RV College of Engineering

DEPARTMENT OF MATHEMATICS

Academic year 2023-2024 (Even Semester 2023)

<table>
<tr><td>Date</td><td>13/05/2024</td><td>Time</td><td colspan="2">10:00 to 11:30 AM</td></tr>
<tr><td>TEST</td><td>Test-I</td><td>Maximum Marks</td><td colspan="2">50</td></tr>
<tr><td>Course Title</td><td colspan="2">Number Theory, Vector calculus and Computational methods</td><td>Course Code</td><td>MA221TC</td></tr>
<tr><td>Semester</td><td>II</td><td>Programs</td><td colspan="2">B. E. (AIML,BT,CD,CS,CY,IS)</td></tr>
</table>

Instructions: Answer all questions.

<table>
<tr><th>Sl. No.</th><th>Questions</th><th>M</th><th>BT</th><th>CO</th></tr>
<tr><td>1(a)</td><td>From the following table estimate the number of students who obtained marks between 50 and 75 using appropriate interpolation formula
<table>
<tr><td>Marks</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60 -70</td><td>70 - 80</td></tr>
<tr><td>Number of students</td><td>31</td><td>73</td><td>124</td><td>159</td><td>190</td></tr>
</table></td><td>6</td><td>L2</td><td>3</td></tr>
<tr><td>1(b)</td><td>Using Lagrange Interpolation Formula fit a polynomial for the given data:
<table>
<tr><td>x</td><td>0</td><td>1</td><td>3</td><td>5</td></tr>
<tr><td>$y(x)$</td><td>1</td><td>5</td><td>49</td><td>231</td></tr>
</table></td><td>4</td><td>L2</td><td>2</td></tr>
<tr><td>2</td><td>The following table gives the values of pressure P and specific volume V of saturated steam:
<table>
<tr><td>V</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td></tr>
<tr><td>P</td><td>304.5</td><td>270.4</td><td>250.2</td><td>204.6</td><td>184.9</td></tr>
</table>
Find the value of pressure P at volume V= 45. Also find the rate of change of pressure with respect to volume at V= 50 and $\frac{d^2P}{dV^2}$ at V=70.</td><td>10</td><td>L3</td><td>3</td></tr>
<tr><td>3(a).</td><td>If $F(D) = (D^4 - \omega^4)$, where D is the linear differential operator, with $D = \frac{d}{dz}$. Obtain the general solution of $F(D)v = 0$.</td><td>4</td><td>L1</td><td>1</td></tr>
<tr><td>3(b)</td><td>Find the particular solution of the initial value problem $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 18e^{-t}$, given $y(0) = 0$ and $y'(0) = 4$.</td><td>6</td><td>L2</td><td>2</td></tr>
<tr><td>4</td><td>Solve $\frac{d^2y}{dx^2} - y = e^{-2x}\cos(e^{-x})$ by method of variation of parameters.</td><td>10</td><td>L2</td><td>2</td></tr>
<tr><td>5</td><td>Obtain the radial displacement x in a rotating disc at a distance s from the axis is given by the differential equation $2s^2\frac{d^2x}{ds^2} + 3s\frac{dx}{ds} - x = \frac{\cos(logs)}{s} - 2s$.</td><td>10</td><td>L3</td><td>4</td></tr>
</table>

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	4	20	16	10	4	26	20	--	--	--

....All the best....

RV College of Engineering®
Mysore Road, RV Vidyaniketan Post,
Bengaluru - 560059, Karnataka, India

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Department of Mathematics

Academic Year 2023-2024 (Even Semester 2023)

Date	18/06/2024	**Time**	10:00 to 11:30 AM
Test	Test-II	**Maximum Marks**	50
Course Title	Number Theory, Vector Calculus and Computational Methods	**Course Code**	MA221TC
Semester	II	**Programs**	B.E. (AIML, BT, CD, CS, CY, IS)

Instructions: Answer all questions.

Sl. No.	Questions	M	BT	CO
1(a)	By using the Euclidean algorithm, find the greatest common divisor d of 1166 and 256, and find integers x and y to satisfy $1166x + 256y = d$.	6	L2	2
1(b)	Find the remainder when 16^{53} is divided by 7.	4	L1	1
2	Given the public key $(e, n) = (7,51)$, encrypt plain text LIV, where the alphabets $A, B, C, \ldots X, Y, Z$ are assigned the numbers 3,4,5, ⋯ ,26,27,28. Give the cipher text and find the private key d.	10	L3	4
3(a)	Find all solutions of linear congruence $18x \equiv 30(mod\ 42)$.	5	L2	2
3(b)	A particle moves along the curve whose parametric equation is given by $x = (t^3 + 1)$, $y = t^2, z = (2t + 5)$, where 't' is time. Find the component of its acceleration at time $t = 2$ in the direction of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.	5	L1	2
4(a)	Find the values of a and b such that the surfaces $5x^2 - 2yz = 9z$ and $ax^2 + by^3 = 4$ cut orthogonally at $(1, -1, 2)$.	5	L2	3
4(b)	Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = \lvert\vec{r}\rvert$. Evaluate $\nabla^2 r^n$. Reduce the final answer in terms of r.	5	L2	1
5	Find the curl of the vector field $\vec{F} = (y \sin z + y\cos(xy))\hat{\imath} + (x \sin z + x\cos(xy))\hat{\jmath} + xy \cos z\,\hat{k}$. If $\vec{F}$ is irrotational then find the scalar potential ϕ of $\vec{F}$ such that $\nabla\phi = \vec{F}$.	10	L3	3

