FA:



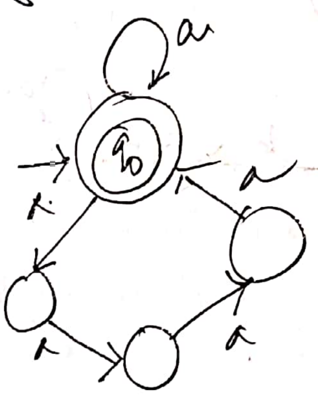


17. $(aa+aaa)^*$

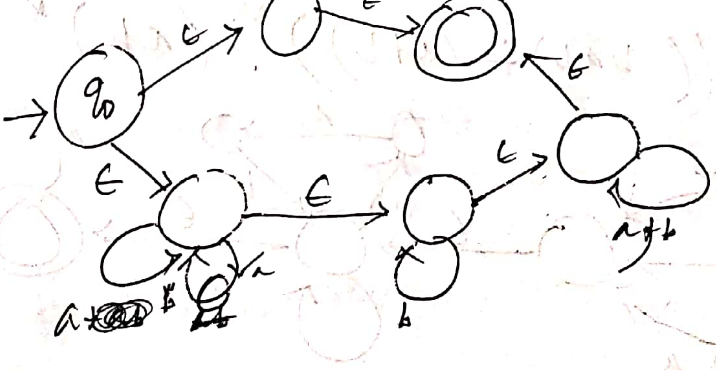
FA:



