Game Theory & Its Applications (BITS F314)

Second Semester: 2021-22

Term-paper assignment

Title of the Term-Paper

Game Theory in Pit stop Strategy in Motorsport

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INTRODUCTION

Formula One is a motor racing competition that takes place all over the world. A Formula 1 race is held to decide the race winner, who is the first driver to cross the finish line after completing a set number of laps.

A pit stop is a halt in motorsports for refuelling, installing fresh tyres, repairs, mechanical changes, driver change or any combination of these. These pit stops take place in an area known as the pit, which is usually accessed via a pit lane that runs parallel to the track's start/finish straight and is connected to it at both ends. When a driver enters the pit lane, he is supposed to slow down the speed of his car which makes him lose around 25 sec relative to his competitors. A row of garages (usually one per team or car) line this lane, outside of which the work is done in a pit box. Depending on the series regulations, a pit crew of up to twenty mechanics perform pit stop work, while the driver frequently waits in the car. It's critical for each team to figure out their best strategy throughout the race in order to get the greatest potential result.

In this paper we will be using game theory to determine the optimum pit stop strategy for the drivers which will assist them in winning the race. There are a variety of strategies that can be used. Because tyres can only last a portion of the race distance, drivers must pit during the n-lap race to replace them with new sets. Drivers can make use of any of the three different types of tyres available to them during the race. These are the following:

- Soft tyres
- Hard tyres
- Medium tyres

While the hard tyre will survive longer, the time lost relative to the soft tyre makes it far too slow to beat the rivals. As a result, the best strategy would be to make several stops, each time putting on a new pair of soft tyres, and then use the hard tyre during the last stint of the race to fulfil the requirement of using two tyres.

Drivers and teams are given points based on where they finish in a race. The winner earns 25 points, the runner-up receives 18 points, and places 3 through 10 receive 15, 12, 10, 8, 6, 4, 2 and 1 points, respectively.

Tyre and Tyre Compounds

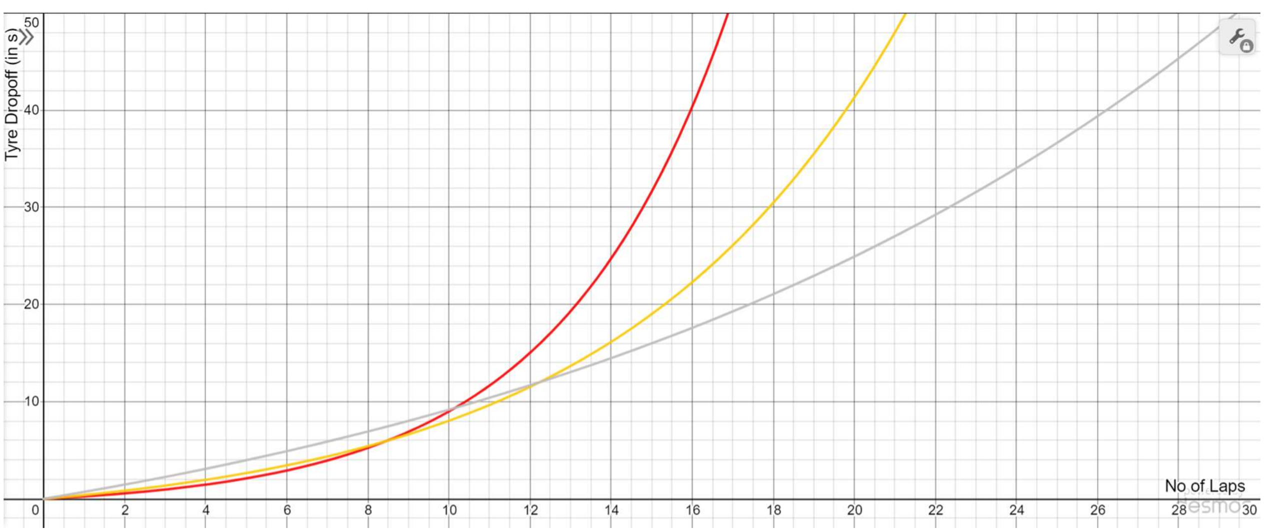
There are three types of tyres used in F1 racing cars. These are soft (marked with red), medium (marked with yellow) and hard (marked with grey). The synthetic rubber of which these tyres are made of, heats because of friction. This heat generated causes the rubber to wear off and thus time is lost i.e. now it takes more time for the car to complete a lap due to the inefficiency of the degraded tyre. Each of these three tyres behaves differently to different temperatures and thus has different rates of degradation. Hence a different amount of time is lost due to each tyre per lap. Interpolating the lap times around different tracks, we get the following equations for the three different tyres. Here the variable y stands for time lost in seconds and x represents the number of laps.

$Y = 0.923\,(e^{0.2375x} - 1)$: For Soft Tyres

$Y = 2.5630\,(e^{0.142x} - 1)$: For Medium Tyres

$Y = 12.82\,(e^{0.054x} - 1)$: For Hard Tyres

Plotting the same equations gives us this graph:



Here the colour red is for soft tyres, yellow for medium and grey for hard. From the above graph we can clearly see that soft tyres are initially the fastest as least time is lost per lap as compared to mediums and hard tyres. But after the eighth lap, the soft tyre starts degrading quickly and makes the car slower as compared to one with medium tyres. Further degradation of it after lap 10 makes it even slower than the car with hard tyres. After the eleventh lap we can also see that the medium tyre degrades faster than hard and thus becomes slower. From this we can conclude that initially soft tyres are fastest and the hards are slowest but due to the high rate of degradation of the fasts and the slow rate of degradation of the hards, at the end hards are fastest whereas softs become slowest, while mediums are always in between hards and softs.

ASSUMPTIONS

1) The first two players (cars) are much faster than the rest of the players. Hence the existence of others can be ignored.
2) Both the players (drivers) are equally skilled and both the cars are equally fast enough.
3) At the start of the race, Player 1 will start ahead of player 2 (we consider player 1 to be the fastest in qualifying, qualifying is where the order in which the drivers start the race is decided) and hence will lead the player 2 by a time difference of 0.5 seconds.
4) Only 1 safety car is allowed per race.
5) Both the players will start with medium tyres first which is the optimum starting tyre.
6) Each player can use only one set of hard, two sets of medium and one set of soft tyres. Considering that they have used the rest of the allotted tyres for practice and Qualifying Sessions.
7) The race will be of 50 laps.
8) All teams complete their pit stops in equal time (25s in normal conditions and 18 seconds under safety car).
9) Both the drivers drive their car perfectly without making any mistakes

One of the rules of Formula 1 that would be applicable to our game:

Each of the two players have to use at least two different types of tyres. A player cannot finish a race only using soft, medium or a hard tyre.

1. Pit stop Strategy

One of the drivers will be leading the race giving him the choice to pit or not. This will affect the other driver's decision thus making it a sequential game. Hence we will solve the sequential game to solve for the equilibrium.

1.1: Finding the Optimal Strategies

Since the optimum starting tyre for most of the race tracks will be medium tyres, we assume both the drivers to be starting on the medium tyres. Drivers have the choice of choosing between a 1-Pitstop Strategy and a 2-Pitstop Strategy to complete the 50 laps of the race. We need to find the optimal tyres to be used for one stop and the optimum lap to pit and change tyres. We can find this by optimizing and differentiating the tyre functions. For the one stop strategy the choice would be starting with the medium tyres and then change to the hard tyres as neither the medium nor the soft tyres would last that long. Hence optimizing the following function we get the optimum lap for changing the tyres.

Y = 2.5630 (e^ {0.142x} - 1) + 12.82 (e^ {0.054(50-x)} - 1)

Equating, $\frac{d}{dx}$2.5630 (e^ {0.142x} - 1) + 12.82 (e^ {0.054(50-x)} - 1) to 0, we get x to be 17

Strategy 1				
TYRES	MEDIUM	PIT	HARDS	TOTAL
LAPS	1-17	17th lap	18-50	--------
TIME LOST (secs)	26.088	25	63.353	114.441

Similarly optimizing the two stop strategy we get the following condition and the laps at which the two stops should happen.

