Optimization of Police Facility Locationing

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This paper presents an application of optimization and geographic information systems techniques for deploying police facilities subject to budgetary and feasibility constraints. The objectives include minimizing the distances and maximizing the coverage of police stations over potential crime spots. Two optimization models were investigated: maximum coverage and *p*-median problems. The optimization was integrated with geographic spatial analysis techniques to allocate limited police resources in the Greater London Area. In addition, the optimal results were compared with existing police deployment in London. Finally, the resilience of optimal solutions was evaluated by applying them to the scenario of the terrorist attack that occurred on July 7, 2005. This study contributes to the study of urban sustainability and security.

London, as the capital of the United Kingdom, attracts thousands of visitors worldwide coming to London every day for various purposes, including traveling, studying, and commercial activities. It makes London a busy, multicultural city (*1*). The first-ever crime plan, the Police and Crime Plan 2013–2016, was set out by the Mayor's Office for Policing and Crime in 2013 (*2*). The plan aims to cut neighborhood crime by 20% and reduce operational costs by 20% as well, by 2016. More than 80% of reports are made by phone rather than at police stations (*2*). Following that observation, it was suggested that 57 of the existing police stations could be shut down before 2016 to reduce operational costs (*3*). With the reduced number of police facilities and the reduced budget, it is vital to deploy all available resources effectively to keep the city safe.

This study explores the application of location optimization to police facility management, with the city of London adopted as the study area. Location optimization has been used in the public and the private sector (*4*). There are some previous examples in the literature showing applications of location models and optimization to police resource management and law enforcement. The location models are used mainly for the design patrol areas and the deployment of police officers in the area (*5*). There are also several notable examples of applying location optimization to crime prevention. Mitchell presents a *p*-median problem formulation for minimizing travel distances between service and expected calls in the area of Anaheim, California (*6*). While no proven optimal solutions were found, the heuristic algorithm employed in Mitchell (*6*) based on Maranzana (*7*) led to a 13% to 24% reduction in average response distances. Aly and Litwhiler formulate a distance minimization problem solved with an interchange heuristic to allocate police briefing stations to districts by shift (*8*). Jong et al. adopt a *p*-median formulation to seek an optimal location plan with the application of geographic information systems (GIS) for a region in South Africa (*9*). What is missing in the literature, however, is analysis of the resilience of optimal deployment plans under extreme events such as terrorist attacks.

In the present study, following the specification of the Greater London Authority, London is divided into a total of 624 zones or "wards" as shown in Figure 1. Two location models, *p*-median and maximum coverage, are applied. The *p*-median optimization model is set up with the objective of minimizing the total weighted distance between the center of the wards and police stations. The distances between wards are determined from a GIS shapefile of the city of London with the use of ArcGIS software. The distances are then weighted by the associated crime rate statistics during the 2009 to 2014 period (i.e., annual average of crime events that occurred in each ward), which are obtained from the London Metropolitan Police database. The maximum coverage model is used to determine the optimal patrol area. The crime distribution is visualized as shown in Figure 2. It can be seen that wards located in the central London area generally have a higher crime rate. The existing locations of the 73 police stations in the area are illustrated by the stars in Figure 2. The figure shows that some of the areas with a high crime rate are not adequately covered. An aim of this research is to explore whether the deployment of police facilities can be improved through location optimization. Once the optimal police facility locations are determined, the resilience of the location plan with respect to extreme events is also tested with the scenario of the terrorist attack that occurred on July 7, 2005. Moreover, the study demonstrates the integration of optimization and GIS, which is seen relatively rarely in the literature.

The paper is organized as follows: the maximum coverage and *p*-median problems are reviewed next. The algorithms are then applied to a case in the Greater London Area, and the results are discussed. Next, the resilience issue is evaluated by applying optimal solutions derived in the previous section to the scenario of the London July 7, 2005 terrorist attack. Finally, some concluding remarks end the paper. This paper presents an integration of GIS and operational research techniques, and it will be a contribution to the area of urban security and resilience management.

LOCATION OPTIMIZATION

In this paper, two different formulations of location optimization are adopted: *p*-median and maximum covering models.

p-Median Problem

The *p*-median model locates *p* facilities over *n* locations (or nodes) such that the average distance between the facilities and the nodes is minimized (*10*). The *p*-median problem can be formulated as

$$\min_{x,y} Z = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i x_{ij} d_{ij}$$

