CIVIL ENGINEERING

INTRODUCTION

Civil engineering is a broad field of engineering that focuses on the design, construction, and maintenance of infrastructure and the built environment. It encompasses a wide range of disciplines and applications, including structural engineering, geotechnical engineering, transportation engineering, environmental engineering, and construction management

It is one of the oldest engineering professions, and ancient engineering achievements due to civil engineering include the pyramids of Egypt and road systems developed by the Romans.

American society of civil engineering defines civil engineering as the profession in which a knowledge of the mathematical and physical sciences gained by study, experience and practice is applied with judgement to develop ways to utilize economically the materials and forces of the nature for the progressive well being of man

Overall, civil engineering plays a crucial role in shaping the built environment and supporting economic development, social progress, and environmental sustainability. Civil engineers apply scientific principles, technological innovations, and interdisciplinary approaches to address complex challenges and improve the quality of life for communities around the world

ROLES OF CIVIL ENGINEERING

i. Plan new townships and extension of existing towns.
ii. Providing shelter to people in the form of low cost houses to high rise apartments.
iii. Laying ordinary village roads to express highways.
iv. Constructing irrigation tanks, multipurpose dams & canals for supplying water to agricultural fields.
v. Supplying safe and potable water for public & industrial uses.
vi. Protecting our environment by adopting sewage treatment & solid waste disposal techniques.
vii. Constructing hydro-electric & thermal-power plants for generating electricity.
viii. Providing other means of transportation such as railways, harbor & airports.
ix. Constructing bridges across streams, rivers and also across seas.
x. Tunneling across mountains & also under water to connect places easily & reduce distance

FEW CIVIL ENGINEERING STRUCTURES ACROSS THE WORLD


Banda Worli Sea Link


Statue of Unity


Chenab Bridge


Colosseum


Palm Island


Leaning tower of Pisa

DIFFERENT BRANCHES OF CIVIL ENGINEERING

1. Surveying

Surveying is an art and science of measuring the object on , above and below the earth. It's a fundamental aspect of civil engineering, construction, land development, and many other fields.

Here's an overview of surveying:

i. **Land Surveying**: Land surveying involves measuring and mapping the Earth's surface, including natural and man-made features such as mountains, rivers, roads, and property boundaries. Land surveyors use various instruments such as total stations, GPS receivers, and drones to collect accurate measurements of distances, angles, and elevations.

ii. **Types of Surveys**: There are different types of land surveys, including boundary surveys, topographic surveys, construction surveys, and geodetic surveys. Boundary surveys determine property lines and legal boundaries, while topographic surveys map the terrain and features of a site. Construction surveys help guide the construction process by providing precise measurements and layout information.

iii. **Surveying Instruments**: Surveyors use a range of instruments and technologies to collect data accurately and efficiently. Traditional instruments include theodolites, levels, and measuring tapes, while modern techniques utilize GPS, laser scanning, and drones for remote sensing and data capture.

iv. **Data Processing and Analysis**: Once data is collected in the field, surveyors process and analyze it to create maps, plans, and digital models of the surveyed area. This may involve using specialized software for data processing, CAD (Computer-Aided Design) software for drafting, and GIS (Geographic Information Systems) for spatial analysis.

v. **Applications**: Surveying has diverse applications across various industries. In civil engineering and construction, surveys are essential for site planning, design, and layout. In land development, surveys help determine property boundaries, assess terrain, and plan infrastructure projects. Surveying is also used in environmental monitoring, resource management, archaeology, and urban planning

Surveying using Total station

Surveying using Auto level

2. Structural Engineering

Introduction

Structural engineering is primarily concerned with designing and constructing buildings and structures that are safe and capable of withstanding the elements to which they will be exposed, as well as improving the structural integrity of existing buildings. Structural engineering depends upon a detailed knowledge of applied mechanics, materials science, and applied mathematics to understand and predict how structures support and resist self-weight and imposed loads.

History

Historically, Structural Engineering, though intuitive, can be traced back in parallel with Civil Engineering. There are numerous examples of outstanding structures, such as the monumental structures including the Taj Mahal in India, and, many Roman structures, throughout this early period of non-formal engineering. It is of interest to see how this early engineering evolved to enable the construction of the modern day bridges and buildings of imposing dimensions, such as the Burj Khalifa, Dubai, and the recently opened 1915 Çanakkale Bridge in Turkey, known as Burj Bridge. This is the world's longest suspension bridge span and would perhaps have been unthinkable even a hundred years ago.

Different types of structures

Civil structural engineering includes all structural engineering related to the built environment. It includes: Bridges, Dams, Earthworks Foundations, Offshore structures ,Pipelines ,Power stations ,Railways ,Retaining structures and walls, Roads ,Tunnels, Waterways ,Reservoirs, Water and wastewater infrastructure

Key aspects of structural engineering:

1. **Design**: Structural engineers use principles of physics and mathematics to design structures that can withstand loads such as gravity, wind, snow, earthquakes, and soil pressure. They consider factors such as materials, shape, and geometry to optimize the performance and efficiency of the structure.
2. **Analysis:** Before construction begins, structural engineers perform detailed analyses to predict how a structure will behave under various conditions. This involves using computer simulations and mathematical models to assess factors such as stress, strain, and deflection.
3. **Materials:** Structural engineers work with a variety of materials including concrete, steel, wood, and composites. They select materials based on factors such as strength, stiffness, cost, and environmental impact.
4. **Construction**: Structural engineers collaborate closely with architects, contractors, and other professionals during the construction phase to ensure that the design is implemented correctly and that the structure meets all safety and quality standards.
5. **Maintenance and Inspection:** After construction is complete, structural engineers may be involved in ongoing maintenance and inspection activities to ensure that the structure remains safe and functional throughout its lifespan. This includes conducting periodic inspections, assessing structural integrity, and recommending repairs or upgrades as needed

Different types of structures.

3. Building materials and Construction

Materials which are required for construction of buildings are referred as building materials. Building materials are classified into three types

i. Natural Materials: - These are naturally occurring materials namely wood, stone, mud or clay etc.
ii. Artificial materials: - These are man-made materials namely bricks, cement, glass & steel.
iii. Composite materials: - These are combination of two or materials namely concrete, plywood

Building materials are essential components of civil engineering, encompassing a diverse range of materials utilized in construction projects worldwide. Concrete, a ubiquitous material in civil engineering, is renowned for its compressive strength, versatility, and durability. Examples include reinforced concrete, precast concrete, and high-strength concrete. Steel, prized for its high tensile strength and ductility, is widely used in structural applications such as beams, columns, and reinforcement. Wood, valued for its renewable nature and aesthetic appeal, finds applications in framing, flooring, and siding. Masonry materials, including brick, concrete block, and stone, offer durability and fire resistance, suitable for walls, facades, and structural elements. Additionally, asphalt is utilized for road construction and roofing, providing a durable and waterproof surface. Glass, prized for its transparency and aesthetic qualities, is used in windows, doors, and curtain walls. Composite materials, such as fiberglass and carbon fiber, offer high strength-to-weight ratios, corrosion resistance, and design flexibility, suitable for structural components and architectural features. The selection of building materials depends on factors such as project requirements, structural design, environmental considerations, and cost constraints.

Different building materials

4. Geotechnical Engineering

Geotechnical engineering, also known as geotechnics, is the branch of civil engineering concerned with the engineering behavior of earth materials. It uses the principles of soil mechanics and rock mechanics to solve its engineering problems

This knowledge is applied to the design of foundations, retaining walls, earth dams, clay liners, and geosynthetics for waste containment. The goals of geotechnical engineers could range from the design of foundations and temporary excavation support, through route selection for railways and highways, to the increasingly important areas of landfill disposal of wastes and groundwater contamination. As such, the geotechnical engineer is involved in field and laboratory investigations to determine the engineering properties of site soils and other geomaterials and their subsequent use in the analytical study of the problem at hand.

Recent computational and computer advances are extending our ability to predict the behaviour of soil and soil-water systems under a wide variety of conditions. In recent years, the activities of geotechnical engineers have also involved geoenvironmental engineering. Geoenvironmental engineers design strategies for the clean-up of contaminated soils and groundwater and develop management systems for contaminated sites

Why Geotechnical Engineering is Important

Geotechnical engineering is important as it helps prevent damage to other buildings and structures as a result of subsurface conditions. The calculations and tests undertaken by these engineers help ensure safety and stability for structures and can mitigate against earthquakes, slope stability shifts, ongoing earth settlement and more

Excavation of soil for engineering purpose

5. Water resource and irrigation engineering

Irrigation and water resource engineering is a branch of civil engineering that focuses on the management, distribution, and utilization of water resources for agricultural, industrial, municipal, and environmental purposes.

Here's an overview of irrigation and water resource engineering:

1. **Water Resources Management**: Irrigation and water resource engineering involves the planning, development, and management of water resources, including surface water bodies such as rivers, lakes, and reservoirs, as well as groundwater aquifers. This includes assessing water availability, conducting hydrological studies, and implementing strategies to ensure sustainable use and conservation of water resources.
2. **Irrigation Systems**: One of the primary applications of water resource engineering is irrigation, which involves the artificial application of water to agricultural crops to supplement natural rainfall and meet crop water requirements. Irrigation systems vary in complexity and scale, ranging from simple gravity-fed systems to advanced pressurized irrigation systems such as drip irrigation and sprinkler irrigation.
3. **Hydraulic Structures**: Water resource engineers design and construct various hydraulic structures to control and manage water flow, storage, and distribution. These structures include dams, reservoirs, weirs, barrages, canals, channels, and drainage systems. Hydraulic structures are essential for regulating water supply, flood control, sedimentation management, and navigation.
4. **Water Supply Systems**: In addition to irrigation, water resource engineering encompasses the design and management of water supply systems for domestic, industrial, and municipal use. This includes sourcing water from natural surface and

groundwater sources, treatment processes to ensure water quality and safety, and distribution networks to deliver water to end-users.

5. **Flood Control and Management**: Water resource engineers play a crucial role in mitigating the impacts of floods by designing and implementing flood control measures such as levees, floodwalls, embankments, and drainage systems. They also conduct flood risk assessments, develop floodplain management plans, and provide early warning systems to minimize flood damage and protect lives and property.

Storage of water in dam and supply water to crops

6. Transportation Engineering

Transportation engineering is a branch of civil engineering that focuses on the planning, design, construction, operation, and maintenance of transportation infrastructure and systems. It encompasses a wide range of modes of transportation, including roads, highways, railways, airports, ports, and public transit systems. Here's an overview of transportation engineering:.

1. **Public Transit Systems**: Transportation engineers are involved in the planning, design, and operation of public transit systems such as buses, trains, subways, and light rail systems. They work to optimize routes, schedules, and service levels to provide efficient, affordable, and accessible transportation options for urban and suburban communities.
2. **Highway Engineering**: Highway engineering focuses on the design, construction, and maintenance of highways, expressways, and freeways. Transportation engineers design roadways and pavements to accommodate various traffic volumes and loads while ensuring safety and durability. This includes pavement design, materials selection, geometric design, and roadside safety features.
3. **Railway Engineering**: Railway engineering involves the planning, design, construction, and maintenance of railway infrastructure, including tracks, stations, signals, and rolling stock. Transportation engineers work on projects ranging from high-speed rail systems to urban commuter rail networks, ensuring efficient and safe rail transportation.

4. **Airport Engineering**: Airport engineering encompasses the planning, design, construction, and operation of airports and aviation facilities. Transportation engineers design airport runways, taxiways, aprons, terminals, and other facilities to accommodate aircraft operations while ensuring safety, efficiency, and compliance with regulatory requirements.

Different modes of transportation

7. Environmental engineering

Environmental engineering, the development of processes and infrastructure for the supply of water, the disposal of waste, and the control of pollution of all kinds. These endeavours protect public health by preventing disease transmission, and they preserve the quality of the environment by averting the contamination and degradation of air, water, and land resources.

Environmental engineering is a field of broad scope that draws on such disciplines as chemistry, ecology, geology, hydraulics, hydrology, microbiology, economics , and mathematics. It was traditionally a specialized field within civil engineering and was called sanitary engineering until the mid-1960s, when the more accurate name *environmental engineering* was adopted.

Projects in environmental engineering involve the treatment and distribution of drinking water ; the collection, treatment, and disposal of wastewater; the control of air pollution and noise pollution; municipal solid-waste management and hazardous-waste management; the cleanup of hazardous-waste sites; and the preparation of environmental assessments, audits, and impact studies. Mathematical modeling and computer analysis are widely used to evaluate and design the systems required for such tasks. Chemical and mechanical engineers may also be involved in the process. Environmental engineering functions include applied research and teaching; project planning and management; the design, construction, and operation of facilities; the sale and marketing of environmental-control equipment; and the enforcement of environmental standards and regulation

8. Town Planning and Architecture

Town planning is the process of managing land resources. It involves the control of existing and new developments, as well as strategy preparation to ensure manage future requirements. It is a dynamic process that changes in response to policy, development proposals and local needs.

Town planners must try and balance the demands of landowners and developers, with the needs and concerns of the community and the policy framework. If planning is successful, it can provide protection for the environment, can promote and faciltiate regeneration, can help create and sustain communities, and can create new and exciting places.

