$$Ratio\ of\ tensions\ in\ the\ belt: \frac{T_1}{T_2} = e^{\mu\theta} = e^{0.28 \times 2.794} = 2.187$$

$$i.e.\ \frac{1000}{T_2} = 2.187 \Rightarrow T_2 = 457.24N$$

$$Initial\ tension\ in\ the\ belt\ T_0 = \left(\frac{T_1 + T_2}{2}\right) = \left(\frac{1000 + 457.24}{2}\right) = 728.62N$$

Gears

Gears are toothed wheels used to transmit power from one shaft to another when a constant velocity ratio is desired and the distance between shafts is relatively small.

Gears are classified as follows:

(i) According to relative position of shaft axes:

Parallel axes: Spur gear, helical gear

Intersecting axes: Bevel gears

Non parallel, Non intersecting: Worm gears

(ii) According to peripheral velocity (v) of gears:

V< 3 m/sec: Low velocity gears

3<V< 15 m/sec: Medium velocity gears

V>15 m/sec: High velocity gears

(iii) According to type of gearing:

Gears mesh externally & hence rotate in opposite directions: External gearing

Gears mesh internally & hence rotate in same directions: Internal gearing

(iv) According to position of the teeth on gear surface:

Straight teeth: Spur gears

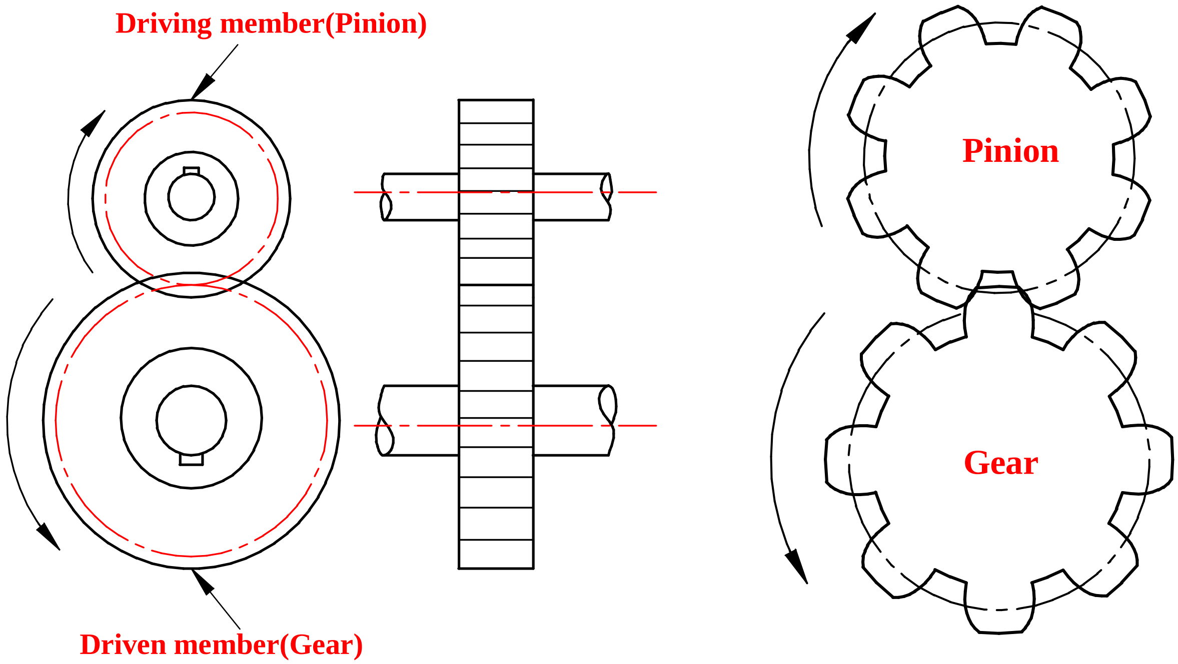
Inclined teeth: Helical gears

Skewed (curved) teeth: Spiral gear

Spur Gears

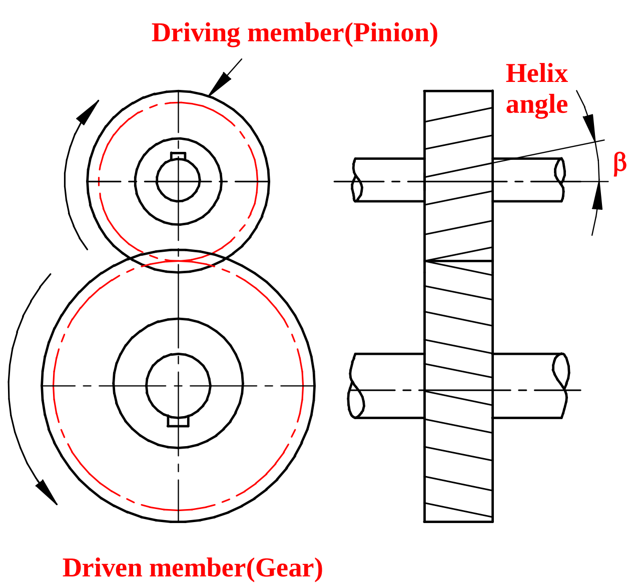