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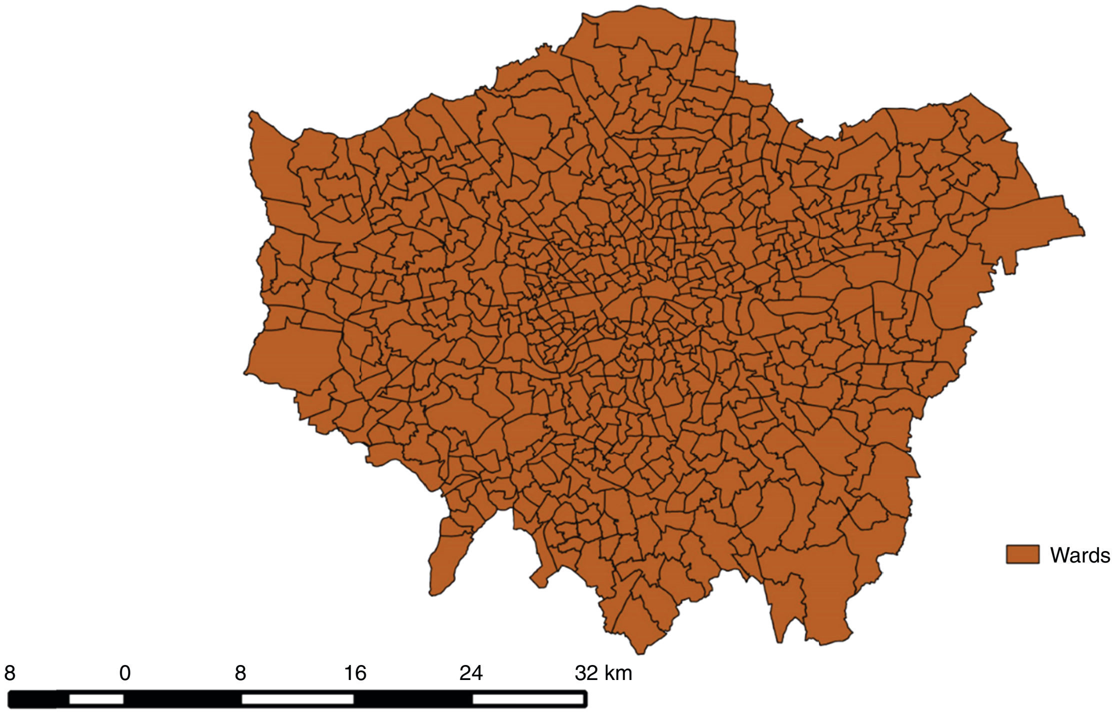

FIGURE 1 Ward boundaries in Greater London.

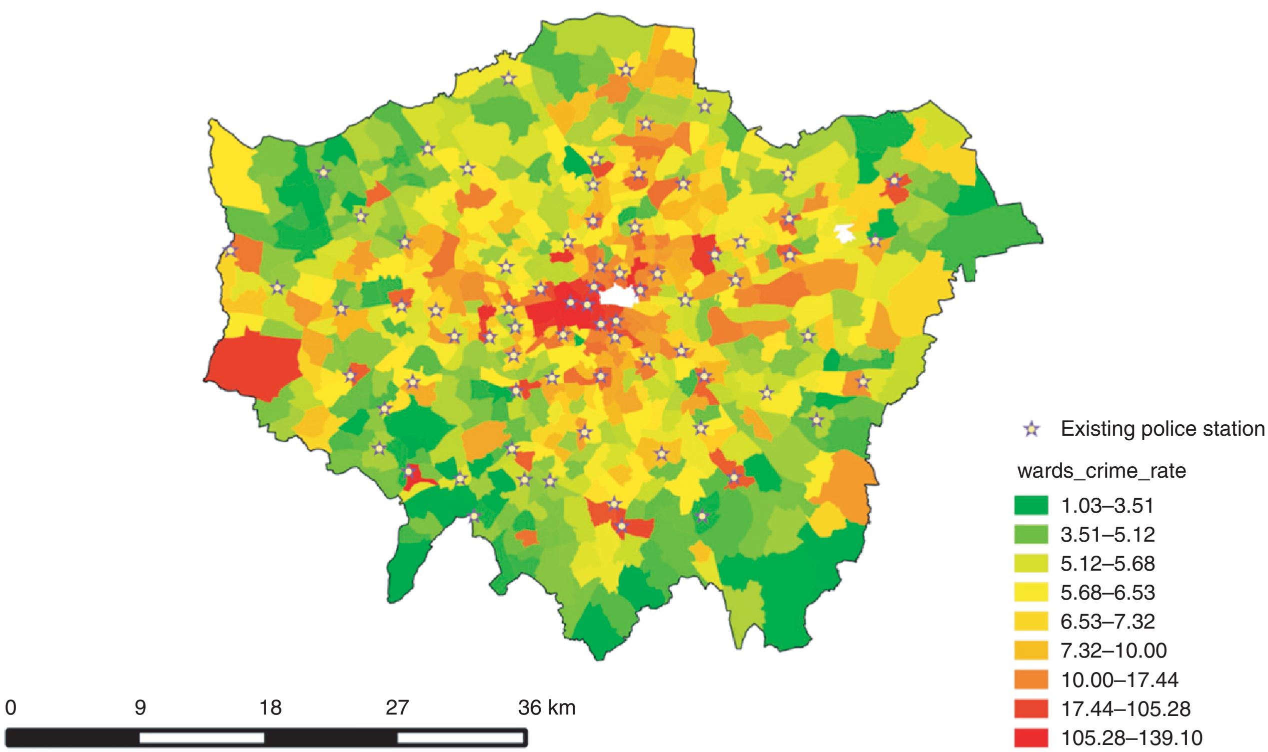

FIGURE 2 Distribution of crime and police stations.