9. Earthquake Engineering:

Earthquake engineering is an interdisciplinary branch of engineering that designs and analyses structures, such as buildings and bridges, with earthquakes in mind. Its overall goal is to make such structures more resistant to earthquakes

EFFECT OF INFRASTRUCTURE FACILITIES ON SOCIO – ECONOMIC DEVELOPMENT OF COUNTRY

Infrastructure is supposed to facilitate and increase economic growth by providing better connectivity and enhancing productivity and efficiency.

The infrastructure and its facilities play a vital role in the socio-Economic development of a country and are mentioned as follows:

i. Increase in food production
ii. Protection from famine
iii. Safe domestic and industrial water supply
iv. Improvement in communication and transportation
v. Generation of hydro-electric power ‘
vi. Improvement in per-capita income and standard of living
vii. Increase in prosperity of people

UNIT I

INTRODUCTION TO ENGINEERING MECHANICS

1.1 Introduction

Mechanics is a science that describes and predicts the conditions of rest or motion of bodies under the action of forces. Mechanics can be broadly classified as mechanics of solids and mechanics of fluids. Further divisions of mechanics are shown in Fig. 1.1

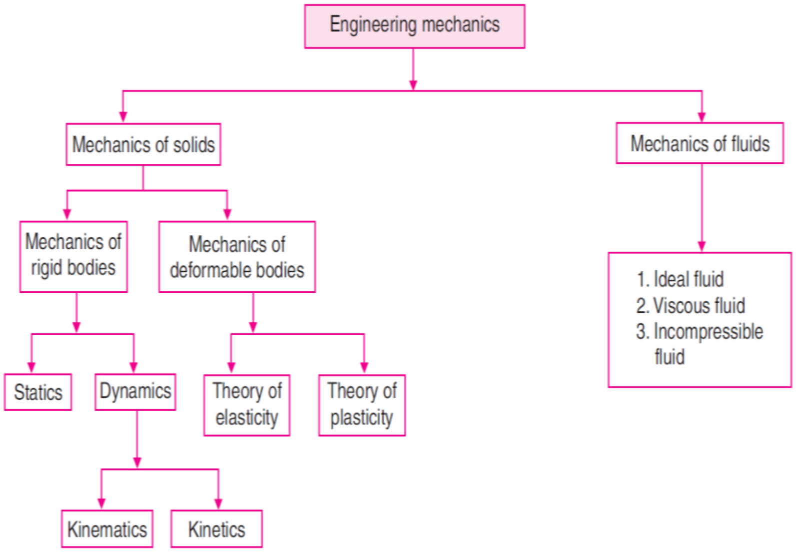


Fig. 1.1 Branches of Mechanics

Engineering mechanics deals with mechanics of rigid bodies which is further classified as statics and dynamics. **Statics** treats the equilibrium of stationary bodies under the influence of various kinds of forces. **Dynamics**, on the other hand, includes the motion of bodies and forces that causes it. Study of motion of bodies without considering force causing the motion is called **Kinematics**. On the other hand study of motion of bodies considering the force causing the motion is called **Kinetics**.

If the internal stresses developed in a body are to be studied, the deformation of the body should be considered. This field of mechanics is called **Mechanics of Deformable Bodies/Strength of Materials/Solid Mechanics**. This field may be further divided into **Theory of Elasticity** and **Theory of Plasticity.**

Liquids and gases deform continuously with application of very small shear forces. Such materials are called **Fluids.** The mechanics dealing with behaviour of fluids is called **Fluid Mechanics.**

Mechanics of ideal fluids, mechanics of viscous fluid and mechanics of incompressible fluids are further classification in this area.

1.2 Idealization in Mechanics

A number of ideal conditions are assumed to exist while applying the principles of mechanics to practical problems. In fact without such assumptions it is not possible to arrive at practical solutions. The following idealizations are usually made in engineering mechanics.

a) The body is rigid.

b) The body can be treated as continuum.

c) If the size of the body is small compared to other distances involved in the problem, it may be treated as a particle.

d) If the area over which force is acting on a body is small compared to the size of the body, it may be treated as a point force.

Continuum

A body consists of several matters. It is a well known fact that each particle can be subdivided into molecules, atoms and electrons. It is not possible to solve any engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter. In other words, the body is treated as **continuum.**

Rigid Body

A body is said to be rigid, if the relative positions of any two particles in it do not change under the action of the forces.

Particle

A particle may be defined as an object which has only mass and no size. Such a body cannot exist theoretically. However in dealing with problems involving distances considerably larger compared to the size of the body, the body may be treated as particle, without sacrificing accuracy.

1.3 Basic Principles of Mechanics

The following are the fundamental laws of mechanics:

i. Newton's first law
ii. Newton's second law
iii. Newton's third law
iv. Newton's law of gravitation
v. Law of transmissibility of forces, and
vi. Parallelogram law of forces

1.3.1 Newton's First Law

It states that everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by external agency acting on it. This leads to the definition of force as the external agency which changes or tends to change the state of rest or uniform linear motion of the body.

1.3.2 Newton's Second Law

It states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it. Thus Second law gives the magnitude of force. ie., F= m . a where, F= resultant force, m= mass of the particle, a = acceleration of the particle.

1.3.3 Newton's Third Law

It states that for every action there is an equal and opposite reaction. Consider the two bodies in contact with each other. Let one body applies a force F on another. According to this law, the second body develops a reactive force R which is equal in magnitude to force F and acts in the line same as F but in the opposite direction.

1.3.4 Newton's Law of Gravitation

The force of attraction between any two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them.

$$F = G\,\frac{m_1 m_2}{d^2}$$

Where 'G' is the constant of proportionality and is known as constant of gravitation.

1.3.5 Law of Transmissibility of Force

According to this law the state of rest or motion of the rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force (Fig. 1.2) where, P= Force

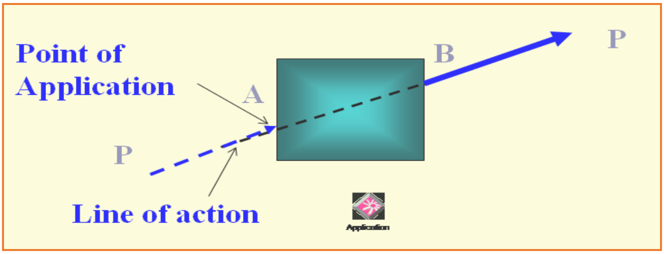


Fig.1.2 Transmissibility of Forces

Principle of Physical Independence of Forces

It states that the action of forces on bodies are independent, in other words the action of forces on a body is not influenced by the action of any other force on the body.

Principle of Superposition of Forces

It states that net effect of a system of forces on a body is same as the combined effect of individual forces acting on the body (Fig. 1.3).

Where, P_1 and P_2 = Forces; a_1 and a_2 = Acceleration due to force

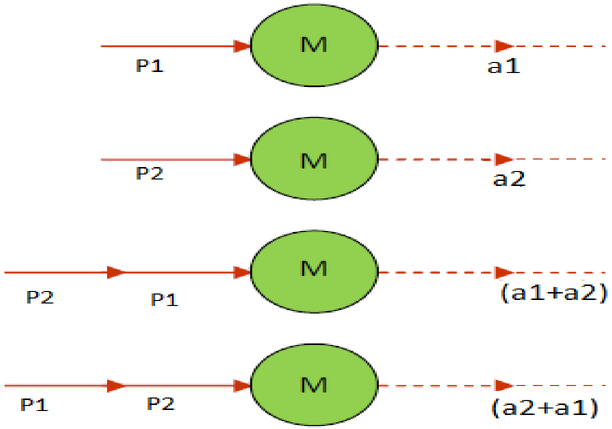


Fig.1.3 Principle of Superposition

1.3.5 Parallelogram Law of Forces

This law states that "if two forces acting simultaneously on a body at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces".

1.4 Concepts of force

Force: Force is the action exerted by one body upon another. Force tends to change the state of rest or motion of a body to which it is applied. Gravitational force, Push or Pull are the examples of force.

A force is characterized by its points of application, magnitude and direction. A force is represented by vector. Mass is the property of the body which measures the resistance of the body to a change of motion.

1.5 Characteristics of a force

From Newton's first law, we defined the force as the agency which tries to change state of rest or uniform motion of the body. From Newton's second law, we arrived at practical definition of force. Thus 1 Newton is the force required to produce an acceleration of 1 m/sec^2 in a body of 1 kg mass. It may be noted that a force is completely defined only when the following four characteristics are specified

i. Magnitude
ii. Point of application
iii. Line of action, and
iv. Direction

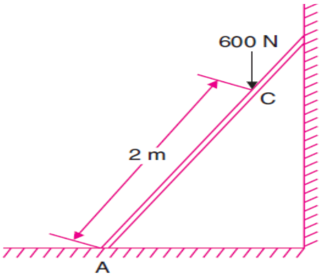


Fig. 1.4 Characteristics of a force

In Fig. 1.4, AB is a ladder leaning against a wall. At point ‘C’, a person weighing 600 N is standing. The force applied by the person on the ladder has the following characters:

i. Magnitude is 600 N
ii. The point of application is at C which is 2 m from ‘A’
iii. The line of action is vertical, and
iv. The direction is downward.

1.6 System of Forces

When number of forces act simultaneously on a body, they are called the system of forces.

Depending upon the orientation of the forces acting on a body, the system of forces is classified as

i) Coplanar force system
ii) Non-coplanar force system.

Coplanar force system consists of a set of forces with their lines of action lying in the same plane. **Non-coplanar force system** consists of a set of forces whose lines of action do not lie in the same plane. Further classifications are indicated in Fig. 1.5 and Fig. 1.6 (a) and Fig. 1.6(b). The types of force system with their characteristics and typical example is listed in Table 1.1(a), Table 1.1 (b) and Table 1.1 (c).

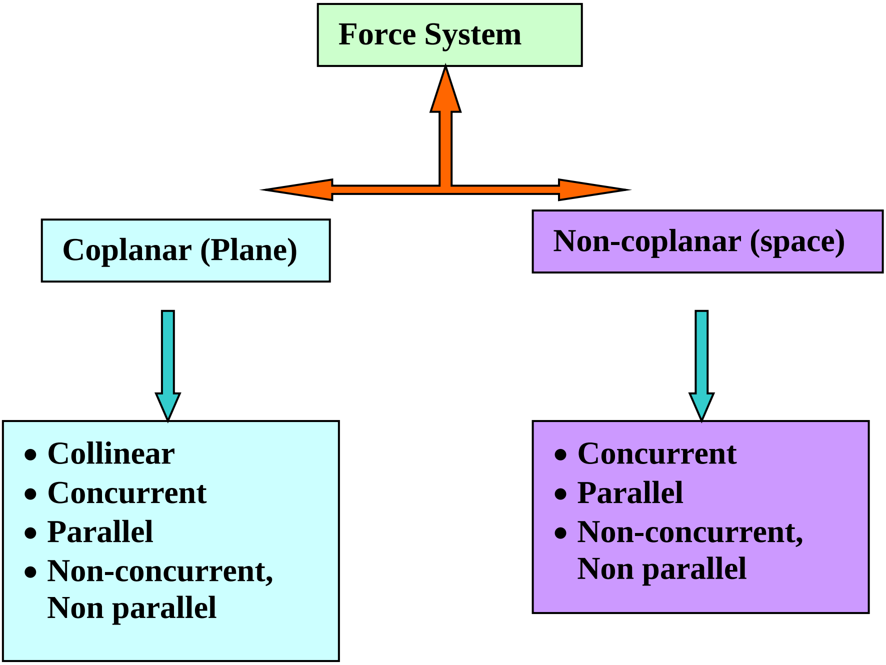


Fig.1.5 Classification of Forces

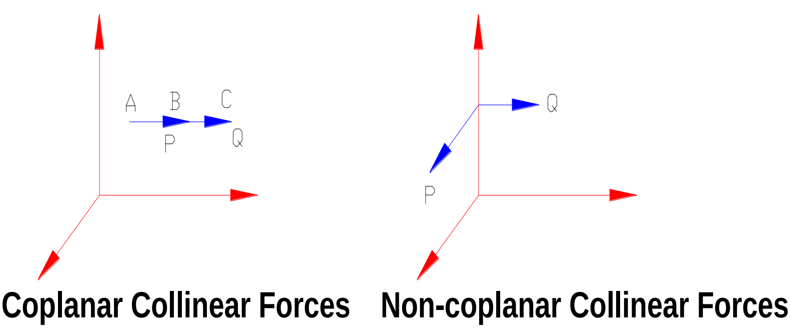
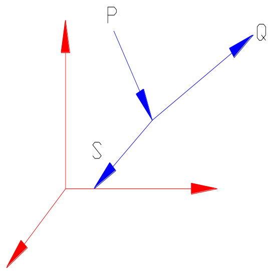
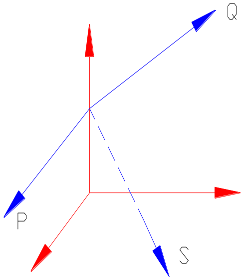


