K-Maps [Karnaugh - Maps]

* Graphical method gives us a systematic approach for simplifying a Boolean expression.

* It contains boxes called cells.

Each of the cell represents one of the 2^n possible products that can be formed from 'n' variables.

2 Variable map $\Rightarrow 2^2 = 4$ cells.
   (n = 2)

3 Variable map $\Rightarrow 2^3 = 8$ cells.
   (n = 3)

4 Variable k map $\Rightarrow 2^4 = 16$ cells.
   (n = 4)

$2^1 \, 2^0$
$\bar{0}.\bar{0} \rightarrow 0$
$0 \; 1$
$1 \; 0$
$1 \; 1$
$2^1 + 2^0$

2 - Variable k-map

A\B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	$\bar{A}\bar{B}$ 0	$\bar{A}B$ 1
A 1	$A\bar{B}$ 2	AB 3

3 - Variable k-map

A\BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	$\bar{A}\bar{B}\bar{C}$ 0	$\bar{A}\bar{B}C$ 1	$\bar{A}BC$ 3	$\bar{A}B\bar{C}$ 2
A 1	$A\bar{B}\bar{C}$ 4	$A\bar{B}C$ 5	ABC 7	$AB\bar{C}$ 6

4 Variable k-map

AB\CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0	$\bar{A}\bar{B}\bar{C}D$ 1	$\bar{A}\bar{B}CD$ 3	$\bar{A}\bar{B}C\bar{D}$ 2
01	$\bar{A}B\bar{C}\bar{D}$ 4	$\bar{A}B\bar{C}D$ 5	$\bar{A}BCD$ 7	$\bar{A}BC\bar{D}$ 6
11	$AB\bar{C}\bar{D}$ 12	$AB\bar{C}D$ 13	$ABCD$ 15	$ABC\bar{D}$ 14
10	$A\bar{B}\bar{C}\bar{D}$ 8	$A\bar{B}\bar{C}D$ 9	$A\bar{B}CD$ 11	$A\bar{B}C\bar{D}$ 10

* Representation of Truth table on k-map.

ExOR :-

A	B	Y
0	0	0
0	1	1 ✓
1	0	1 ✓
1	1	0
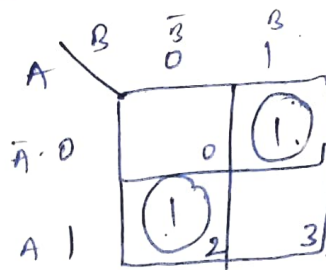
$$Y = \bar{A}B + A\bar{B}$$

full adder (Cout) :-

A	B	C_{in}	Cout
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1 ✓
1	0	0	0
1	0	1	1 ✓
1	1	0	1 ✓
1	1	1	1 ✓
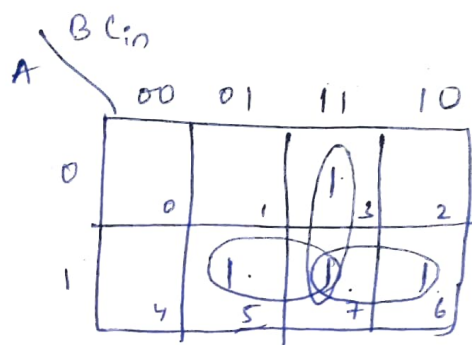
$$Y = AB + BC_{in} + AC_{in}$$

Grouping in k-mAp :-

$$2^3 \quad 2^2 \quad 2^1 \quad 2^0$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$8 \quad 4 \quad 2 \quad 1$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$\text{Octet} \quad \text{Quad} \quad \text{Pair} \quad \text{single}$$

1)
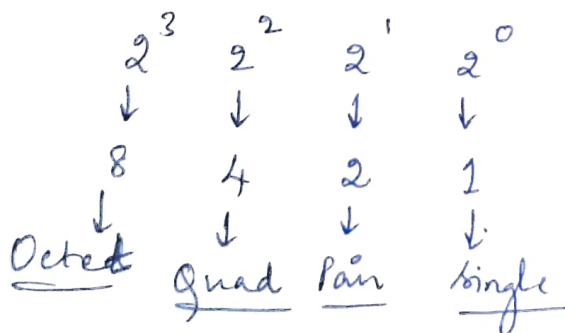
A\BC	00	01	11	10
0	0	(1)	1	2
1	4	5	7	6

⇒ Grouping of 2 ones. (Pair)

$$Y = \bar{A}\,C$$

(2)

A\BC	00	01	11	10
0	0	1	3	2
1	(1)	5	7	(1)

$$Y = A\bar{C}$$

(3)

A\BC	00	01	11	10
0	0	(1)	(1)	2
1	4	5	(1)	6

$$Y = \bar{A}\,C + BC$$

(4) $$Y = \bar{A}\,\bar{B}\,C + \bar{A}\,B\,C + ABC + AB\bar{C}$$

A\BC	00	01	11	10
0	0	(1)	(1)	2
1	4	5	(1)	6

→ (Pair not required)
(Redundant)

$$Y = \bar{A}\,C + AB$$

(5)

