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Answers

1. b) We know that,

Newton Raphson Method is a root finding method that can be repeated untill a desired accuracy is reached.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$; where,

$f'(x)$ is 1^{st} derivative of $f(x)$

ATQ,

$x\sin(x) + \cos(x) = 0$ with $x_0 = \pi$

Let; $f(x) = x\sin(x) + \cos(x)$

$f'(x) = x\cos(x) + \sin(x) - \sin(x)$

ATF,

$$x_{n+1} = x_n - \frac{x\sin(x) + \cos(x)}{x\cos(x) + \sin(x) - \sin(x)}$$

Case-I

If $x_0 = \pi$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \pi - \frac{f(\pi)}{f'(\pi)}$$

$$= \pi - \left(\frac{\pi\sin\pi - \cos\pi}{\pi\cos\pi}\right)$$

$$\Rightarrow x_1 = \pi - \frac{1}{\pi} = 2.8233$$

Case - II

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \cancel{2.8233}.$$

$$= 2.8233 - \frac{[(2.823 \sin 2.823) + \cos(2.823)]}{2.823 \cos(2.823)}$$

$$\Rightarrow x_2 = 2.7986$$

Case - III

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$ where $x_2 = 2.7986$

$$\therefore x_3 = 2.7989$$

Case - IV

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$, where $x_3 = 2.7989$

$$\Rightarrow x_4 = 2.7989$$

$\therefore$ Required real root is 2.7989

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2. a) $$\int_0^6 \frac{dx}{1+x^2} = ?$$

$$\int_a^b f(x)\,dx = \frac{h}{3} \cdot \{ (y_0 + y_n) + 4(y_1 + y_3 + \dots y_{n-1}) + 2(y_2 + y_4 + \dots y_{n-2}) \}$$

(Acc. To Simpson's 1/3rd rule)

$$\int_0^6 \frac{dx}{(1+x^2)} dx$$

$n = 6$ iterals

$$h = \frac{b-a}{n}$$

$$= \frac{6-0}{6}$$

$$\Rightarrow \boxed{h = 1}$$

$x_1 = 1$, $y_1 = \frac{1}{1+1} = \frac{1}{2}$

$x_2 = 2$, $y_2 = \frac{1}{1+4} = \frac{1}{5}$

$x_3 = 3$, $y_3 = \frac{1}{1+9} = \frac{1}{10}$

$x_4 = 4$, $y_4 = \frac{1}{1+16} = \frac{1}{17}$

$x_5 = 5$, $y_5 = \frac{1}{1+25} = \frac{1}{26}$

$x_6 = 6$, $y_6 = \frac{1}{1+36} = \frac{1}{37}$

$$\therefore \int_0^6 \frac{dx}{(1+x^2)} dx = \frac{1}{3}\left\{\left(1 + \frac{1}{37}\right) + 4\left(\frac{1}{2} + \frac{1}{10} + \frac{1}{26}\right) + 2\left(\frac{1}{5} + \frac{1}{17}\right)\right\}$$

$$= \frac{1}{3}\left\{(1 + 0.027) + 4(0.638) + 2(0.258)\right\}$$

$$= \frac{1}{3}\left[1.027 + 2.55 + 0.5\right]$$

$$= 1.359$$

3. A) Acc. To Trapezoidal Rule,

$$T_6 = \cancel{f_{\Delta}} \frac{h}{2}\left[f(x_0) + 2f(x_1) + \cdots 2 \cdot f(x_5) + f(x_6)\right]$$

$$\int_0^1 e^{-x^2} dx \quad , \quad f(x) = e^{-x^2}$$

$a = 0 , b = 1$

$\frac{x}{36}$

$$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

$f\left(\frac{0}{6}\right) = e^{-0} = 1$

$f\left(\frac{1}{6}\right) = e^{-(1/36)} = 1.025$

$f\left(\frac{2}{6}\right) = e^{-1/9} = 1.118$

$f\left(\frac{3}{6}\right) = e^{-9/36} = e^{-1/4} = 1.284$

$f\left(\frac{4}{6}\right) = e^{-4/3} = 1.560$

$f\left(\frac{5}{6}\right) = e^{-25/36} = 2.002$

$f\left(\frac{6}{6}\right) = f(1) = e^{-1} = 0.368$

$$\therefore T_6 = \frac{1}{12}\left[1 + 2\times 1.025 + 2\times 1.118 + 2\times 1.284 + 2\times 1.56 + 2\times 2.002 + 0.368\right]$$