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	9	16	15	10	9	21	20	--	--	--

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Department of Mathematics
Academic Year 2023-2024 (Even Semester 2024)

Date	01/07/2024	**Time**	10:00 AM to 12 NOON
Test	Improvement Test (Quiz & Test)	**Maximum Marks**	10+50=60
Course Title	Number Theory, Vector Calculus and Computational Methods	**Course Code**	MA221TC
Semester	II	**Programs**	B.E. (AIML, BT, CD, CS, CY, IS)

Instructions: Answer all questions

PART - A

S.No.	Questions	M	BT	CO
1	If the general solution of a differential equation is $y = c_0 + c_1 e^{-x} \sin x + c_2 e^{-x} \cos x$, then the differential equation is ______.	2	L1	1
2	The Wronskian of $y_1 = \cos 2x\,, y_2 = \sin 2x$ is ______.	2	L1	1
3	Reduce the Cauchy's equation $x\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \frac{1}{x}$ to a differential equation with constant coefficients ______.	2	L2	2
4	The value of $\frac{1}{\pi} \oint_C \left(3y - e^{\cos(x^2)}\right)dx + \left(7x + \sqrt{y^4 + 11}\right)dy$ where $C: x^2 + y^2 = 9$ oriented positively is ______.	2	L2	2
5	Let S be the portion of the plane $2x + y = 4$ in the first octant bounded by $z = 1$ and $z = 4$. The surface integral $\iint_S (x\,\hat{\imath} + y\hat{\jmath} + z\hat{k}) \cdot \hat{n}\, dS$, over the region R in terms of double integral with the limits is given by______, where R being the projection of S taken on yz −plane.	2	L3	3

PART - B

S.No.	Questions	M	BT	CO
1a	Obtain the particular integral of the differential equation $y'' - y = x \cos x$.	4	L2	2
1b	Determine the displacement x at time $t > 0$ from the equilibrium position of the mass-spring system governed by the differential equation $t^2x'' + tx' + x = 2\cos^2(\log t)$.	6	L3	1
2	Using the method of variation of parameters solve the differential equation: $$\frac{d^2y}{dx^2} + y = \frac{1}{1+\sin x}.$$	10	L2	2
3a	Find the work done by the force $\vec{F}(x, y) = 2xy\,\hat{\imath} + 4y^2\hat{\jmath}$ acting along the piecewise curve consisting of the line segments from $(-2, 2)$ to $(0, 0)$ and from $(0, 0)$ to $(2, 3)$.	6	L3	3
3b	Evaluate the line integral $\int_C \vec{F}.\hat{T}\, ds$, where $\vec{F} = 2xy\,\hat{\imath} + x^2\hat{\jmath}$ being the conservative field, along any smooth curve C joining the point $(0, 0)$ to $(1, 1)$.	4	L2	4
4	Verify Green's theorem for $\oint_C (x^2 - y^2)\, dx + (2y - x)\, dy$, where C consists of the boundary of the region in the first quadrant bounded by the graphs $y = x^2$ and $y = x^3$.	10	L3	3
5	Use the surface integral in Stokes' theorem to determine the circulation of the field $\vec{F} = y\,\hat{\imath} + xz\,\hat{\jmath} + x^2\,\hat{k}$ around the boundary of the triangle cut from the plane $x + y + z = 1$ by the first octant, counterclockwise when viewed from above.	10	L4	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	10	18	18	14	4	22	24	10	--	--

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DEPARTMENT OF MATHEMATICS

Course: Number theory, Vector calculus and Computational methods	CIE-I SCHEME AND SOLUTION	Date: 13-05-2024
Course code: MA221TC	II Semester	

Q.No	Answer all questions	M

1(a)

Less than (x)	No of students (y)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
40	31				
		73			
50	104		51		
		124		-16	
60	228		35		12
		159		-4	
70	387		31		
		190			
80	577				

2

$$x_n = 80, x = 75$$

$$p = \frac{75 - 80}{10} = -0.5$$ 1

$$y_{75} = y_{n} + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \ldots\ldots\ldots\ldots \frac{p(p+1)\ldots\ldots(p+n-1)}{n!}\nabla^n y_n$$

$$y_{75} = 577 - 0.5(190) - \frac{0.5(0.5)(31)}{2!} - \frac{0.5(0.5)(1.5)(-4)}{3!} - \frac{0.5(0.5)(1.5)(2.5)(12)}{4!}$$ 1