- This is the simplest form of gears for transmitting power between two parallel shafts. The teeth are straight & parallel to the axis.
- Spur gears impose only radial loads on bearings.
- Because of the instantaneous line contact during meshing, the drive will be noisy.
- Spur gear drive is widely used in machine tools, automobile gear boxes, etc.

Spur Gears

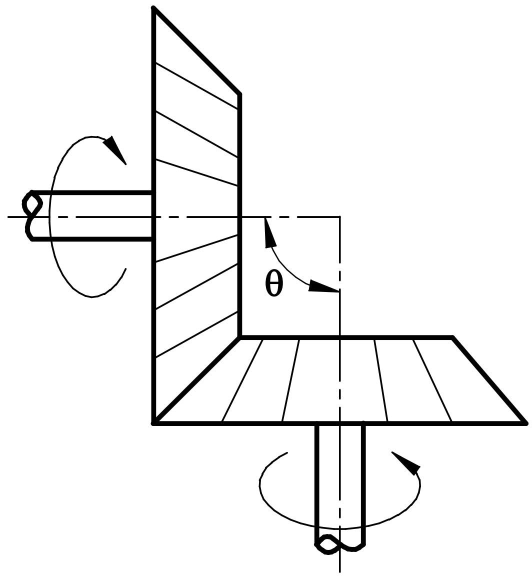


Helical Gears

- Helical gears are used to transmit power between parallel shafts.
- In these gears, the teeth are inclined to the axis of the shaft at an angle known as ***Helix angle (15^0 to 45^0).***
- Helical gears are preferred to spur gears as their operation is quiet due to progressive engagement of teeth.
- The disadvantage of helical gears is it produces an axial thrust. Hence double helical gears (herringbone gears) are used.

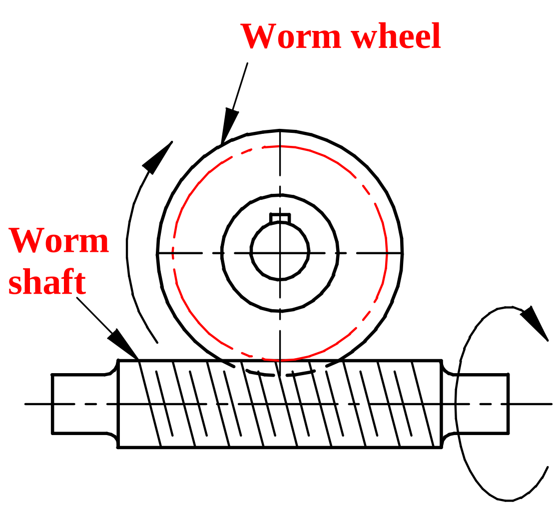


Bevel gears



- Bevel gears are most commonly used for transmitting power between intersecting shafts.
- The pitch surfaces of bevel gears are rolling cones. The tooth section becomes gradually smaller as the apex of the cone is approached.
- They impose thrust as well as radial loads on the bearings supporting the shaft.
- When two ***equal*** bevel gears have their axes ***at right angles***, they are called ***Miter bevel*** gears.