Y = 2.5630 (e^ {0.142x} - 1) + 12.82 (e^ {0.054z} - 1) + 2.5630 (e^ {0.142(50-x-z)} - 1)

Equating, $\frac{d}{dx}$ 2.5630 (e^ {0.142x} - 1) + 12.82 (e^ {0.054z} - 1) + 2.5630 (e^ {0.142(50-x-z)} - 1)

To 0, we get x to be 15

Now,

Equating, $\frac{d}{dz}$ 2.5630 (e^ {0.142x} - 1) + 12.82 (e^ {0.054z} - 1) + 2.5630 (e^ {0.142(50-x-z)} - 1)

To 0, we get z to be 18

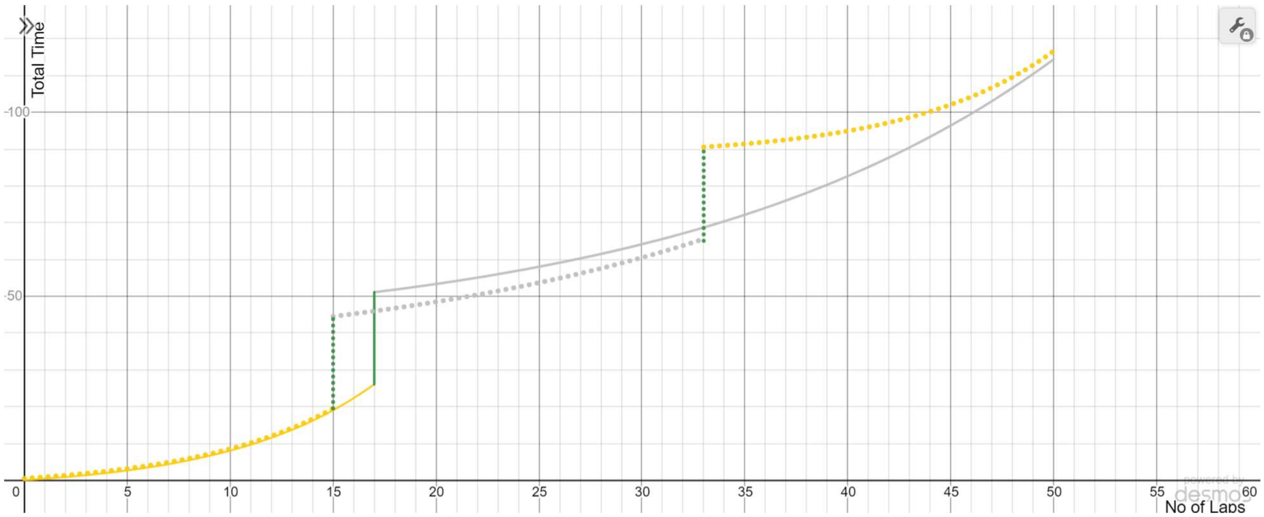
Strategy 2						
TYRES	MEDIUM	PIT	HARDS	PIT	MEDIUM	TOTAL
LAPS	1-15	15th lap	16-33	33rd lap	34-50	--------
TIME LOST (secs)	19.004	25	21.066	25	26.088	116.158

We can see from the tables that the optimized two stop strategy would be slower to the optimized one stop strategy by 1.717 secs

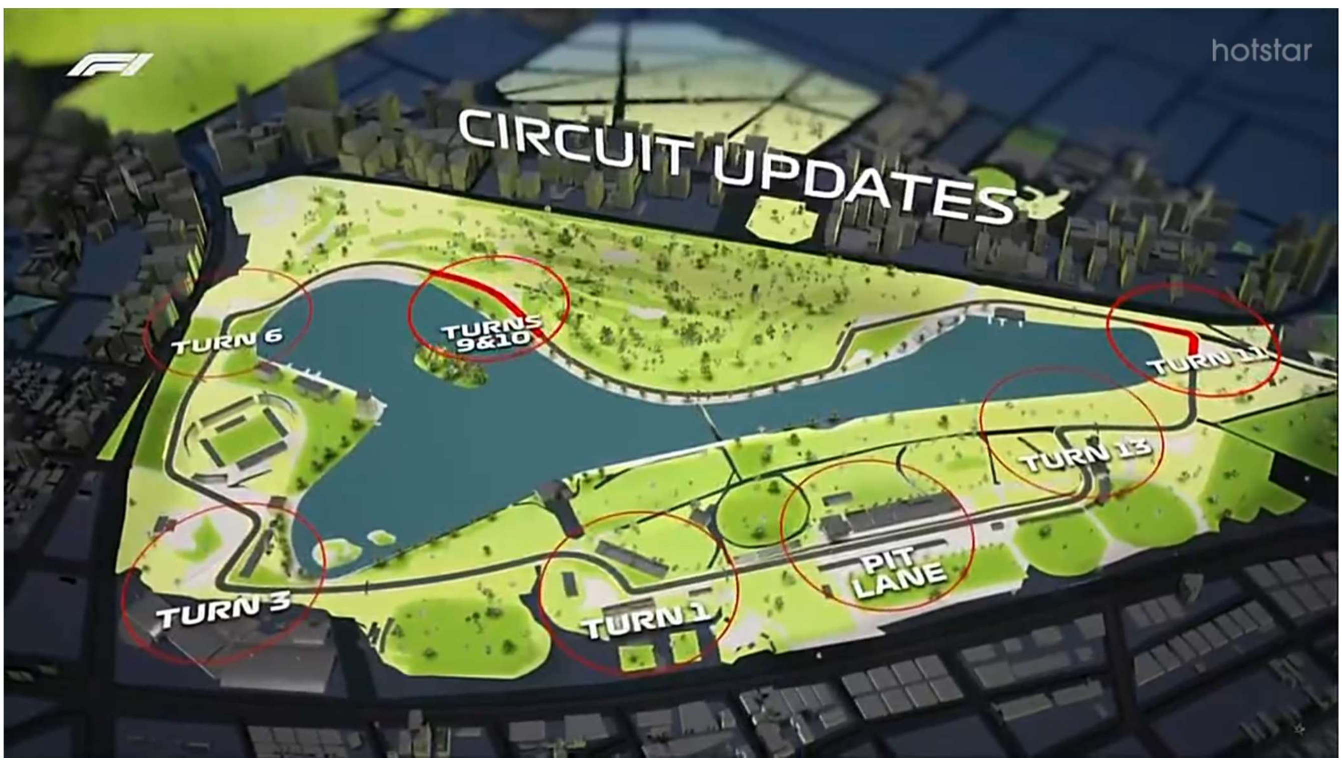
The following would be the graph for the two strategies:

Y axis = Total time lost due to tyre degradation and pit stops

X axis = No. of Laps



The dotted line indicates the two stop strategy and the solid line indicates the one stop strategy. 25 seconds are lost with every pit stop which is denoted by the green line.



This is map or a circuit layout showing exactly where the pit stop begins

1.2 Normal Form of the Game

<u>Normal Form of the game</u>:

Players: N= [1, 2]

Terminal history: {(One-stop, One-stop), (One-stop, Two-stop), (Two-stop, Two-stop), (Two-stop, One-stop)}

Player Functions:

P (φ) = Driver 1

P (Two-stop) = Driver 2

P (One-stop) = Driver 2

Strategy set:

Player1: S1= {One-stop, Two-stop}

Player2: S2= {One-stop, Two-stop}

Payoffs:

We consider the points that the driver receives as the payoff of the following game.