A\BC	00	01	11	10
0	0	1	3	2
1	(1)	1	1	1)

$$Y = A$$

(6)

AB\CD	00	01	11	10
00	0	1	3	2
01	4	(1)	1	6
11	12	13	1)	14
10	8	9	11	10

→ $$Y = BD.$$

↘ $$Y = A\bar{D}$$

(7)

$$y = \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,B\,C\,\overline{D} + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,C\,\overline{D}$$

CD / AB Karnaugh map with cells 0,1,3,2 / 4,5,7,6 / 12,13,15,14 / 8,9,11,10

$$y = \overline{B}\,\overline{D}$$

(8) $y = A\,B\,\overline{C} + A\,D + A\,C$

(8)

$$y = \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,\overline{C}\,D + \overline{A}\,B\,C\,D +$$
$$A\,B\,\overline{C}\,D + A\,B\,C\,D + A\,\overline{B}\,\overline{C}\,D + A\,\overline{B}\,C\,D$$

$$y = D$$

(9)

$$y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,\overline{C}\,D +$$
$$\overline{A}\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,C\,D + \overline{A}\,B\,C\,\overline{D}$$

$$y = \overline{B}$$
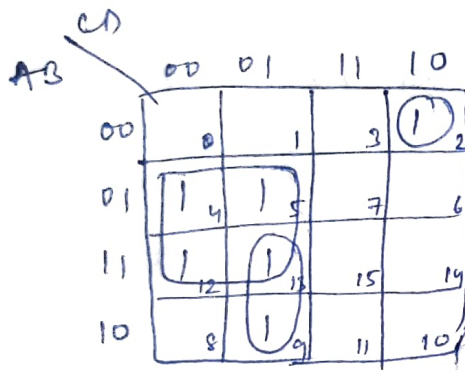
(10), Solve the Exp^n using k-map.

$$y = A\bar{B}C + \bar{A}BC + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}B\bar{C}$$

BC / A

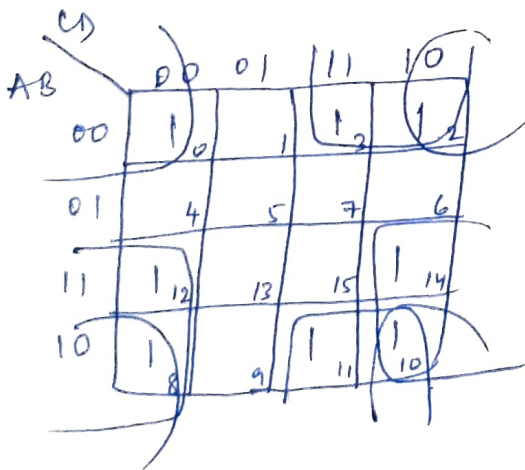|	00	01	11	10
0	1 (0)	(1) (1)	(1) (3)	(2)
1	1 (4)	1 (5)	(7)	(6)
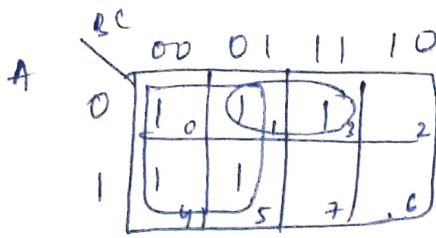
$$y = \bar{A}C + B$$

(11) $$y = \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}D$$
$$+ \bar{A}\bar{B}C\bar{D}$$

CD / AB

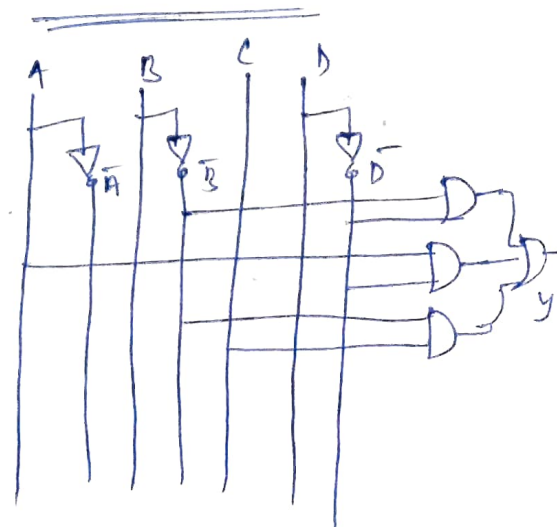|	00	01	11	10
00	(0)	(1)	(3)	(1) (2)
01	1 (4)	1 (5)	(7)	(6)
11	1 (12)	(1) (13)	(15)	(14)
10	(8)	(1) (9)	(11)	(10)

$$y = \bar{A}BC\bar{D} + A\bar{C}D + B\bar{C}$$

(12) $$y = \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}\bar{D}$$
$$+ AB\bar{C}\bar{D} + \bar{A}BCD + \bar{A}B\bar{C}\bar{D}, \text{ Realize using}$$
$$\text{Logic circuit}$$