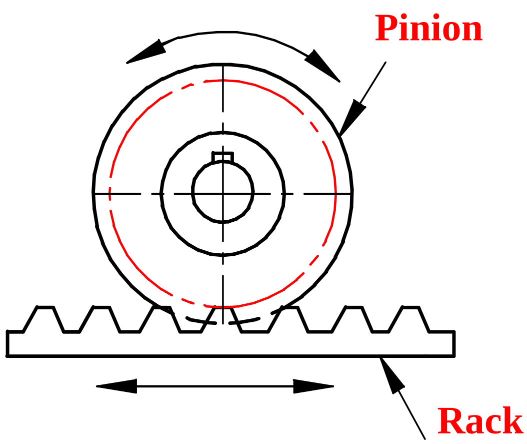
Worm gears



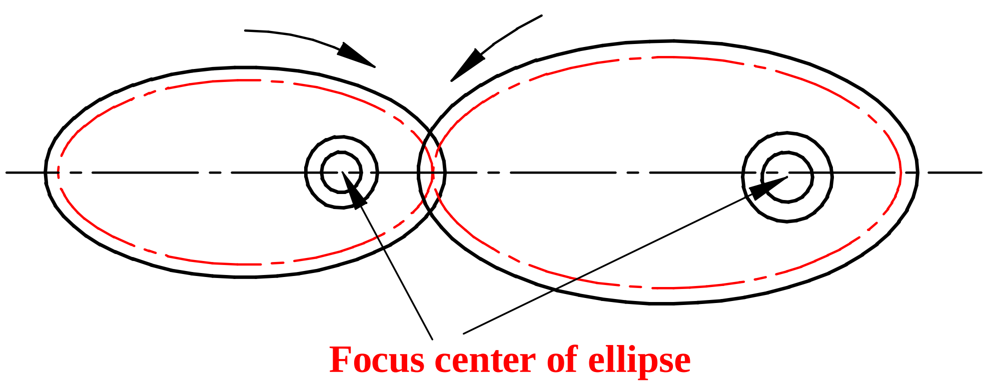
- Worm gears are used to transmit power between two non parallel, non intersecting shafts.
- A worm drive consists of a ***worm shaft*** with helical grooves which meshes with a gear called ***worm wheel.***
- Worm gear drives are used for high speed reduction as high as 60:1.
- The worm gear drive may be made self ***locking,*** i.e. does not allow the reversal in the direction of the drive.

Rack & Pinion

- When a rotary motion is to be converted into a linear motion, rack & pinion arrangement is used.
- Theoretically rack is a straight gear of infinite diameter.



Elliptical Gears



- Elliptical gears are used when there is need for varying speeds of the driven gear in each revolution.
- In each revolution of the driven shaft, there are four different speeds, two maximum & two minimum.
- They are used in printing machines, packaging machines, quick return motion mechanisms, etc.

Gear tooth profiles

- Gears are mainly used for transmission of motion & power and must be of accurate profile to obtain exact velocity ratio.
- Two commonly used profiles of gear teeth are the *Involute profile* & the *Cycloidal profile*
- Involute is defined as the path described by a point on an inextensible cord which is unwound from a stationary cylinder.
- Cycloid is defined as the curve traced by a point on the rim of a circle which rolls without slipping on a fixed straight line.

Advantages of Gear drives

1. They are positive drives and used to connect closely spaced shafts.
2. High efficiency, compactness, reliability, longer life, less maintenance.
3. They can transmit heavier loads.

Disadvantages of Gear drives

1. Not suitable for large center distances.
2. High production cost.
3. Due to errors and inaccuracies in manufacture, the drive may become noisy and produce vibrations at high speeds.

