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DEPARTMENT OF MATHEMATICS

Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS	TEST-I	Maximum marks: 50
Course code: 22MA21C	Second semester 2022-2023 Physics Cycle Branch: AI, BT, CD, CS, CY, IS, SPARK-C	Time: 9:30AM-11:00AM Date: 10-07-2023

Sl. No.	Questions	M	BT	CO
1	Details regarding marks scored by 280 candidates in an examination are given by the following table. Using Newton- Gregory interpolation formula estimate the number of candidates who secured marks between 45 and 65. Marks: Below 30, 30-40, 40-50, 50-60, 60-70, 70-80 Number of Students: 35, 49, 62, 74, 40, 20	10	L2	2
2. (a)	The following table gives the relation between steam pressure and temperature. T°C: 361, 367, 378, 387, 399 P: 154.9, 167.9, 191, 212.5, 244.2 Using suitable interpolation formula find the pressure at the temperature 372°C.	6	L2	3
2. (b)	Given the following table of values of x and y, find using inverse interpolation the value of x when $y = 100$. x: 1, 3, 5, 8 y: 10, 24, 54, 129	4	L2	3
3. (a)	The following table gives corresponding values of pressure p and specific volume v of superheated steam: v: 2, 4, 6, 8, 10 p: 105, 42.7, 25.3, 16.7, 13 Find the rate of change of p with respect to v at $v = 4$ and $v = 8$.	6	L2	1
3. (b)	Solve $\frac{d^3x}{dt^3} - 8\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 50x = 0$.	4	L1	1
4	Obtain the general solution of the differential equation: $2\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = e^{\frac{x}{2}} + \sin^2(2x) + x^2$.	10	L3	2
5. (a)	Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = e^{2x}\sin(3x)$.	5	L3	2
5. (b)	Solve the initial value problem $\frac{d^2x}{dt^2} + \mu x = 0$ $(\mu > 0)$ given that $x = a$ and $\frac{dx}{dt} = 0$ when $t = \frac{\pi}{\sqrt{\mu}}$.	5	L2	3

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	10	25	15	00	04	31	15	--	--	--

DEPARTMENT OF MATHEMATICS

Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS	**CIE-II**	**Maximum marks: 50**
Course code: 22MA21C	**Second semester 2022-2023** **Physics Cycle** **Branch: AI, BT, CD, CS, CY, IS, SPARK-C**	**Time: 10:00AM-11:30AM** **Date: 21-08-2023**

Sl. No.	**Questions**	M	BT	CO
1	Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan(x)$.	10	L3	3
2	Reduce the differential equation $x\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4\frac{y}{x} = \log_e(x)$, where $x > 0$, to a linear differential equation with constant coefficients and hence solve.	10	L3	3
3. (a)	The current in an LRC circuit is governed by the differential equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E(t)$. A circuit in series has an electromotive force given by $E(t) = 0V$, a resistor of 10Ω, an inductor of $0.25H$ and a capacitor of 0.001F. If the initial current and the initial charge on the capacitor are both zero, determine the charge on the capacitor at any time $t > 0$.	6	L2	2
3. (b)	Find all the solutions of the linear congruence $6x \equiv 15\ (mod\ 21)$.	4	L2	2
4. (a)	By using the Euclidean algorithm, determine the greatest common divisor d of 2947 and 3997 and find integers x and y to satisfy $2947x + 3997y = d$.	7	L1	1
4. (b)	Compute the last two digits of the number 87^{474}.	3	L2	2
5	Given the public key $(e, n) = (11,65)$, encrypt plain text J B E, where the alphabets $A, B, C, \ldots X, Y, Z$ are assigned the numbers $2,3,\cdots,26,27$. Give the cipher text. Find the private key d.	10	L3	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	7	13	20	10	7	13	30	--	--	--

DEPARTMENT OF MATHEMATICS

Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS	**IMPROVEMENT CIE**	**Maximum marks: 50**
Course code: 22MA21C	**Second semester 2022-2023** **Physics Cycle** **Branch: AI, BT, CD, CS, CY, IS, SPARK-C**	**Time: 02:00PM-3:30PM** **Date: 06-09-2023**