$\Rightarrow T_6 = 1.27933$

$\therefore \boxed{T_6 = 1.280}$ A

4. A) Here,

$$f(x,y) = \sqrt{x^2+y},$$

$$x_0 = 0$$

$$y_0 = 0.8$$

$$h = 0.2$$

$$\therefore k_1 = hf(x_0, y_0) = 0.2 f(0, 0.8)$$

$$= 0.2\sqrt{0^2+0.8}$$

$$= 0.17889$$

$$\therefore k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= 0.2 f(0.1, 0.88944)$$

$$= 0.2\sqrt{(0.1)^2 + 0.88944}$$

$$= 0.18968$$

$$\therefore k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.2 f(0.1, 0.8984)$$

$$= 0.2\sqrt{(0.1)^2 + 0.89484}$$

$$= 0.19025$$

$$\therefore k_4 = \cancel{hf}\ hf(x_0 + h, y_0 + k_3)$$

$$= 0.2 f(0.2, 0.99025)$$

$$= 0.2\sqrt{(0.2)^2 + 0.99025}$$

$$= 0.20300$$

$$Y_1 = Y(x_0 + h)$$

$$= Y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 0.8 + \frac{1}{6}(0.17889 + 12 \times 0.1868 + 2 \times 0.19025 + 0.2030)$$

$$= 0.99209$$

$$\therefore Y(0.2) = 0.99029$$

Now, $x_1 = 0.2$, $Y_1 = 0.99029$, $h = 0.2$

$$\therefore k_1 = h f(x_1, y_1) = 0.2 f(0.2, 0.99029)$$

$$= 0.21742$$

$$k_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2})$$

$$= 0.2 f(0.3, 1.09180)$$

$$= 0.21742$$

$$k_3 = h f(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2})$$

$$= 0.2 f(0.3, 1.09901)$$

$$= 0.21808$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$= 0.2 f(0.4, 1.20038)$$

$$= 0.23396$$

$$\therefore Y_2 = Y(x_1 + h)$$

$$= Y_1 + \frac{1}{6}(k_2 + 2k_2 + 2k_3 + k_4)$$

$$= 1.28032$$

$$\boxed{\therefore Y(0.4) = 1.2083}$$

7A) $f(x) = x^3 - 5x - 7$

Take $x_0 = 2$, $x_1 = 3$ such that

$$f(x_0) = -9 \; ; \; f(x_1) = 5$$

Acc. To regula falsi iteration formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f(x_n) - f(x_{n-1})} (x_n - x_{n-1})$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f(x_1) - f(x_0)} (x_1 - x_0)$$

$$\Rightarrow x_2 = 3 - \frac{5}{5+9} (3-2)$$

$$\boxed{x_2 = 2.642857}$$

No. of iteration (n)	x_{n-1} $(f(x_{n-1})<0)$	x_n $(f(x_n)<0)$	$f(x_{n-1})$	$f(x_n)$	x_{n+1}	$f(x_{n+1})$
1	2	3	−9	5	2.642852	−1.7597
2	2.642857	3	−1.759740	5	2.735635	−0.2055
3	2.7365635	3	−0.205506	5	2.746072	−0.02247
4	2.746072	3	−0.22479 ~~−0.205506~~	5	2.747204	−0.00025
5	2.747208	3	−0.002444	5	2.747332	−0.00026
6	2.747332	3	−0.000257	5	2.747345	−0.00005

∴ Real root of given equation

$= 2.7473$ (upto 4 decimal place)

5. A) $\int_0^{\pi/2} \sqrt{\sin x}\, dx$

$\boxed{y = \sqrt{\sin x}}$

$h = \dfrac{\pi/2 - 0}{8} = \pi/16$

$n = 8$

$x_0 = 0, \quad y_0 = \sqrt{\sin 0} = 0$

$x_1 = \pi/16, \quad y_1 = \sqrt{\sin(11.25)} = 0.441690$

$x_2 = 2\pi/16, \quad y_2 = \sqrt{\sin(22.5)} = 0.6186141$

$x_3 = 3\pi/16, \quad y_3 = \sqrt{\sin(33.75)} = 0.74533658$

$x_4 = 4\pi/16, \quad y_4 = \sqrt{\sin(45)} = 0.840896$

$x_5 = 5\pi/16$, $y_5 = \sqrt{\sin(56.25)} = 0.911849$

$x_6 = 6\pi/16$, $y_6 = \sqrt{\sin(67.5)} = 0.9611865$

$x_7 = 7\pi/16$, $y_7 = \sqrt{\sin(78.75)} = 0.990346$

$x_8 = 8\pi/16$, $y_8 = \sqrt{\sin(90)} = 1.00000$

Acc. To Trapezoidal Rule,

$$\int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi/16 \left[\frac{0+1}{2} + 0.441690 + 0.618691 + 0.7453658 + 0.890896 + 0.911849 + 0.9611865 + 0.990346\right] \times 2 \times \frac{1}{2}$$

$$= \frac{1}{16_8} \times \frac{22^4}{7} \left\{\frac{1}{2} + 5.5099471\right\}$$

$$= 1.18052538 2$$

Ans