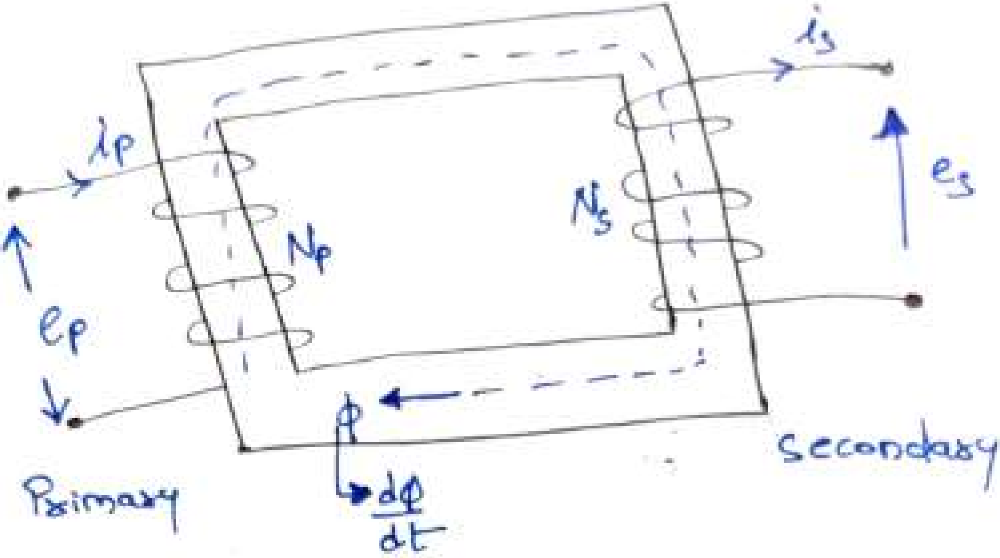
TRANSFORMERS:



$e_p = N_p \frac{d\Phi}{dt}$ → FARADAY'S LAW ← $e_s = N_s \cdot \frac{d\phi}{dt}$

→ The ratio of two voltages:

$$\frac{e_s}{e_p} = \frac{N_s}{N_p}$$

$$\Rightarrow \boxed{e_s = \left(\frac{N_s}{N_p}\right) \cdot e_p}$$ ⇒ Pottential variable (effort)