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \text{for all } i = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} y_j = p$$

$$x_{ij} \leq y_j \qquad \text{for all } i, j = 1, 2, \ldots, n$$

$$x_{ij} = 0 \text{ or } 1 \qquad \text{for all } i, j = 1, 2, \ldots, n$$

$$y_j = 0 \text{ or } 1 \qquad \text{for all } j = 1, 2, \ldots, n$$

where

- n = total number of nodes in area; node can be centroid generating crime, feasible location for police facility to be placed, or both;
- x_{ij} = 0–1 binary decision variable representing whether potential crime-generating centroid i should be taken care of by police facility located at j;
- y_j = 0–1 binary decision variable representing whether police facility should be located at j;
- w_i = potential crime rate generated at centroid i;
- d_{ij} = travel distance (or cost) between centroid i and facility at node j; and
- p = total number of facilities available.

It is easy to verify that the total number of possible solutions to a p-median problem with n nodes is $C_p^n = n!/p!(n-p)!$. The implication is that the time for solving the optimization problem grows factorially as the number of locations and facilities increases.

Maximum Coverage Problem

The maximum coverage problem deploys a fixed number of facilities to maximize their coverage over a given number of centroids (*11*). The maximum coverage problem can be formulated as

$$\max_{x,y} Z = \sum_{i=1}^{n} w_i \alpha_i$$

subject to

$$\sum_{j \in N_i} y_j \geq \alpha_i \qquad \text{for all } i = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} y_j = p$$

$$\alpha_i = 0 \text{ or } 1 \qquad \text{for all } i = 1, 2, \ldots, n$$

$$y_j = 0 \text{ or } 1 \qquad \text{for all } j = 1, 2, \ldots, n$$

where α_i is a 0–1 binary variable representing whether centroid i is covered by a facility and N_i is the set of facilities that are within a predefined distance (D) from centroid i. Similar to the p-median problem, it can be verified that the total number of possible solutions to the maximum coverage problem grows factorially as the number of centroids and facilities increases.

Solution Strategies

Innovative solution heuristics are needed for solving p-median and maximum coverage problems because of their complexity. A number of solution strategies are proposed for solving p-median problems. These solution methods can be categorized into two classes: construction algorithms and improvement algorithms (*12*). Construction algorithms attempt to build a good solution from the start. An example is the myopic algorithm presented in Daskin (*13*). A popular improvement algorithm is the substitution (swap-based) algorithm first introduced by Teitz and Bart (*14*). This method is based on swapping predetermined facilities with nonfacilities. If there is an improvement in the objective function after swapping, the locations of the facility will be swapped and the searching procedure proceeds. The improvement method can be completed in polynomial time. Whitaker introduces a fast interchange heuristic, which is an improvement to the substitution algorithm (*15*). The key advantage of this algorithm is that it locates the best candidate to substitute in O(n) time, where n is the number of facilities available. Another improvement heuristic is the neighborhood search algorithm first proposed by Maranzana (*7*). This algorithm locates the optimal centers of a set of nearby facilities called "neighborhoods." A network is broken down into several neighborhoods. Since locating an optimal center can be done by total enumeration, this method is therefore viable. A downside of the neighborhood search method in contrast with the substitution method is that the effect of relocating a facility is considered only within the neighborhood, neglecting the potential benefits to nodes outside the neighborhood (*13*). Another method to solve p-median problems is the Lagrangian relaxation algorithm (*16*). Beasley presents the use of Lagrangian relaxation and subgradient optimization in the solution of p-median problems (*16*). This method gives upper and lower bounds for the optimal solution, which can be used iteratively to find the optimal solution. In each iteration, the lower or upper bound is brought closer to the optimal solution. More recent developments in solving p-median problems are focused on developing metaheuristic algorithms. Talbi proposes that metaheuristics are general purpose algorithms that can be applied to solve almost any optimization problem (*17*). They may be viewed as upper level general methods that can be used as a guiding strategy in designing underlying heuristics (*17*). Uses of metaheuristics for solving p-median problems include simulated annealing, Tabu search, and genetic algorithms (*18*).

Two common methods for solving the maximum coverage problem are the greedy algorithm and the substitution algorithm (*13*). Lagrangian relaxation is sometimes also used (*19*). In Lagrangian heuristics, one defines upper bounds by using vertex addition or a substitution method and lower bounds by using a subgradient algorithm. Gendreau et al. develop a Tabu search heuristic for their double coverage ambulance location problem with randomly generated data and real data that provide near-optimal solutions in a modest computing time (*20*). More recently, Berman and Krass proposed an integer programming (IP) approach (*21*). Their new formulation leads to an improvement in computation time. To solve even larger problems, Jia et al. develop a genetic algorithm method (*22*). Berman et al. extend the maximal covering formulation in which some cost functions can be negative to account for some points being considered as obnoxious facilities (*23*). See the review by Farahani et al. for further discussion on solution methods for the maximum coverage problem (*24*).

CASE STUDY

The *p*-median and maximum coverage models are now applied to optimize police facility locations in the Greater London Area. Two different types of data sets are collected. For the *p*-median method, the crime rate in January 2014 for all wards in Greater London is considered. The data are obtained from the London Metropolitan Police. A 624 × 624 distance matrix is created by ArcGIS; the matrix determines the distances between each pair of wards. In regard to the crime statistics, the data collected have more than an average of 90,000 incidents monthly that occurred at about 19,777 locations. All the patrol checkpoints are assumed to be placed in the center of wards, and their service radius is 2 km.