Velocity ratio in Gear Drives :

$$\frac{n_2}{n_1} = \frac{d_1}{d_2} = \frac{z_1}{z_2}, \text{ where } n_1 = \text{Speed of driving pulley}, n_2 = \text{Speed of driven pulley}$$

$$d_1 = \text{Pitch circle diameter (PCD) of driver gear}, d_2 = \text{PCD of of driven gear}$$

$$z_1 = \text{No of teeth on driver gear}, z_2 = \text{No of teeth on driven gear}$$

Gear Trains

A gear train is an arrangement of two or more successively meshing gears through which power can be transmitted between the driving & driven shafts.

Train Value:

Train value is the ratio of speed of the driven gear to that of the driving gear. It is the reciprocal of the velocity ratio.

Direction of rotation

When gears mesh externally they rotate in the opposite direction and when they mesh internally, they rotate in the same direction.

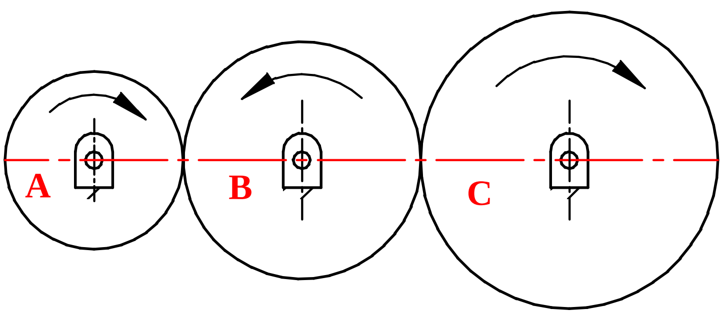
Types of Gear Trains

A gear train may be broadly classified into the following;

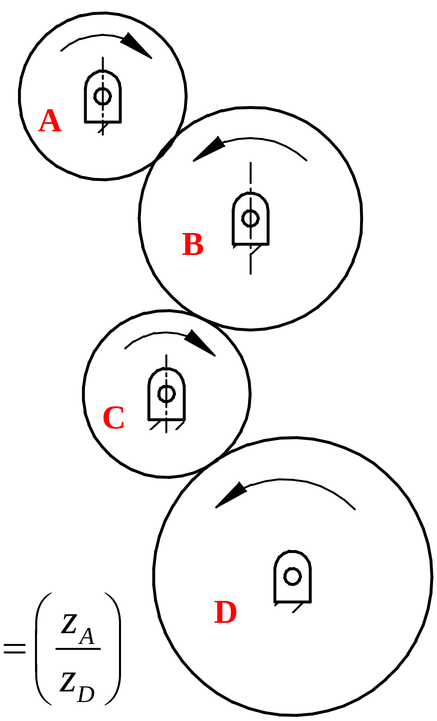
1. *Simple Gear Train*
2. *Compound Gear Train*
3. *Reverted Gear Train*
4. *Epicyclic Gear Train*

Simple gear train

- A **simple gear train** is one in which ***each shaft carries only one gear.***
- From the fig, gear A is the driving gear and gear D is the driven gear. B & C are the intermediate gears or ***Idler gears.***
- The idler gears do not affect the velocity ratio but simply bridge the gap between the driver & the driven gears.
- Also if ***odd*** number of intermediate gears are used, the driver & the driven gears rotate in the ***same direction***.
- If ***even*** number of intermediate gears are used, the driver & the driven gears rotate in the ***opposite directions***.