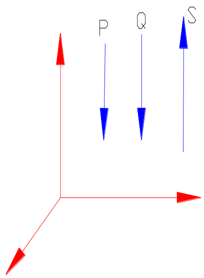
Fig. 1.6 (a) Force Systems



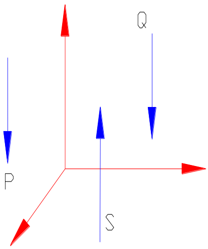
Coplanar Concurrent Forces



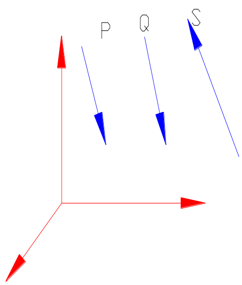
Non-coplanar Concurrent Forces



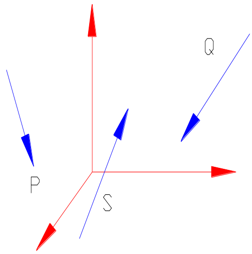
Coplanar Parallel Forces



Non-coplanar Parallel Forces



Coplanar Non-Parallel Forces



Non-coplanar Non-Parallel Forces

Fig. 1.6 (b) Force Systems

Table 1.1(a) Types of force systems

Force System	Characteristics	Examples	Illustration
Collinear forces	Line of action of all the forces act along the same line	Forces on a rope in a tug of war	
Coplanar Parallel forces	All forces are parallel to each other and lie in a single plane	System of forces acting on a beam subjected to vertical loads	
Coplanar like Parallel forces	All forces are parallel to each other and lie in a single plane and act in the same direction	Weight of a stationary train on a rail when the track is straight	

Table 1.1(b) Types of force systems

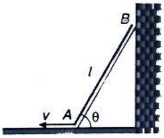
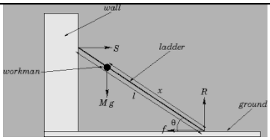
Coplanar concurrent forces	Line of action of all forces pass through a single point and forces lie in the same plane	Forces on a rod resting against a wall at point of contact	
Coplanar non concurrent forces	All forces do not meet at a point but lie in a single plane	Forces on a ladder resting against a wall when a person stands on a rung which is not at its centre of gravity	

Table 1.1(c) Types of force systems

Non coplanar parallel forces	All the forces are parallel to each other but not in the same plane	The weight of benches in a class room	
Non coplanar concurrent forces	All forces do not lie in the same plane ,but their lines of action pass through a single point	A tripod carrying a camera	
Non coplanar non concurrent forces	All forces do not lie in the same plane and their lines of action do not pass through a single point	Forces acting on a moving bus	

1.7 Moment of a Force

A force applied on a body can create i) a tendency to translate and /or ii) tendency to rotate about an axis or a point. The effect of a force on a body to rotate it about a fixed point is called **Moment**. Moment is measured as the product of force and distance between point of rotation and line of action of force. The point of rotation is called as **Moment Centre** and distance between point of rotation and line of action measured perpendicular to the force is called as **Arm of force** or **Lever Arm**.

For example the force applied by a wrench to a nut can rotate the nut (Fig. 1.7). Similarly the force applied at a door (to open or close) allows it to rotate on its hinge. (Fig. 1.7).

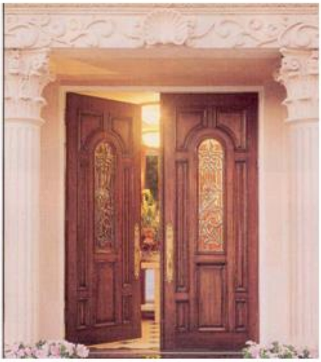


Fig 1.7 Moment of a Force

1.7.1 Moment about a point

The magnitude of moment or tendency of the force to rotate the body about the axis O-O (Fig. 2.8) perpendicular to the plane of the body is proportional to both the magnitude of the force and to the moment arm d, therefore magnitude of the moment is defined as the product of force and moment arm.

Moment = Force x moment arm

M = F×d

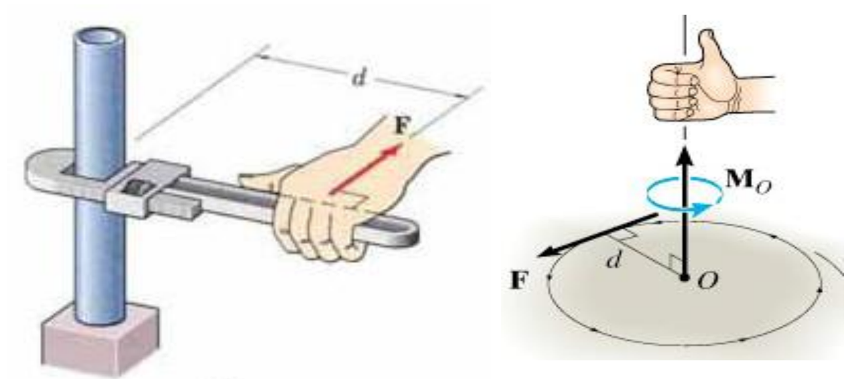


Fig. 1.8 Moment about a point

The moments are classified according to the direction in which the force tends to rotate the body about a fixed point

i. Clockwise and
ii. Anticlockwise moment

Clockwise Moment

When the force tends to rotate the body in the same direction in which the hands of clock move is called clockwise moment.

Anticlockwise Moment

When the force tends to rotate the body in the opposite direction in which the hands of clock move is called anti clockwise moment.

Example 1.1

Find the moment of force F = 600N about A as shown in the Fig. 1.9

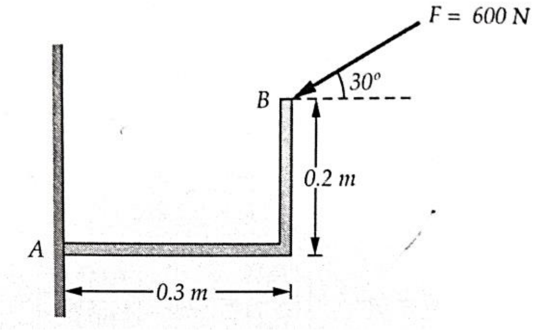


Fig. 1.9

Solution:

Moment of the force can be obtained by resolving the given force as shown in Fig. 1.10

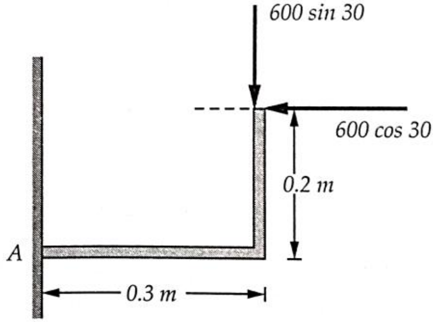


Fig. 1.10

$M_A = (600 \text{ Cos } 30^o)(0.2) - (600 \text{ Sin } 30^o)(0.3)$

M_A = 13.92 N-m (Anticlockwise Moment)

Example 1.2

A 200 N force acts on the bracket as shown in Fig. 1.11 Determine the moment of the force about point A.

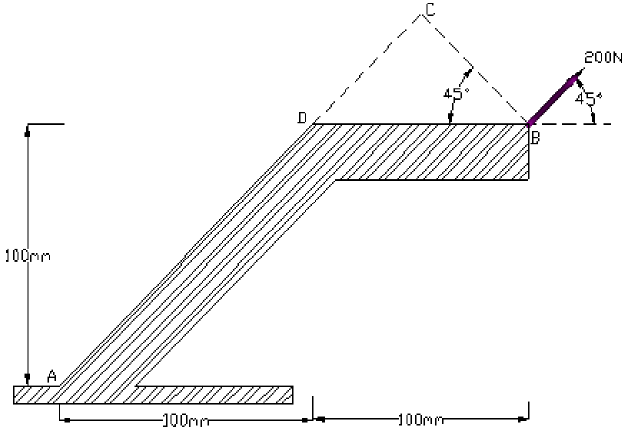


Fig. 1.11

Solution:

Moment about A = M_A= Force x perpendicular distance

$$=200 \text{ x CB} (\quad)$$

From triangle BCD CB= DB x cos 45° = 70.71 mm

M_A= 200 x 70.71 = 14142 N-mm = 14.142 N-m

Alternatively:

The force at B is resolved into x and y components and the moment about A is computed using Varignon’s theorem

+ve M_A=200 cos 45° x 100 – 200 sin 45° x 200

= -14.142 N-m ( )

1.8 Couple

Two parallel forces of same magnitude but opposite direction separated by a distance‘d’ as shown in Fig 1.12 form couple.

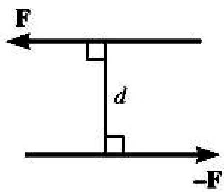


Fig. 1.12 Couple of Force

As matter of fact a couple does not produce any straight-line motion but produces rotation in the body on which it acts. So couple can be defined as unlike parallel forces of same magnitude but opposite direction which produce rotation about a specific direction and whose resultant is zero.

1.8.1 Moment of couple

The moment of the couple is the product of the force (one of the force of the two equal and opposite parallel forces) and the arm of the couple.

Magnitude of the couple is equal to the product of the force and the distance between the parallel forces. M= F x d.

Moment of couple = force x arm of couple

Moment of couple = F x d

Moment of a couple about any point is same.

Let us find the moment of couple about a point 'O' on the couple arm AB as shown in Fig. 1.13

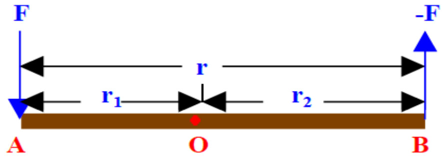


Fig. 1.13 Moment of couple

Moment of Couple about 'O'

$= F \times r_1 + F \times r_2 = F(r_1 + r_2) = F \times r(Anticlockwise)$.

Moment of Couple about 'A'= $F \times r(Anticlockwise)$.

Moment of Couple about 'B'=. $F \times r(Anticlockwise)$

1.11 Composition of Forces

It is the process of combining a number of forces into a single force such that the net effect produced by the single force is equal to the algebraic sum of the effects produced by the individual forces. The single force in this case is called the resultant force which produces the same effect on the body as that produced by the individual forces acting together.

Sign conventions:

Following sign conventions are used in determining the resultant force

1. Horizontal forces:

i. Left to Right + ve

i. Right to Left - ve

2. Vertical forces:

i. Upward + ve

ii. Downward – ve

1.12 Resultant of coplanar concurrent force system

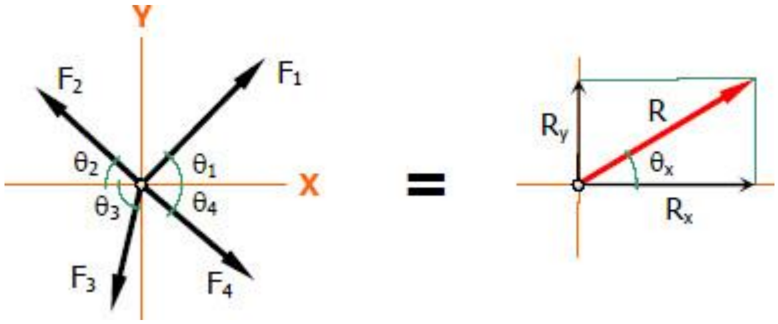
The line of action of each forces in coplanar concurrent force system are on the same plane. All of these forces meet at a common point, thus concurrent. In x-y plane, the resultant can be found by the following formulas.

Resultant of a force system is a force or a couple that will have the same effect to the body, both in translation and rotation, if all the forces are removed and replaced by the resultant.

The equation involving the resultant of force system are the following

1. $R_x = \Sigma F_x = F_{x1} + F_{x2} + F_{x3} +$
   The x-component of the resultant is equal to the summation of forces in the x-direction.

2. $R_y = \Sigma F_y = F_{y1} + F_{y2} + F_{y3} + \ldots$
   The y-component of the resultant is equal to the summation of forces in the y-direction.



$R_x = \Sigma F_x$

$R_y = \Sigma F_y$

$R = \sqrt{\Sigma Fx^2 + \Sigma Fy^2}$

$\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x}$

~~But the Area of ΔACD = Area of ΔABD = Area of ΔAOB~~

~~Now Area of triangle AOD = Area of triangle AOC + Area of~~ triangle AOB

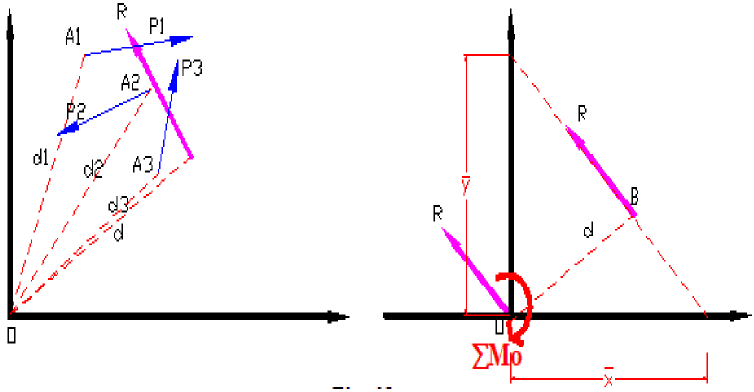
~~Moment of force R about O = Moment of force P about O + Moment of force Q about~~ O

1.14 Resultant of coplanar non-concurrent force system

If two or more forces are acting in single plane, but not passing through the single point, such a force system is known as coplanar non-concurrent force system.