= 477.9063 ~ 478 students 1

$\therefore$ 50-75 marks obtained by 478-104 =374 1

1(b)

Lagrange Interpolation

$$f\,(x) = \frac{(x-x_1)\,(x-x_2)\ldots\ldots\ldots..(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots\ldots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\ldots\ldots\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots\ldots.(x_1-x_n)}y_1 + \cdots$$

$$+ \frac{(x-x_0)(x-x_1)\ldots\ldots\ldots.(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots\ldots\ldots..(x_n-x_{n-1})}\;y_n$$

$$=\frac{(x-1)\,(x-3)(x-5)}{(-1)(-3)(-5)}+\frac{(x-0)\,(x-3)(x-5)}{(1-0)(1-3)(1-5)}5+\frac{(x-0)\,(x-1)(x-5)}{(3-0)(3-1)(3-5)}49$$ 2

$$+\frac{(x-0)(x-1)\,(x-3)}{(5-0)(5-1)(5-3)}231$$

$$=\frac{9}{4}x^3-3x^2+\frac{19}{4}x+1$$ 2

$$=2.25x^3-3x^2+4.75x+1$$

2

v	P	First diff	Second diff	Third diff	Fourth diff
40	304.5				
		-34.1			
50	270.4		13.9		
		-20.2		-39.3	
60	250.2		-25.4		90.6
		-45.6		51.3	
70	204.6		25.9		
		-19.7			
80	184.9				

3

$$p=\frac{45-40}{10}=0.5$$ 1

$$v_{45}=P_0+\text{p}\Delta P_0++\frac{p(p-1)}{2!}\Delta^2P_0+\frac{p(p-1)(p-2)}{3!}\ \Delta^3P_0+\ldots\ldots$$ 1

$$=304.5+0.5(-34.1)+\frac{0.5(-0.5)(13.9)}{2!}+\frac{0.5(-0.5)(-1.5)(-39.3)}{3!}$$

$$+\frac{0.5(-0.5)(-1.5)(-2.5)(90.6)}{4!}$$ 1

=279.7172

$$\left(\frac{dP}{dv}\right)_{v=50}=\frac{1}{h}\left[\Delta P_0-\frac{1}{2}\Delta^2P_0+\frac{1}{3}\Delta^3P_0-\frac{1}{4}\Delta^4P_0+\cdots\right]$$ 1

$$=\frac{1}{10}\left[-20.2-\frac{1}{2}(-25.4)+\frac{1}{3}(51.3)\right]$$ 1

= 0.96

$$\left(\frac{d^2P}{dv^2}\right)_{v=70}=\frac{1}{h^2}\left[\nabla^2P_n+\nabla^3P_n+\frac{11}{12}\nabla^4P_n+\frac{5}{6}\nabla^5P_n+\frac{137}{180}\nabla^6P_n+\cdots\right]$$

$$=\frac{1}{100}\left[-25.4-39.3\right]$$ 1

=-0.647 1

3(a)	$(D^4 - \omega^4)v = 0$	
	$m^4 - \omega^4 = (m^2 - \omega^2)(m^2 + \omega^2) = 0$	
	$m = \pm\omega, \pm i\omega$	2
	$v = C_1 e^{\omega z} + C_2 e^{-\omega z} + C_3 cos\omega z + C_4 sin\omega z$	2
3(b)	$(D^2 + 4D + 4)y = 18e^{-t}$	
	$m^2 + 4m + 4 = 0$	1
	$m = -2, -2$	
	$y_c = (C_1 + C_2 t)\, e^{-2t}$	1
	$y_p = \frac{18e^{-t}}{((-1)^2 - 4 + 4)}$	
	$y_p = \frac{18e^{-t}}{(D^2 + 4D + 4)}$	
	Replace D by -1 $\quad y_p = 18e^{-t}$	1
	$y = y_c + y_p$	
	$y = (C_1 + C_2 t)e^{-2t} + 18e^{-t}$	
	$y' = -2(C_1 + C_2 t)e^{-2t} + C_2 e^{-2t} - 18e^{-t}$	1
	$y(0) = 0 = C_1 + 18$	
	$C_1 = -18$	1
	$y'(0) = 4 = -2(-18) + C_2 - 18$	
	$C_2 = -14$	1
	$y = -(18 + 14t)e^{-2t} + 18e^{-t}$	
4	A E $\quad m^2 - 1 = 0, m = \pm 1$	1
	$y_c = C_1 e^x + C_2 e^{-x}$	1
	$y_p = uy_1 + vy_2$	
	$y_1 = e^x, y_2 = e^{-x}$	
	$w = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -2$	1
	$u = -\int \frac{y_2 X}{w} dx = -\int \frac{e^{-x} e^{-2x} \cos(e^{-x})}{-2} dx$	1
	$= -\frac{1}{2}\int e^{-2x} \cos(e^{-x})(-e^{-x})dx$	
	Put $z = e^{-x}, dz = -e^{-x}dx$	
	$= -\frac{1}{2}\int z^2 \cos z\, dz$	1