CD / AB

	00	01	11	10
00	1 (0)	(1)	1 (2)	1 (3)
01	(4)	(5)	(7)	(6)
11	1 (12)	(13)	1 (15)	1 (14)
10	1 (8)	(9)	1 (11)	1 (10)

$$y = B\bar{D} + A\bar{D} + \bar{B}C$$

(13) $y = \bar{A}\,\bar{B}\,\bar{C}\,D + \bar{A}\,B\,\bar{C}\,D + \bar{A}\,B\,C\,D + \bar{A}\,B\,C\,\bar{D} +$

$\qquad A\,B\,\bar{C}\,\bar{D} + A\,B\,\bar{C}\,D + A\,B\,C\,D + A\,\bar{B}\,C\,D$.

$y = \bar{A}\,\bar{C}\,D + \bar{A}\,B\,C + A\,B\,\bar{C} + A\,C\,D$

(14) $f(A,B,C,D) = \bar{A}\,\bar{B}\,D + A\,B\,\bar{C}\,\bar{D} + \bar{A}\,B\,D + A\,B\,C\,\bar{D}$.

$\qquad = \bar{A}\,\bar{B}\,D\,(C+\bar{C}) + A\,B\,\bar{C}\,\bar{D} + \bar{A}\,B\,D\,(C+\bar{C}) + A\,B\,C\,\bar{D}$

$\qquad = \bar{A}\,\bar{B}\,D\,C + \bar{A}\,\bar{B}\,\bar{C}\,D + A\,B\,\bar{C}\,\bar{D} + \bar{A}\,B\,C\,D + \bar{A}\,B\,\bar{C}\,D$

$\qquad\qquad + A\,B\,C\,\bar{D}$

$y = A\,B\,\bar{D} + \bar{A}\,D$

(15) $f(A,B,C,D) = \sum m(0,1,4,8,9,10)$

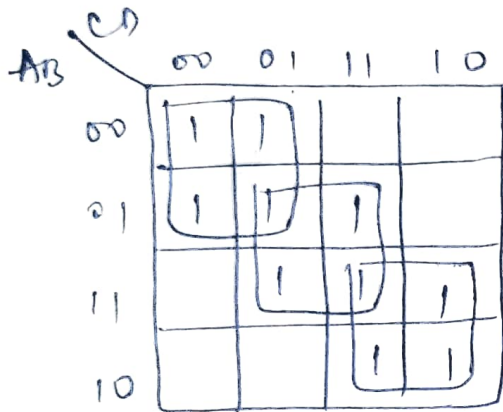$f = \bar{A}\,\bar{C}\,\bar{D} + A\,\bar{B}\,\bar{D} + \bar{B}\,\bar{C}$

(19) $y = \sum (0, 2, 6, 7, 8, 10, 14, 15)$

Ans : $\bar{B}\bar{D} + BC = y$
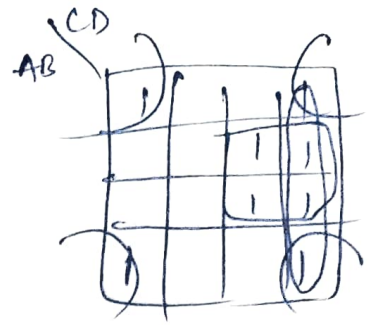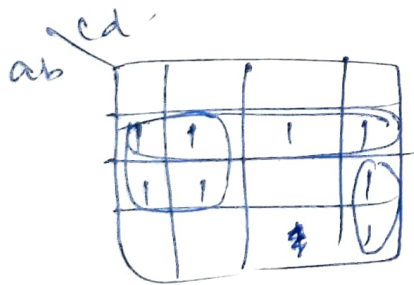

(20) Simplify the Exp using k-map
and realize using Basic Gates

$y = \sum m(0, 1, 4, 5, 7, 10, 11, 13, 14, 15)$

$y = \bar{A}\bar{C} + BD + AC$

(21) $y(a, b, c, d) = \sum (4, 6, 12, 13, 14, 15, 7, 10)$

$y = \bar{a}b + \bar{b}c + ac\bar{d}$

realize using gates

(22) $y = ABCD + AB\bar{C}D + \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}BC\bar{D} + \bar{A}\bar{B}C\bar{D} + ABC\bar{D} + A\bar{B}\bar{C}\bar{D}$

(23) $y = \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + ABCD + A\bar{B}C\bar{D}$

(24) $f(x) = \sum (0, 2, 8, 10, 13, 15) + d_c(5, 7) \rightarrow$ Ans : $B\bar{D} + \bar{B}\bar{D}$