Consider a system of forces P_1, P_2, P_3, … etc., acting on a rigid body at the points A_1, A_2, A_3, etc., located at a distance d_1, d_2, d_3 …. etc., as shown in Fig.1.29 (a)

Each force is replaced by a force of same magnitude, direction and a couple perpendicular to the force at 'O'. Forces P_1, P_2, P_3…etc acting at A1, A2, A3 ….etc., respectively are replaced by respective force and corresponding couple as shown in Fig. 1.29 (b) At 'O' the forces are replaced by the resultant R and net couple ΣM_o. Further this couple ΣM_o and resultant R at O can be reduced to a single force R at the point B such that $\Sigma M_o = R \times d$.



(a) **(b)**

Fig. 1.29

Therefore, $d = \dfrac{\sum M_o}{R}$, where, $R = \sqrt{R_x^2 + R_y^2}$, $R_x = \Sigma F_x$ and $R_y = \Sigma F_y$. The x and y intercept of the resultant on x and y axes is computed using theorem of moments as $\bar{x} = \dfrac{M_o}{R_y}$ and $\bar{y} = \dfrac{M_o}{R_x}$.

In coplanar non-concurrent force system, we can calculate the magnitude, direction and position of the resultant force as follows:

i) Magnitude of resultant using the formula

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

ii) Direction of the resultant using the formula

$$\theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$$

iii) Position of resultant

Position of the resultant means the calculation of d, or x and y intercepts as shown in the figure

R x d = algebraic sum of moments of number of forces about that point.

R x d = ΣM_o

$$d = \frac{\Sigma M}{R}$$

X- intercept:

$$x = \left|\frac{\Sigma M}{\Sigma F_y}\right|$$

Y- intercept:

$$y = \left|\frac{\Sigma M}{\Sigma F_x}\right|$$

Let x = Perpendicular distance between B and the line of action of the resultant force.

Now taking moments of the resultant force about B and equating the same,

$1.732\,P \times x = 3P \times 100 \sin 60° = 3P \times (100 \times 0.866) = 259.8\,P$

$$x = \frac{259.8}{1.732} = 150 \text{ mm}$$

1.15 Equilibrium of Forces

A body is said to be under equilibrium, under external forces, when it continues to be at rest or under motion.

1.15.1 Equilibrium of Concurrent force systems:

For a body to be under equilibrium, the resultant of force system should be zero. If a resultant exists, then the resultant of force system should be zero. If a resultant exists, then the body can be under equilibrium by applying a force of equal magnitude and opposite. This force is called as **equilibrant**. For equilibrium $R = \sqrt{\sum F_x^{\,2} + \sum F_y^{\,2}} = 0$. To satisfy this condition sum of the components along both directions taken separately should be zero, ie., $\Sigma F_x=0$ and $\Sigma F_y=0$

Equations of equilibrium for coplanar Concurrent force systems:

$\Sigma F_x=0$ and $\Sigma F_y=0$ and,

Additionally, for the case of coplanar Non concurrent force systems we have: $\Sigma M_z=0$

1.16 Lami's Theorem:

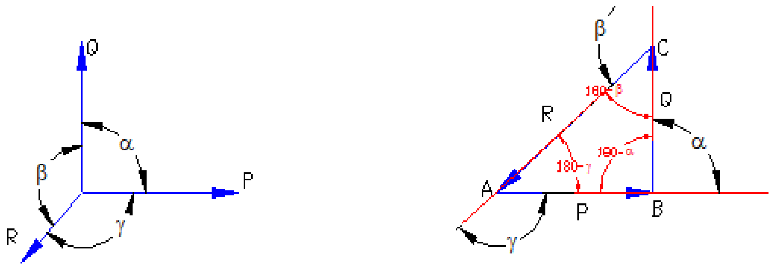


Fig 1.40

Statement: If a body is in equilibrium under action of three forces, each force is proportional to the sine of the angle between the other two forces. Referring to Fig 1.40

P $\propto$ sin β; Q $\propto$ sin γ; R $\propto$ sin α; or $\dfrac{P}{\sin\beta} = \dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin\alpha}$

Note: In the above equation all the three forces are considered to be of similar nature, ie., either all are acting away (Tensile) or *acting towards the point (Compressive). If any of the force is unlike, then that force is considered as negative.*

Proof: Applying sine rule to the triangle ABC in Fig. 1.40 the following expression is evaluated

$$\frac{AB}{\sin\angle BCA} = \frac{BC}{\sin\angle CAB} = \frac{CA}{\sin\angle ABC};$$

$$\frac{P}{\sin(180-\beta)} = \frac{Q}{\sin(180-\gamma)} = \frac{R}{\sin(180-\alpha)}$$

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$$

Because sin (180-θ) = sinθ

1.17 Free Body Diagram (FBD)

In practice the problem is derived from actual physical situation. A sketch showing the physical condition of the problem is known as space diagram. In order to obtain solution for this problem, the body under consideration and all the forces acting on the body is considered. Free body diagram is a sketch of isolated body which shows the external forces on the body and the reactions exerted on it by the removed elements. Some examples of writing free body diagram is shown in Fig. 1.41

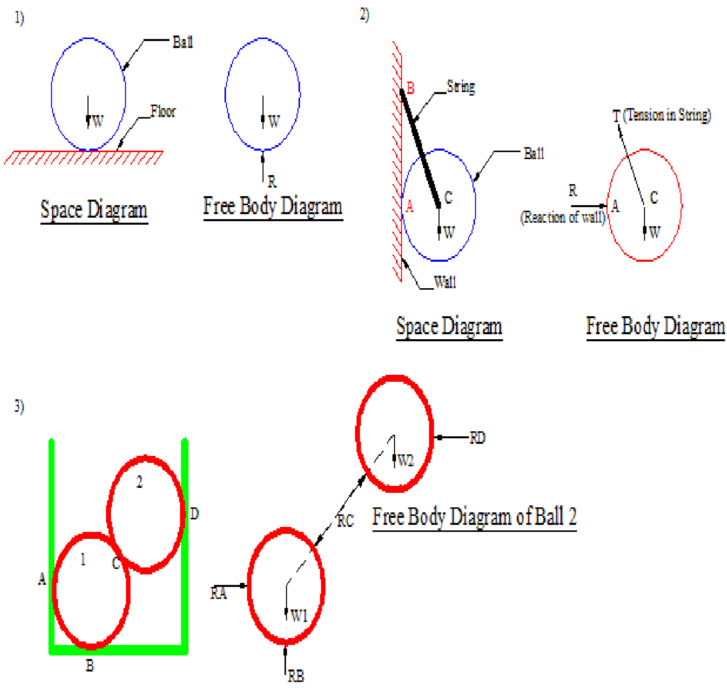


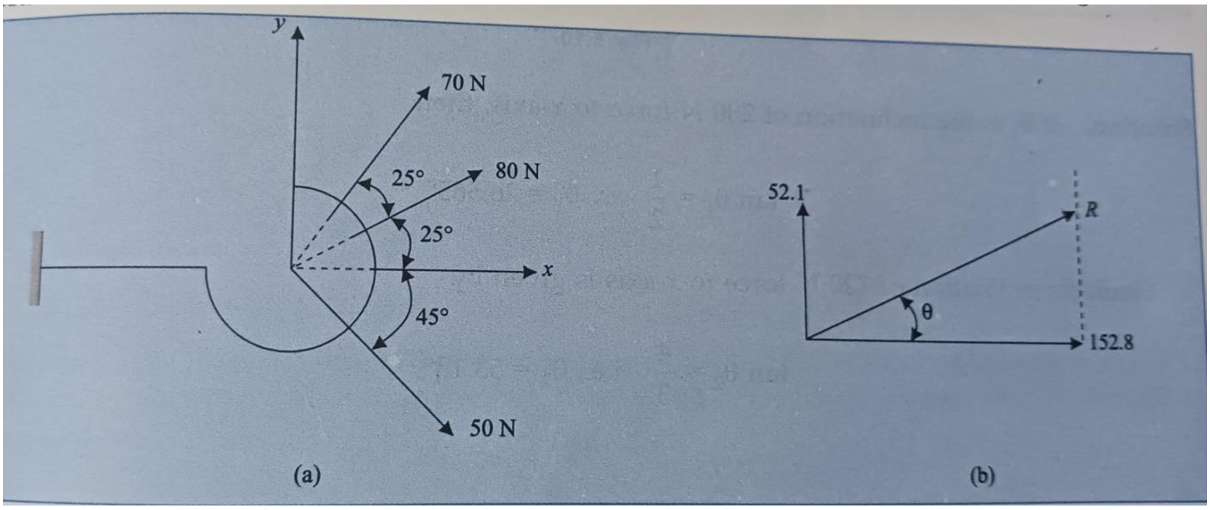
Fig. 1.41

Example 1.19

A sphere of weight 100 N is tied to a smooth wall by a string as shown in Fig. 1.42 Find the tension T in the string and reaction of the wall.

NUMERICAL ON COPLANAR CONCURRENT FORCE SYSTEM

1. Determine the resultant of the three forces acting on a hook shown in the Figure below?



Force	x- component	y- component
F1 =70 N	70 cos 50 = 45.00	70 sin 50 = 53.6 N
F2 = 80 N	80 cos 25 = 72.5 N	80 sin 25 = 33.8 N
F3 = 50 N	50 cos 45 = 35.4 N	- 50 sin 45 = -35.4 N

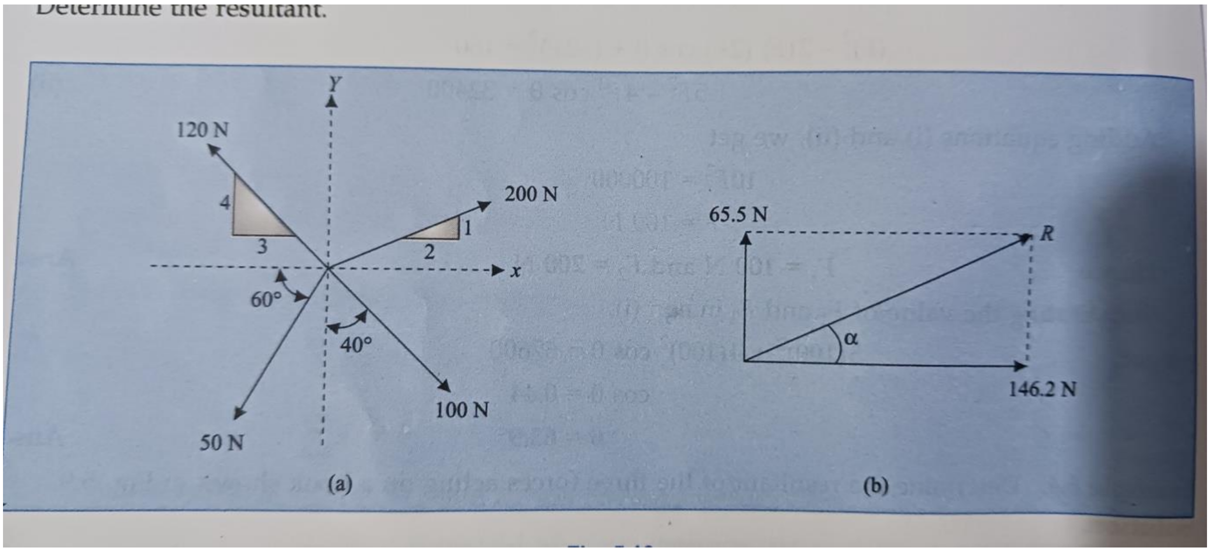
$$\Sigma Fx = 45.0 + 72,5 + 35.4 = 152.8 \text{ N}$$

$$\Sigma Fy = 53.6 + 33.8 - 35.4 = 52.1 \text{ N}$$

$$R = \sqrt{1528^2 + 52.1^2} = \mathbf{161.5\ N}$$

$$\alpha = \tan^{-1} \frac{52.1}{158.8} = \mathbf{18.83°}$$

2. A system of four forces acting at a point on a body is as shown in Figure below. Determine the resultant?



If θ1, is the inclination of 200 N force to x-axis, then

$\tan\theta_1 = 1/2 \;\therefore\; \theta_1 = 26.565°$

Similarly, inclination of 120 N force to x-axis is given by

$\tan\theta_2 = 4/3$ i.e., $\theta_2 = 53.13°$.

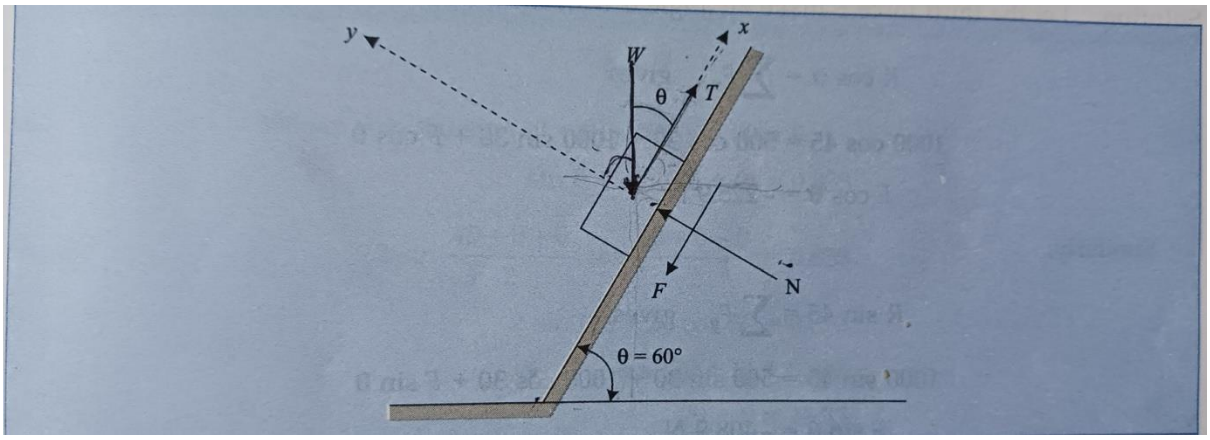
$\Sigma F_x = 200\cos 26.565 - 120\cos 53.13 - 50\cos 60 + 100\sin 40 = 146.2$ N

$\Sigma F_y = 200\sin 26.565 + 120\sin 53.13 - 50\sin 60 - 100\cos 40 = 65.5$ N

$R = \sqrt{146.27^2 + 65.52^2} = 160.2$ N

$\alpha = \tan^{-1} = \frac{65.5}{146.2} = \mathbf{24.1°}$

3. A system of forces acting on a body resting on an inclined plane is as shown in Fig below. Determine the resultant force if θ = 60°, W = 1000 N, vertically downward, N = 500 Newton acting normal to the plane, F = 100 N, acting down the plane and T = 1200 N, acting parallel to the plane.