	$= -\frac{1}{2}[z^2 \sin z - 2z(-\cos z) + 2(-\sin z)]$	1
	$= -\frac{1}{2}e^{-2x} \sin e^{-x} - e^{-x}(\cos e^{-x}) + \sin e^{-x}$	1
	$v = \int \frac{y_1 X}{w} dx = \int \frac{e^x e^{-2x} \cos(e^{-x})}{-2} dx$	
	$= \frac{1}{2}\int \cos(e^{-x})(-e^{-x})dx = -\frac{1}{2}\int \cos z \, dz$	1
	$= \frac{1}{2}\sin z = \frac{1}{2}\sin e^{-x}$	
	$y_p = uy_1 + vy_2$	
	$= -\frac{1}{2}e^{-x} \sin e^{-x} - (\cos e^{-x}) + e^x \sin e^{-x} + \frac{e^{-x}}{2}\sin e^{-x}$	1
	$= -\cos e^{-x} + e^x \sin e^{-x}$	1
	$y = C_1 e^x + C_2 e^{-x} - \cos e^{-x} + e^x \sin e^{-x}$	
5	Dividing by 2,	
	$s^2 x'' + \frac{3}{2}sx' - \frac{1}{2}x = \frac{\cos(logs)}{2s} - s$	
	Put $s = e^t \ or \ t = logs, \ sx' = Dx, s^2x'' = D(D-1)x$	1
	Equation becomes.	
	$\left[D(D-1)\frac{3}{2}D - \frac{1}{2}\right]x = \frac{\cos t}{2e^t} - e^t$	1
	A.E $m^2 + \frac{1}{2}m - \frac{1}{2} = 0$	
	$m = -1, \frac{1}{2}$	1
	C.F is $C_1 e^{-t} + C_2 e^{\frac{t}{2}} = C_1 s^{-1} + C_2 s^{\frac{1}{2}}$	1
	P I is	
	$\frac{1}{D^2+\frac{1}{2}D-\frac{1}{2}}(\frac{e^{-t}\cos t}{2} - e^t) = \frac{1}{2}\frac{1}{\left(D^2+\frac{1}{2}D-\frac{1}{2}\right)}(e^{-t}\cos t) - \frac{1}{(D^2+\frac{1}{2}D-\frac{1}{2})} \ e^t$	
	PI 1: consider $\frac{1}{2}\frac{1}{\left(D^2+\frac{1}{2}D-\frac{1}{2}\right)}e^{-t}\cos t = \frac{1}{2}e^{-t}\frac{1}{((D-1)^2+\frac{1}{2}(D-1)-\frac{1}{2})}\cos t$	1
	$= \frac{1}{2}e^{-t}\frac{1}{\left(-1-\frac{3}{2}D\right)}\cos t = -\frac{1}{2}e^{-t}\frac{(1-\frac{3}{2}D)}{(1-\frac{9}{4}D^2)}\cos t = -\frac{1}{2}e^{-t}\frac{1}{\left(1-\frac{9}{4}D^2\right)}\left(\cos t + \frac{3}{2}\sin t\right)$	1+1
	$= -\frac{e^{-t}}{13}[2\cos t + 3\sin t]$	
	PI 2: consider $\frac{1}{(D^2+\frac{1}{2}D-\frac{1}{2})} \ e^t = e^t$	1
	$\therefore \frac{1}{2(D^2+\frac{1}{2}D-\frac{1}{2})}(e^{-t}\cos t - 2e^t) = -\frac{1}{13s}[2\cos(\log s) + 3\sin(\log s)] - s$	1
	$y = C_1 s^{-1} + C_2 s^{\frac{1}{2}} - \frac{1}{13s}[2\cos(\log s) + 3\sin(\log s)] - s$	1

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Department of Mathematics
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Date	18/06/2024	**Time**	10:00 to 11:30 PM	
Test	Test-II	**Maximum Marks**	50	
Course Title	Number Theory, Vector Calculus and Computational Methods		**Course Code**	MA221TC
Semester	II	**Programs**	B.E. (AIML, BT, CD, CS, CY, IS)	

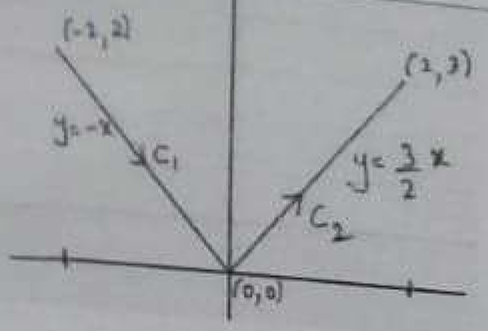
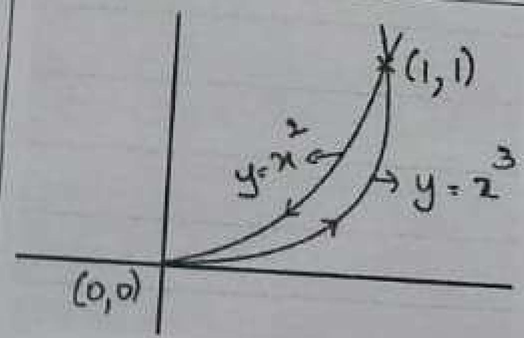
SCHEME AND SOLUTION