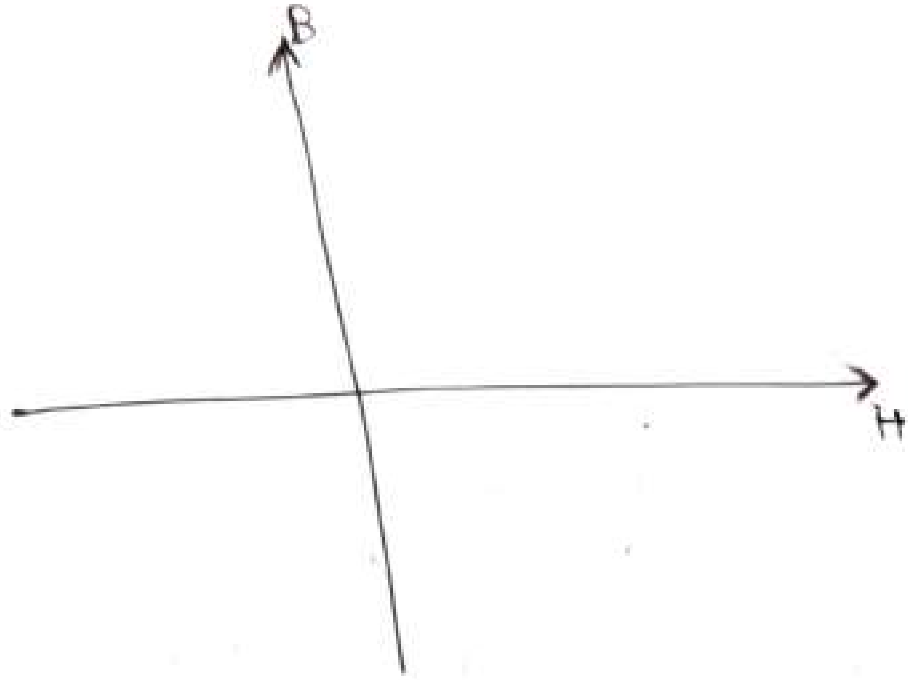
→ Power balance equation is given as

$$e_p i_p = e_s \cdot i_s$$

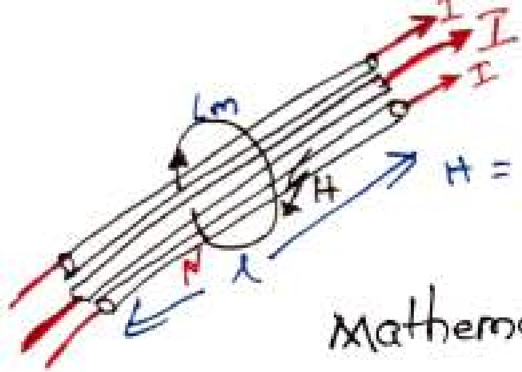
$$\Rightarrow \cancel{e_p} \cdot i_p = \left(\frac{N_s}{N_p}\right) \cdot \cancel{e_p} \cdot i_s$$

$$\Rightarrow \frac{i_p}{i_s} = \frac{N_s}{N_p} \Rightarrow \boxed{i_p = \frac{N_s}{N_p} \cdot i_s}$$

→ Kinetic variable (flow)



Amperes law:



$\Rightarrow$ H = magnetic field intensity

Mathematically

$$\int_0^{l_m} H \cdot dl = mmf = NI \text{ (Ampereturns)}$$

$\Rightarrow NI$

Flux density: (B)