If the driver finishes 1st he gets 25 points

If the driver finishes 2nd he gets 18 points

Driver has to make the first pit stop at lap 15 for 2 stop strategy

Driver has to make the first pit stop at lap 17 for 1 stop strategy

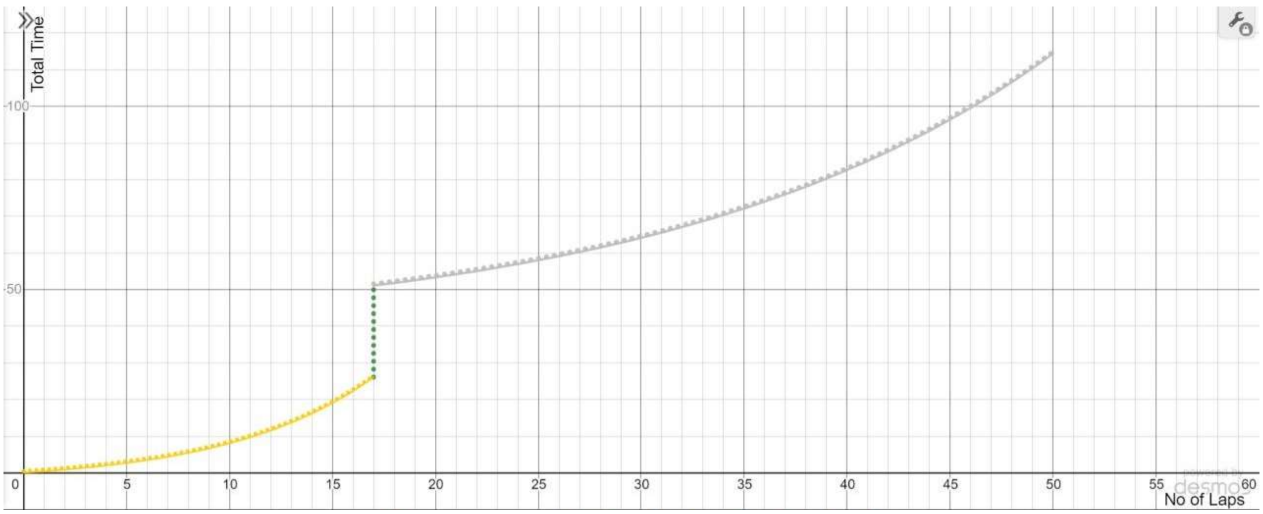
As shown above, our game is a form of sequential game. We had assumed previously that player 1 will be ahead of player 2 at the beginning of the race. Since there is only one place to pit in a racetrack, player 1 will get the first chance to pit (i.e. which strategy to choose)? If a player wants to go for strategy 2 (i.e. two pit stops), the first pit stop has to be made at lap number 15.Now since in our game, player 1 is ahead of 2 at the start, player 1 has to decide by the lap number 14 whether he wants to pit at lap 15 (choose strategy 2) or not. Once Player 1 crosses the pit entry on lap 15, then the strategy chosen by player 1 will be locked. After this the player 2 will get a chance to choose a strategy (he has to do this again before he crosses the pit lane entry on lap 15).

We examine the different cases:

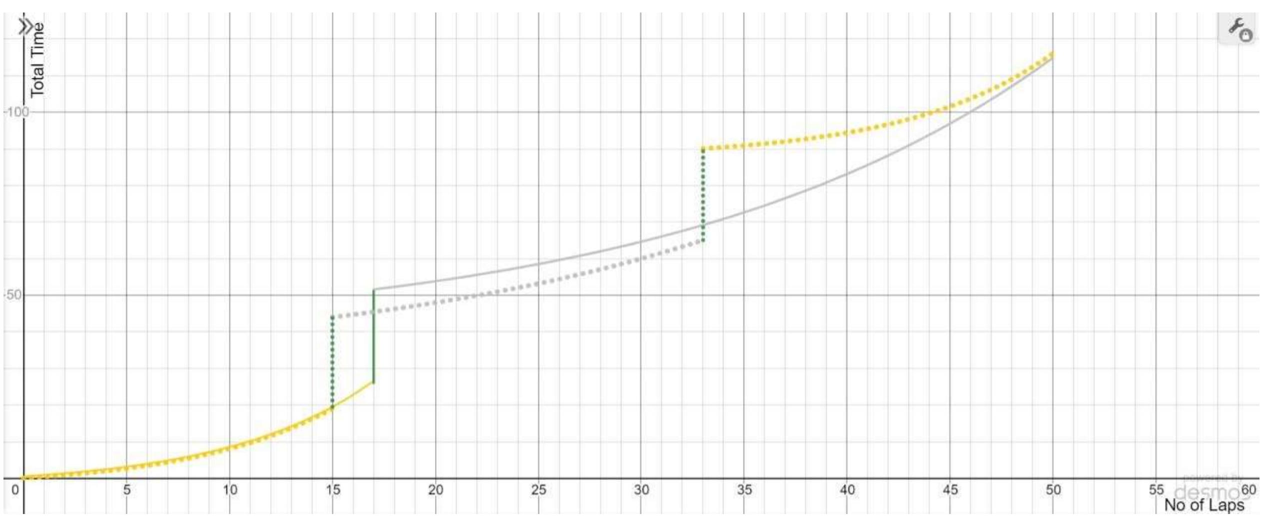
Driver 1 will be denoted by the solid line

Driver 2 will be denoted by the dotted line

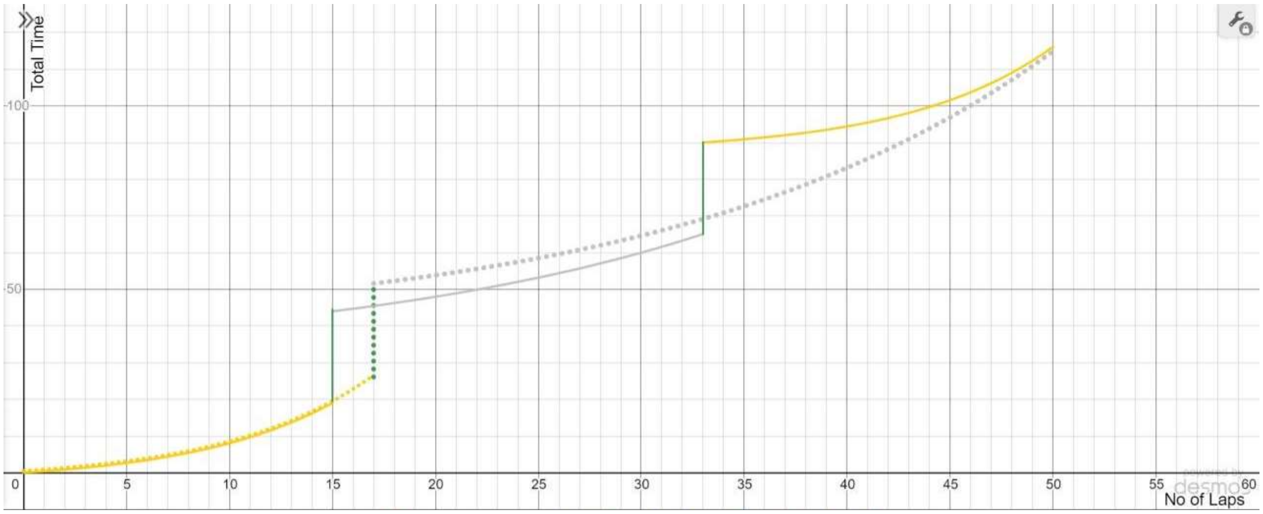
i) (One-stop, One-stop) = (114.441, 114,941) **Driver 1 wins**



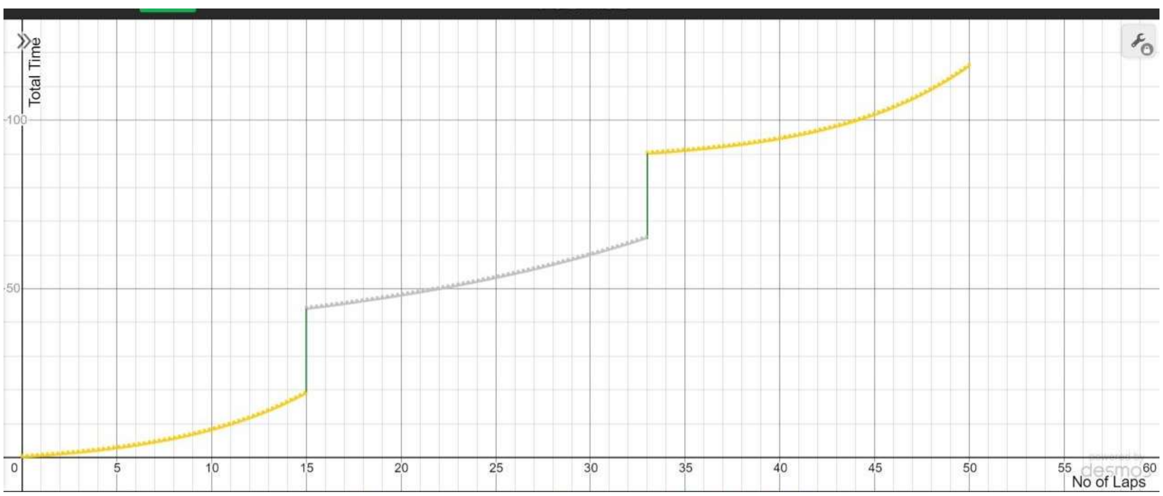
ii) (One-stop, Two-stop) = (114.441, 116.658) **Driver 1 wins**