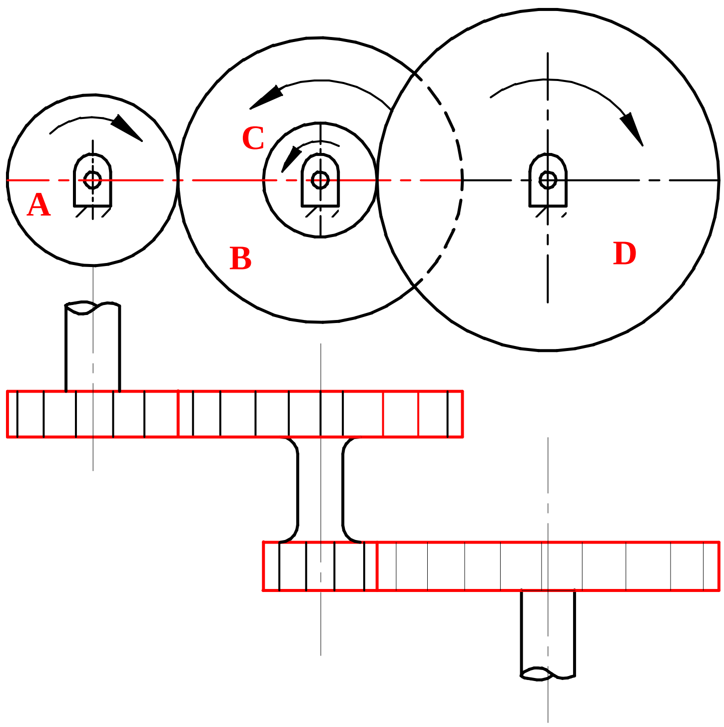
$$\text{Velocity ratio } \left(\frac{N_C}{N_A}\right) = \left(\frac{z_A}{z_C}\right)$$



$$\text{Velocity ratio } \left(\frac{N_D}{N_A}\right) = \left(\frac{z_A}{z_D}\right)$$

Compound gear train

- In a **compound gear train *the intermediate shaft carries two or more gears*** which are keyed to it.
- Compound gears are used when a high velocity ratio is required in a limited space.
- The intermediate gears will have an effect on the overall velocity ratio.



From the fig, $\left(\dfrac{N_B}{N_A}\right)=\left(\dfrac{z_A}{z_B}\right)$ and also $\left(\dfrac{N_D}{N_C}\right)=\left(\dfrac{z_C}{z_D}\right)$

$$\therefore \left(\frac{N_B}{N_A}\right)\times\left(\frac{N_D}{N_C}\right)=\left(\frac{z_A}{z_B}\right)\times\left(\frac{z_C}{z_D}\right)$$

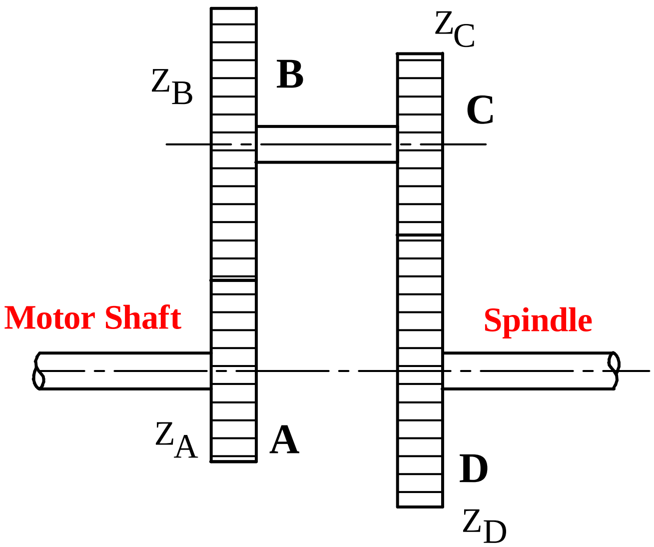
As gears B & C are on same shaft, $N_B = N_C$

$$\Rightarrow \left(\frac{N_D}{N_A}\right)=\left(\frac{z_A\times z_C}{z_B\times z_D}\right)$$

$$i.e.\ \frac{\text{Speed of last driven shaft}}{\text{Speed of the first driving shaft}}=\frac{\text{Product of no of teeth on driver}}{\text{Product of no of teeth on driven}}$$

Reverted gear train

- A reverted gear train is a compound gear train in which the first & the last gears are on the same axis.
- Hence, in a reverted gear train, the center distances for the two gear pairs must be same.
- Reverted gear trains are used in automotive transmissions, lathe back gears, and in clocks.