From Amperes law

$$\int_0^{l_m} H \cdot dl = NI = mmf$$

$$H \int_{0}^{l_m} dl = NI$$

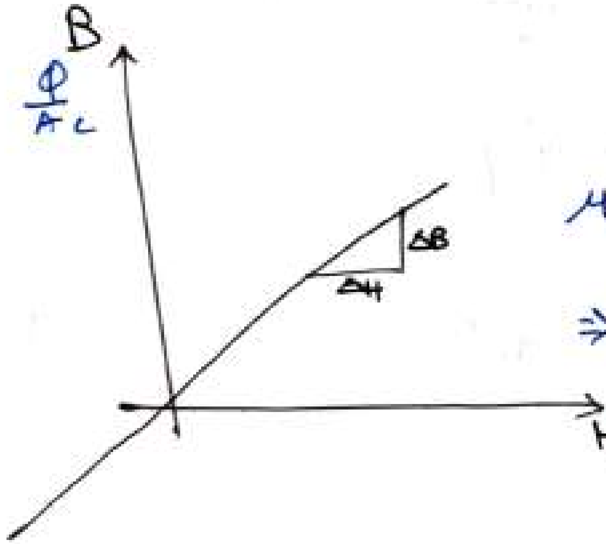
$$\Rightarrow H \cdot [l_m] = NI$$

$$\Rightarrow \boxed{H = \frac{NI}{l_m}} \text{ Amp/m}$$

⇒ The flux density is given by

$$\boxed{B = \frac{\phi}{A_c}} \text{ Tesla [Flux per unit area]}$$

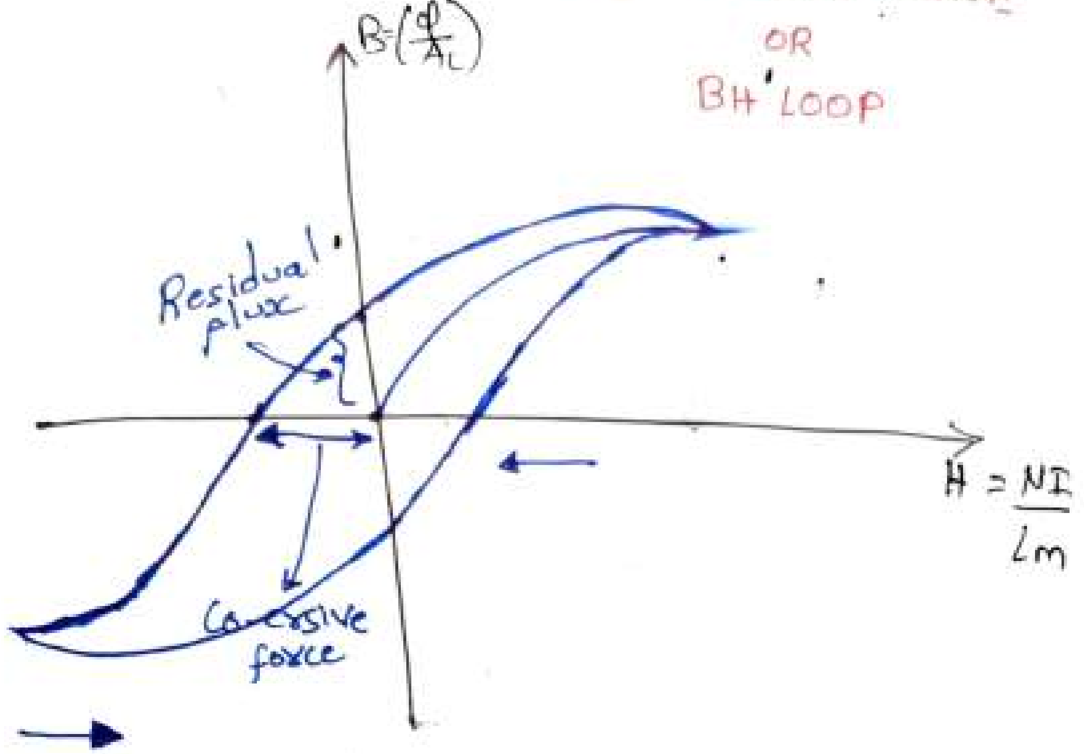
↳ weber/m²



$$\mu = \frac{\Delta B}{\Delta H} = \frac{B}{H}$$

$$\Rightarrow \boxed{B = \mu H}$$

HYSTERISIC CURVE
OR
BH LOOP



The equations are:

