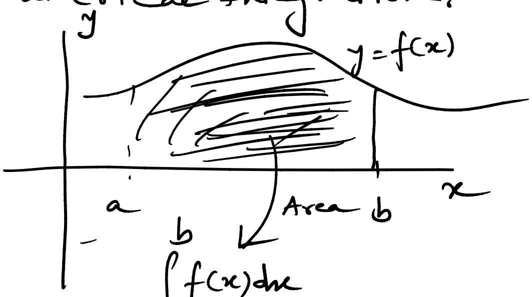
✓ Trapezoidal Method

✓ Simpson's $\frac{1}{3}$ method

Weddle's Method (If time permits)

☐ Numerical Integration :-



$$\int_a^b f(x)\,dx$$

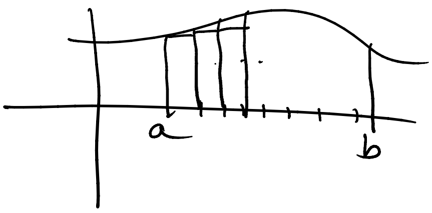
Some fns e^{x^2}

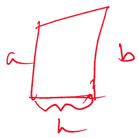
$$\int_1^2 e^{x^2} dx$$

Cannot integrate

↓

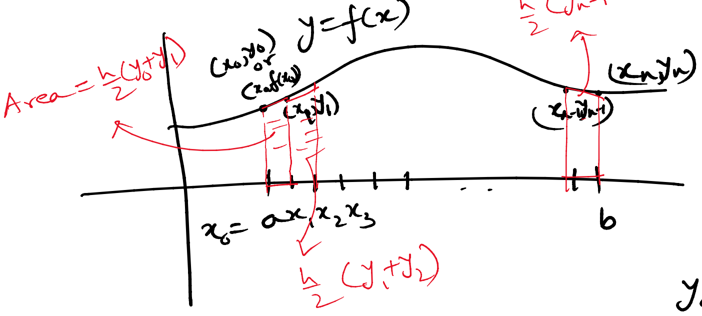
But we need the value for some purpose





Area = $(a+b)\frac{h}{2}$

☐ Trapezoidal Method :-



1st divide $[a,b]$ in n equal sub-intervals.

So length of each interval $= \frac{b-a}{n} = h$ say.

$x_0 = a$, $x_1 = x_0 + h$, $x_2 = x_1 + h$, ..., $x_n = x_{n-1} + h$

$y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$, ..., $y_n = f(x_n)$

$\int_a^b f(x)\,dx \triangleq$ sum of areas of the trapeziums.

$$= \frac{h}{2}(y_0+y_1) + \frac{h}{2}(y_1+y_2) + \cdots + \frac{h}{2}(y_{n-1}+y_n)$$

$$= \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

↳ formula for Trapezoidal Method.

e.g. Approximate $\int_0^2 x^2\,dx$ by Trapezoidal method taking 5 equal sub-intervals

Sol^n :

$f(x) = x^2$, $a = 0 = x_0$, $b = 2 = x_5$

$h = \frac{2-0}{5} = 0.4$

$\int \frac{x^3}{3}\Big|_0^2 = \frac{8}{3} = 2.666$

$x_0 = 0$, $x_1 = 0 + 0.4 = 0.4$, $x_2 = x_1 + 0.4 = 0.8$

$x_3 = x_2 + 0.4 = 1.2$, $x_4 = x_3 + 0.4$

$x_0 = 0,\ x_1 = 0 + 0.4 = 0.4,\ x_2 = x_1 + 0.4 = 0.8$

$x_3 = x_2 + 0.4 = 1.2,\ x_4 = x_3 + 0.4 = 1.6$

$x_5 = x_4 + 0.4 = 2$ (last limit)

x :	0	0.4	0.8	1.2	1.6	2
$x^2 = f(x)$:	0	0.16	0.64	1.44	2.56	4
	$\downarrow y_0$	$\downarrow y_1$	$\downarrow y_2$	$\downarrow y_3$	$\downarrow y_4$	$\downarrow y_5$

So the approximate value of the integration is

$$\frac{h}{2}\left[y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4)\right] = \frac{0.4}{2}\left[0 + 4 + 2(0.16 + 0.64 + 1.44 + 2.56)\right]$$

$$= 2.72$$

Q. $\int_0^1 \frac{dx}{1+x^2}$, take $n=6$ & hence approximate the value of π. (by Trapezoidal Method)

Sol^n:- $f(x) = \frac{1}{1+x^2}$, $n=6$, $h = \frac{1-0}{6} = \frac{1}{6}$, $x_0 = 0$, $x_1 = \frac{1}{6}$, $x_2 = x_1 + \frac{1}{6} = \frac{1}{3}$

$f\left(\frac{1}{6}\right) = \frac{1}{1+\frac{1}{36}}$ $\qquad f(0) = \frac{1}{1+0} = 1$

x :	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$f(x)$:	1	0.973	0.9	0.8	0.6923	0.5902	0.5
	$\downarrow y_0$	$\downarrow y_1$	y_2	$\downarrow y_3$	$\downarrow y_4$	$\downarrow y_5$	$\downarrow y_6$

$$\int_0^1 f(x)\,dx \triangleq \frac{h}{2}\left[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= 0.7842$$

$$\int_0^1 \frac{dx}{1+x^2} = \tan^{-1}x\Big|_0^1 = \frac{\pi}{4} \triangleq 0.7842$$

So $\pi \triangleq 0.7842 \times 4 = 3.1270$ . .

Q. Evaluate $\int_{-1}^{0} x e^x\,dx$ by Trapezoidal method $(n=6)$.

$a = -1$
$b = 0$

x :	-1	$-\frac{5}{6}$	$-\frac{4}{6}$	$-\frac{3}{6}$	$-\frac{2}{6}$	$-\frac{1}{6}$	0
$f(x)$:	-0.3678	-0.3622	-0.3404	-0.3033	-0.2388	-0.1411	0

$h = \frac{1}{6}$

$$\int_{-1}^{0} f(x)\,dx \triangleq -0.2616$$

☐ Practice Problems

Ans. 0.69315 (1) Evaluate $\int_0^1 \frac{dx}{1+x}$ by Trapezoidal method $(n=11)$ & then approximate the value of $\ln 2$ correct to 5 significant digits.

2.6353

approximate the value of $\ln 2$ correct to 5 significant digits.

Ans. 2.6353 (2) Evaluate $\int \frac{x}{\sin x} dx$ in $(0, 2)$ $(n=8)$. (Hint $\lim_{x \to 0} \frac{x}{\sin x} = 1$

Use radian for sinx.