Statistics (I)

SLIDES BY:

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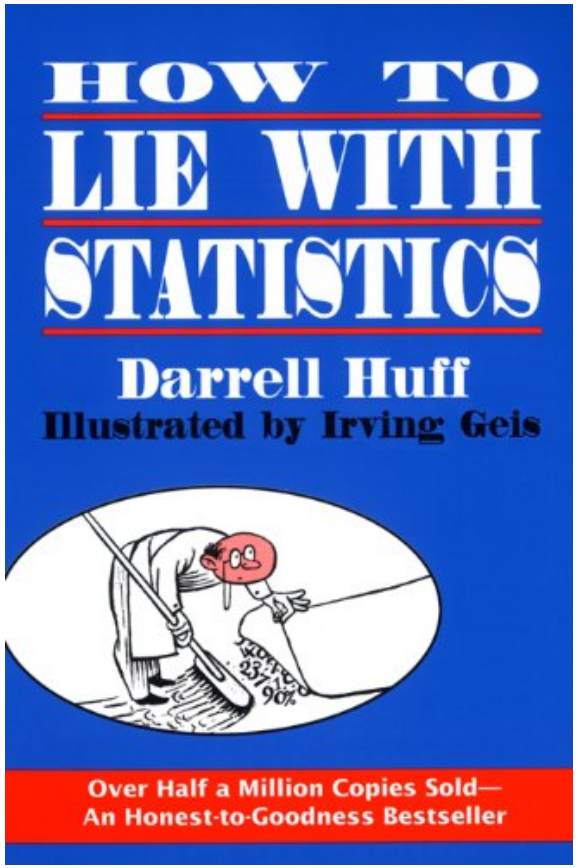
Why Should You Care?



> “There are three kinds of lies: lies, damned lies, and statistics”

Simpson's paradox

Is UC Berkeley gender biased?

	Applicants	Admitted
Men	8442	44%
Women	4321	35%

Simpson's paradox

Is UC Berkeley gender biased?

Department	Men		Women	
	Applicants	Admitted	Applicants	Admitted
A	825	62%	108	**82%**
B	560	63%	25	**68%**
C	325	**37%**	593	34%
D	417	33%	375	**35%**
E	191	**28%**	393	24%
F	373	6%	341	**7%**

NO!

Women tended to apply to competitive departments with low rates of admission

Outline

Statistical Thinking

Descriptive Statistics

Inferential Statistics

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Statistical Thinking

1. Data is just a sample

2. You goal is to infer a population

3. Think about how to go "backwards" from the sample to the population

Example 1. Image Classification

Is it a dog or a cat?

Dataset: 1000 images collected from the Web

Without Statistical Thinking

Treat the 1000 images as the population

> Train a model on the data

> Evaluate a model on the same data

> Model accuracy: 95%

With Statistical Thinking

What is the population?

- All the images in the Web

What is your dataset?

- A sample of 1000 images drawn from the Web

What should you do?

- Split the dataset into a training dataset and a test dataset
- Train the model on the training dataset
- Evaluate the model on the test dataset

Example 2. Poll Prediction

Who will win the election?

Dataset: A survey of 100 people

Without Statistical Thinking

Treat the 100 people as the population

> Count the number of people who wants to vote for Hillary, e.g., 52

> Count the number of people who wants to vote for Trump, e.g., 48

> Hillary will win the election

With Statistical Thinking

What is the population?

- All the people who will vote in the election day

What is your dataset?

- A sample of 1000 people before the election day

Analysis result

Hillary: 52% ±3%
Trump: 48% ± 2%

Assumption: People have not changed their votes since the time of the poll

Summary

Statistical Thinking

- Sample, Population and Their Connection
- With vs. Without Statistical Thinking

Descriptive Statistics

Inferential Statistics

Outline

Statistical Thinking

Descriptive Statistics

Inferential Statistics

Descriptive vs. Inferential Statistics

Descriptive Statistics: e.g., Median

- *Why?* Aim to understand the data
- *How?* Data summarization, data visualization, etc.

Inferential Statistics: e.g., A/B Testing

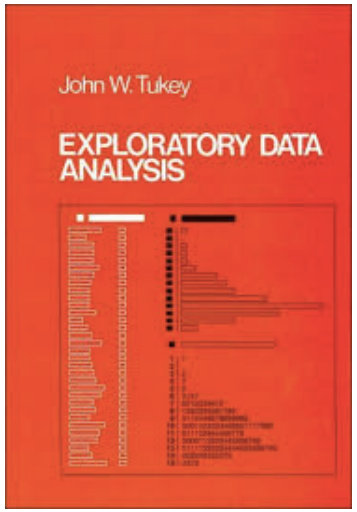
- *Why?* Aim to use the data (i.e., sample) to learn about a population
- *How?* Estimation, confidence intervals, hypotheses testing, etc.