1. $H = \frac{NI}{Lm} = \frac{mmf}{Lm}$

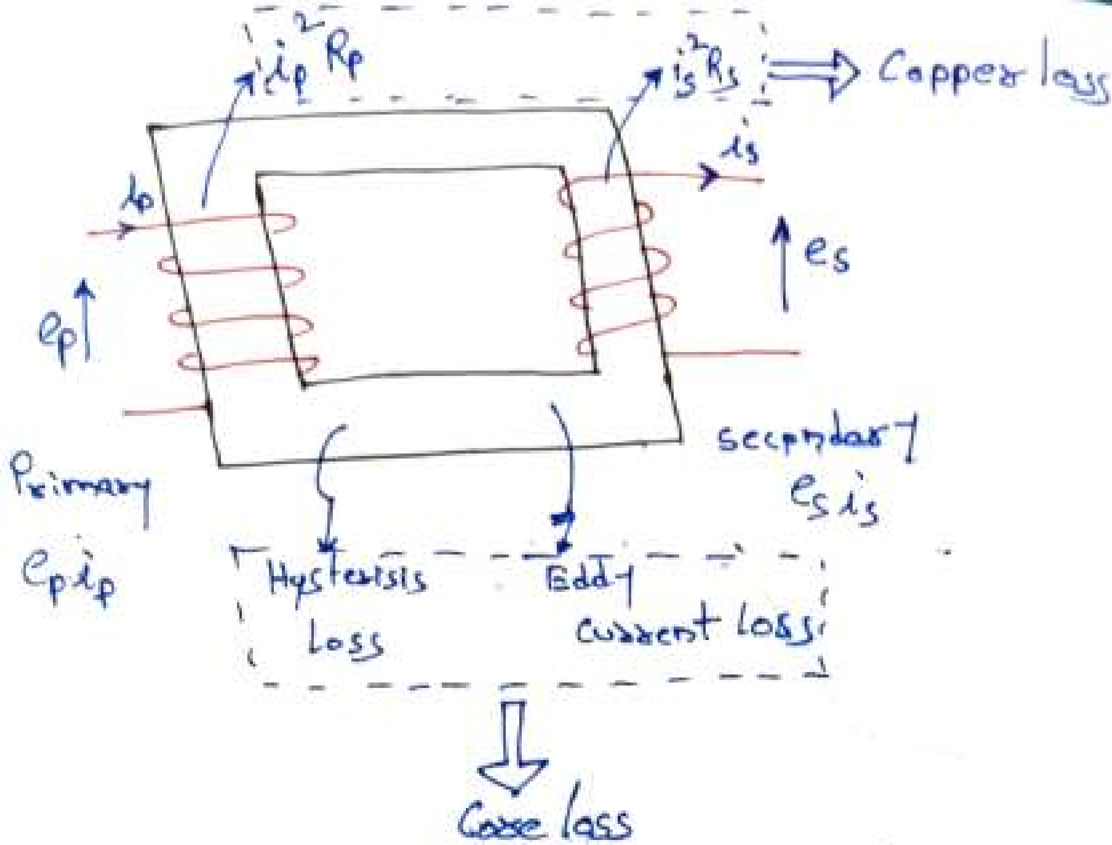
2. $B = \frac{\phi}{A_c}$

3. $B = \mu H$

Losses in Practical transformer:

1. Hysterisis loss

2. Eddy current loss.



Hysterisis loss:

The flux density is given by

$$B = \mu H = \mu \cdot \frac{NI}{l_m}$$

$$\Rightarrow I = \frac{B \cdot l_m}{N \cdot \mu} \longrightarrow (1) \quad \text{From Amperes law}$$

From Faradays law

$$V = N \cdot \frac{d\phi}{dt}$$

but

$$\phi = B \cdot A_c$$

$$\therefore V = N \cdot A \cdot \frac{dB}{dt} \longrightarrow (2)$$

The instantaneous energy

$$dE = V \cdot I \cdot dt$$

$$\Rightarrow dE = \underbrace{\left(N A_c \frac{dB}{dt}\right)}_{V} \cdot \underbrace{\left(\frac{B l_m}{\mu N}\right)}_{I} \cdot dt \quad \text{Joules}$$

$V_c = A_c l_m =$ Core volume

$$\therefore dE = \cancel{N} V_c \cdot B \cdot \frac{dB}{\mu \cancel{N}}$$

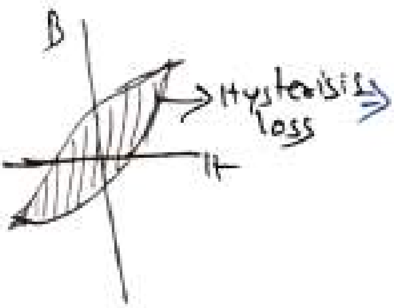
$$\Rightarrow dE = \frac{V_c \cdot B}{\mu} \cdot dB \quad \text{joules}$$

The energy per unit volume

$$dE = \left(\frac{B}{\mu}\right) dB \quad \text{joules}$$

$$\Rightarrow dE = H \cdot dB$$

(or)



$$\Rightarrow \int dE = \int H \cdot dB$$ energy per unit volume traversed for one cycle.

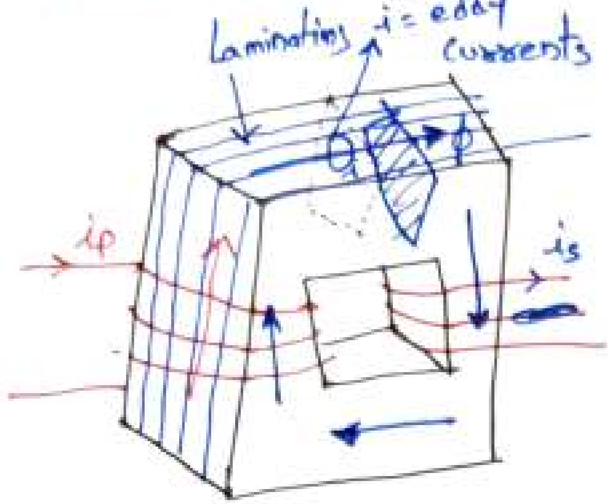
$$\therefore \int dE = \int \frac{B}{\mu} \cdot dB = \frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2} BH$$