Solving the *p*-Median Problem

To solve the *p*-median problem, the myopic and substitution methods are used. The myopic algorithm is a construction algorithm and is used to locate the first set of locations (i.e., the initial solution). A flowchart of this myopic algorithm is shown in Figure 3. This initial set of locations is often suboptimal, and thus a substitution algorithm is used to refine the solution. It takes the locations of existing solutions and exchanges them with other points and checks whether the objective function is improved. When all *p* facilities have been allocated, the algorithm stops. A flowchart of this substitution algorithm is shown in Figure 4.

Solving Maximum Coverage Problem

The greedy algorithm is used for solving the maximum coverage model. The algorithm is chosen for its simplicity and polynomial computation time. More important, this method is known to work well particularly when many sites are to be located (*13*). However, the trade-off is that a global optimal solution cannot be guaranteed. The algorithm starts with identifying a facility location where most potential crime spots could be covered. Following this step, the next facility is then located such that the most crime spots that remain uncovered could be covered. The algorithm is repeated until the prespecified number of facilities, *p*, have all been located. The algorithm is summarized in Figure 5.

Results

The optimal police station locations are determined by solving the *p*-median model (where $p = 73$, which is the number of police stations) with myopic and substitution algorithms; the results are shown as the solid stars in Figure 6. With the same number of police stations, the stations are distributed closer to areas with a high crime rate when compared with the existing locations.

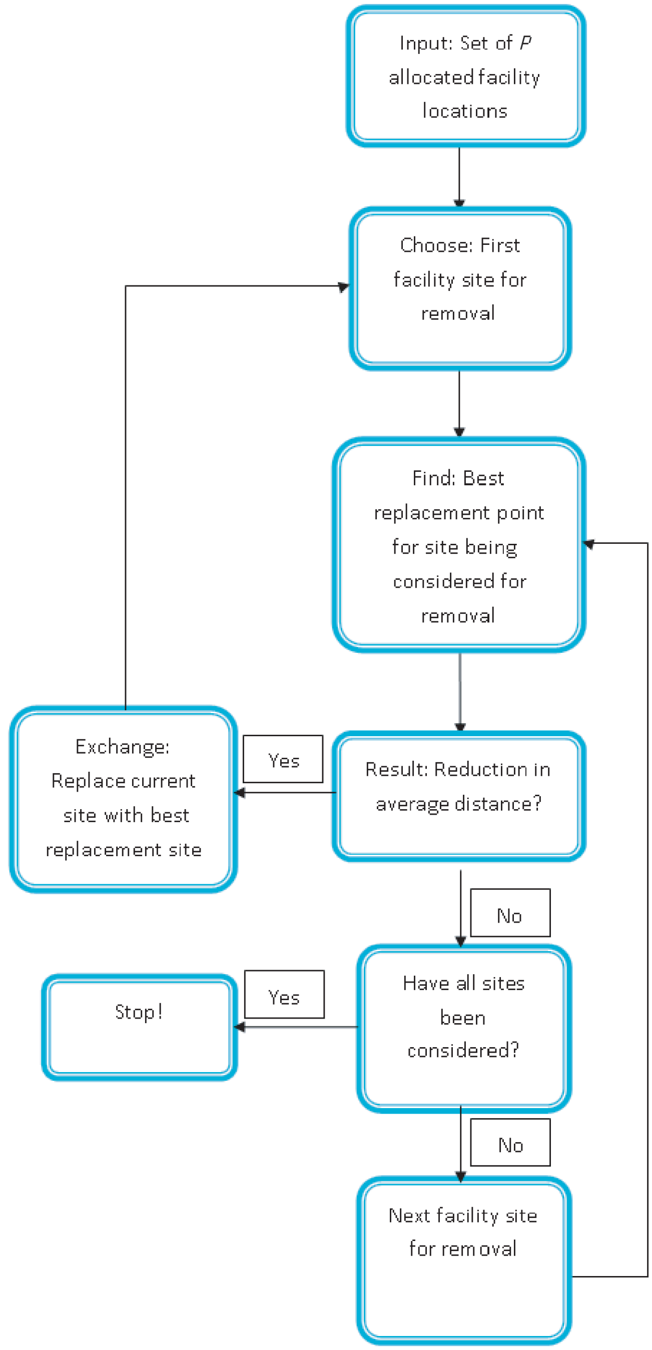

FIGURE 3 Myopic solution algorithm for *p*-median problem.

The existing total weighted distance (i.e., crime rate × distance) between police stations to the centroid of each ward is 1,866 km–crime. The weighted distance is reduced to 1,559 km–crime through optimization. This 19% reduction in travel distance is expected to have a positive impact on police response time. Of course, relocating all police stations according to the solutions obtained from the *p*-median model is impossible in reality. Nevertheless, police service can still be improved through deploying mobile police patrol units according to the optimal plan.

A sensitivity analysis exploring the relationship between the distance and number of police stations is conducted (see Figure 7).