iii) (Two-stop, One-stop) = (116.158, 114.941) **Driver 2 wins**



iv) (Two-stop, Two-stop) = (116.158, 116.658) **Driver 1 wins**

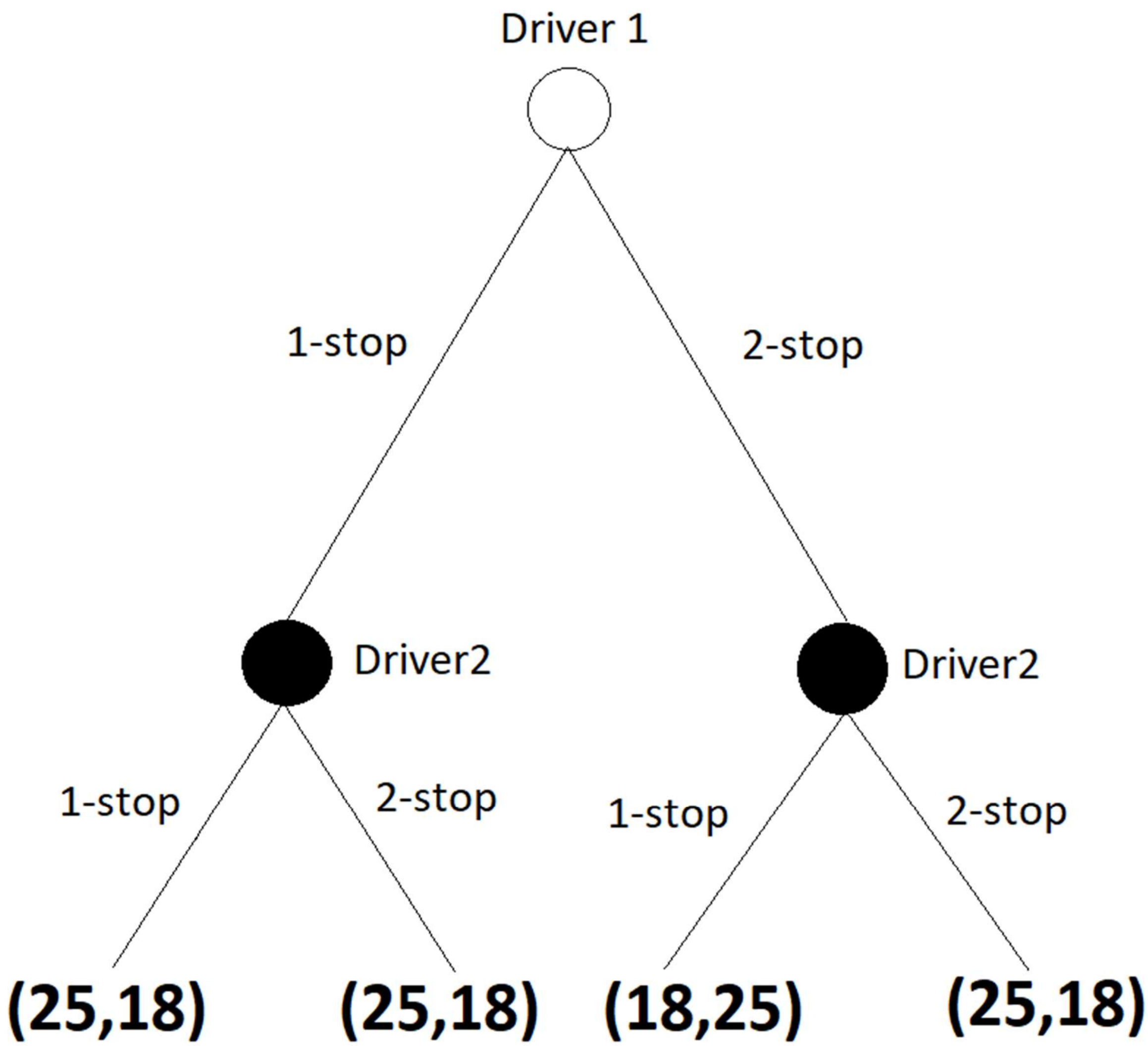


O (One-Stop, One-Stop) = (25, 18)

O (One Stop, Two-Stop) = (25, 18)

O (Two-Stop, One-Stop) = (18, 25)

O (Two-Stop, Two-Stop) = (25, 18)



Now if player 1 chooses strategy 2 (2 pit stops), then player 2 will have two choices: 1) to pit once (strategy 1) and get a payoff of 25 or 2) to pit twice (strategy 2) and get a payoff of 18. It is obvious that player 2 will go for strategy 1 as that is his best response. Since player 1 also has common knowledge he will also think that if he chooses strategy 2, then player 2 will definitely go for strategy 1 which is its best response and hence player 2 will win. Therefore player 1 will never choose strategy 2. **Now if player 1 chooses strategy 1, player two is indifferent on choosing strategy 1 or 2 since it has equal payoffs for both i.e. 18**.

Therefore there are two Nash equilibriums namely, (One-Stop, One-Stop) and (One-stop, Two-stop)

	One-Stop	Two-Stop
One-Stop	25,18	25,18
Two-Stop	18,25	25,18

Therefore there are two Nash equilibriums namely, (One-Stop, One-Stop) and (One-stop, Two-stop).

Both are Credible Nash Equilibriums.

1.3 The safety car effect

Whenever there is an accident or a crash on a racetrack, racing (overtaking) is paused for some time until the debris is cleared. This is done by introducing a car named safety car. All the racing cars have to form a line and follow the safety car slowly. We know that when a car chooses to pit, it loses a time of 25 seconds compared to other cars as it has to go out of the race track and drive slowly on a different path called pit lane. If there is an accident on the track at the same time, a safety car comes out, then all the other cars will have to follow the safety car at a lower speed. Hence the player will lose only 18 seconds (less than 25) as compared to the other cars if he decided to pit at that time. This is because other cars are also now moving at low speed due to the safety car.

Conclusion: If a player is pitting and a safety car comes in around the same time, the player will lose only 18 seconds as compared to losing 25(if the safety car would not have come).

If Driver 2 choose to go with the 2 stop Strategy, The following cases may arise with the arrival of the safety car and after analysing the different time lost curves and the tyres that the teams have we get the following results:

1. If safety car comes *before 15 laps*:

   Both the drivers will pit hence both will follow the same strategy

   Outcome: Driver 1 will win.

2. If safety car comes *between laps 15 and 17*:

   The pit stop time for Driver 1 will decrease whereas Driver 2 would have already pitted. As a result Driver 1's lead will further increase.

   Outcome: Driver 1 will win.

3. If safety car comes *between laps 18 and 31*:

   Driver 1 will choose not to pit as pitting would make him lose. Driver 2 will also choose not to pit as it would lead to him falling further behind.

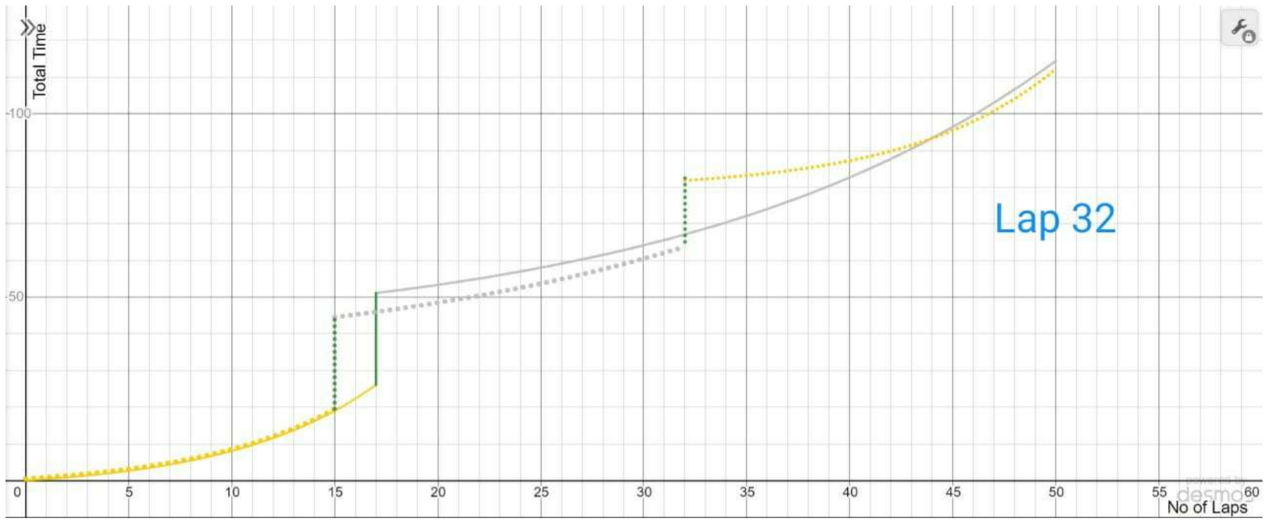
   Outcome: Driver 1 will win.