Exploratory Data Analysis (EDA)

The process of doing descriptive statistics

John W. Turkey

- Professor at Princeton University
- Founding chairman of the Princeton statistics department in 1965
- Worked on EDA at Bell Labs since 60's
- Wrote a book entitled "Exploratory Data Analysis" in 1977



EDA is like detective work

From John Turkey

> “ Exploratory data analysis is an attitude, a state of flexibility, a willingness to look for those things that we believe are not there, as well as those that we believe to be there ”

EDA with Dataprep

EDA Solutions in Python

Solutions	APIs	Efficiency	Out-of-memory Data
Pandas + Matplotlib[1]	Designed for plotting	Slow	Not supported
Pandas-profiling[2]	Designed for profiling	Slow	Not supported
Dataprep.eda[3]	Designed for EDA	Fast	Supported

1. https://pandas.pydata.org/pandas-docs/version/0.13/visualization.html
2. https://github.com/pandas-profiling/pandas-profiling
3. http://dataprep.ai/

EDA Tasks vs Dataprep API

Task 1. Understand the distributions of all columns

- plot(df)

Task 2. Dive into the distribution of a single column

- plot(df, x)

Task 3. Dive into the relationship between two columns

- plot(df, x, y)

EDA Tasks vs Dataprep API

Task 4. Understand the missing values of all columns

- plot_missing(df)

Task 5. Dive into the missing values w.r.t. a single column

- plot_missing(df, x)

Task 6. Dive into the missing values between two columns

- plot_missing(df, x, y)

EDA Tasks vs Dataprep API

Task 7. Understand the correlation for all pairs of columns

- plot_correlation(df)

Task 8. Dive into the correlation w.r.t. a single column

- plot_correlation(df, x)

Task 9. Dive into the correlation between two columns

- plot_correlation(df, x, y)

Correlation Analysis

Correlation

- It is a measure of relationship between two variables

Why is correlation analysis useful?

- For understanding data better
- For making predictions better

Case Study: How to do correlation analysis

Height and weight are correlated

	height	weight	age	male
2	151.765	47.8256065	63	1
3	139.7	36.4858065	63	0
4	136.525	31.864838	65	0
5	156.845	53.0419145	41	1
6	145.415	41.276872	51	0
7	163.83	62.992589	35	1
8	149.225	38.2434755	32	0

Source: *Think Stats -- Exploratory Data Analysis in Python*

Idea 1. Visualization

Scatter Plot

	height	weight	age	male
2	151.765	47.8256065	63	1
3	139.7	36.4858065	63	0
4	136.525	31.864838	65	0
5	156.845	53.0419145	41	1
6	145.415	41.276872	51	0
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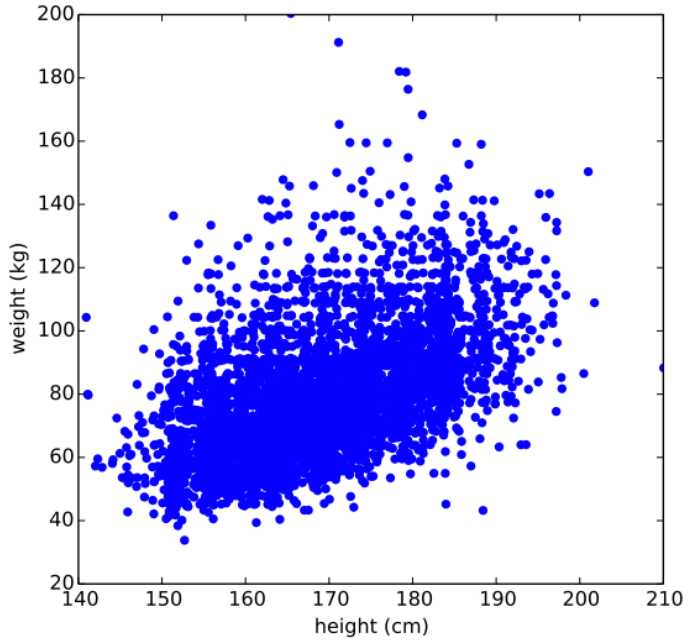