Hysterisis energy ~~per~~ for complete core

$$\epsilon = \frac{1}{2} BH \cdot V_c$$

$$\boxed{P = \frac{1}{T}\varepsilon_h = \frac{1}{2} BH V_c \cdot f}$$

Eddy Current loss:



→ By laminations the cross sectional area for circulating is increased.

$$P_e \propto B_m^2 f^2$$

Core loss = Hysterisis loss + Eddy current loss

$$P_{core} = P_h + P_e$$

$$P_{cu} = i^2 R_{cu}$$

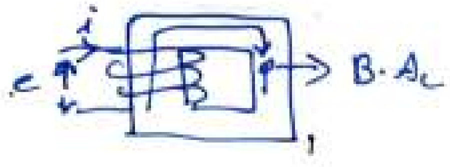
$P_{core} + P_{cu}$ = total losses in the transformer

VOLT. SEC BALANCE:

From Faraday law of electromagnetism

$$e = N \cdot \frac{d\phi}{dt}$$

$$= N A_c \frac{dB}{dt}$$



$\Rightarrow B = \frac{1}{NA_c}\int e\,dt$

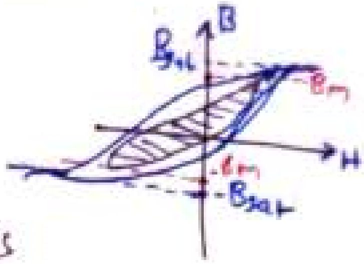
$\int v\cdot dt = V\cdot t$

$\Rightarrow \boxed{B \propto \int \underset{V}{e}\cdot \underset{sec}{dt}}$

$\int V_m \sin\omega t = \frac{V_m}{\omega}\cos\omega t$

$\Rightarrow$ For sinusoidal waveforms

$$e = NA_c \cdot \frac{dB}{dt}$$



$B_m < B_{sat}$

$\rightarrow$ Flux density swings

$-B_m \rightarrow +B_m - B_m$ in one cycle.

$$\underbrace{\frac{1}{(T/2)}\int_0^{(T/2)} e\cdot dt}_{E_{avg}} = \frac{1}{T/2} NA_c \cdot B$$

$\because (-B_m \rightarrow +B_m)$ in time of 0 to $T/2$

$\therefore E_{avg} = \frac{2}{T} NA_c\, 2B_m$

$\Rightarrow \boxed{E_{avg} = 4\cdot N\cdot A_c\cdot f\cdot B_m}$

$$\frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} \qquad \frac{\pi\cdot V_m}{2\sqrt{2}\,V_m}$$

