Type of bifurcations. feigen baum Constant (4.669)

$$X_{n+1} = \gamma X_n (1 - X_n).$$

dripping faucet
Mandelbrot set
population of rabbits
~~th~~ermal convection in a fluid
the firing of neurons in your brain

$$X_{n+1} = \gamma X_n (1 - X_n)$$

$X_{n+1} = \gamma X$ → ~~the~~ rabbits this year

↳ growth (let's say 2).

↳ this would mean the population would double every year.

↙ this mean exponential growth.

so add $(1 - X)$

constraints of the environment

$X \Rightarrow$ is a percentage of the theoretical maximum so (so it goes from 0 to 1 and as it appraches that maximum then, $(1 - X)$ becomes 0 and the constraint ↳ stops the population.

the populations.

$\gamma = 2.6$ $X_n = 0.4$.

$X_{n+1} = 0.624$.

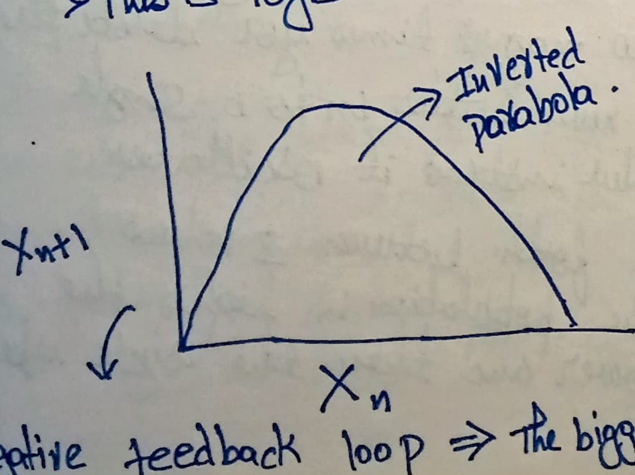
$X_{n+1} = 0.6100$

0.6185
0.6134
0.6165
0.6146
0.6157
0.6151
0.615

$$X_{n+1} = \gamma X_n (1 - X_n)$$

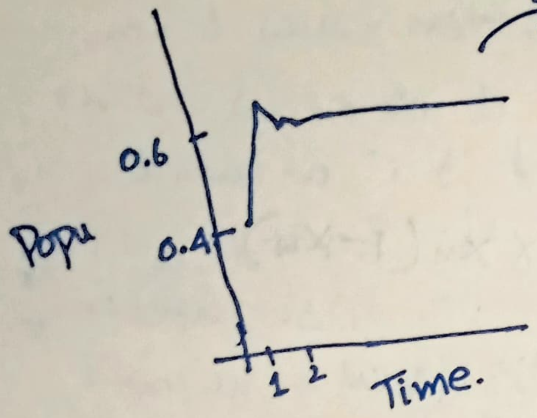
↳ This is logistic map. (SO).

here you see that population doesn't really change it has stabilized which matches what we see in the wild population often remain the same as long as births and deaths are balanced.



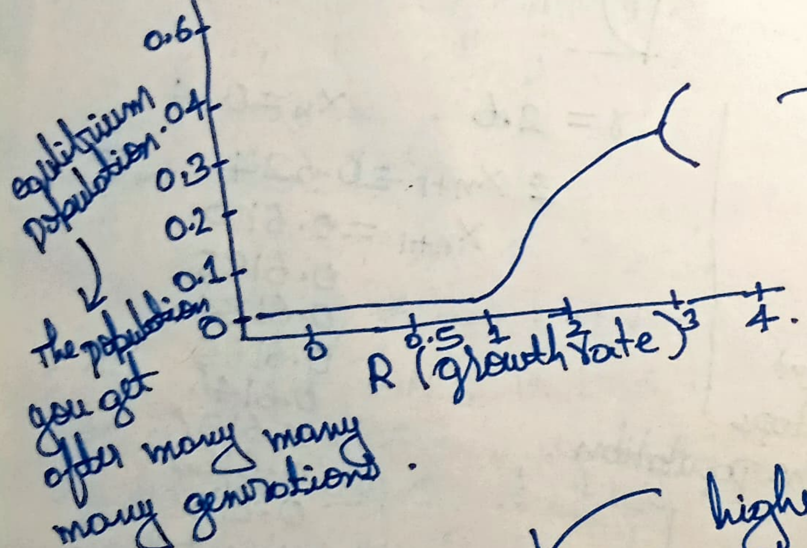
negative feedback loop ⇒ The bigger the population gets over X_n the smaller it will be following years

It reached an equilibrium value 6.15.



Now what will happen if I change the initial population. what you see is the first few years change but the equilibrium population remains the same.

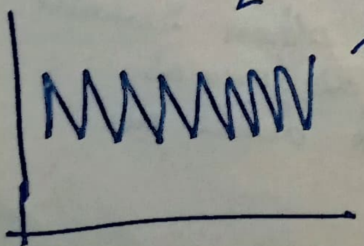
Now, let see how equilibrium population vary depending on r (growth rate * If we lower the growth rate, the equilibrium postion decreases that make sense and in fact if R goes below 1, the population drops and eventually goes extinct.