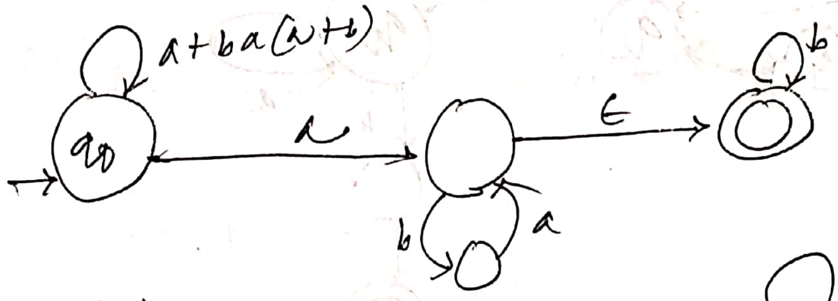
18. $(a+aaaa)^*$

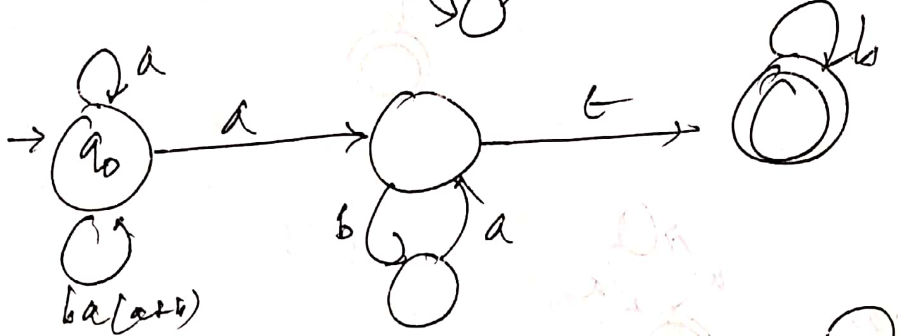


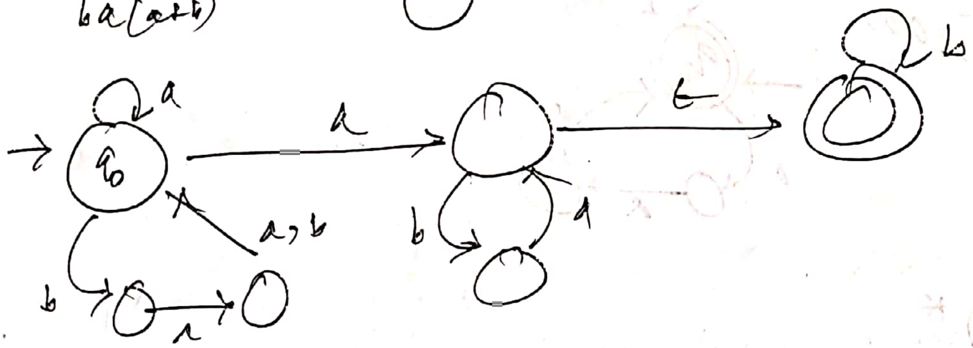
19. $(ab)^* + (a+ab)^* b^* (a+b)^*$



(20) $[a+ba(a+b)]^* a(ba)^* b^*$

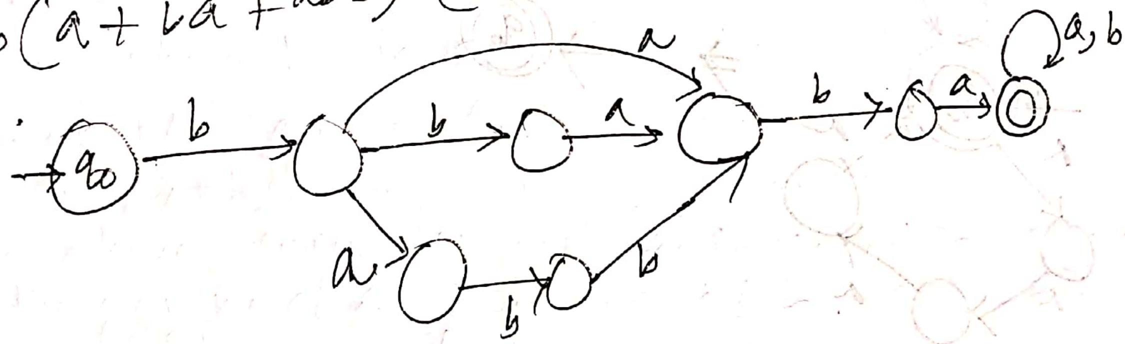






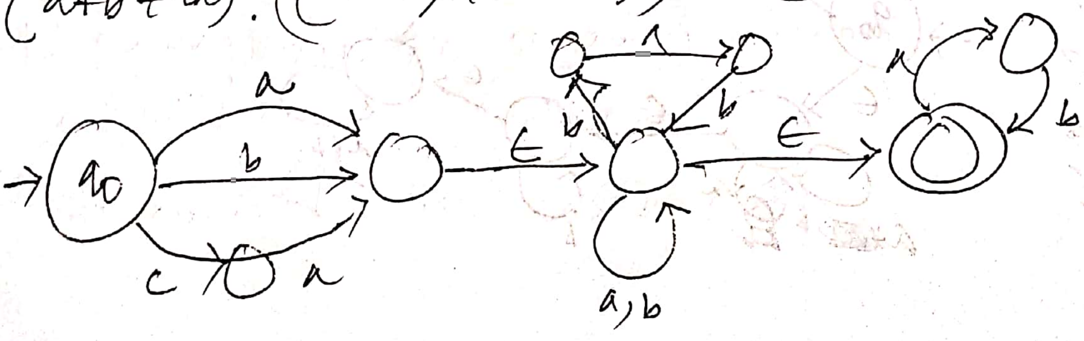
(21) $b(a+ba+abb)(ba(a+b)^*)$

FA:



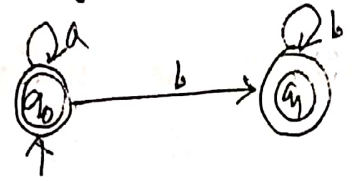
(22) $(a+b+ca)((bab)^*+(a+b)^*)^*(ab)^*$

$(a+b+ca).((bab)+(a+b))^*.(ab)^*$



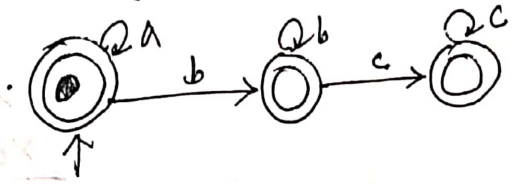
* How to identify whether a 'L' is regular or not?

① $L = \{a^m b^n \mid m, n \geq 0\}$



FA can remember the order.
It can remember the order by changing the state.