Scatter Plot (with transparency)

	height	weight	age	male
2	151.765	47.8256065	63	1
3	139.7	36.4858065	63	0
4	136.525	31.864838	65	0
5	156.845	53.0419145	41	1
6	145.415	41.276872	51	0
7	163.83	62.992589	35	1
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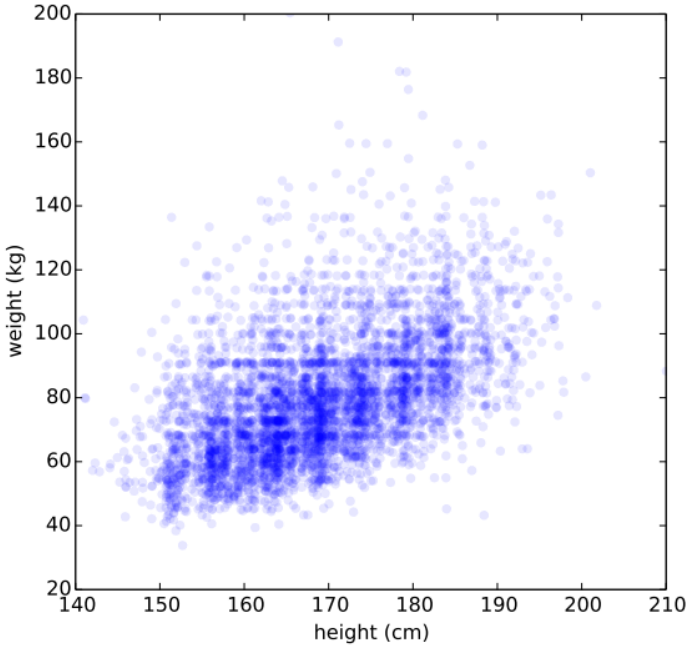

Hexbin Plot

	height	weight	age	male
1				
2	151.765	47.8256065	63	1
3	139.7	36.4858065	63	0
4	136.525	31.864838	65	0
5	156.845	53.0419145	41	1
6	145.415	41.276872	51	0
7	163.83	62.992589	35	1
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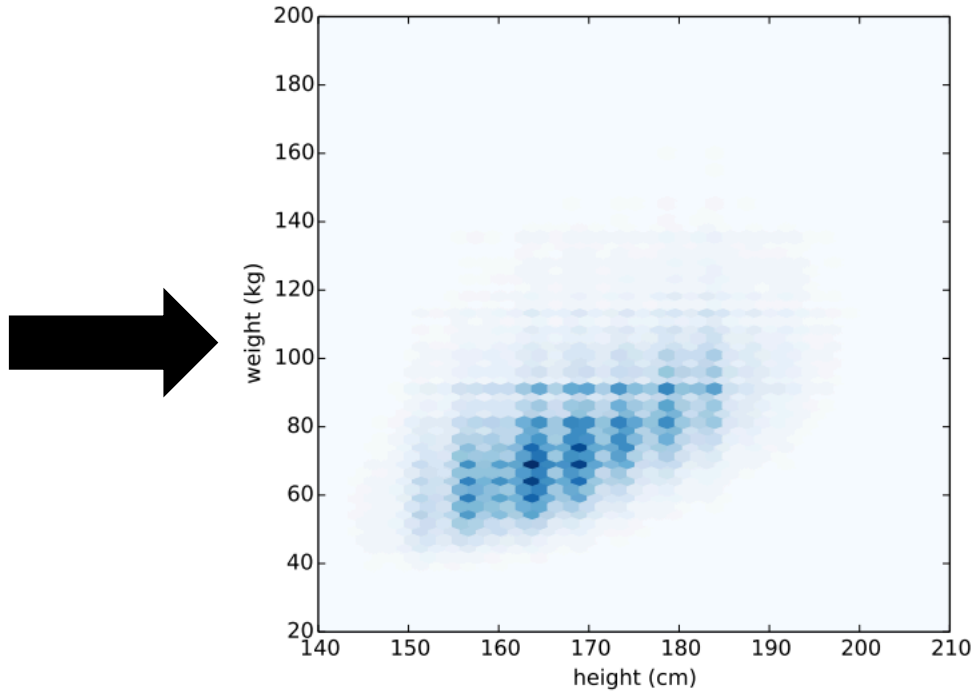

Characterizing relationships

	height	weight	age	male
1				
2	151.765	47.8256065	63	1
3	139.7	36.4858065	63	0
4	136.525	31.864838	65	0
5	156.845	53.0419145	41	1
6	145.415	41.276872	51	0
7	163.83	62.992589	35	1
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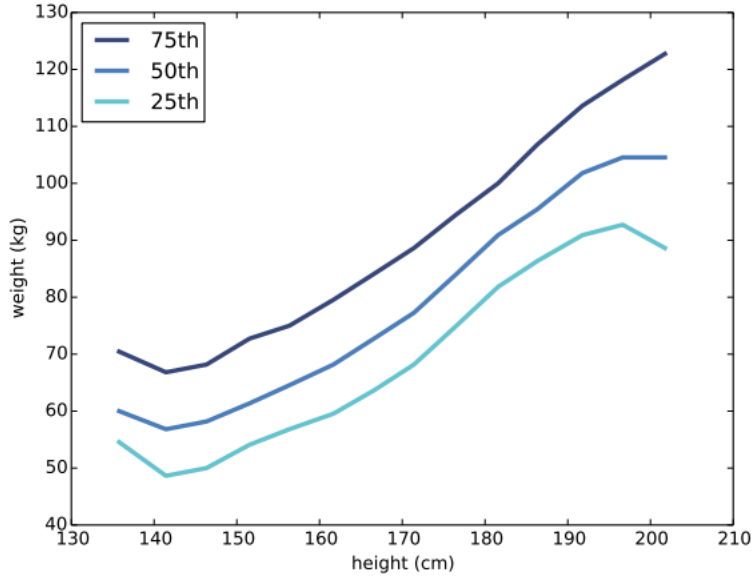