So for low value of r we see the population alway go extinct, so the equilibrium value is zero but once our r hits 1 the population stabilizes on to a constant value the the higher R is the higher the equilibrium population.

So for all good.

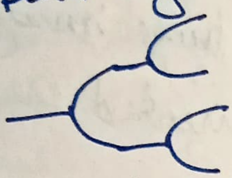
once r passes 3 the graph splits in two why?



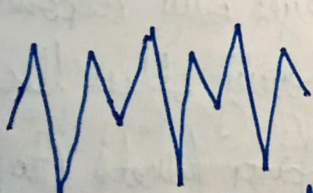
No matter how many times you iterate the equation it never settles on to a single constant value instead it oscillates back and forth between 2 values, one year the population is higher the year its lower, and then the cycle repeats.

the cyclic nature of population is observed in nature too, one year there might be more rabbits and the fewer the next year and more again year after as our.

as r continous to increase the fork spreads apart and the one splits again.



Now instead of oscillating back and forth between 2 values populations go through a four year cycle befer-reapeating.

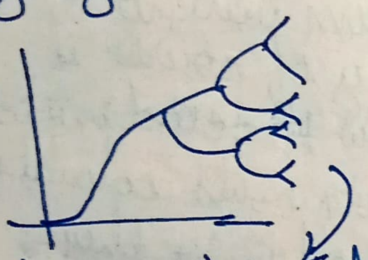


Since the length of the cycle or period has doubled these are known as period doubling bifurcation. and as R increases further there are more period doubling bifurcation, they come faster and faster leading to cycles of 8 16 32 64 and ...

at R equals 3.57 chaos the population never settles down at at all it bounces around as if at random in fact this equation provides one of the first methods of generating random numbers on computers.

It was a way to get something unpredictable from a deterministic machine there is no pattern there no repeating of course if you did know the exact initial conditions you could calculate the values exactly. so they are considered only pseudo-random numbers now might expect the equation to be chaotic from here on out but as R increases order returns...

but as R increases order returns there are these windows of stable periodic behavior amid the chaos for example at R equals 3.83 there is a stable cycle with a period of 3 years and as R continues to increase it splits into 6 12 24 and so on before returing to chaos. in fact this one equation contains periods of every length 37, 50, 1052 whatever you like if you just have the right value r.



looking at this bifurcation diagram you may notice that it look like a fractal → the large-scale features look to be repeated on smaller and smaller scales. It is in fact a fractal.

arguably the most famous fractal is mandelbrot set the. plot twist here is that the bifurcation diagram is acutally part of ~~of~~ the mandelbrot set. how does that work? ↳ $Z_{n+1} = Z_n^2 + C$

but if you're more practically minded you may be asking but does this equation actually model populations of animals? and the answer is yes. particularly in the controlled environment, scientists have set up labs. what I find even more amazing is how this one simple equation applies a huge range of totally unrelated areas of science.

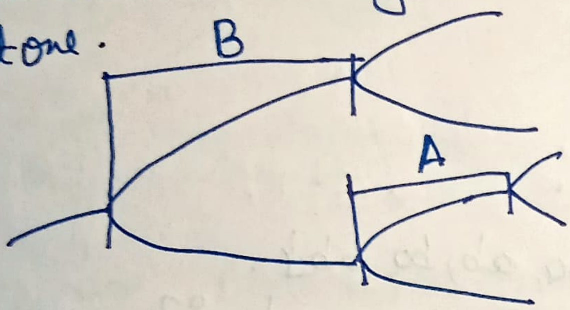
* our eyes and responce to flickering lights.
  (we find period doubling)
  once the light reaches a certain rate of flickering our eyes only respond to every other flicker.

* dripping faucets (tap water), once the flow rate is increases a little bit you get period doubling so now the drips comes 2 at a time. and eventually. you can get chaotic behavior. just by adjusing the flow rate.

* population of rabbits
* convection in a fluid
* the firing of neurons in your brain.
* mandelbrot set.

~~mitchell feo~~

there was this physicist michell feigenbaum who was looking at when the bifurcations occurs he divided the width of each bifurcation section by the next one.



$$\frac{B}{A} = 4.669\ldots$$

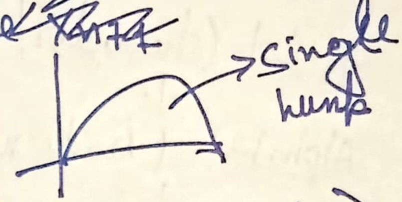
feigenbaum constant.

he found that ration closed in on this number 4.669. which is now called the feigenbaum constant.

The bifurcations come faster and faster but in a ration that approaches that fixed value and no one knows where this constant comes from it doesn't seem to relate to any other known physical constant so it is itself a fundametal constant of nature what's even crazier is that it doesn't have to be the particular form of the equation

$$x_{n+1} = r\, x_n(1-x_n)$$

any equation that has a single hump, ~~if you iterate it the way that we have so you could use~~ $\cancel{x_{n+1}}$



$$x_{n+1} = r.\sin(x_n)$$

if you iterate that one again and again and again you will also see bifurcations and ratio will 4.669.

∴ any single hump function interated will give you that fundamental constant 4.669...

References:

Jame Gleick, Chaos
↓
book name.