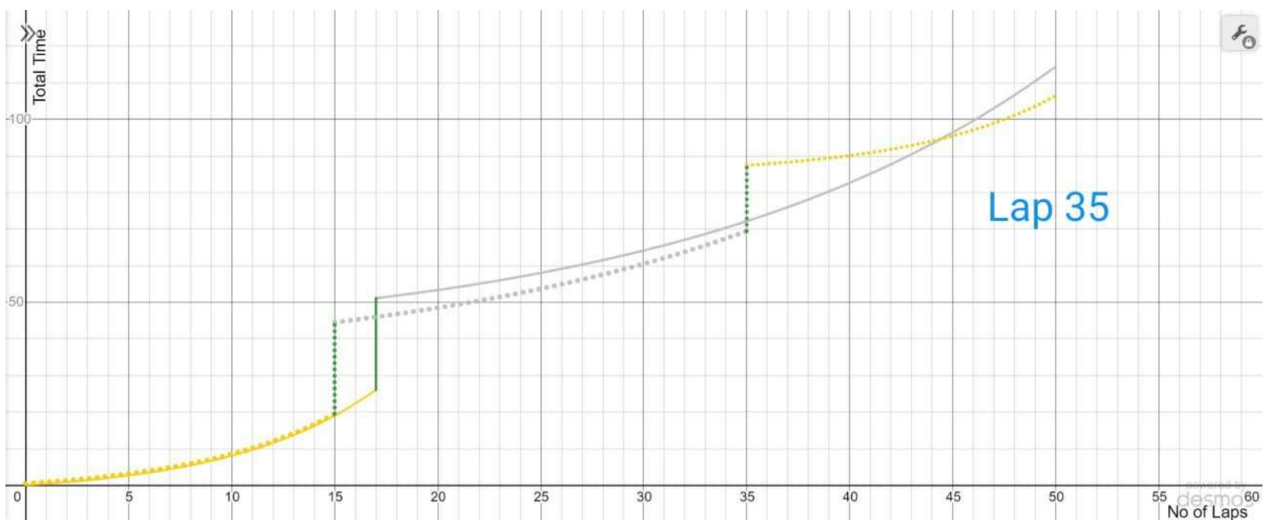
4. If safety car comes *between laps 32 and 42*:

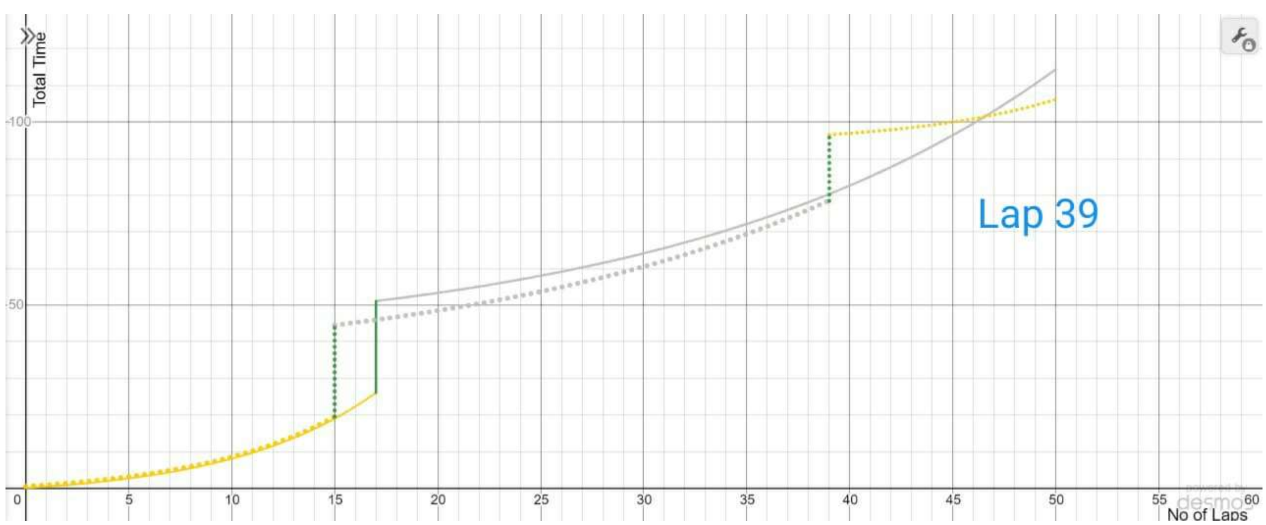
   If Driver 2 pits to medium tyres, he will win the race no matter what Driver 1 does as Driver 2 leads Driver 1 at this point and can pit to better tyres under a short pit stop.

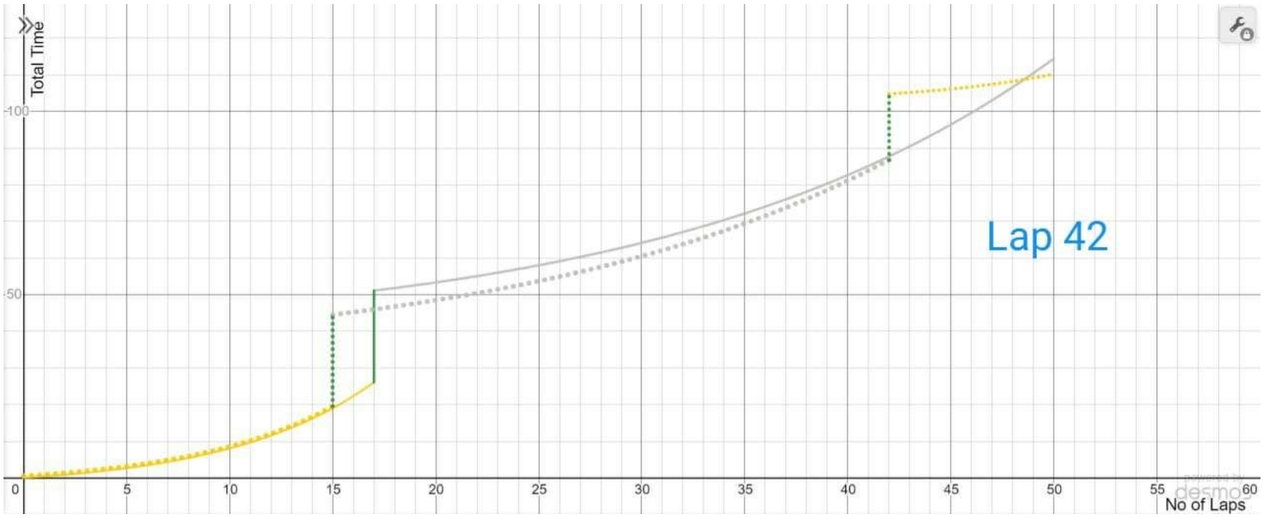
   Outcome: Driver 2 will win.

The following Graphs illustrate driver 2 winning the race if the safety car comes between the above mentioned laps.


