Unit 3

ALGEBRA AND LOGIC CIRCUITS:

Binary numbers, Number base conversion and Hexadecimal Numbers, Complements, Basic definitions, Basic theorems and properties of Boolean Algebra, Boolean functions, Canonical and Standard forms, Digital Logic gates, Demorgan's Laws, Ex-OR realization using NAND and NOR, Kmaps (Upto 4 variable)

COMBINATIONAL LOGIC:

Introduction, Design procedure, Adders-Half adder, Full adder

Number System

Number System is a way to represent numbers in computer architecture.

Four different types of the number system:

1. Binary number system (base 2)
2. Octal number system (base 8)
3. Decimal number system(base 10)
4. Hexadecimal number system (base 16)

Binary Number System:

- According to digital electronics and mathematics, a binary number is defined as a number that is expressed in the binary system or base 2 numeral system.
- It describes numeric values by two separate symbols; 1 (one) and 0 (zero). The base-2 system is the positional notation with 2 as a radix.
- The binary system is applied internally by almost all latest computers and computer-based devices because of its direct implementation in electronic circuits using logic gates.
- Every digit is referred to as a **bit**.
- A binary number consists of several bits. Examples are:

EX: 10101 is a five-bit binary number

101 is a three-bit binary number

100001 is a six-bit binary number

Decimal to Binary

Convert 4 in binary number system.

Step 1: Divide the number 4 by 2. Use the integer quotient obtained in this step as the dividend for the next step. Continue this step, until the quotient becomes 0.

Dividend	Remainder
4/2 = 2	0
2/2 = 1	0
1/2 = 0	1

Step 2: write the remainder in reverse chronological order. (i.e from bottom to top)

Binary to Decimal

1. What is binary number 1.1 in decimal

Step 1: 1 on the left-hand side is on the one's position, so it's 1.

Step 2: The one on the right-hand side is in halves, so it's 1 × ½

Step 3: so, 1.1 = 1.5 in decimal.

2.Write 10.11_2 in Decimal?

S1:10.11 = $1 \times (2)^1 + 0\ (2)^0 + 1\ (½)^1 + 1(½)^2$

= 2 + 0 + ½ + ½

= 2.75

- So, 10.11 is 2.75 in Decimal.

Octal Number System

- Has a base of eight and uses the numbers from 0 to 7.
- Any number with base 8 is an octal number like 24_8, 109_8, 55_8, etc
- The octal numbers, in the number system, are usually represented by binary numbers when they are grouped in pairs of three.
- For example, an octal number 12_8 is expressed as 001010_2 in the binary system, where 1 is equivalent to 001 and 2 is equivalent to 010.
- Solve an octal number, each place is a power of eight.

$124_8 = 1 \times 8^2 + 2 \times 8^1 + 4 \times 8^0$

- Only 3 bits are used to represent Octal Numbers. Each group will have a distinct value between 000 and 111

Note: Beyond 7, such as 8 and 9 are not octal digits. For example, 19 is not an octal number.

Decimal to Octal Number

the octal dabble method is used- decimal number is divided by 8 each time, it yields or gives a remainder.

The first remainder we get is the least significant digit(LSD) and the last remainder is the most significant digit(MSD)

- Ex: 560 is a decimal number, convert it into an octal number.

Solution: If 560 is a decimal number, then,

560/8 = 70 and the remainder is 0

70/8 = 8 and the remainder is 6

8/8 = 1 and the remainder is 0

And 1/8 = 0 and the remainder is 1

So the octal number starts from MSD to LSD, i.e. 1060

Therefore, $560_{10} = 1060_8$

Octal Digital Value	Binary Equivalent
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Decimal to Octal Number

Convert 0.52 into an octal number.

- **Solution:** The fraction part of the decimal number has to be multiplied by 8.
- 0.52 × 8 = 0.16 with carry 4
- 0.16 × 8 = 0.28 with carry 1
- 0.28 × 8 = 0.24 with carry 2
- 0.24 × 8 = 0.92 with carry 1
- So, for the fractional octal number, we read the generated carry from up to down.
- Therefore,4121 is the octal number.

Octal to Decimal

- multiply each digit of the given octal with the reducing power of 8.