In this this problem coordinates are taken parallel to and perpendicular to inclined plane, Since they are more convenient. Noting that W makes an angle θ with y-axis.

$$\Sigma F_x = T - F - W \sin \theta$$

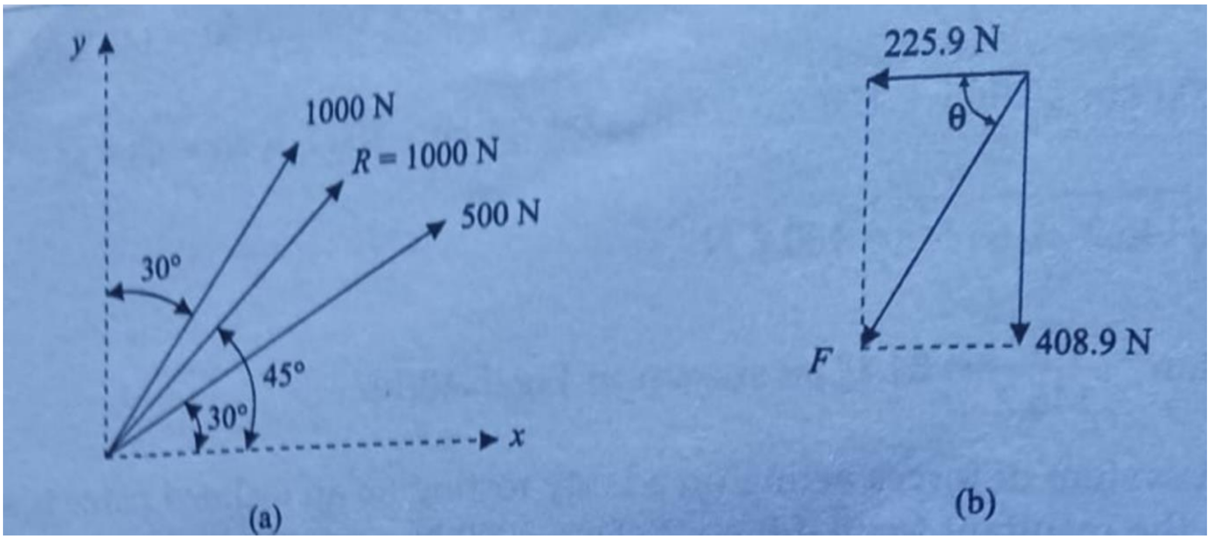
$$= 1200 - 100 - 1000 \sin 60$$

$$= 234.0 \text{ N}$$

$$\Sigma F_y = N - W\cos 60 = 500 - 1000 \cos 60 = 0$$

∴ Resultant is a force of magnitude 234 N directed up the plane.

4. Two forces acting on a body are 500 N and 1000 N as shown in Figure below .Determine the third force F such that the resultant of all the three forces is 1000 N, directed at 40° to x-axis.



Let the third force F make an angle θ with the x-axis. Then,

$$R \cos \alpha = \Sigma F x \text{ gives}$$

$$1000 \cos 45 = 500 \cos 30 + 1000 \sin 30 + F \cos \theta$$

$$F \cos \theta = -225.9 \text{ N}$$

Similarly,

$$R \sin 45 = \Sigma F y \text{ gives,}$$

$$1000 \sin 45 = 500 \sin 30 + 1000 \cos 30 + F \sin \theta$$

$$F \sin \theta = -408.9 \text{ N}$$

$$F = \sqrt{225.92 + 408.92} = \mathbf{467.2\ N}$$

and

$$\tan \theta = \frac{-408.9}{-225.9} = 1.810$$

$$\boldsymbol{\theta = 61.08^\circ} \textbf{ as shown in Figure}$$

5. A truck is pulled along a straight rodd with the help of two ropes A and B shown in figure below the resultant of two ropes is given by 12KN and angle b/w A and B is 60°. Find the force in rope A if rope B exerts a force of 5KN and also find resultant Inclination.

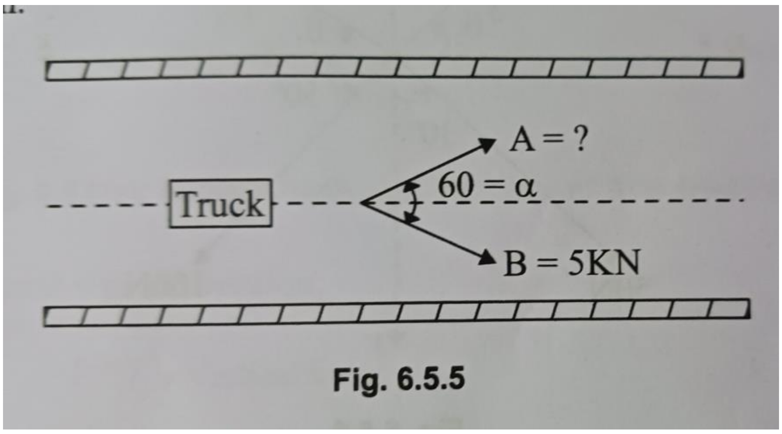


Fig. 6.5.5

By Parallelogram law.

$$R^2 = P^2 + Q^2 + 2PQ \cos \alpha$$

$$(12)^2 = A^2 + (5)^2 + 2A(5) \cos 60^\circ$$

$$144 = A^2 + 25 + 10A \cos 60^\circ$$

$$A^2 + 5A - 119 = 0 \text{ on solving}$$

$$\mathbf{A = 8.69KN}$$

Resultant 12KN Inclination with A = 8.69KN is given by

$$\tan\theta = \frac{Q \sin\alpha}{P + Q\cos\alpha} = \frac{5 \sin 60°}{8.69 + 5 \cos 60°}$$

$$\theta = 21.15°$$

6. Two cables attached at the top of tower carries a gay cable AB. Determine the tension in gay cable such that the resultant of the forces in all three cables acts vertically down. Also find the resultant force.

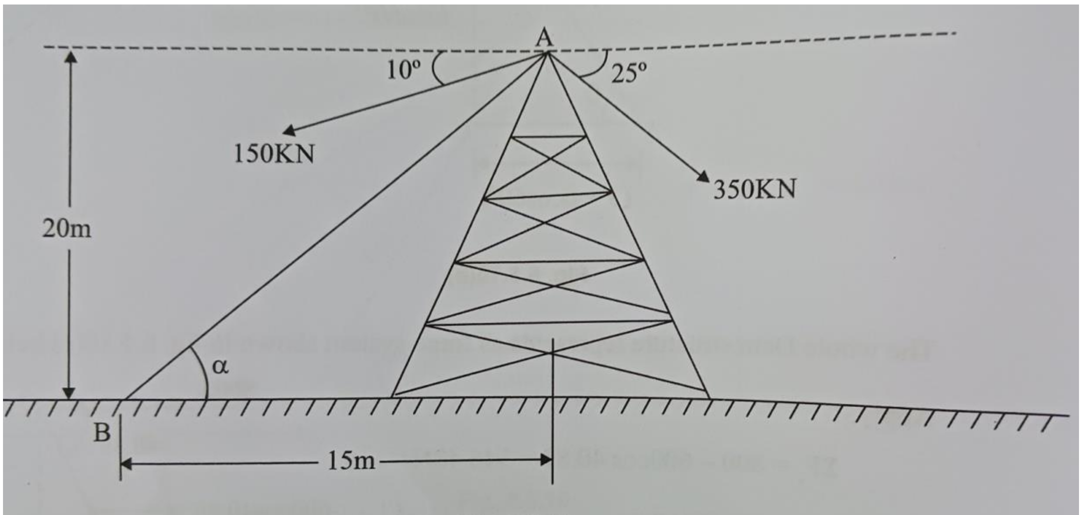


Fig. 6.5.11

Solution

Let AB be the guy cable, with that the angle of the cable force with Horizontal can be obtained by drawing a vertical line form 'A' shown in figure.

$\therefore \tan\alpha = \dfrac{20}{15}$

[$\alpha = 53.13°$]

Apply sing convention $\uparrow^{+ve}$, $\downarrow^{-ve}$

$\rightarrow^{+ve}$, $\leftarrow^{-ve}$

$\therefore$ $\Sigma Fx = 350\cos 25° - 150\cos 10° - T\cos 53.13°$ ……….(1)

$\Sigma Fy = -350\sin 25° - 150\sin 10° - T\sin 53.13°$ ……….(2)

The Resultant in three cables acts vertically down therefore

$\Sigma Fy = R$ and $\Sigma Fx = 0$

Eqn. (1) and (2) will becomes

From Eqn. (1),

$T\cos 53.13° = 169.48$

[$T = 282.47$ KN]

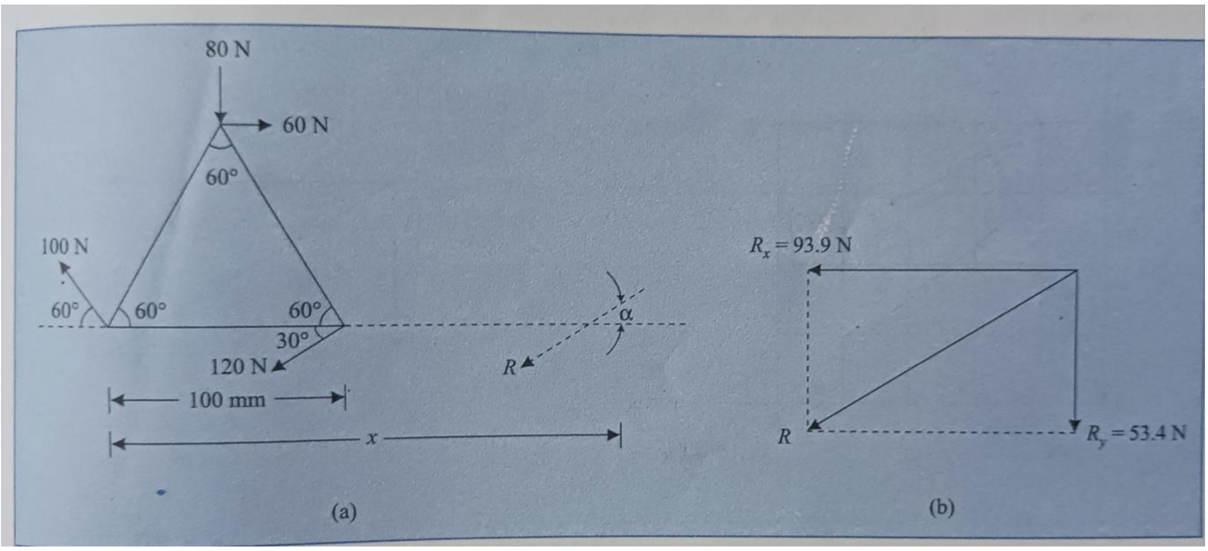
From Eqn. (2),

$R = -350\sin25° - 150\sin 10° - 282.47\sin53.13°$ [But, $T = 282.47$KN]

[$R = 400$KN]

NUMERICAL ON NON CONCURRENT FORCE SYSTEM

1. Find the resultant of the force system shown in Fig. 68(a) acting on a lamina of equilateral triangular shape.



$R_x = \Sigma Fx = 60 - 100 \cos 60 - 120 \cos 30$

$= -93.9 \text{ N} = \overleftarrow{93.9} \text{ N}$

$R_y = \Sigma Fy = -80 + 100 \sin 60 - 120 \sin 30$

$= -53.40 = 53.40 \text{ N} \downarrow$

$\therefore R = \sqrt{93.92^2 + 53.40^2}$

i.e., **R = 108.0 N**

$$\alpha = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{53.40}{93.9} = -2$$

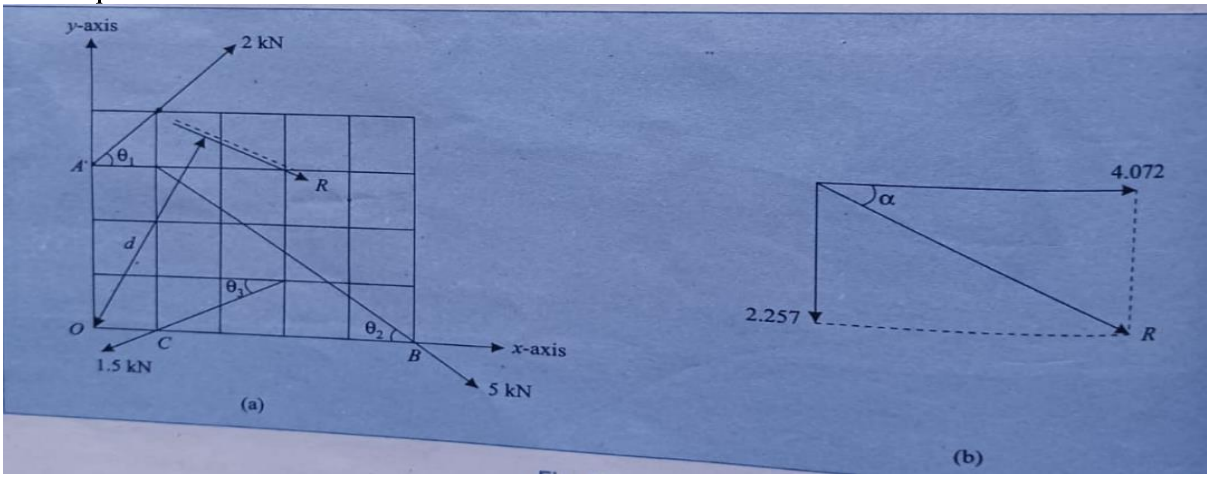
i.e., $\alpha = 29.60°$, as shown in Fig. 1(b)

Let x be the intercept on x-axis from A. Then taking moment about A,

$$x = \frac{\Sigma M_A}{R_y} = \frac{80 \times 100 \cos 60 + 60 \times 100 \sin 60 + 120 \sin 30 \times 100}{53.40}$$

i.e., x = 284.6 mm, as shown in Fig. 1(a).