Sl. No.	Questions	M
1(a)	$1166 = 4 \times 256 + 142$; $256 = 1 \times 142 + 114$ $142 = 1 \times 114 + 28$; $114 = 4 \times 28 + 2$; $28 = 14 \times 2 + 0$ $\gcd(1166,256) = 2$ $2 = 5 \times 114 - 4 \times 142 = 41 \times 256 - 9 \times 1166$	1+1 1 1 1+1
1(b)	$16 \equiv 2\ (mod\ 7)$, $2^3 \equiv 1\ (mod\ 7)$, $2^{53} = 2^{17\times3+2} = (2^3)^{17} \times 2^2 \equiv 1 \times 2^2\ (mod\ 7)$ $16^{53} \equiv 2^{53}\ (mod\ 7) \equiv 4\ (mod\ 7)$	1 2 1
2	$\phi(51) = 32$, Plain text $L = 14$, $I = 11$, $V = 24$ Cipher Text: L: $14^7 \equiv 23\ (mod\ 51)$ $\quad L \longrightarrow U$ I: $11^7 \equiv 20\ (mod\ 51)$ $\quad I \longrightarrow R$ V: $24^7 \equiv 12\ (mod\ 51)$ $\quad V \longrightarrow J$ Cipher text is URJ Private key $(d, 32)$ $7d \equiv 1\ (mod\ 32) \Longrightarrow d = 23$	1 2 2 2 1 2
3(a)	$\gcd(18,42) = 6$, Therefore, the linear congruence has 6 incongruent solutions. Solutions are $x \equiv 4\ (mod\ 42)$, $x \equiv 11\ (mod\ 42)$, $x \equiv 18\ (mod\ 42)$, $x \equiv 25\ (mod\ 42)$, $x \equiv 32(mod\ 42)$, $x \equiv 39(mod\ 42)$	1 2 1+1
3(b)	$\vec{r}(t) = (t^3 + 1)\boldsymbol{i} + t^2\boldsymbol{j} + (2t + 5)\boldsymbol{k}$ $\vec{v}(t) = \frac{d\vec{r}}{dt} = 3t^2\boldsymbol{i} + 2t\boldsymbol{j} + 2\boldsymbol{k}$, $\vec{a}(t) = 6t\boldsymbol{i} + 2\boldsymbol{j}$ $\vec{a}(2) = 12\boldsymbol{i} + 2\boldsymbol{j}$ $\vec{u} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, $\hat{u} = \frac{1}{\sqrt{3}}(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ Component of acceleration= $\vec{a}(2) \cdot \hat{u} = \frac{14}{\sqrt{3}}$	1 1 1 1 1
4a	Let $\phi_1 = 5x^2 - 2yz - 9z$ and $\phi_2 = ax^2 + by^3$ $\nabla\phi_1 = 10x\boldsymbol{i} - 2z\boldsymbol{j} - (2y + 9)\boldsymbol{k}$, $\nabla\phi_2 = 2ax\boldsymbol{i} + 3by^2\boldsymbol{j}$ At $(1, -1,2)$ $\vec{n}_1 = \nabla\phi_1 = 10\boldsymbol{i} - 4\boldsymbol{j} - 7\boldsymbol{k}$, $\vec{n}_2 = \nabla\phi_2 = 2a\boldsymbol{i} + 3b\boldsymbol{j}$ $\vec{n}_1 \cdot \vec{n}_2 = 0 \Longrightarrow 20a - 12b = 0 \Longrightarrow 5a = 3b$ As $(1, -1,2)$ lies on $ax^2 + by^3 = 4$, $a - b = 4 \Longrightarrow 3a - 3b = 12 \Longrightarrow a = -6, b = -10$ or The point $(1, -1,2)$ is not on the surface ϕ_1	1 1 1 1 1 5
4b	$r = \sqrt{x^2 + y^2 + z^2}$, $\nabla^2 r^n = \frac{\partial^2}{\partial x^2}(r^n) + \frac{\partial^2}{\partial y^2}(r^n) + \frac{\partial^2}{\partial z^2}(r^n)$ $\frac{\partial}{\partial x}(r^n) = nr^{n-1}\frac{\partial r}{\partial x} = nxr^{n-2}$ $\frac{\partial^2}{\partial x^2}(r^n) = \frac{\partial}{\partial x}(nxr^{n-2}) = nr^{n-2} + nx(n-2)r^{n-3}\frac{\partial r}{\partial x} = nr^{n-2} + n(n-2)x^2r^{n-4}$ Similarly	 1 2

P.T.O.