As $\left(\dfrac{d_A + d_B}{2}\right) = \left(\dfrac{d_C + d_D}{2}\right)$

But d=mz, and the module 'm' is same for all gears,

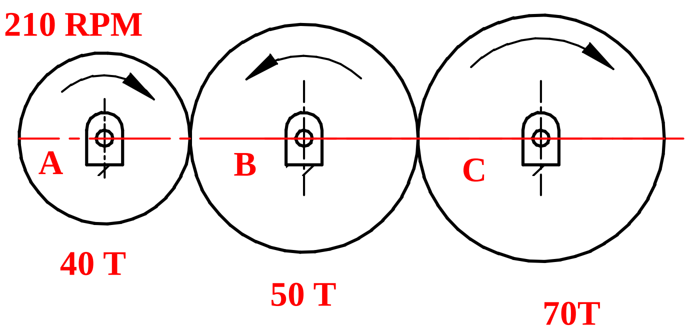
$\therefore z_A + z_B = z_C + z_D$

EPICYCLIC GEAR TRAIN

- An epicyclic gear train is one in which the axis of one or more gears moves relative to the frame.
- Large speed reductions are obtained with an epicyclic train.
- They are compact in size and automobile differential.

Problem 1

A simple train of wheels consists of successively engaging three wheels having number of teeth 40, 50 & 70 respectively. Find its velocity ratio. If the driving wheel having 40 teeth runs at 210 rpm clockwise, find the speed of the driven wheel and its direction of rotation.



From the fig, $\left(\frac{N_B}{N_A}\right)=\left(\frac{z_A}{z_B}\right)$ and also $\left(\frac{N_C}{N_B}\right)=\left(\frac{z_B}{z_C}\right)$

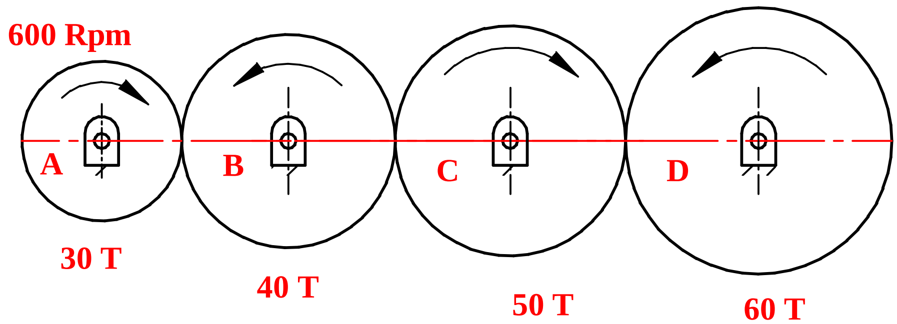
$$\therefore \left(\frac{N_B}{N_A}\right)\times\left(\frac{N_C}{N_B}\right)=\left(\frac{z_A}{z_B}\right)\times\left(\frac{z_B}{z_C}\right) \Rightarrow \left(\frac{N_C}{N_A}\right)=\left(\frac{z_A}{z_C}\right)$$

The speed of gear C, $N_C=\left(\frac{z_A}{z_C}\right)\times N_A \Rightarrow N_C=\left(\frac{40}{70}\right)\times 210 = 120\ rpm.$

As there is odd number of idler gears, the driven gear rotates at 120 rpm clockwise. (i.e. same as that of driving gear)

Problem 2

In a simple gear train consists of four wheels having number of teeth 30, 40, 50 & 60 teeth respectively. Determine the speed and the direction of rotation of the last gear if the first makes 600 rpm, clockwise.



From the fig, $\left(\frac{N_D}{N_A}\right) = \left(\frac{z_A}{z_B}\right) \times \left(\frac{z_B}{z_C}\right) \times \left(\frac{z_C}{z_D}\right)$