2. Find the resultant of the system of coplanar forces acting on a lamina as shown in Fig. 2(a). Each square has a side of 10 mm.



if θ_1 , θ_2 & θ_3 are the inclinations of forces 2 kN, 5 kN and 1.5 kN with respect to x-axis, then

$\tan \theta_1 = \frac{10}{10} = 1$ $\quad\therefore \theta_1 = 45°$

$\tan \theta_2 = \frac{30}{40}$ $\quad\therefore \theta_2 = 36.87°$

$\tan \theta_3 = \frac{10}{20}$ $\quad\therefore \theta_3 = 26.565°$

$R_x = \Sigma Fx = 2 \cos 45 + 5 \cos 36.87 - 1.5 \cos 26.565$

$= 4.072$ kN

$R_y = \Sigma Fy = 2 \sin 45 - 5 \sin 36.87 - 1.5 \sin 26.565$

$= -2.257$ kN

$R = \sqrt{4.072^2 + (-2.257)^2}$ = **4.655 kN**

$\alpha = \tan^{-1} \frac{2.257}{4.072}$ **= 29°, as shown in Fig. 2(b)**

Resolving the forces into their x and y components at A, B and C as shown in Fig. 2(a) and then finding their moment about 'O', we get,

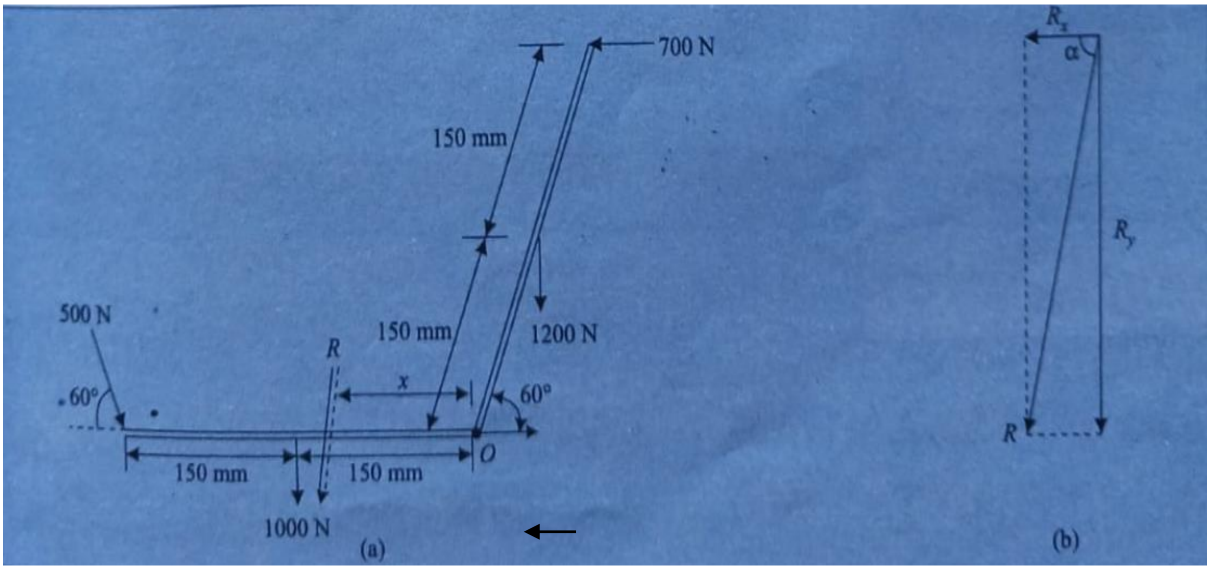
$\Sigma M_o = 2 \cos 45 \times 30 + 5 \sin 36.87 \times 50 + 1.5 \sin 26.565 \times 10$

$= 199.13$ kN-mm

∴ Distance d of the resultant from O is given by

$d = \frac{199.13}{R} = \frac{199.13}{4.655}$ = **42.8 mm as shown in Fig. 2(b).**

3. The system of forces acting on a bell crank is shown in Fig. 3(a). Determine the magnitude, direction, and the point of application of the resultant.



$R_x = \mathbf{\Sigma}Fx = 500 \cos 60 - 700 = -450 \text{ N} = 450 \text{ N}$

$R_y = \mathbf{\Sigma}Fy = -500 \sin 60 - 1000 - 1200 = -2633 \text{ N} = 2633 \text{ N} \downarrow$

$R = \sqrt{450^2 + 2633^2} = \mathbf{2671.2\ N}$

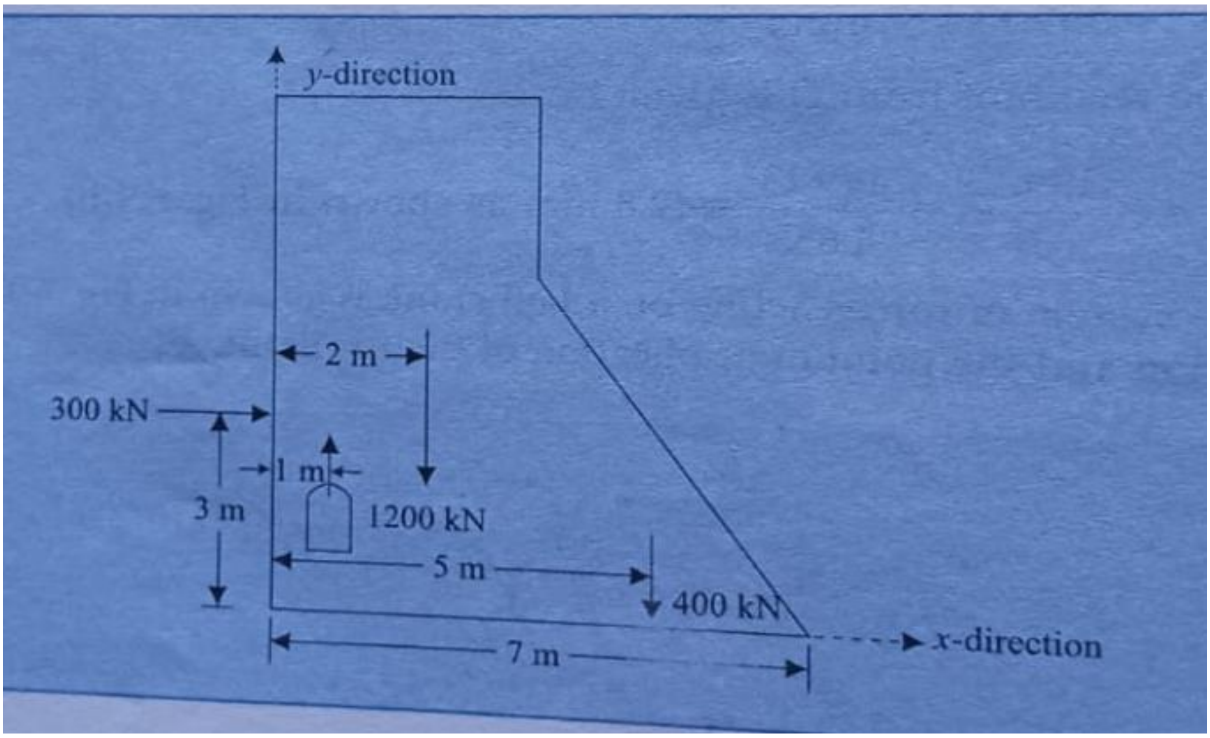
$\alpha = \tan^{-1} \dfrac{2633}{450} = 80.30°$, as shown in Fig 3(b)

Let the point of application of the resultant be at a distance x from 'O' along the horizontal arm. Then

$$x = \frac{\mathbf{\Sigma}M_o}{R_y} = \frac{-500 \sin 60 \times 300 - 1000 \times 150 + 1200 \times 150 \cos 60 - 700 \times 300 \sin 60}{2633}$$

∴ x = 141.2 mm as shown in Fig. 3(a)

4. Various forces to be considered for the stability analysis of a dam are shown in Fig.4. The dam is safe if the resultant of forces passes through middle third of the base. Verify whether the dam is safe.



$R_x = \Sigma Fx = 300$ kN

$R_y = \Sigma Fy = 100 - 1200 - 400 = -1500$ kN $= 1500$ kN ↓

Let the resultant pass through the base at a distance x from 'O. Then

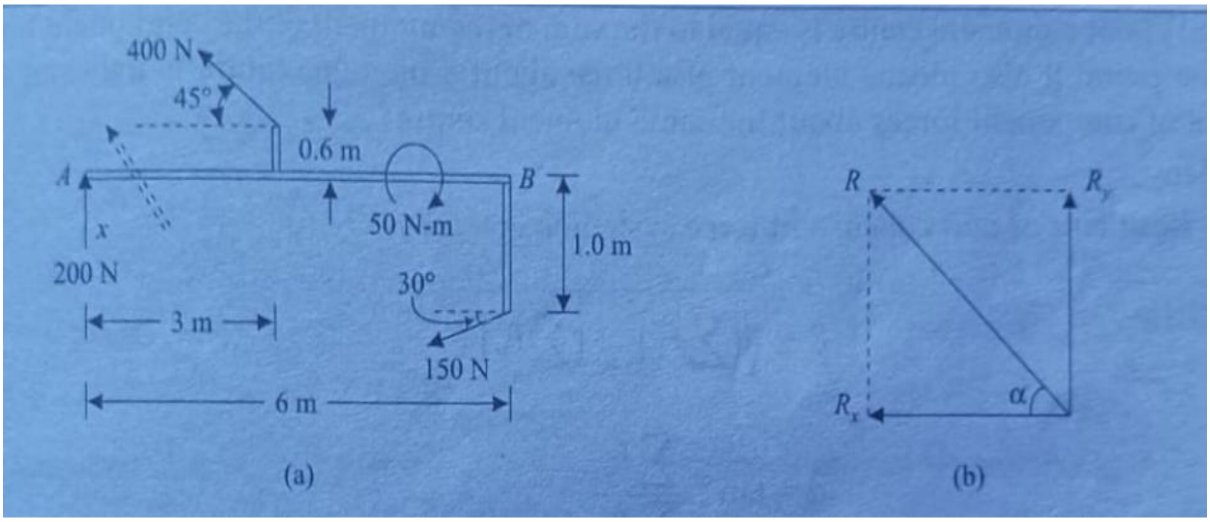
$$x = \frac{\Sigma M_o}{R_y} = \frac{300 \times 3 - 1000 \times 1 + 1200 \times 2 + 400 \times 5}{1500}$$

$= 3.467$ m.

The resultant lies in the middle third of the base (i.e.., x is between 7/3 and (2 x 7)/3).

Hence the dam is safe.

5. A bracket is subjected to three forces and a couple as shown in Fig. 5(a).Determine magnitude, direction and the line of action of the resultant.



$R_x = \Sigma Fx = -400 \cos 45 - 150 \cos 30$

$= -412.7\ N = 412.7\ \overleftarrow{N}$

$R_y = \Sigma Fy = 200 + 400 \sin 45 - 150 \sin 30$

$= 407.8\ N$

$R = \sqrt{(412.7)^2 + (407.8)^2} =$ **580.2 N**

$\alpha = \tan^{-1}(R_y / R_x) = \tan^{-1} \dfrac{407.8}{412.7}$ = **44.66°, as shown in Fig. 5(b)**

Let the resultant intersect arm AB at a distance x from A.