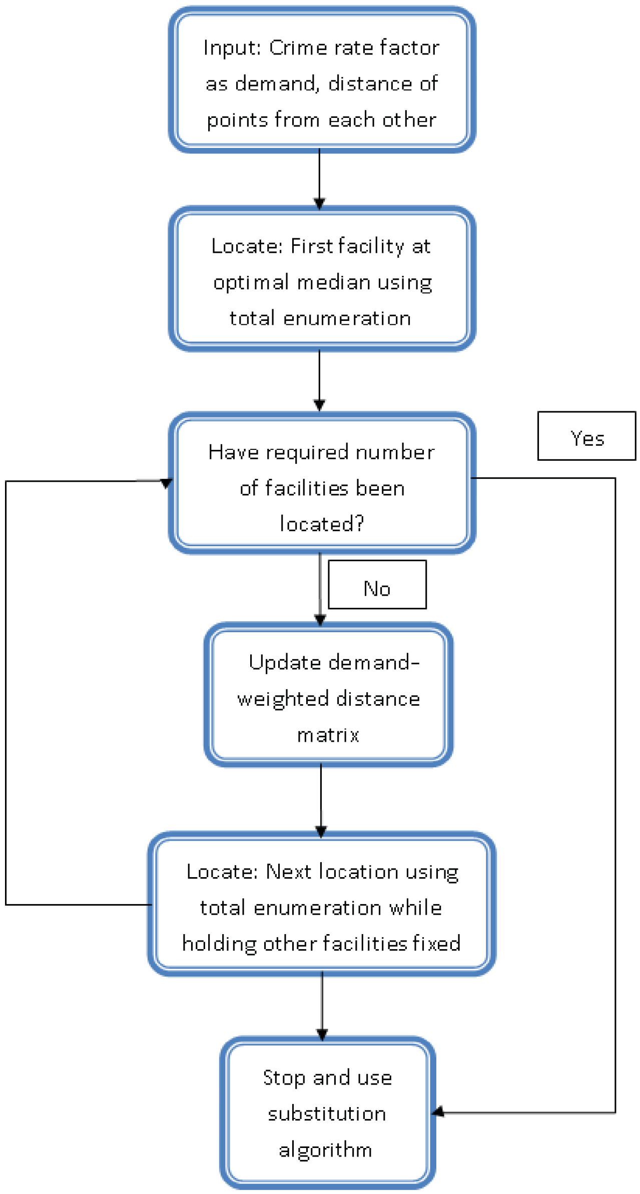

FIGURE 4 Substitution algorithm for solving *p*-median problem.

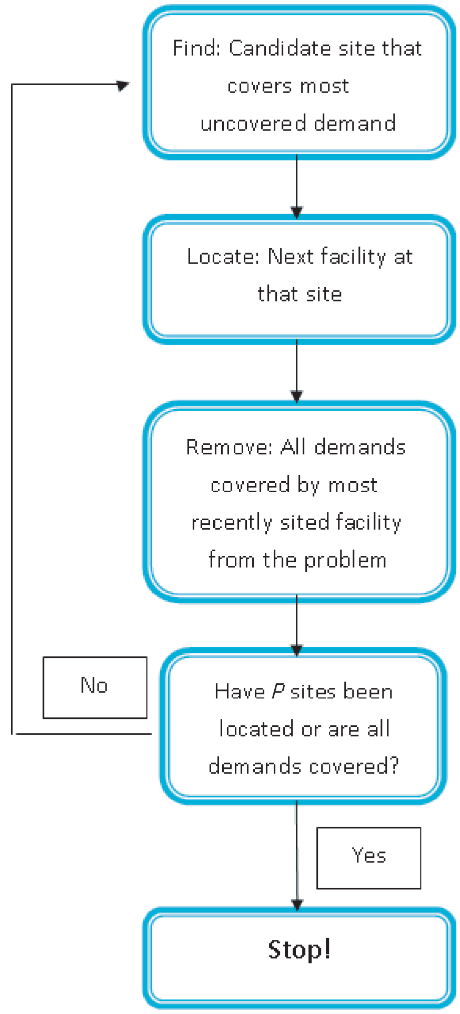

FIGURE 5 Greedy algorithm for solving maximum coverage problem.

As shown, the more police stations, the shorter the distance between police stations and crime spots, which is understandable. However, the relationship is nonlinear; the marginal reduction in average distance decreases as the number of stations increases. In the present study, the reduction in average distance becomes insignificant with respect to the increase in the number of police stations when the number of stations goes beyond 70. The implication could be that 70 police stations will be enough as a further increase in police stations does not provide significant extra benefit but increases operating costs.

Following the determination of the police station locations, the maximum coverage model is used to determine the optimal police patrol bases. The coverage distance (D) of one patrol base is predefined to be 2 km following the specification set by the London Metropolitan Police. According to the London Mayor's Police and Crime Plan 2013–2016, there will be 217 patrol bases from which police patrol vehicles will be deployed. The existing locations of these 217 patrol bases are shown in Figure 8. With the current locations of patrol bases, 91.99% of the crime spots can be covered within the 2-km distance from the corresponding patrol bases. Figure 9 shows the locations of the patrol bases determined by the present maximum coverage optimization model. An apparent difference between the optimal and existing locations is that the patrol bases are more evenly spaced after optimization and placed nearer to the crime hot spots. The coverage percentage increases from the existing 91.99% to a nearly perfect 99.82%.