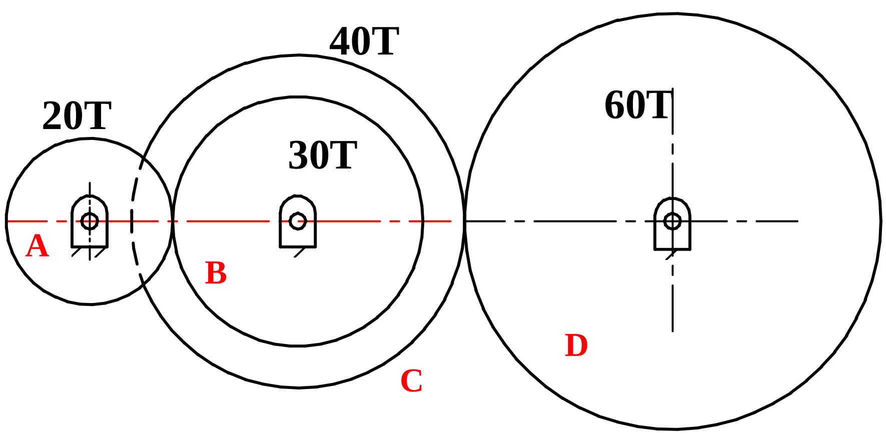
$$\Rightarrow \quad \left(\frac{N_D}{N_A}\right) = \left(\frac{z_A}{z_D}\right)$$

The speed of gear D, $N_D = \left(\frac{z_A}{z_D}\right) \times N_A \Rightarrow N_C = \left(\frac{30}{60}\right) \times 600 = 300 \text{ rpm.}$

As there is even number of idler gears, the driven gear rotates at 300 rpm counter clockwise. (i.e. opposite to that of driving gear)

Problem 3

A compound gear train consists of 4 gears, A, B, C & D and they have 20, 30, 40 & 60 teeth respectively. A is keyed to the driving shaft, and D is keyed to the driven shaft, B & C are compound gears. B meshes with A & C meshes with D. If A rotates at 180 rpm, find the rpm of D.



From the fig, $\left(\frac{N_B}{N_A}\right) = \left(\frac{z_A}{z_B}\right)$ and also

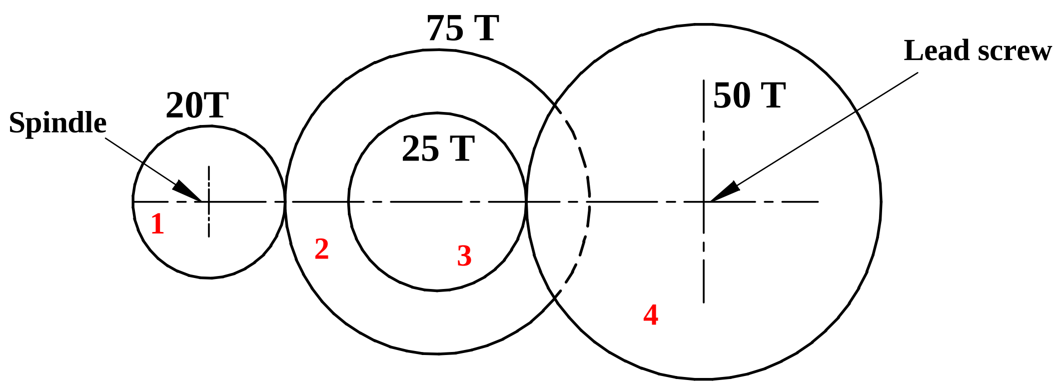
$$\left(\frac{N_D}{N_C}\right) = \left(\frac{z_C}{z_D}\right) \quad \therefore \left(\frac{N_B}{N_A}\right) \times \left(\frac{N_D}{N_C}\right) = \left(\frac{z_A}{z_B}\right) \times \left(\frac{z_C}{z_D}\right)$$

As gears B & C are on same shaft, $N_B = N_C \Rightarrow \left(\frac{N_D}{N_A}\right) = \left(\frac{z_A \times z_C}{z_B \times z_D}\right)$

$$\therefore N_D = \left(\frac{20 \times 40}{30 \times 60}\right) \times 180 = 80 \text{ RPM}$$

Problem 4

Fig shows a train of gears from the spindle of a lathe to the lead screw used for cutting a screw thread of a certain pitch. If the spindle speed is 150 rpm, what is the lead screw speed? Gears 2 & 3 form a compound gear.