Total Time
100
50
0
5
10
15
20
25
30
35
40
45
50
55
60
No of Laps
Lap 32

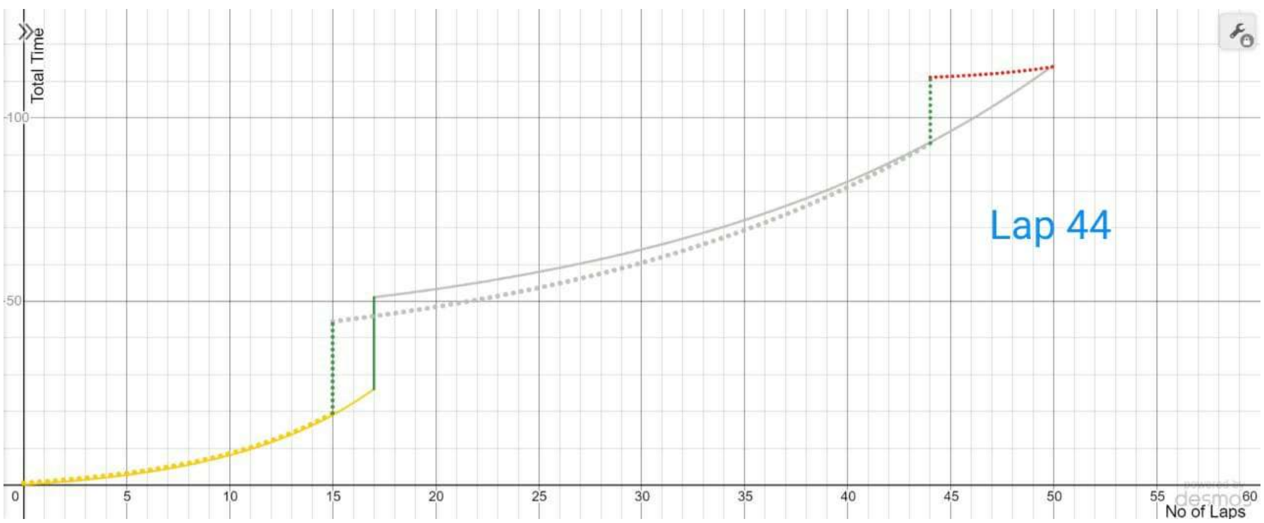

Total Time
100
50
0
5
10
15
20
25
30
35
40
45
50
55
60
No of Laps
Lap 35


Total Time
100
50
0
5
10
15
20
25
30
35
40
45
50
55
60
No of Laps
Lap 39



5. If safety car comes *between laps 43 and 44*:

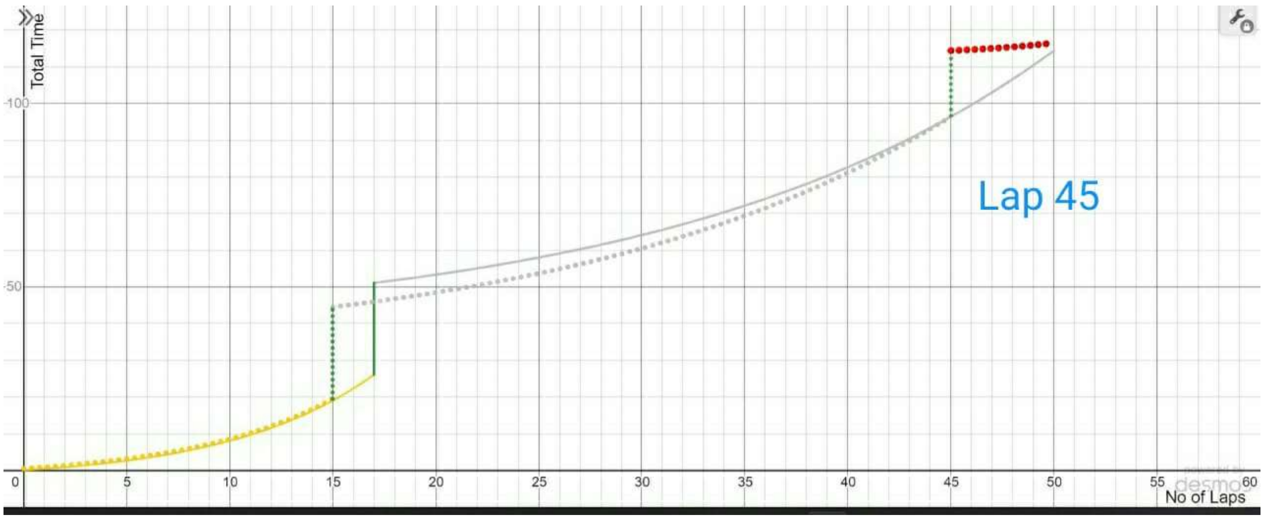
   If Driver 2 pits to soft tyres, he will win the race no matter what Driver 1 does as Driver 2 leads Driver 1 at this point. This is illustrated by the Graph given below:



   Outcome: Driver 2 will win.

6. If safety car comes *between laps 45 and 50*:

   Driver 1 has overtaken Driver 2 at this point hence whatever Driver 2 does, he will lose and there are very few laps for Driver 2 to catch Driver 1 even after pitting under the safety car.



Outcome: Driver 1 will win.

1.4: Game with the Probability of Safety Car

Normal Form of the game:

Players: N= [1, 2]

Terminal history: {(One-stop, One-stop), (One-stop, Two-stop), (Two-stop, Two-stop), (Two-stop, One-stop)}

Player Functions:

P (φ) = Driver 1

P (Two-stop) = Driver 2

P (One-stop) = Driver 2

Strategy set:

Player1: S1= {One-stop, Two-stop}

Player2: S2= {One-stop, Two-stop}

Payoffs:

We consider the points that the driver receives as the payoff of the following game.

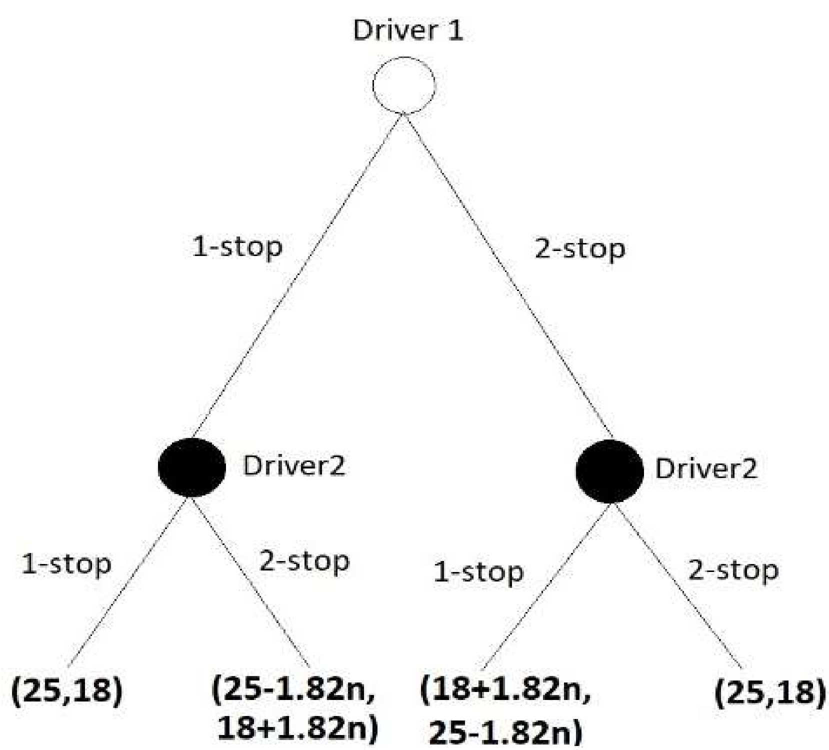
If the driver finishes 1st he gets 25 points

If the driver finishes 2nd he gets 18 points

Driver has to make the first pit stop at lap 15 for 2 stop strategy

Driver has to make the first pit stop at lap 17 for 1 stop strategy

Here n denotes the probability that a safety car appears during the race, the more dangerous the track, the higher the chance of a crash and higher the chance of a safety car and higher the value of n. This n is track specific and generally varies from .5 to .7. Now if the safety car comes in between laps 1-31, the result of the race will be Driver 1 winning. However if the safety car comes in between laps 32-44 then the outcome of the race changes and Driver 2 wins. Again if the safety car comes in between laps 45-50, Driver 1 wins. Hence there is .26 chance that the safety car when it comes, comes in between laps 32and 44, Summarizing this we get that .26n is the probability that a safety car appears specifically between laps 32 and 44.



Now since n is a probability value, its value will always lie between [0, 1]

Now if n=E where E>0 then 25-1.82n <25, 18+1.82n>18

While if n=1 then 25-1.82n=23.18, 18+1.82n=19.82

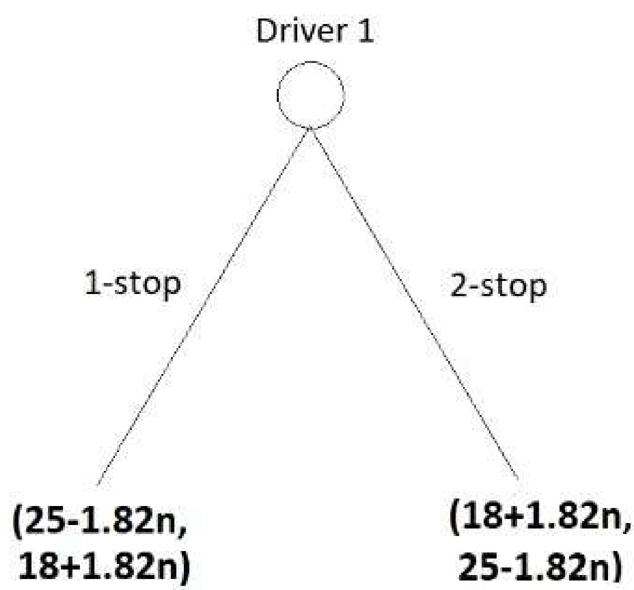
Therefore by backward induction,

Driver 2 has to make first choice

We will get that driver 2 chooses 2-stop if driver 1 choose 1-stop as (18+1.82n is always > 18),

While driver 2 chooses 1-stop if driver 1 choose 2-stop as (25-1.82n is always > 18)

Simplifying us get the following game tree:



Now driver 1 has to choose his strategy

Since (25-1.82n is always greater than 18+1.82n) therefore driver 1 will choose 1-stop

Hence the Nash equilibrium for the drivers now is just (1-stop, 2-stop)

2. Wet Weather Strategy

The tyres for F1 cars come in two types: wet tyres and dry tyres. **The softs, mediums and hards come under the dry category.** Dry tyres are the ones which have a smooth outer surface and can be used in dry weather. Wet tyres have a rough outer layer to generate additional friction when it rains heavily. Dry tyres if used on wet tracks might cause accidents due to slippery conditions. And wet tyres (the ones with rough surfaces) move slowly on dry tracks as compared to dry tyres. In case of a slight drizzle, both wet as well as dry tyres work with equivalent efficiency.