IMPLICATION FOR RESILIENCE

Now the implications of the location optimization for urban resilience under extreme circumstances such as terrorist attacks are investigated. Terrorism is one of the biggest concerns in major cities such as London. A way to evaluate the resilience of the location optimization models is to consider a terrorist attack scenario. This study adopts the terrorist attack event that occurred on July 7, 2005. On that day London was subject to a series of bomb attacks. Three of the attacks were on the London Underground, and the other was on a bus. The locations were

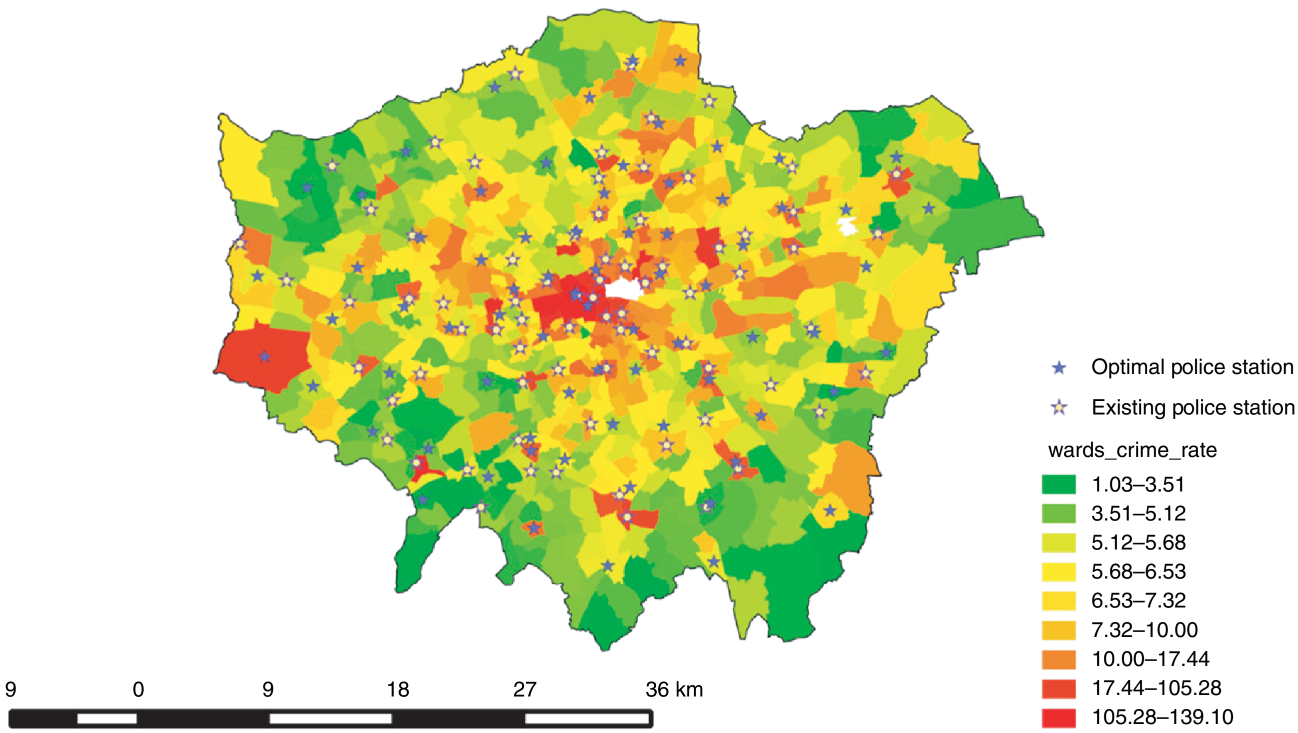

FIGURE 6 Locations of police stations determined by *p*-median model.

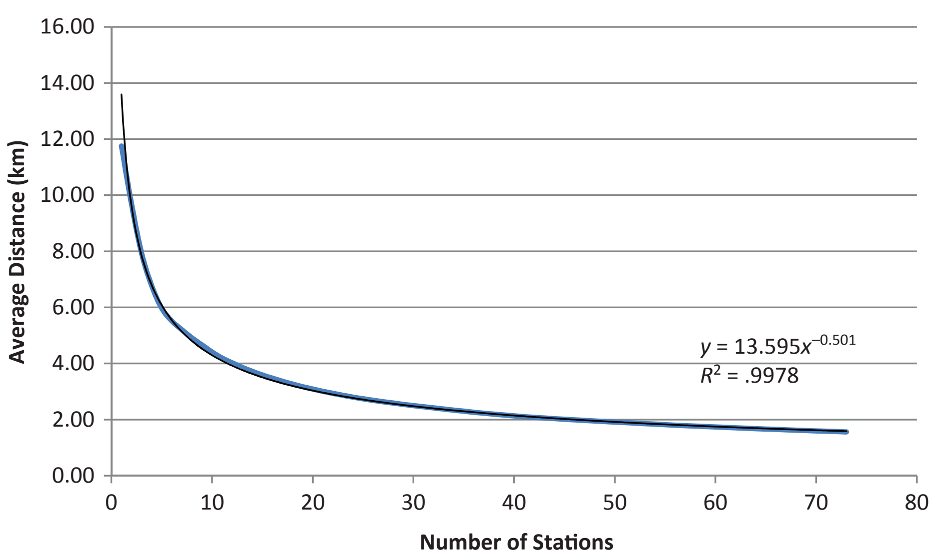

FIGURE 7 Sensitivity analysis of optimal average distance between police stations and crime spots with respect to number of stations.