$\rightarrow \frac{RMS}{AVG} = 1.11 \Rightarrow RMS = (1.11)\,Avg$

$$\boxed{E_{RMS} = 4.44\, NA_c f B_m}$$

where

N = number of turns

f = frequency of applied voltage

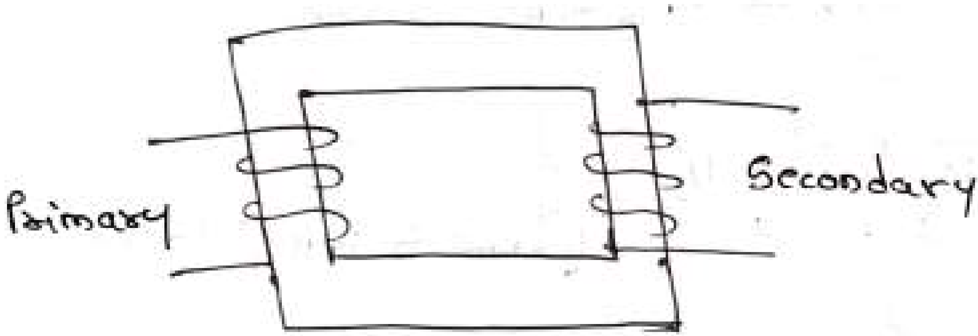
B_m = max. flux density allowed

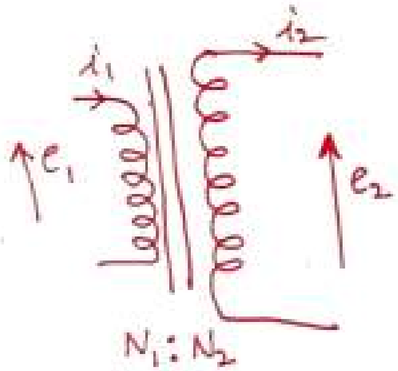
* Bm for Ferrites: 0.25T ; $B_{sat} = 0.3T$

* Bm for CRGO = 1T ; $B_{sat} = 1.2T$
[Cold Rolled Grain Oriented]

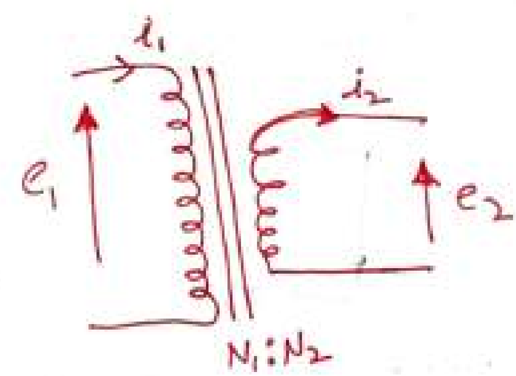
$$\frac{E}{f} = 4.44\, \underbrace{N}_{\text{constant}}\, \underbrace{A_c}_{\text{constant}}\, \underbrace{B_m}_{\text{constant}}$$

$\therefore$ E/f = constant $\Rightarrow$ Volt-sec is constant

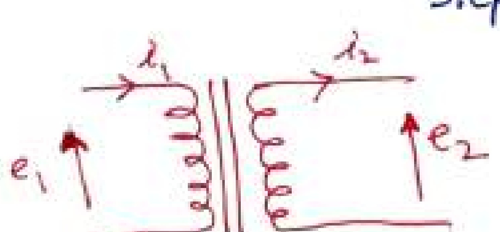




$N_1 < N_2$
Step up



$N_1 > N_2$
Step down



1:1
Galvanic Isolation transformer

Transformer Efficiency:

The efficiency of a transformer at a particular load and power factor is defined as ratio of power output to power input.

$$\therefore \text{Efficiency} = \frac{\text{Output}}{\text{input}}$$

$$= \frac{\text{output}}{\text{output} + \text{losses}}$$

$$= \frac{\text{output}}{\text{output} + \text{Cu loss} + \text{Iron loss}}$$

(or)

$$\text{Efficiency} = \frac{\text{input} - \text{losses}}{\text{input}} = 1 - \frac{\text{losses}}{\text{input}}$$

Condition for maximum efficiency:

Iron losses:

P_i = hysteresis loss + eddy current loss

$= P_h + P_e$

Copper losses:

$P_c = I_1^2 R_{01}$ (or) $I_2^2 R_{02}$

Considering primary side:

$$\text{Input} = V_1 I_1 \cos\phi_1$$

$$\therefore \eta = \frac{V_1 I_1 \cos\phi_1 - \text{losses}}{V_1 I_1 \cos\phi_1}$$

$$= \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{01} - P_i}{V_1 I_1 \cos\phi_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos\phi_1} - \frac{P_i}{V_1 I_1 \cos\phi_1}$$