	$\frac{\partial^2}{\partial y^2}(r^n) = nr^{n-2} + n(n-2)y^2r^{n-4}$ and $\frac{\partial^2}{\partial z^2}(r^n) = nr^{n-2} + n(n-2)z^2r^{n-4}$ $\nabla^2 r^n = n(n+1)r^{n-2}$	1 1
5	$curl\,\vec{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y\sin z + y\cos(xy) & x\sin z + x\cos(xy) & xy\cos z \end{vmatrix}$ $= [x\cos z - x\cos z]\boldsymbol{i} - [y\cos z - y\cos z]\boldsymbol{j} + [\sin z + \cos(xy) - xy\sin(xy) - \sin z - \cos(xy) + xy\sin(xy)]\boldsymbol{k}$ $= \boldsymbol{0}$ Hence $\vec{F}$ is irrotational. Let ϕ be the potential of $\vec{F}$. $\frac{\partial\phi}{\partial x} = y\sin z + y\cos(xy), \frac{\partial\phi}{\partial y} = x\sin z + x\cos(xy), \frac{\partial\phi}{\partial z} = xy\cos z$ $d\phi = [y\sin z + y\cos(xy)]dx + [x\sin z + x\cos(xy)]dy + xy\cos z\,dz$ $= d(xy\sin z) + d(\sin(xy))$ $\phi = xy\sin z + \sin(xy) + c$	1 1+1+1 1 1 1 2 1

Note: Appropriate marks maybe awarded for the alternative methods.

		1
	$W=\int_C \vec{F}.\overrightarrow{dr}=\int_{C_1}\vec{F}.\overrightarrow{dr}+\int_{C_2}\vec{F}.\overrightarrow{dr}$	
	Along C_1: $y=-x \Rightarrow dy=-dx, 0\le x\le -2$	1
	$\int_{C_1} 2xy\,dx+4y^2dy=\int_{-2}^{0}-2x^2dx-4x^2dx=-2^4=-16$	
	Along C_2: $y=\frac{3}{2}x \Rightarrow dy=\frac{3}{2}dx$	2
	$\int_{C_2} 2xy\,dx+4y^2dy=\int_0^2(3x^2dx+\frac{27}{2}x^2dx)=44$	
	$\therefore \boldsymbol{W=-16+44=28}$	2
3b	Since F is conservative, scalar potential $\phi=\boldsymbol{x^2y}$	
	$\therefore \int_C \vec{F}.\hat{T}ds=\phi]_{0,0}^{1,1}=1$	2
4		2 1
	Along C_1: $y=x^3 \Rightarrow dy=3x^2dx$	
	$\int_{C_1}(x^2-y^2)dx+(2y-x)dy=\int_0^1(x^2-x^6)dx+(2x^3-x)3x^2dx=\frac{37}{84}$	2
	Along C_2: $y=x^2 \Rightarrow dy=2x\,dx$	
	$\int_{C_2}(x^2-y^2)dx+(2y-x)dy=\int_1^0(x^2-x^4)dx+(2x^2-x)2x\,dx=-\frac{7}{15}$	2
	$\therefore \int_C(x^2-y^2)dx+(2y-x)dy=\frac{37}{84}-\frac{7}{15}=-\frac{11}{420}$	1
	RHS:	
	$\int_R\int(-1+2y)\,dy\,dx=\int_{x=0}^{1}\int_{y=x^3}^{x^2}(-1+2y)\,dy\,dx=\int_0^1(-x^2+x^4+x^3-x^6)\,dx=-\frac{11}{420}$	2+2
5	Stokes' theorem	
	$\oint_C \vec{F}.\hat{n}\,ds=\iint_S \nabla\times\vec{F}.\hat{n}\,dS=\iint_{R_{xy}}\nabla\times\vec{F}.\frac{\nabla\phi}{\lvert\nabla\phi\cdot\hat{k}\rvert}\,dA.$	1
	Where $\nabla\times\vec{F}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ y & xz & x^2\end{vmatrix}=-x\hat{\imath}-2x\hat{\jmath}+(z-1)\hat{k}$	2
	$\phi=x+y+z-1,\ \nabla\phi=\hat{\imath}+\hat{\jmath}+\hat{k}$	
	$\nabla\times\vec{F}\cdot\nabla\phi=-4x-y$ and $\nabla\phi\cdot\hat{k}=1$	2 1
	We take projection on the $xy-$plane, $R_{xy}=\{(x.y)\mid 0\le x\le 1,\ 0\le y\le 1-x\}$.	
	Thus, $\iint_{R_{xy}}\nabla\times\vec{F}.\frac{\nabla\phi}{\lvert\nabla\phi\cdot\hat{k}\rvert}\,dA=\int_0^1\int_0^{1-x}(-4x-y)\,dy\,dx=-\frac{5}{6}$.	2 2

Note: Alternate correct methods may be awarded appropriate marks.