Now,

$\Sigma M_A = -400 \sin 45 \times 3 - 400 \cos 45 \times 0.6 + 50 + 150 \sin 30 \times 6 + 150 \cos 30 \times 1$

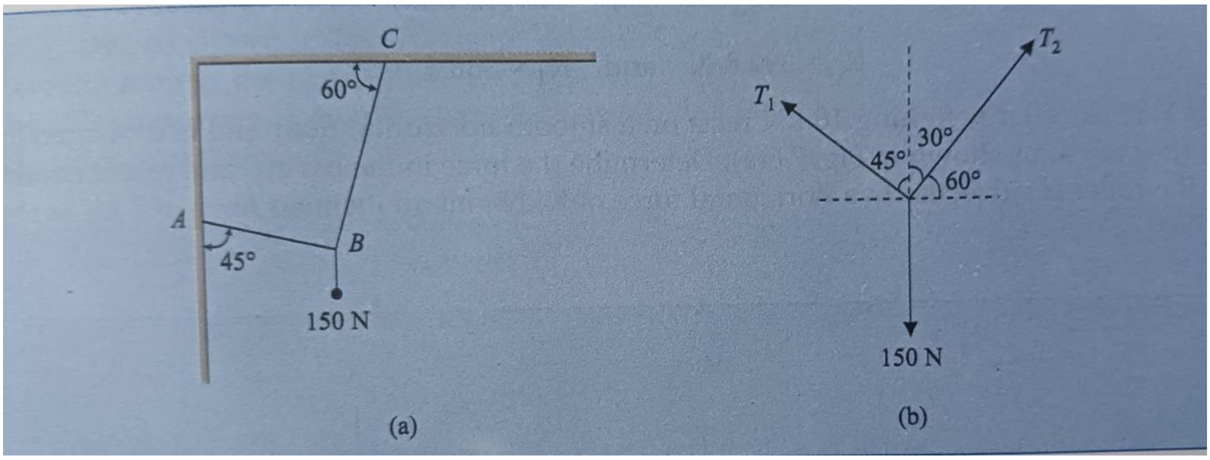
$= -438.3$ N-m

$= 438.3$ N-m, anticlockwise

$x = \dfrac{M_A}{R_y} = \dfrac{438.3}{407.8}$ = **1.074 m, as shown in Fig. 5(a)**

Lami's Theorem

1. Find the forces developed in the wires, supporting an electric fixture as shown in the Figure.



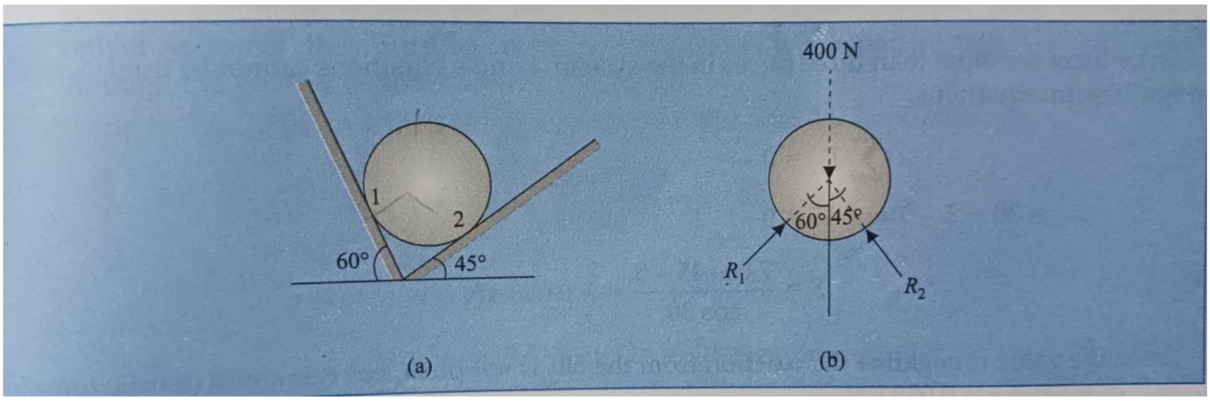
Let the forces developed in the wires BA and BC be T_1 and T_2 as shown in Figure above

Applying Lami's theorem to the system of forces, we get

$$\frac{T_1}{\sin(90+60)} = \frac{T_2}{\sin(180-45)} = \frac{150}{\sin(45+30)}$$

T_1 = 77.6 N and T_2 = 109.8 N

2. A 200 N sphere is resting in a trough as shown in Figure below. Determine the reactions developed at contact surfaces. Assume all contact surfaces are smooth.



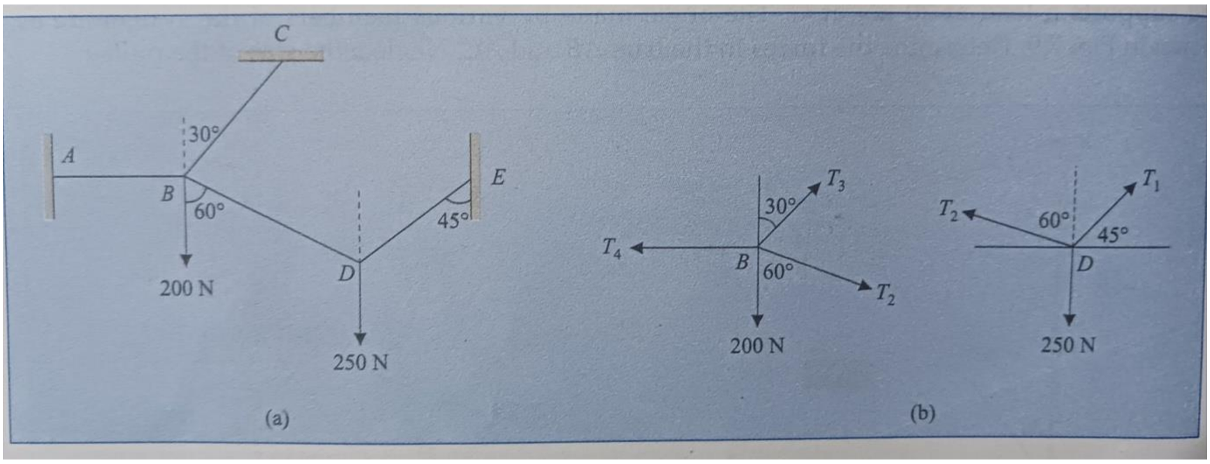
At contact point 1, the surface of contact is making 60° to horizontal. Hence the reaction R_1 which is normal to it makes 60° with the vertical. Similarly, the reaction R_2 at contact point 2 makes 45° to the vertical. FBD is as shown in Figure 2(b).

Applying Lami's theorem to the system of forces, we get

$$\frac{R_1}{\sin (180 - 45)} = \frac{R_2}{\sin (180 - 60)} = \frac{400}{\sin (60 + 45)}$$

R, = 292.8 N and R2 = 358.6 N

3. A system of connected flexible cables shown in Fig, 3(a)is supports to vertical forces 200 N and 250 N at points B and D. Determine the forces in various segments of the cable.



Free body diagrams of points B and D are as shown in Figure 3(b). Le the forces in the members be as shown in the figure.

Applying Lami's theorem to the system of forces at point D, we get

$$\frac{T_1}{\sin (180 - 60)} = \frac{T_2}{\sin (90 + 45)} = \frac{250}{\sin (60 + 45)}$$

∴ T, = 224.1 N and T, = 183 N

Now, consider the system of forces acting at B.

$\Sigma F_V = 0 \rightarrow$

$T_3 \cos 30 - T_2 \cos 60 - 200 = 0$

$T_3 \cos 30 = T_2 \cos 60 + 200 = 183 \cos 60 + 200 = 291.6$

∴ $\mathbf{T_3 = 336.6\ N}$

$\Sigma F_H = 0 \rightarrow$

- T, + T, sin 30 + T sin 60 = 0

∴ $T_4 = - 336.6 \sin 30 + 183 \sin 60 =$ **326.8 N**

ADDITIONAL NUMERICAL

Example 1.11

Four parallel forces 2 kN, F1, F2 and 10 kN have a downward resultant of 5 kN that acts at a distance of 1.8m to the right of 2kN force as shown in Fig. 1.30. Determine F1 and F2.

Fig. 1.30

Solution

From Fig $\sum F_y = -2 + F_1 - F_2 + 10 = -5$ kN

$F_1 - F_2 = -13$ ---(1)

Summation of Moment of all forces about 'A' is

$\sum M_A = -F_1 + 3F_2 - 40$ ------(2)

Moment of the Resultant about 'A' is $= 5X1.8$ ----(3)

According to varignon's theorem, Moment of the Resultant is equal to the sum of the moments of component forces.

Equating (2) and (3)

$-F_1 + 3F_2 = 49$ -------(4)

Solving (3) and (4)

F_2=18kN F_1=5kN

Example 1.12

Determine the magnitude, direction and position of the resultant force with respect to C as shown in Fig. 1.31.

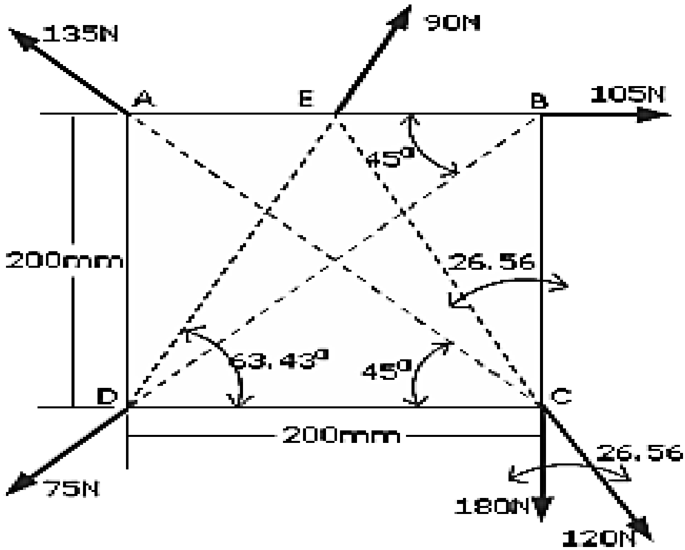


Fig. 1.31

ΣFx = -135 cos 45+ 90 cos 63.43-105+120 cos 63.43-75 cos 45

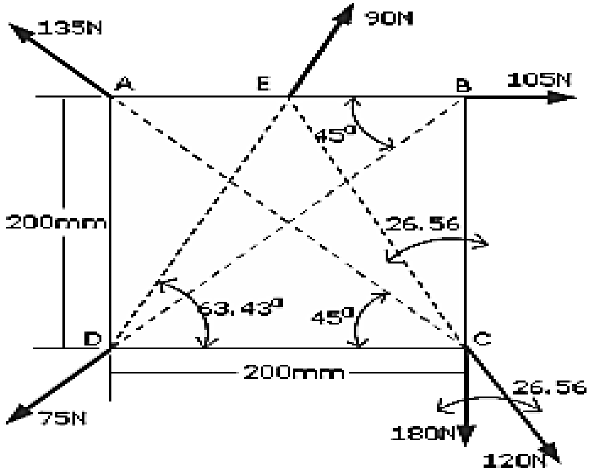
ΣFx =-159.56 N

ΣFy = 135sin45+90sin 63.43- 120 cos 63.43+180-75sin45

ΣFy =249.25 N

$$R = \sqrt{(-159.65)^2 + (249.25)^2}$$

$$R = 295.95N$$



$$\theta = \tan^{-1}\frac{249.25}{159.65} = 57.37^0$$

ΣM$_C$ = (105x200) - (75 sin 45x200) - (135cos 45x200) + (135 sin 45x200) + (90cos 63.43x 200) + (90sin63.43x100)

ΣM$_C$ = 26494.13 N-mm.

$$d = \frac{\Sigma M}{R} = \frac{26494.13}{295.95} = \mathbf{89.52}\boldsymbol{mm}$$

$$x = \frac{26494.13}{249.25} = \mathbf{106.29}\boldsymbol{mm}$$

$$y = \frac{26494.13}{159.56} = \mathbf{166.04}\boldsymbol{mm}$$

Example 1.13

Determine the resultant of the three forces acting on the dam shown in Fig. 1.32 and locate its intersection with the base AB. For a good design, this intersection should occur within the middle third of the base. Comment whether it is a good design or not.

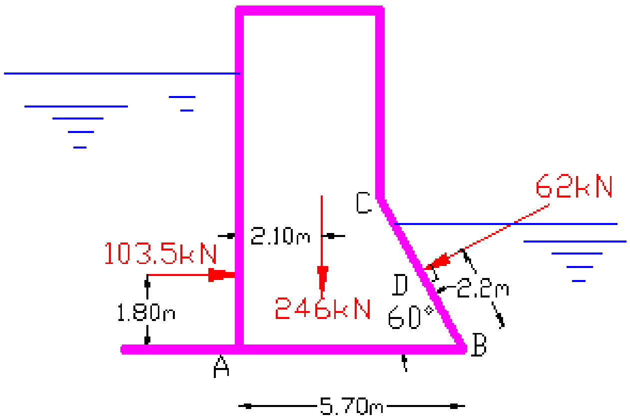


Fig. 1.32

Solution:

$\longrightarrow +^{ve}\ \Sigma F_x\ 103.5 - 62\cos 30 = 49.8$ N

$\uparrow +^{ve} \Sigma F_y = \quad -246 - 62 \sin 30 = -277$N

$$R = \sqrt{\sum F_x{}^2 + \sum F_y{}^2} = 281.44\text{N},\ \tan\alpha = \frac{\sum F_y}{\sum F_x} = 5.56\ \ \alpha = 79.8^\circ$$

$$\bar{x} = \frac{M_A}{\sum F_y}$$

M_A=103.5x1.8+246x2.1-62xcos30x2.2sin60+62sin60x(5.7-2.2cos60)=847.5 kN-m

$\therefore \bar{x} = \frac{847.5}{277} = 3.05m$. For safe design the resultant should act within the middle third. For this the position should be in the range of 1.9m and 3.8 m. Hence the given design is safe.