Optimal tyres for respective track conditions:

Dry track: Dry tyres (Wet tyres are much slower)

Partially wet track: Dry or wet tyres

Wet track: Wet tyres (Dry tyres are much slower and there is a chance of a crash)

2.1: Wet weather Game

When it starts to drizzle the strategies and the game changes completely, now the drivers have to decide between changing to wet tyres and staying on the current strategy (includes only dry tyres). Since, it's just drizzling both wet tyres and soft tyres currently are equally fast, if the rain becomes worse only the drivers who pitted to wets would be fast and would overtake the drivers who are on dry tyres and vice versa if it doesn't rain further and the track dries up. For the following game we take two assumption

1) The leading driver would be called driver 1 and the lagging driver will be called driver 2

2) The drivers on the slower tyres (dry or wet tyre depending on whether the track is dry and rain became heavier) would get overtaken.

Since driver 1 is ahead of driver 2 at the start of the race, he has the first chance to decide whether to pit or not (i.e. to change to wet tyres or continue with the current tyre). After this, the second driver will have to choose whether to pit or to continue with the same tyre.

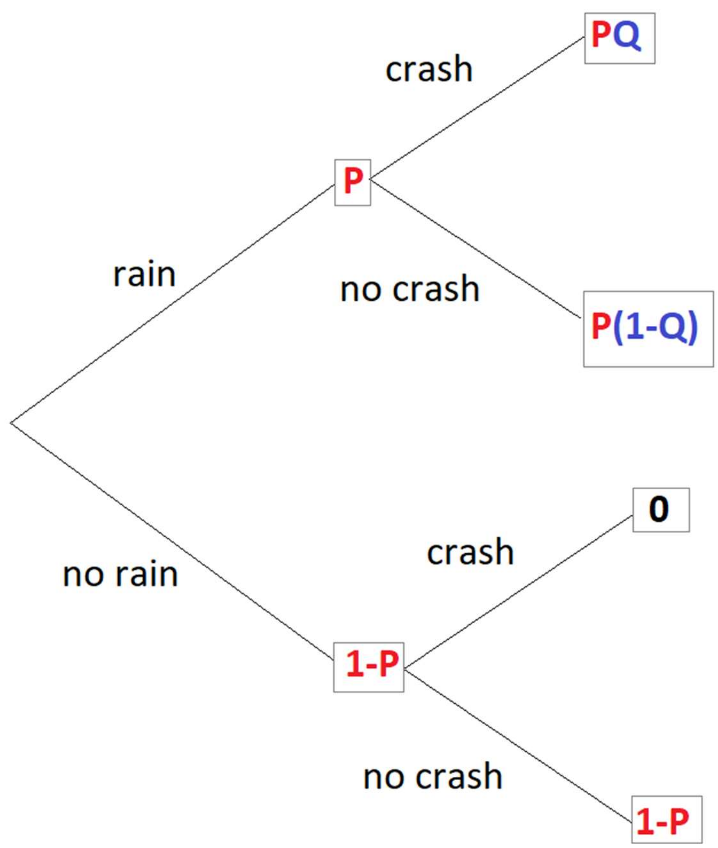
Initially both the players start with dry tyres.

Driver 1 leads Driver 2

P represents the probability that rain will get heavier.

Q represents the probability that the car will crash.

Shown below is a probability chart outlining the 4 cases that might occur?



Note (Assumptions):

1) If a driver is on wet tyres and it is raining heavily, then there is no chance of him crashing.
2) If a player crashes, he is not able to complete the race and hence his payoff is zero.
3) The leading driver would be called driver 1 and the lagging driver will be called driver 2
4) The drivers on the slower tyres (dry or wet tyre depending on whether the track is dry and rain became heavier) would get overtaken.

In total there will be four cases:

Driver 1 Driver 2

1) Driver 1 pits (chooses wet tyres) and driver 2 also pits (chooses wet tyres)

P (Pits to wet tyres, Pits to wet tyres) = [25, 18]

2) Driver 1 pits (chooses wet tyres) and driver 2 does not pit (continues with current dry tyres)

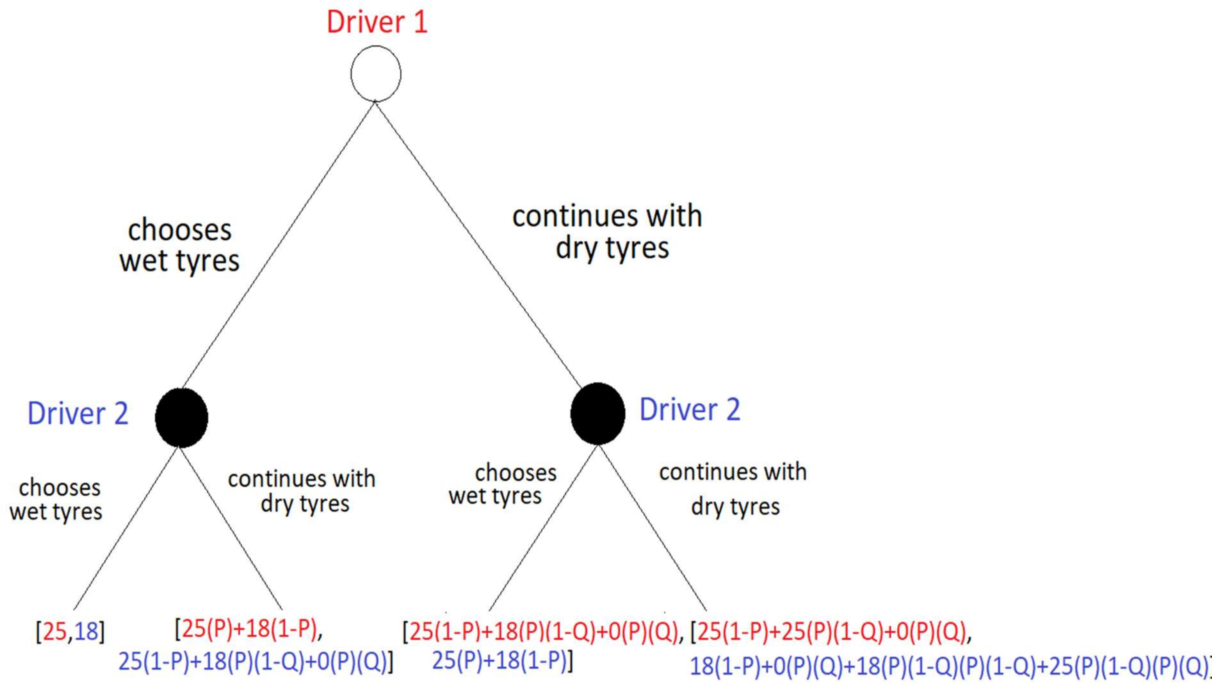
P (Pits to wet tyres, Continues with dry tyres) = [25(P) +18(1-P), 25(1-P) +18(P) (1-Q) +0(P) (Q)]

3) Driver 1 does not pit (continues with current dry tyres) and driver 2 pits (chooses wet tyres)

P (Continues with dry tyres, Pits to wet tyres) = [25(1-P) +18(P) (1-Q) +0(P) (Q), 25(P) +18(1-P)]

4) Driver 1 does not pit (continues with current dry tyres) and driver 2 does not pit (continues with current dry tyres)

P(Continues with dry tyres, Continues with dry tyres)= [25(1-P)+25(P)(1-Q)+0(P)(Q),18(1-P)+0(P)(Q)+18(P)(1-Q)(P)(1-Q)+25(P)(1-Q)(P)(Q)]



2.2: Real Life Example

From the sequential game tree, we can see that the Nash equilibrium of the game would depend on the different values of the probabilities of the rain becoming heavier (P) and the probability of crashing when on dry tyres in wet weather (Q).