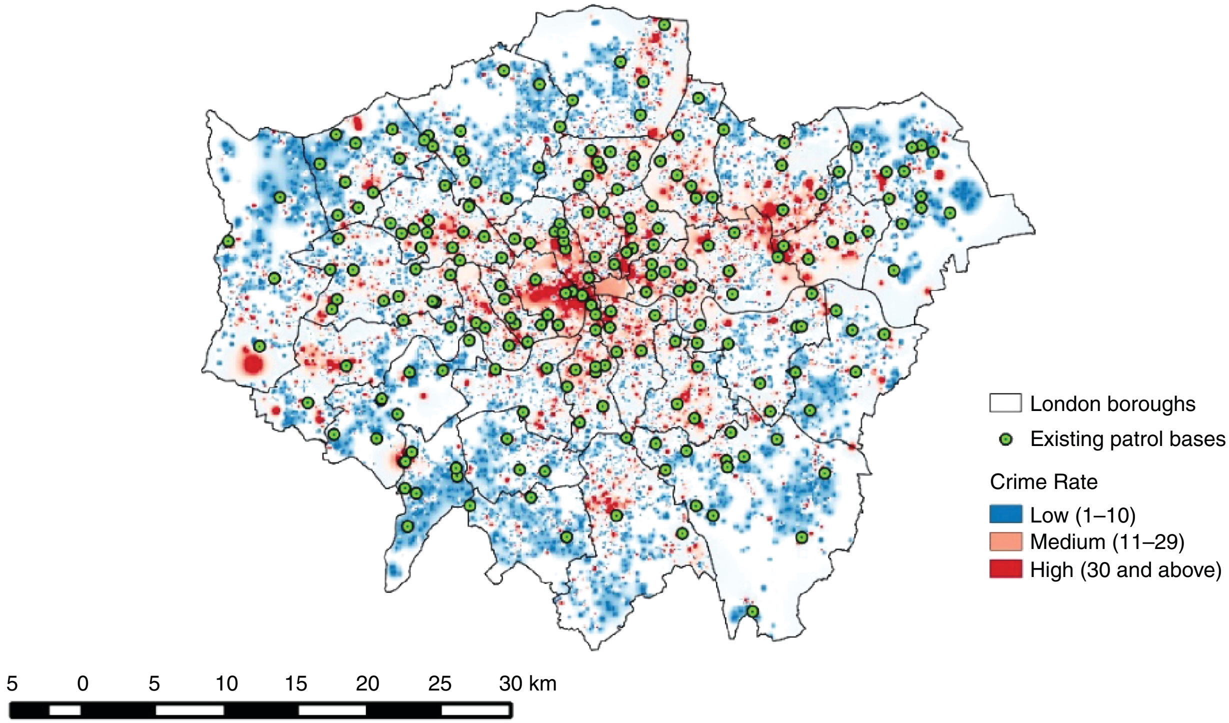

FIGURE 8 Existing police patrol bases in Greater London Area.