Differentiating both sides w.r.t I_1

$$\frac{d\eta}{dI_1} = 0$$

$$\Rightarrow \frac{R_{01}}{V_1 \cos\phi_1} = \frac{P_i}{V_1 I_1^2 \cos\phi_1}$$

$$\Rightarrow P_i = I_1^2 R_{01} \text{ (or) } I_2^2 R_{02}$$

$$\Rightarrow \boxed{\text{Copper losses} = \text{Iron losses}}$$

∴ Output current corresponding to maximum efficiency is

$$\boxed{I_2 = \sqrt{\frac{P_i}{R_{02}}}}$$

→ Efficiency at any load is given by

$$\eta = \frac{x \times \text{full-load kVA} \times \text{P.f.}}{(x \times \text{full-load kVA} \times \text{P.f.}) + x^2 W_{cu} + W_i} \times 100$$

where

x = ratio of actual to full-load kVA

W_i = Iron losses in kW;

W_{cu} = Cu loss in kW;

Cu loss @ x% of load $= x^2 \cdot$ (Cu loss)

Example:

A 600 KVA, 1-phase transformer when working at u.p.f has an efficiency of 92% at full load and also at half load. Determine its efficiency when it operates at unity P.f and 60% of full load.

Sol:- At ~~half~~ full load:

output = 600 kW

$$\text{Input} = \frac{\text{output}}{\eta} = \frac{600 \times 1000}{0.92}$$

$$= 652.2 \text{ kW}$$

$\therefore$ Total loss = Input − output

$$= 52.2 \text{ kW}$$

Let x = Iron loss → it remains constant at all loads

y = F.L. Cu loss → it is $\propto (\text{kVA})^2$

$$\therefore \boxed{x + y = 52.2} \rightarrow (1)$$

At half-load:

output = 300 kW

$$\text{Input} = \frac{300}{0.92} = 326.08$$

$\therefore$ losses = 26.1 kW

@ half load Cu losses $= \frac{1}{4}(\text{Cu loss @ F.L})$

$$\therefore \boxed{\tfrac{1}{4} \cdot y + x = 26.1} \rightarrow 2$$

$\therefore$ By solving (1) & (2)

$$\boxed{x = 17.4 \text{ kW}} ; \boxed{y = 34.8 \text{ kW}}$$

At 60% of full load

$$\text{Output} = 0.6 \times 600$$
$$= 360 \text{ kW}$$

$$\text{Total loss} = W_{cu} + W_i$$

where

$$W_i = \text{Iron loss} = 17.4 \text{ kW}$$

$$W_{cu} = x^2 (W_{cu} @ F.L)$$
$$= (0.6)^2 \times 34.8$$
$$= 12.53 \text{ kW}$$

$$\therefore \frac{\text{output}}{\text{input}} = \eta \Rightarrow \eta = \frac{360}{360 + 17.4 + 12.53}$$

$$= \frac{360}{389.93} = 0.93$$

$$\Rightarrow \boxed{\eta\% = 93\%}$$

Example 02:

A 600 KVA, 1-phase transformer has an efficiency of 92% both at full & half load at unity power factor. Determine its efficiency at 60% of full-load at 0.8 power factor lag.

$$\therefore \eta = \frac{x \times KVA \times \cos\phi}{(x \times KVA) \times \cos\phi + W_i + x^2 W_{cu}} \times 100$$

$x \rightarrow$ % of full load

At F.L. u.p.f

here $x = 1$

$$\therefore 92 = \frac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_i + 1^2 W_{cu}} \times 100$$

$$\Rightarrow \boxed{W_i + W_{cu} = 52.174 \text{ kW}} \rightarrow (1)$$

At half of F.L., UPF, here $x = 1/2$

$$92 = \frac{(1/2) \times 600 \times 1}{(1/2) \times 600 \times 1 + W_i + (1/2)^2 W_{cu}} \times 100;$$

$\therefore \boxed{W_i + 0.75\, W_{cu} = 76.087 \text{ kW}} \rightarrow (2)$

From (1) & (2)

$W_i = 17.39$ kW;

$W_{cu} = 34.78$ kW;

At 60% F.L. 0.8 p.f (lag)

here, $x = 0.6$

$$\eta = \frac{0.6 \times 600 \times 0.8 \times 100}{(0.6 \times 600 \times 0.8) + 17.39 + (0.6)^2 34.78}$$

$\Rightarrow \boxed{\eta = 85.9\%}$