Sl. No.	Questions	M	BT	CO
1. (a)	A particle moves along the curve $x = t^3 + 1,\ y = t^2,\ z = 2t + 3$, where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $\hat{\imath} + \hat{\jmath} + 3\hat{k}$.	5	L1	1
1. (b)	If $\vec{f} = \nabla(2x^3y^2z^4)$, then find $div(\vec{f})$ at $(1,2,-1)$.	5	L2	2
2. (a)	Find the values of the constants a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1,2,-1)$ has maximum of magnitude 64 in a direction parallel to the z-axis.	6	L3	3
2. (b)	If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = \lvert\vec{r}\rvert$, then show that $\nabla r^n = nr^{n-2}\vec{r}$.	4	L3	3
3	Find the values of the constants a, b, c such that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is conservative. Also find its scalar potential.	10	L3	4
4	The following table gives the temperature θ of a cooling body at different instant of time t (in seconds) <table><tr><td>t</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr><tr><td>θ</td><td>85.3</td><td>74.5</td><td>67</td><td>60.5</td><td>54.3</td></tr></table>Calculate θ at $t = 2$ and $t = 8$ using suitable interpolation formula. Also find approximate rate of cooling at $t = 9$ seconds.	10	L2	2
5. (a)	Using suitable interpolation formula find $y(11)$ for the following data <table><tr><td>x</td><td>6</td><td>7</td><td>10</td><td>12</td><td>15</td></tr><tr><td>y</td><td>3</td><td>10</td><td>43</td><td>75</td><td>138</td></tr></table>	6	L2	3
5. (b)	Given the following table of values of x and y, find by using inverse interpolation the value of x when $y = 100$. <table><tr><td>x</td><td>1</td><td>3</td><td>5</td><td>8</td></tr><tr><td>y</td><td>10</td><td>24</td><td>54</td><td>129</td></tr></table>	4	L2	3

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	5	15	20	10	05	25	20	--	--	--

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DEPARTMENT OF MATHEMATICS

Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS	TEST-II	Maximum marks: 50
Course code: 22MA21C	Second semester 2022-2023 Physics Cycle Branch: AI, BT, CD, CS, CY, IS, SPARK-C	Time: 10:00AM-11:30AM Date: 21-08-2023

Scheme and Solutions

Q.No	PART -B	Marks
1.	The auxiliary equation is $m^2 - 2m + 2 = 0$ and roots are $m = 1 \pm i$	1
	$CF = e^x(c_1 \cos(x) + c_2 \sin(x))$ and $PI = Au + Bv$	1
	$W = \begin{vmatrix} e^x \cos(x) & e^x \sin(x) \\ e^x(-\sin(x) + \cos(x)) & e^x(\sin(x) + \cos(x)) \end{vmatrix} = e^{2x}$	2
	$A = -\int \frac{vf(x)}{W} dx = -\int \frac{e^x \sin(x)\, e^x \tan(x)}{e^{2x}} dx$	
	$A = -\int \frac{\sin^2(x)}{\cos(x)} dx = -[\log_e(\sec(x)) + \tan(x)) - \sin(x)]$	3
	$B = \int \frac{uf(x)}{W} dx = \int \frac{e^x \cos(x)\, e^x \tan(x)}{e^{2x}} dx = \int \sin(x)\, dx = -\cos(x)$	2
	$y = e^x(c_1 \cos(x) + c_2 \sin(x)) - e^x \cos(x) \log_e(\sec(x) + \tan(x))$	1
2.	Given equation is converted to linear differential equation with constant coefficients by substituting $x = e^z$ or $z = \log_e(x)$ and $xD = D_1, x^2D^2 = D_1(D_1 - 1)$ where $D_1 = \frac{d}{dz}$	2
	$(D_1^2 + 4D_1 + 4)y = ze^z$	
	The auxiliary equation is $m^2 + 4m + 4 = 0$ and roots are $m = -2, -2$	
	$CF = \frac{(c_1 + c_2 \log_e(x))}{x^2}$	2
	$PI = \frac{1}{D_1^2 + 4D_1 + 4} e^z z = e^z \frac{1}{D_1^2 + 6D_1 + 9} z = \frac{e^z}{9}\left(1 + \frac{D_1}{3}\right)^{-2} z = \frac{e^z}{9}\left(z - \frac{2}{3}\right)$	1+3+1
	$PI = \frac{x}{9}\left(\log_e(x) - \frac{2}{3}\right)$	
	$y = CF + PI$	1
3(a)	$q'' + 40q' + 4000q = 0$	1
	Auxiliary equation $m^2 + 40m + 4000 = 0$	
	Roots: $m = -20 \pm 60i$	1
	$q = e^{-20t}(c_1 \cos(60t) + c_2 \sin(60t))$	1
	$c_1 = 0, c_2 = 0$	1+1
	$q(t) = 0$	1