Department of Mathematics

Academic Year 2023-2024 (Even Semester 2024)

Scheme and solution

Course: Number Theory, Vector Calculus and Computational Methods	Improvement CIE (QUIZ & TEST)	**Maximum marks: 10+50=60**
Course code: MA221TC	Second semester 2023-2024 Physics Cycle : AIML, BT, CD, CS, CY, IS	

Q.No	PART A – Quiz	Marks
1.1	$m(m^2-(-1+i-1-i)m+(-1+i)(-1-i))=0$ $D(D^2+2D+2)y=0$	1+1
1.2	$W(y_1,y_2)=2$	2
1.3	$(D^3-2D^2+D)y=e^t$	2
1.4	$I=\frac{1}{\pi}(7-3)9\pi=36$	2
1.5	$\int_1^4\int_0^4 2\,dy\,dz$	2

Q.No	PART B	Marks
1a	$x\cos x=Re(xe^{ix})$	
	$P.I.=e^{ix}\frac{1}{(D+i)^2-1}x=e^{ix}\frac{1}{D^2+2iD-2}x$	1
	$P.I.=-\frac{e^{ix}}{2}\left(1-\frac{D^2+2iD}{2}\right)^{-1}x=-\frac{e^{ix}}{2}\left[1+\frac{D^2+2iD}{2}+..\right]x$	1
	$P.I=-\frac{1}{2}e^{ix}(x+i)=-\frac{1}{2}(\cos x+i\sin x)(x+i)$	1
	$\therefore P.I.=-\frac{1}{2}(x\cos x-\sin x)$	1
1b	Let $t=e^z\Rightarrow z=\log t$	1
	The given equation becomes $(D^2+1)x=2\cos^2 z$	
	$\therefore m=\pm i$	
	$x_c=c_1\cos z+c_2\sin z=c_1\cos(\log t)+c_2\sin(\log t)$	2
	$x_p=\frac{2\cos^2 z}{D^2+1}=\frac{1+\cos 2z}{D^2+1}$	1
	$x_p=\frac{1}{D^2+1}+\frac{\cos 2z}{D^2+1}=1-\frac{\cos 2z}{3}=1-\frac{\cos 2(\log t)}{3}$	2
	$\therefore x=x_c+x_p$	
2	$A.E.: m^2+1=0\Rightarrow m=\pm i$	
	$y_c=c_1\cos x+c_2\sin x$	3
	$W(\cos x,\sin x)=1$	1
	$A(x)=-\int\frac{\sin x}{1+\sin x}dx=-\int\frac{\sin x-\sin^2 x}{1-\sin^2 x}dx=-\int\frac{\sin x}{\cos^2 x}dx+\int\tan^2 x\,dx=-\sec x+\tan x-x$	2
	$B(x)=\int\frac{\cos x}{1+\sin x}dx=\log(1+\sin x)$	2
	$y_p=(-\sec x+\tan x-x)\cos x+\log(1+\sin x)\sin x$	1
	Therefore, $y=y_c+y_p$	1

MA221TC / 22MA21C

USN | 1 | R | V | 2 | 3 | C | D | 0 | 0 | 3 |

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

II Semester B. E. Regular / Supplementary Examinations Aug-2024

(Common to AI, BT, CS, CY, CD & IS)

NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Mathematics hand book to be provided.

PART-A

			M	BT	CO
1	1.1	The sum of positive divisors of 864 is ________.	02	1	1
	1.2	Compute 6^{100} *modulo* 7.	02	2	2
	1.3	If $\phi(x,y,z) = xy^2z^3 - x^3y^2z$ then find $\nabla\phi$.	02	2	2
	1.4	Show that the vector function $\vec{f} = 2xyz\,\hat{\imath} + (xy - y^2z)\hat{\jmath} + (x^2 - zx)\hat{k}$ is solenoidal.	02	1	2
	1.5	If $\vec{F}$ represents the velocity of a fluid then $\int_c \vec{F}.d\vec{r}$ represents ________ of $\vec{f}$ around C and if $\vec{F}$ is force then $\int_c \vec{F}.d\vec{r}$ represents ________.	02	1	1
	1.6	The value of $\int_c (y^2 + 2yx)dx + (2xy + x^2)dy$ where C is the circle $x^2 + y^2 = 9$ is ________.	02	1	1
	1.7	Solve the differential equation $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0$.	02	1	2
	1.8	Find the particular integral of $(D^2 + 4)y = 2\sin x \cos x$.	02	2	2
	1.9	Construct the forward difference table: $\begin{array}{\|c\|c\|c\|c\|c\|}\hline x & 0 & 2 & 4 & 6 \\ \hline y & -1 & 3 & 8 & 11 \\ \hline\end{array}$	02	1	1
	1.10	Find x at $y = 2$ given: $\begin{array}{\|c\|c\|c\|c\|}\hline x & 0 & 1 & 3 \\ \hline y & 1 & 3 & 4 \\ \hline\end{array}$	02	2	2