Idea 2. Correlation Coefficient

Covariance

Covariance is a measure of the **tendency** of two variables to vary together.

$$\operatorname{cov}(X,Y) = \operatorname{E}\left[(X - \operatorname{E}[X])(Y - \operatorname{E}[Y])\right]$$

$$\operatorname{cov}(X,Y) = \operatorname{E}[XY] - \operatorname{E}[X]\operatorname{E}[Y]$$

Hard to interpret

113 kilogram-centimeters

Pearson's correlation

Pearson's correlation is a measure of the linear relationship between two variables

$$\rho_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

Easy to Interpret

- [-1, 0) → Negative Correlated
- (0,+1] → Positive Correlated
- -1 or +1 → Perfectly Correlated

What about non-linear relationship?

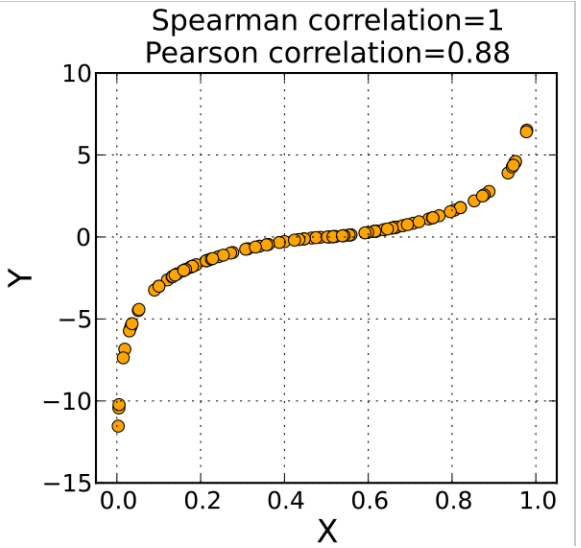
Spearman's rank correlation

Spearman's rank correlation is a measure of **monotonic relationship** between two variables

$$r_s = \rho_{\mathrm{r}_X, \mathrm{r}_Y} = \frac{\mathrm{cov}(\mathrm{r}_X, \mathrm{r}_Y)}{\sigma_{\mathrm{r}_X} \sigma_{\mathrm{r}_Y}}$$

Advantages

- Mitigate the effect of outliers
- Mitigate the effect of skewed distributions



Summary

Statistical Thinking

Descriptive Statistics

- Descriptive vs. Inferential Statistics
- Exploratory Data Analysis with Dataprep
- Correlation Analysis

Inferential Statistics

Outline

Statistical Thinking

Descriptive Statistics

Inferential Statistics

- Estimation

Estimation

Problem statement

- Estimate a numerical value associated with a population

Examples

- Estimate the percentage of the people in the US who will vote for Trump
- Estimate the median annual income of all households in the US

Example: Median Annual Income

How to estimate the median annual income of all households in the US?

- Randomly select 10,000 households from the US
- Report their median annual income: 50,000USD

- BUT, we need to report something like

50,000 ±500 USD

A Naïve Solution

- Randomly select 10,000 households from the US
- Report their median annual income

Repeat this process for 100 times

50,000 49,600 50,200 … 49,200

You have to survey 1,000,000 million households in total ☹

A Smart Solution: Bootstrapping

Key Idea: Resampling

- Sample with replacement from the original data sample

Population: 1, 1, 8, 2, … 3, 3

Sample: 3, 8, 1, 8, 3

Resample: 8, 3, 3, 3, 1

A Smart Solution: Bootstrapping

- Randomly select 10,000 households from the US
- Draw a resample from the 10,000 households
- Report the median annual income of the resample

Repeat this process for 100 times

You do NOT need to survey any new household. ☺

Notes on Bootstrapping

Start with a large random sample (at least 30)

Replicate the resampling procedure as many times as possible (more than 1000 times)

Does not work for min/max

Conclusion

Statistical Thinking

- Sample, Population and Their Connection
- With vs. Without Statistical Thinking

Descriptive Statistics

- Descriptive vs. Inferential Statistics
- EDA with Dataprep.eda
- Correlation Analysis

Inferential Statistics

- Estimation and Bootstrapping