Example 1: Suppose 215_8 is an octal number, then it's decimal form will be,

$215_8 = 2 \times 8^2 + 1 \times 8^1 + 5 \times 8^0$

$= 2 \times 64 + 1 \times 8 + 5 \times 1 = 128 + 8 + 5$

$= 141_{10}$

Example 2: Let 125 is an octal number denoted by 125_8. Find the decimal number.

$125_8 = 1 \times 8^2 + 2 \times 8^1 + 5 \times 8^0$

$= 1 \times 64 + 2 \times 8 + 5 \times 1 = 64+16+5$

$=85_{10}$

Binary To Octal Number

Convert $(100010)_2$ to an octal number

100---4

010---2

Therefore, $(100010)_2 = 42$

Octal Digital Value	*Binary Equivalent*
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Octal to Hexadecimal Number

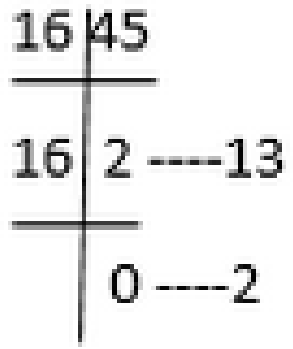
- Conversion of the octal number to hexadecimal requires two steps.
- First, convert octal numbers to decimal numbers.
- Then, convert decimal numbers to hexadecimal numbers.

Example

Convert the octal numbers to decimal

$(55)_8 = (45)_{10}$

Convert $(45)_{10}$ into a hexadecimal number by dividing 45 by 16 until you get a remainder less than 16.



$(45)_{10} = (2D)_{16}$

Or $(55)_8 = (2D)_{16}$

Practice Questions

Convert the following octal numbers into their equivalent decimal number.

1. 23_8 (Answer: 19_{10})
2. 770_8 (Answer: 504_{10})
3. 152_8 (Answer: 106_{10})

Convert the following octal numbers into hexadecimal numbers.

1. 23_8 (Answer: 13_{16})
2. 770_8 (Answer: $1F8_{16}$)
3. 152_8 (Answer: $6A_{16}$)

Hexadecimal Number System

- Has a base value equal to 16. It is also pronounced sometimes as **‘hex’**. Hexadecimal numbers are represented by only 16 symbols. These symbols or values are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F.
- Each digit represents a decimal value. For example, D is equal to base-10 13.

Decimal Numbers	4-bit Binary Number	Hexadecimal Number
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Decimal to Hexadecimal Conversion

- Divide the number by 16
- Take the quotient and divide again by 16
- The remainder left will produce the hex value
- Repeats the steps until the quotient has become 0

Example: Convert $(242)_{10}$ into hexadecimal.

Solution: Divide 242 by 16 and repeat the steps, till the quotient is left as 0.

16	242	
16	15	$2 \rightarrow 2$
	0	$15 \rightarrow F$

$(242)_{10} = (F2)_{16}$

Octal to Hexadecimal Conversion

To convert octal to hex, we have to first convert octal number to decimal and then decimal to hexadecimal.

- **Example:** Convert $(121)_8$ into hexadecimal.
- **Solution:** First convert 121 into decimal number.
- $\Rightarrow 1 \times 8^2 + 2 \times 8^1 + 1 \times 8^0$
  $\Rightarrow 1 \times 64 + 2 \times 8 + 1 \times 1$
  $\Rightarrow 64 + 16 + 1$
  $\Rightarrow 81$

$(121)_8 = 81_{10}$

Now converting 81_{10} into a hexadecimal number.

Therefore, $81_{10} = 51_{16}$

16	81	
16	5	$1 \rightarrow 1$
	0	$5 \rightarrow 5$

Hexadecimal to Binary Conversion

Use only 4 digits to represent each hexadecimal number, where each group has a distinct value from 0000 (for 0) and 1111 (for F= 15 =8 + 4 + 2 + 1)

Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

Binary to Hexadecimal Conversion

- Binary to hexadecimal conversion is a simple method to do. You just have to put the values of the binary number to the relevant hexadecimal number.
- **Example:** Convert $(11100011)_2$ to hexadecimal.
  Solution: From the table, we can write, 11100011 as E3.

Therefore, $(11100011)_2 = (E3)_{16}$