Example 1.14

Determine the magnitude and Y-intercept of the force system acting on the lamina as shown in Fig. 1.33

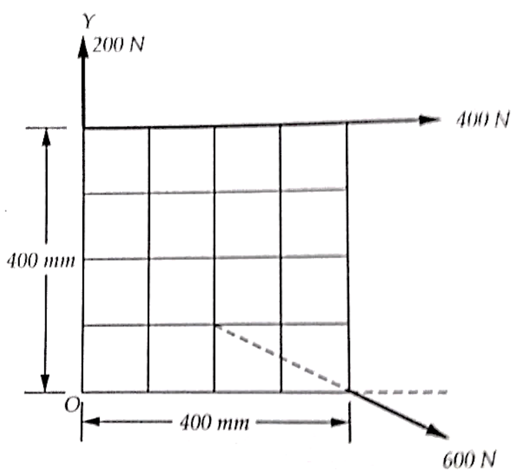


Fig. 1.33

Solution: The angle for the 600N force with horizontal is $\tan^{-1}(\frac{100}{200}) = 26.565°$

$R_X = \sum F_X = 400 + 600\cos 26.565$

$R_X = 936.66N$

$R_y = \sum F_y = 200 - 600\sin 26.565$

$R_y = -68.33N$

$R = \sqrt{R_X^2 + R_Y^2} = \sqrt{936.66^2 + 68.33^2}$

$R = 939.15N$

$\theta = \tan^{-1}(|RY|/|RX|) = \tan^{-1}(68.33/936.66)$

$\theta = 4.17°$

Plot the resultant on Y-axis at distance Y from O as shown in Fig. 1.34 using Varignon's theorem at 'O'

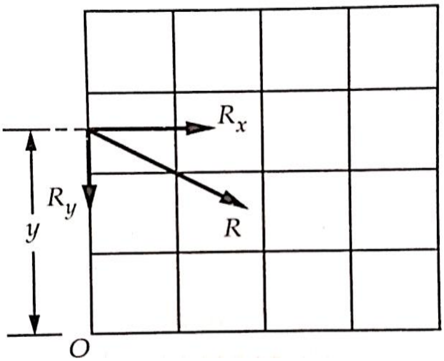


Fig. 1.34

$-(400)(400) - (600\sin 26.565)(400) = -(936.66)(Y)$

$Y = 285.41mm$

Example 1.15

For the non-concurrent coplanar system shown in Fig. 1.35 determine the magnitude, direction and position of resultant force with reference to 'A'.

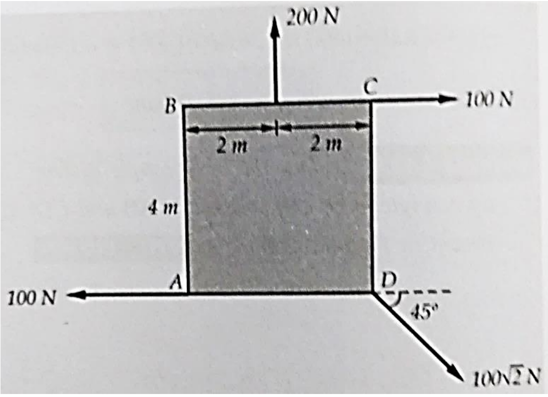


Fig. 1.35

Solution: $R_X = \sum F_X = 100 - 100 + 100\sqrt{2}\cos 45 = 100N$

$R_y = \sum F_y = 200 - 100\sqrt{2}\sin 45 = 100N$

$R = \sqrt{100^2 + 100^2}$

$R = 141.42N$

$\theta = \tan^{-1}(100/100)$

$\theta = 45°$

The perpendicular distance of the resultant from A can be obtained using Varignon's theorem at A

$(200)(2) - (100)(4) - (100\sqrt{2}\ \sin 45)\ (4) = (141.42)(d)$

$d = -2.828m$

As the perpendicular distance is negative, the resultant produces clockwise moment about A. The position of resultant with respect to A is shown in Fig. 1.36

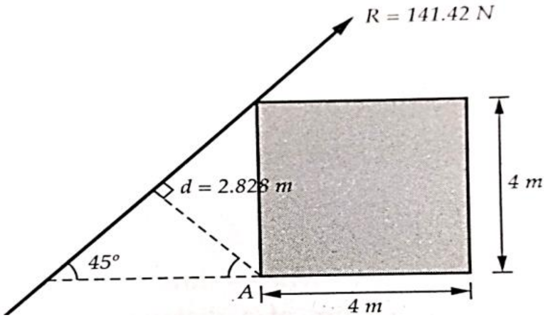


Fig. 1.36

Note the X-intercept will be $\frac{2.828}{sin45}$ = 4m to the left of A and Y-intercept is also 4m above A.

Example 1.16

A rigid plate is subjected to the forces as shown in Fig. 1.37, compute resultant of forces and position of resultant force with respect to centroid point O of the plate.

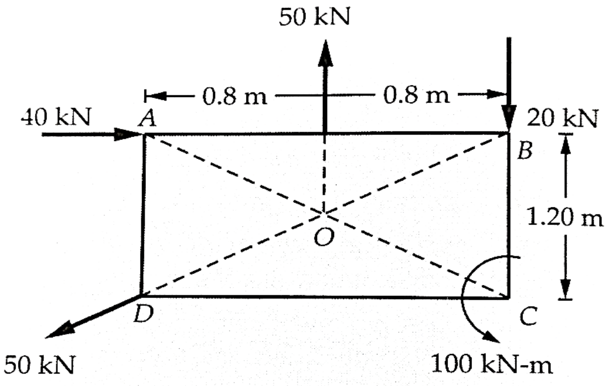


Fig. 1.37

Solution: For the 50kN force at A, the angle with horizontal is $\tan^{-1}(1.2/1.6) = 36.87°$

$R_X = \sum F_X = 40 - 50\cos 36.87 = 0$

$R_y = \sum F_y = 50 - 20 - 50\sin 36.87 = 0$

$R = 0$

As the resultant force is zero, the resultant can be a moment.

$M_O = -(40)(0.6) - (20)(0.8) + 100$

$M_O = 60\text{kN-m}$

The resultant is a couple moment of 60kN-m . This moment can act anywhere in the plane.

Example 1.17

Determine the magnitude and direction of the resultant for the system of coplanar forces shown in Fig. 1.38. Also determine the position of resultant with respect to 'O'

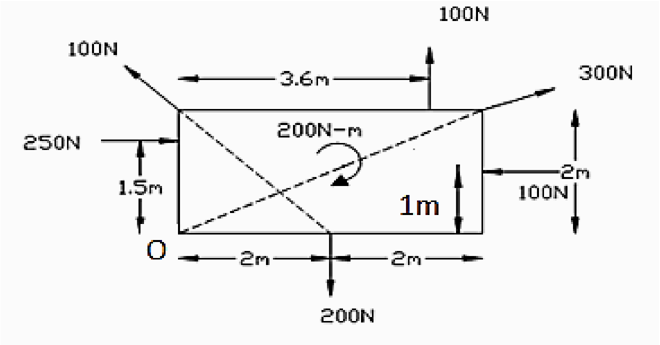


Fig. 1.38

Solution: Referring to the Fig 1.38

$$\sum F_x = 347.62N$$
$$\sum F_y = 104.87N$$
$$R = 363.01N$$
$$\sum M_o = 373.69N - m$$
$$R \times d = \sum M_o$$
$$d = 1.02m$$

Example 1.18 Three forces of 2P, 3P and 4P act along the three sides of an equilateral triangle of side 100 mm taken in order. Find the magnitude and position of the resultant force.

Example 1.20

A spherical ball of weight 75 N is attached to a string and is suspended from the ceiling as shown in Fig. 1.43(a) Find the tension in the string, if a horizontal F is applied to the ball as shown in Fig. 1.43(b) Determine the angle the string makes with the vertical and also the tension in the string if F=150 N.

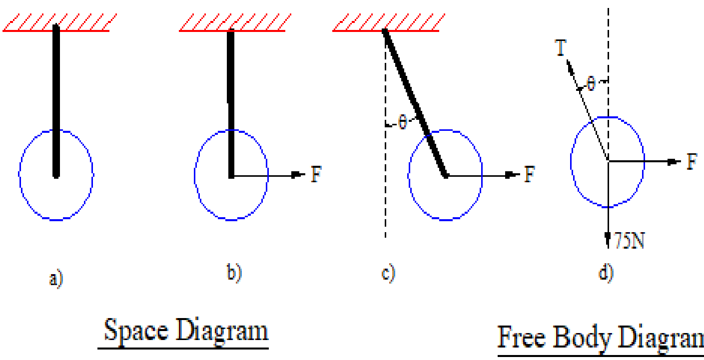


Fig. 1.43(a) Fig. 1.43 (b)

Solution:

i) When the ball is hanging from the ceiling, the tension in the string is equal to the weight of the ball i.e., T=W=75 N.

ii) Free body diagram of ball is shown in Fig. 2.4(b) when a horizontal force F is applied. Magnitude of tensile force and inclination θ of string with vertical can be computed using the method of components and equilibrium condition.

$\xrightarrow{} +^{ve}\ \Sigma F_x = 0;\ -T\sin\theta + 150 = 0$ --(1)

$\uparrow +^{ve}\ \Sigma F_y = 0;\ T\cos\theta - 75 = 0$ --(2)

Equations (1) and (2) can be simplified as T sinθ = 150; T cos θ = 75

Dividing first equation by the second equation;

$$\frac{T\sin\theta}{T\cos\theta} = 2;\ \tan\theta = 2 \text{ and } \theta = \tan^{-1}2 = 63.43^{\circ}$$

Substituting value of θ in (1) the value of T is computed as T=167.7N

Alternatively, using Lami's theorem, the solution can also be obtained as

$$\frac{T}{\sin 90} = \frac{75}{\sin(90+\theta)} = \frac{150}{\sin(180-\theta)}$$

T sin (90+θ) =75 or T cos θ = 75 and T sin(180-θ) = 150 or T sin θ = 150

These two equations are similar to equations (2) and (1) and hence the solution leads to the same result ie θ = 63.43° and T = 167.7 N

Example 1.21

Determine the reactions at contact points for the system shown in Fig. 1.44

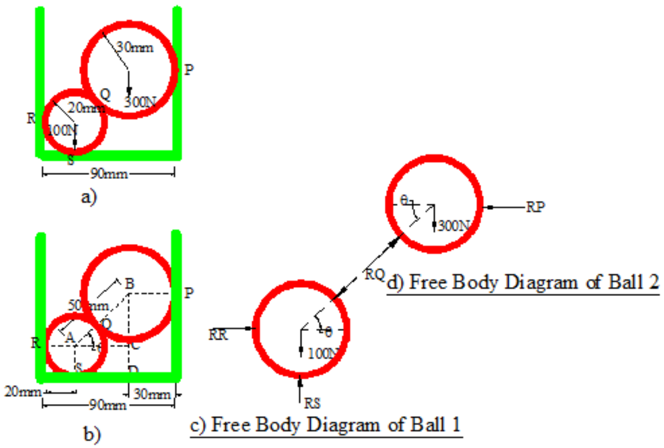


Fig. 1.44

Solution:

The free body diagram of ball 1 and 2 is shown in Fig. 1.44.

From triangle ABC the inclination of AB with AC (θ), is

computed as $\cos\theta = \frac{AC}{AB}$, where AC= 90-20-30 = 40mm. and

AB = 50 mm. $\therefore \theta = 36.87°$

Equations of equilibrium is applied to Free Body Diagram of Ball1 as

$\xrightarrow{}+^{ve}\ \Sigma F_x = 0;\ RR - RQ\cos\theta = 0$ -- (1)

$$\uparrow +^{ve}\Sigma F_y = 0;\ RS - 100 - RQ\sin\theta = 0 \qquad --(2)$$

Equations of equilibrium is also applied to Free Body Diagram of Ball 2 as

$$\rightarrow +^{ve}\ \Sigma F_x = 0;\ RQ\cos\theta - RP = 0 \qquad --(3)$$

$$\uparrow +^{ve}\Sigma F_y = 0;\ RQ\sin\theta - 300 = 0 \qquad --(4)$$

From equation (4) $RQ = \dfrac{300}{\sin 36.87} = 500$ substituting this in equation (3), the value of R_D is computed as

$R_P = 500 \cos 36.87 = 400$ N

Substituting the value of R_Q in (1) and (2);

$R_R = 500 \cos 36.87 = 400$ N; $\rightarrow$

$R_S = 100 + 500 \sin 36.87 = 400$ N $\uparrow$

Example 1.22

Two identical cylinders each weighing 2kN are supported by vertical and inclined plane ash shown in Fig.1.45. Assuming smooth surfaces determine the reactions at A, B and C.