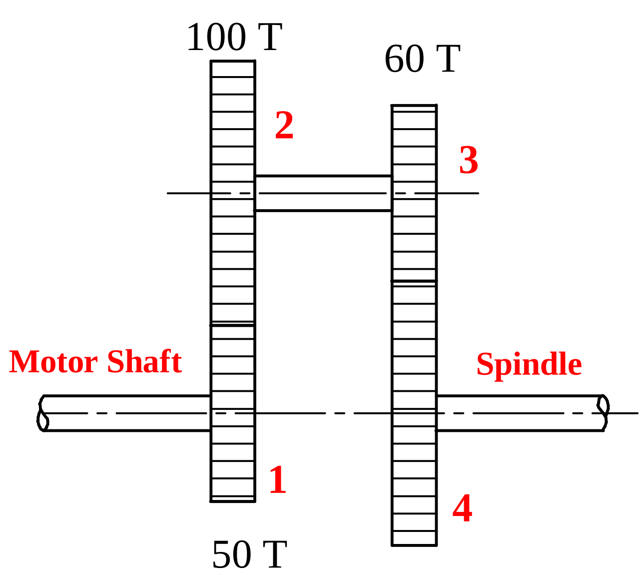
From the fig, velocity ratio

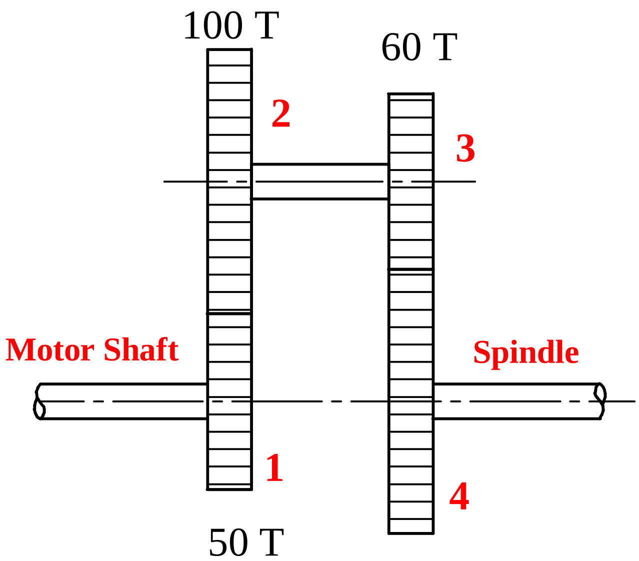
$$\left(\frac{\text{Speed of the driven shaft}}{\text{Speed of the driving shaft}}\right) = \left(\frac{\text{Product of the no of teeth on driver}}{\text{Product of the no of teeth on driven}}\right)$$

$$\Rightarrow \left(\frac{N_4}{N_1}\right) = \left(\frac{z_1 \times z_3}{z_2 \times z_4}\right) \quad \text{As } N_1 = 150\text{rpm}, N_4 = \left(\frac{20 \times 25}{75 \times 50}\right) \times 150 = 20 \text{ RPM}$$

Problem 5

Fig shows a reverted gear train used in a lathe headstock. If the motor runs at 1200 rpm, find the speed of the spindle.





As the center distance between the shafts is same,

$$\left(\frac{d_1+d_2}{2}\right)=\left(\frac{d_3+d_4}{2}\right) \Rightarrow (d_1+d_2)=(d_3+d_4)$$

The circular pitch $=\left(\frac{\pi d}{z}\right)=\pi m \Rightarrow d = mz$ where 'm' is known as module.

For two gears in mesh, circular pitch and hence the module is same.

As $z_1+z_2=z_3+z_4 \Rightarrow 50+100=60+z_4$

No of teeth on gear 4=90 teeth. $\therefore$ Speed of the spindle $N_4=\left(\frac{z_1\times z_3}{z_2\times z_4}\right)\times N_1$

$$\Rightarrow N_4=\left(\frac{50\times 60}{100\times 90}\right)\times 1200 = 400 \text{ rpm}$$