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Paper Name : Formal Language & Automata Theory

Paper Code : PCC-CS401

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Date : 05/05/2021

## Answers

1. A)  $W_1 = (A, C)$  as  $A \rightarrow a$  &  $C \rightarrow b$  are productions with a ~~terminal~~ terminal string on RHS

$$W_2 = \emptyset \{A, C\} \cup \{A, \mid A, \rightarrow \alpha \text{ with } \alpha \in (\Sigma \cup \{A, C\})^*\}$$
$$= \{A, C, S\} \cup \emptyset$$

As  $W_3 = W_2$

$$V_N' = W_2 = \{S, A, C\}$$

$$P' = \{A, \rightarrow \alpha \mid A, ; \alpha \in (V_N' \cup \Sigma)^*\}$$
$$= \{S \rightarrow CA, A \rightarrow a, C \rightarrow b\}$$

Then

$$G_1 = (\{S, A, C\}, \{a, b\}, \{S \rightarrow CA, A \rightarrow a, C \rightarrow b\}, S)$$

Step-2 We have to apply Theorem to  $G_1$ , Thus

$$W_1 = \{S\}$$

As we have production  $S \rightarrow CA$  &  $S \in W_1$ ,  ~~$W_2 = \{S\}$~~

$$W_2 = \{S\} \cup \{A, C\}$$

As  $A \rightarrow a$  &  $C \rightarrow b$  are productions with  $A$   
 $A, C \in W_2$ ,  $W_3 = \{S, A, C, a, b\}$

As  $W_3 = V_N' \cup \Sigma$ ,

$$P'' = \{S \rightarrow a \mid A, C \in W_3\} = P'$$

$\therefore G' = \{S, A, C, a, b\}$

$\therefore G' = (\{S, A, C\}, \{a, b\}, \{S \rightarrow CA, A \rightarrow a, C \rightarrow b\}, S)$

2. A)

3 b)

We have to start with an S-production.  
 At every stage we apply a suitable production  
 which is likely to derive  $w$ .

In this example, the substring is underlin-  
 ed to be replaced by use of production

$$S \Rightarrow \underline{aA_1} A_2 a$$

$$\Rightarrow baa \underline{A_2} a$$

$$\Rightarrow baa \underline{A_1} aba$$

$$\Rightarrow baabab \underline{aA_1} A_2 baba$$

$$\Rightarrow baabbaa \underline{A_2} baba$$

$S \Rightarrow baabba \underline{aA} abbaba$

$\Rightarrow baabbabaaabbaba = w$

$\therefore \boxed{w \in L(G)}$

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2A)  $S \rightarrow AB$ ,  $A \rightarrow BS|b$ ,  $B \rightarrow SA|a$   
Then

$S \rightarrow AB$  is not in GNF. So substitute  
 $A \rightarrow BS|b$  then we get,  
 $S \rightarrow BSB|bB$

However  $S \rightarrow BSB$  is not in GNF, so  
substitute  $B \rightarrow SA|a$  we get

$S \rightarrow SASB|aSB|bB$

Here,  $S \rightarrow SASB$  is left recursion.

So,  $C \rightarrow SA|SAC$ .

then production becomes

$S \rightarrow aSBC|bBC|aSB|bB$



4. A)

$$L = \{a^n b^n \mid n \geq 1\}$$

$$\therefore L = \{ab, aabb, aaabbb, \dots\}$$

As we want to design a PDA, then every time a comes before b, when 'a' comes then push it in stack & if again 'a' comes then also push it. After that, when 'b' comes then pop one 'a' from the stack. each time. So, at the end if the stack becomes empty then we can say that string is accepted by PDA.

Stack transition functions :-

$$\delta(q_0, a, z) \vdash (q_0, az)$$

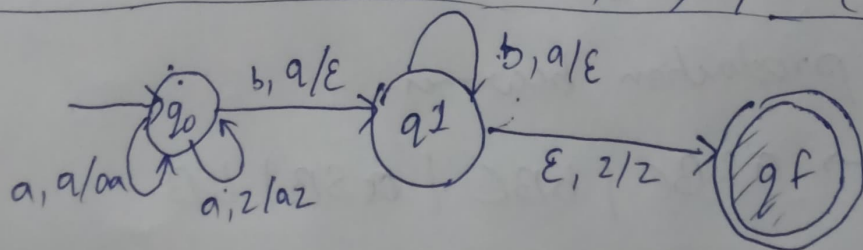
$$\delta(q_0, a, a) \vdash (q_0, aa)$$

$$\delta(q_0, b, a) \vdash (q_1, \epsilon)$$

$$\delta(q_1, b, a) \vdash (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z) \vdash (q_f, z)$$

$q_0 \rightarrow$  initial state  
 $q_f \rightarrow$  final state  
 $\epsilon \rightarrow$  indicates pop operation



$$5. A) S \rightarrow AMB$$

$$M \rightarrow aMb$$

$$M \rightarrow \epsilon$$

Removal of Null production

$$M \rightarrow \epsilon$$

In RHS Misplaced

$$S \rightarrow AMS$$

$$M \rightarrow aMb$$

Case 1 :-  $S \rightarrow AMB$

$$S \rightarrow AB \quad [\because M \rightarrow \epsilon]$$

Case 2 :-  $M \rightarrow aMb$

$$M \rightarrow ab \quad [\because M \rightarrow \epsilon]$$

After removal of null production

$$S \rightarrow AMB / AB$$

$$M \rightarrow aMb / ab$$


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6. A)  $L = \{a^p \mid p \text{ is prime}\}$  is NOT CFL

There is no pattern for prime no. can't be identified by both FSA (Finite State Automation).

We just given the  $p = \text{prime no.}$  & there is no bound on the length. So, we need infinite amount of space which can be provided by Turing machine.

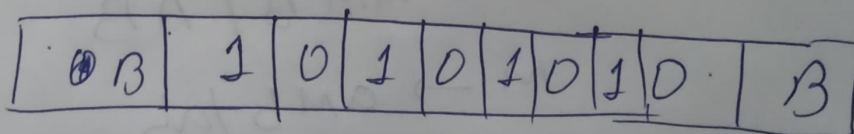
$\therefore a^p$  where  $p$  is prime is a CSL

$$w = xyz$$

$$|xyz| = p \quad |y| \geq 1$$

$$\begin{aligned} |xy'z| &= |xyz| + |y|p \\ &= p + (|y|) \end{aligned}$$

7. B) Input



Output