PART-B

2	a	By using the Euclidean algorithm, find the greatest common divisor d of 1769 and 2378 and then find integers x and y to satisfy $1769x + 2378y = d$.	08	3	3
	b	Find the last two digits of 3333^{4444}.	08	3	3
3	a	A particle moves along the curve $\vec{r} = 2t^2\,\hat{\imath} + (t^2 - 4t)\hat{\jmath} + (3t - 5)\hat{k}$. Find the components of velocity and acceleration in the direction of the vector $\vec{c} = \hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ at $t = 1$.	08	2	2
	b	Find the constants a and b so that the surface $3x^2 - 2y^2 - 3z^2 + 8 = 0$ is orthogonal to the surface $ax^2 + y^2 = bz$ at the point $(-1, 2, 1)$.	08	3	2
		OR			

<table>
<tr><td>4</td><td>a</td><td>Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$, then prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$.</td><td>08</td><td>2</td><td>2</td></tr>
<tr><td></td><td>b</td><td>Show that $\vec{f} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find the function ϕ such that $\vec{f} = grad\ \phi$.</td><td>08</td><td>3</td><td>2</td></tr>
<tr><td>5</td><td>a</td><td>Find the work done in moving a particle in the force filed $\vec{f} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along the straight line $(0,0,0)$ to $(2,1,3)$.</td><td>08</td><td>3</td><td>3</td></tr>
<tr><td></td><td>b</td><td>Show that area enclosed by a simple closed curve C is given by $\frac{1}{2}\oint\{x\,dy - y\,dx\}$. Using this, find the area bounded by the ellipse with axes $2a$ and $2b$.</td><td>08</td><td>3</td><td>4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>6</td><td>a</td><td>Using Divergence theorem, evaluate $$\iint_S \left[(x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}\right].\hat{n}ds$$ over the surface of the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$.</td><td>08</td><td>3</td><td>3</td></tr>
<tr><td></td><td>b</td><td>Evaluate by Stokes theorem $\oint_c (x + y)dx + (2x - z)dy + (y + z)dz$, C is the boundary of the triangular surface with vertices $(0,0,0)$, $(1,0,0)$, and $(1,1,0)$.</td><td>08</td><td>3</td><td>4</td></tr>
<tr><td>7</td><td>a</td><td>Solve the differential equation: $(D^2 - 4D + 4)y = x^2e^{2x} + \sin^2 x$.</td><td>08</td><td>2</td><td>3</td></tr>
<tr><td></td><td>b</td><td>Solve the differential equation $y'' + a^2y = sec(ax)$ using the method of variation of parameters.</td><td>08</td><td>3</td><td>3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>8</td><td>a</td><td>Find the general solution of the differential equation: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = (x^2 + 7x + 5)$</td><td>08</td><td>2</td><td>3</td></tr>
<tr><td></td><td>b</td><td>Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$</td><td>08</td><td>3</td><td>3</td></tr>
<tr><td>9</td><td>a</td><td>The table gives the distance in nautical miles (y) of the visible horizon for the given heights (x) in feet above the earth's surface:
<table>
<tr><td>x</td><td>150</td><td>200</td><td>250</td><td>300</td><td>350</td><td>400</td></tr>
<tr><td>y</td><td>13.03</td><td>15.04</td><td>16.81</td><td>18.42</td><td>19.90</td><td>21.27</td></tr>
</table>
Find the values of y when $x = 160ft$ and $x = 410ft$.</td><td>08</td><td>3</td><td>4</td></tr>
<tr><td></td><td>b</td><td>Find y' and y'' at $x = 1.2$, given:
<table>
<tr><td>x</td><td>1.0</td><td>1.2</td><td>1.4</td><td>1.6</td><td>1.8</td><td>2.0</td></tr>
<tr><td>y</td><td>2.72</td><td>3.32</td><td>4.06</td><td>4.96</td><td>6.05</td><td>7.39</td></tr>
</table></td><td>08</td><td>2</td><td>3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>10</td><td>a</td><td>By using the Lagrange's interpolation formula fit a polynomial to the data given:
<table>
<tr><td>x</td><td>0</td><td>1</td><td>3</td><td>4</td></tr>
<tr><td>y</td><td>−12</td><td>0</td><td>6</td><td>12</td></tr>
</table>
Hence find y when $x = 2$.</td><td>08</td><td>3</td><td>4</td></tr>
<tr><td></td><td>b</td><td>The following table gives the temperature θ (in degree <i>Celcius</i>) of a cooling body at different instants of time t (in <i>seconds</i>)
<table>
<tr><td>t</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr>
<tr><td>θ</td><td>85.3</td><td>74.5</td><td>67.0</td><td>60.5</td><td>54.3</td></tr>
</table>
Find approximately the rate of cooling at $t = 3\ seconds$ and $t = 7\ seconds$.</td><td>08</td><td>2</td><td>3</td></tr>
</table>