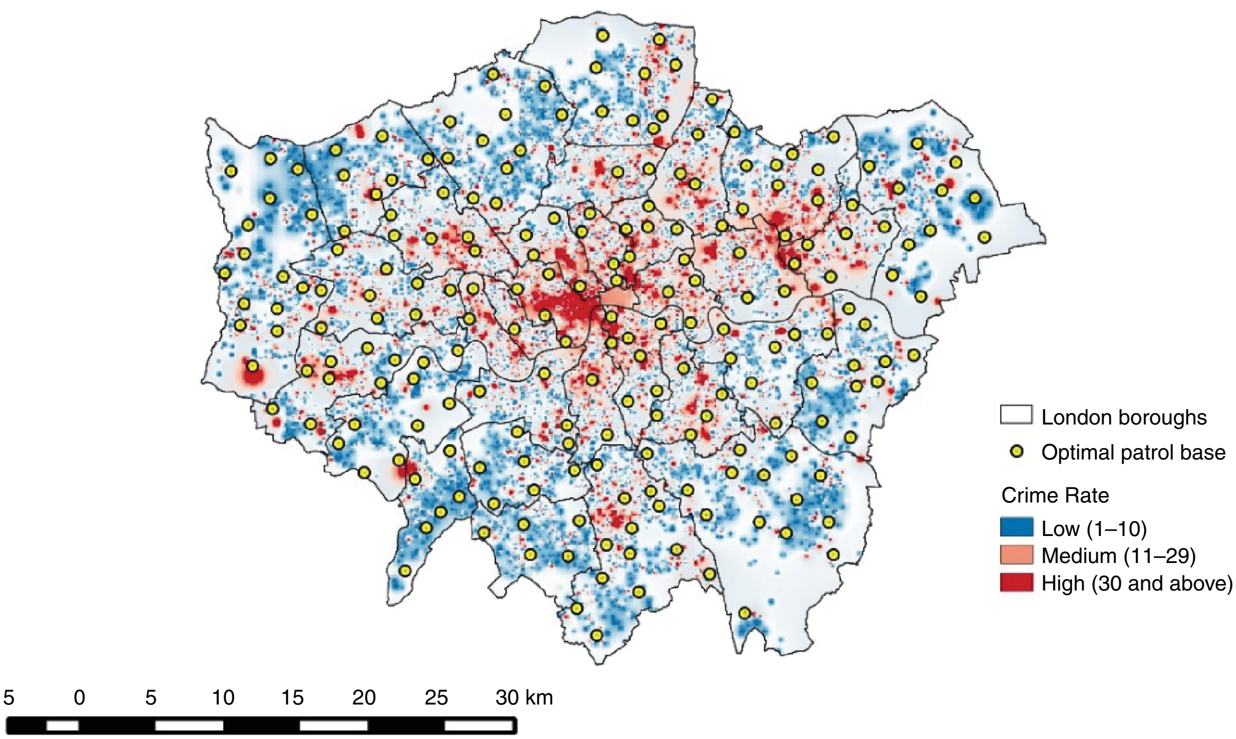

FIGURE 9 Optimal location of police patrol bases.

TABLE 1 Total Weighted Distance Between Police Stations and Potential Crime Spots, by Scenario

Deployment of Police Stations	Total Weighted Distance (km-crime)
Existing scenario with no attack	1,866.76
Existing scenario under attack	2,055.88
Optimized locations with no attack	1,559.43
Optimized locations under attack	1,726.37

Edgware Road, Tavistock Square, King's Cross, Russell Square, and Liverpool Street. Fifty-two civilians were killed and more than seven hundred were injured (*25*). A huge proportion of the police force and resources was used to assist in rescue operations and maintain security. Nevertheless, the efficiency of police operations may drop because of such a redeployment of police resources. In the study presented here, the interest is in the decrease in the police force in other areas as a result of the attack event.

To simulate the situation on July 7, 2005, it is assumed that all members of the police force located in the boroughs where the attack took place would be used for rescue operations. Consequently, the implication is that there will be no police stations or patrol points in the ward where the incident took place; they will all be removed from the model. Four incidents are located in three different wards: Camden, Harrow, and Hackney. There are 12 police stations and 24 patrol points located in these wards in reality; there are 12 police stations and 17 patrol points in the optimization model. When the attacks occur, these facilities are assumed to be busy and unavailable.

Table 1 summarizes the total weighted distances between police stations and potential crime spots under different situations. With the closure of 12 police stations in reality, the total weighted distance is estimated to increase from 1,866.76 to 2,055.88 km–crime (i.e., a 10.1% increase). With the optimized police station locations with the *p*-median model, the total weighted distance is estimated to increase from 1,559.43 to 1,726.37 km–crime (i.e., a 10.7% increase). Table 2 summarizes the coverage of patrol bases over the potential crime spots under different situations. With the closure of 24 patrol points in reality, the coverage percentage is estimated to drop from 91.99% to 86.76%. With the optimized patrol base locations with the maximum coverage model, the coverage is estimated to drop from 99.82% to 85.71%. Both studies suggest that the deployment of police resources becomes less resilient with respect to the existing deployment. Nevertheless, this result should not be surprising as the objective functions of the two models (*p*-median and maximum coverage) do not take resilience into account. The findings here suggest the importance of incorporating measures of resilience into the optimization framework; this topic is left for further investigation.

TABLE 2 Coverage of Patrol Bases, by Scenario

Deployment of Patrol Bases	Coverage (%)
Existing scenario with no attack	91.99
Existing scenario under attack	86.76
Optimized locations with no attack	99.82
Optimized locations under attack	85.71

CONCLUDING REMARKS

This paper presents an application of location optimization to police facility deployment; the Greater London Area is adopted as a case study scenario. The performance of the optimal solutions is compared with the existing deployment of police stations and patrol bases to highlight the potential benefit of the model in regard to covering potential crime spots. With the optimization models, it is shown that the average distance between police stations and potential crime spots is reduced by 19%. This reduction in travel distance is expected to have a positive effect on police response time. Moreover, the coverage percentage of potential crime spots is increased from the existing 91.99% to a nearly perfect 99.82%. Furthermore, an investigation was done on the resilience of the optimal solutions and the existing deployment in the event of an actual terrorist attack such as the one that occurred on July 7, 2005. It is shown that the optimal police resources deployment is less resilient with respect to the existing one. Nevertheless, this should not be surprising as the objective functions of the two models do not take resilience into account. The findings here suggest the importance of incorporating measures of resilience into the optimization framework; this topic is left for further investigation. The solution algorithms adopted in this study are the greedy type, which cannot guarantee global optimality. A possible future extension of the current study is to explore the use of other advanced algorithms, such as Lagrangian relaxation and branch-and-bound search, and see whether better solutions can be achieved. Finally, the study did not have access to the detailed data, including the distribution of different crimes, the level of severity, and the situation of wards. One can derive a better location plan if there is access to this detailed information. It will be interesting to explore how the level of information would affect location deployment plans.

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