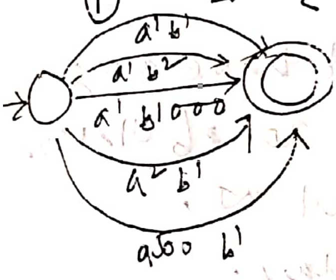
② $L = \{a^m b^n c^q \mid m, n, q \geq 0\}$

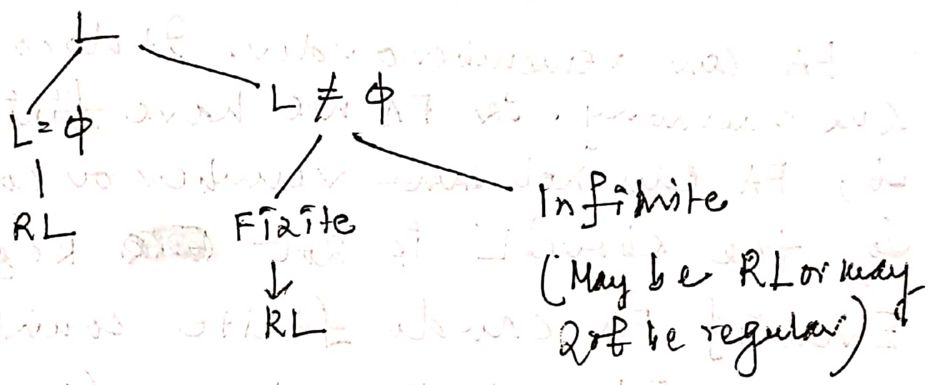


③

$L = \{a^{x_1} a^{x_2} \dots a^{x_{26}} \mid x_i \geq 0;\ 1 \leq i \leq 26\}$

④ $L = \{a^m b^n \mid 1 \leq m \leq 500;\ 1 \leq n \leq 1000\}$





⑤

$w_1, w_2, w_3 \dots, w_n$

For finite no of strings we have finite no of ~~sets~~ paths ~~of~~ from IS to FS. So, for finite no. of strings we can make a FA. Hence the L can be regular.

$L = \{a^m b^n \mid 1 \leq m \leq 500;\ 1 \leq n \leq 1000\}$

We can go ~~for~~ 'm' upto 500 which is a finite no.

We can go for 'n' upto 1000 which is also a finite no.

Hence ④ is a RL.

⑤ $L = \{ a^n b^n \mid 1 \le n \le 10 \}$

upper range $= 10$ = finite

Hence L is RL.

$4 \times 60 = 240$

⑥ $\{ a^n b^n \mid n \geq 0 \} = L$

$\{ \varepsilon, ab, aabb, \ldots \} = L$

10 -> 12 = 2
PL -> 2 = 2

240

Here 'n's lower range is '0'

" n's upper " " '∞'

FA can remember order. It does not have any aux memory. In FA we have finite no. of states. So, FA cannot ~~can~~ remember or count ∞,

So, the above 'L' is not Regular.

Even if FA can do finite counting & " " FA " " " comparison.

In the above example we ~~not~~ need to compare whether ∞ no of a = ∞ no of b. which is an infinite no of comparison.

⑦ $L = \{ a^n b^n \mid 1 \le n \le 2^{29\text{th prime no}} \}$

upper range $= 2^{29\text{th prime no}}$

1st 29th prime no is a finite no.

$2^{\text{finite no}}$ = finite no.

Hence L is Regular.

⑧ $L = \{a^n b^n \mid 1 \leq n \leq 2^{|GATE|}\}$

This 'L' needs comparison because of $a^n b^n$.
'GATE' is a string. $|GATE| = 4$.
$2^4 = 16$ which is a finite no.
Hence L is a RL.

⑨ $L = \{a^m b^n \mid m > n,\ n \geq 0\}$ it is not regular.

⑩ $L = \{a^m b^n \mid m, n \geq 0 \quad m \neq n\}$ it is not regular.

⑪ $L = \{a^m b^n \mid m$ is divisible by $n\}$ it is not regular.

⑫ $L = \{a^m b^n \mid m = n^p \mid p \geq 1\}$ it is not regular.

⑬ $L = \{a^m b^n c^q \mid m = n = q\}$ it is not regular.

⑭ $L = \{a^m b^n c^q \mid m + n = q\}$ it is not regular.

⑮ $L = \{a^m b^n \mid m + n = \text{even}\}$ " " " "

⑯ $L = \{a^m b^n \mid m + n = \text{odd}\}$ " " " "

→

a	b	n
O +	O	e
O +	e	O
e +	O	O
e +	e	e