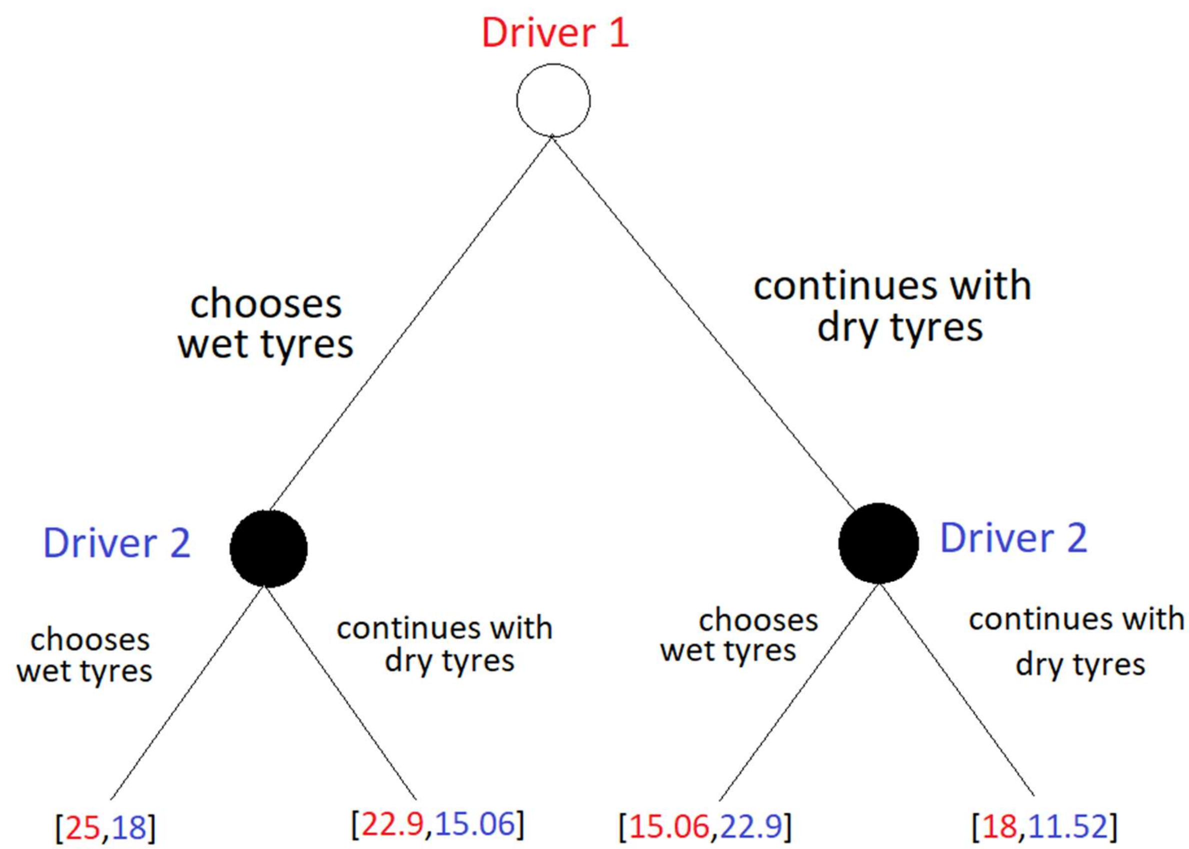
We will be considering the case of the two drivers, Lando Norris and Lewis Hamilton (Driver 2) in the 2021 Russian Grand Prix hosted at the Sochi Autodrom. Lando Norris was the leading driver (Driver 1) in our game until the final 7 laps when it started to drizzle on the race track. The race engineers predicted a 70% chance of the rain getting worse and we consider the probability of crashing or spinning when driving on dry tyres under wet conditions to be 40% by analysing the different wet races in the last two years.

$P = 0.7$

$Q = 0.4$

O (chooses wet tyres, chooses wet tyres) = (25, 18)
O (chooses wet tyres, continues with dry tyres) = (22.9, 15.06)
O (continues with dry tyres, chooses wet tyres) = (15.09, 22.9)
O (continues with dry tyres, continues with dry tyres) = (18, 11.52)

We substitute the following values and this what our sequential game tree would look like:



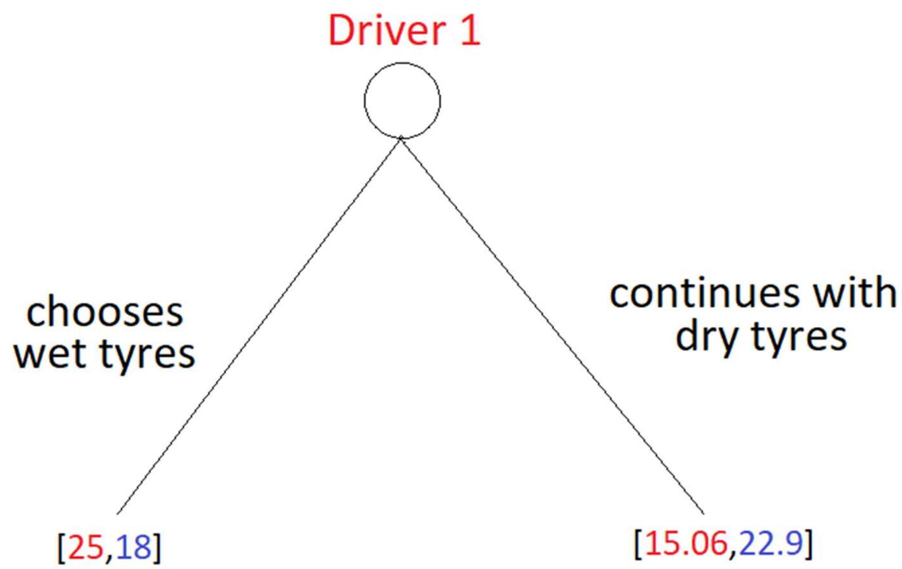
Therefore by backward induction,

Driver 2 has to make first choice

We will get that driver 2 chooses to pit to wet tyres if driver 1 chose to pit to wet tyres as (18 is > 15.06)

While driver 2 chooses to pit to wet tyres if driver 1 chose to continue on dry tyres as (22.9 > 11.52)

Simplifying us get the following game tree:



Now driver 1 has to choose his strategy

Since (25>15.06) therefore **driver 1 will choose to pit to wet tyres**

Hence the Nash equilibrium for the drivers now is (Pits to wet tyres, Pits to wet tyres)

	Chooses wet tyres	**Continue with dry tyres**
Chooses wet tyres	25,18	22.9,15.06
Continue with dry tyres	15.06,22.9	18.11.52

The Nash equilibrium is also credible.

Pitting to wet tyres is the optimum choice for both Lando Norris and Lewis Hamilton. The race strategists of Lando Norris knew this and advised Lando to come into the pits and change to wet tyres through the radio. **Norris decided to ignore the instructions given by his team and decided to stay out on dry tyres**. **Hamilton's race strategists on seeing that Norris didn't change to wet tyres advised Hamilton to change to wet tyres and Hamilton followed their advice changing to wet tyres.** The advice of the race strategists is justified in the sequential game tree.

Hamilton later went on to win the race and got 25 points while Lando had a minor crash but was able to get his car to move and came home in 7th place and ended up getting 6 points.

1	L. Hamilton Mercedes · #44	1:30:41.001
2	M. Verstappen Red Bull · #33	+53.271s
3	C. Sainz Jr. Ferrari · #55	+62.475s
4	D. Ricciardo McLaren · #3	+65.607s
5	V. Bottas Mercedes · #77	+67.533s
6	F. Alonso Alpine · #14	+81.321s
7	L. Norris McLaren · #4	+87.224s
8	K. Räikkönen Alfa Romeo · #7	+88.955s

Even though Norris could have maximized his payoff by pitting to wet tyres, he decided to stay on the dry tyres. Hamilton followed game theory and decided to change to wet tyres after Norris deciding not to pit and hence was able to maximize his payoff and this helped him to win the race.

CONCLUSION

We can conclude from our paper and research that game theory plays a very important role in Formula 1 Races and any other motorsport in deciding pit stop strategy.

We were able to prove how even with a very small variable such as the safety car and weather came into the picture, the Nash equilibrium completely changed. This shows that Motorsport and Pit stop Strategy is an ever changing game with hundreds of different variables which could result in completely different outcomes and Nash Equilibriums with each changing variable. This also shows how important a tool is game theory for a race strategist to optimize the race and help the driver and team to win the race and also make decisions in a game where the drivers are taking risks with their lives driving at very high speeds.

With the wet weather game we were also able to prove that not following game theory in deciding a strategy resulted in disastrous results for the team and the driver.

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