3(b)	$21 \mid 6x - 5 \Rightarrow 6x - 15 = 21k \Rightarrow x = \dfrac{21k + 15}{6}$ $x = 6\ (mod\ 21), \qquad x = 13\ (mod\ 21), \qquad x = 20\ (mod\ 21)$	1 3
4(a)	$3997 = 1 \times 2947 + 1050$ $2947 = 2 \times 1050 + 847$ $1050 = 1 \times 847 + 203$ $847 = 4 \times 203 + 35$ $203 = 5 \times 35 + 28$ $35 = 1 \times 28 + 7$ $28 = 4 \times 7 + 0$ $\gcd(2947,3997) = 7$ Linear combination: $7 = 1 \times 35 - 1 \times 28$ $7 = 1 \times 35 - 1 \times (203 - 5 \times 35)$ $7 = -1 \times 203 + 6 \times 35$ $7 = -1 \times 203 + 6 \times (847 - 4 \times 203)$ $7 = 6 \times 847 - 25 \times 203$ $7 = 6 \times 847 - 25 \times (1050 - 1 \times 847)$ $7 = -25 \times 1050 + 31 \times 847$ $7 = -25 \times 1050 + 31 \times (2947 - 2 \times 1050)$ $7 = 31 \times 2947 - 87 \times 1050$ $7 = 31 \times 2947 - 87 \times (3997 - 1 \times 2947)$ $7 = -87 \times 3997 + 118 \times 2947$ $7 = -87 \times 3997 + 118 \times (2947 + 0 \times 3997)$ $7 = 118 \times 2947 - 87 \times 3997$ $7 = 2947(118) + 3997(-87)$ $x = 118, y = -87$	4 3
4(b)	$\gcd(87,100) = 1$ By Euler's theorem, $87^{\phi(100)} \equiv 1\ (mod\ 100) \Rightarrow 87^{40} \equiv 1\ (mod 100)$ $87^{440} \equiv 1\ (mod 100)$ $87^{3} \equiv 3\ (mod 100)$ $87^{33} \equiv 47 (mod\ 100)$ $87^{474} \equiv 1 \times 47 \times 87\ (mod\ 100)$ $87^{474} \equiv 89\ (mod\ 100)$	1 2
5	$e = 11, \qquad n = 65, \qquad p = 5, \qquad q = 13$ $\phi(65) = 48$ $c_J = 11^{11} (mod\ 65) = 6$ $C_B = 3^{11} (mod\ 65) = 22$ $C_E = 6^{11} (mod\ 65) = 11$ $de \equiv 1\ (mod\ 48)$ $d = 35$	1 1 2 2 2 2

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Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS	TEST-I	Maximum marks: 50
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Scheme and Solutions

<table>
<tr><th>Q.No</th><th>PART -B</th><th>Marks</th></tr>
<tr><td>1.</td><td>
Cumulative frequency table

Marks less than (x)	30	40	50	60	70	80
Number of students (y)	35	84	146	220	260	280

Difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
30	35					
		49				
40	84		**13**			
		62		**-1**		
50	146		12		**-45**	
		74		-46		**105**
60	220		-34		**60**	
		40		**14**		
70	260		**-20**			
		20				
80	280					

$p=\frac{x-x_0}{h}=\frac{45-30}{10}=1.5$ and $p=\frac{x-x_n}{h}=\frac{65-80}{10}=-1.5$

Using Newton's forward interpolation formula, we get

$$y=y_0+p\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0+\cdots$$

$y(45)=111.15\approx 111$

Using Newton's backward interpolation formula, we get

$$y=y_0+p\nabla y_n+\frac{p(p+1)}{2!}\nabla^2 y_n+\frac{p(p+1)(p+2)}{3!}\nabla^3 y_n+\cdots$$

$y(65)=246.0117\approx 246$

The number of students with marks between 45 and 65 is $246-111=135$.
</td><td>1
3
1
2
2
1</td></tr>
<tr><td>2(a)</td><td>

$$y=\frac{(x-367)(x-378)(x-387)(x-399)}{(361-367)(361-378)(361-387)(361-399)}154.9+\frac{(x-361)(x-378)(x-387)(x-399)}{(367-361)(367-378)(367-387)(367-399)}167.9+\frac{(x-361)(x-367)(x-387)(x-399)}{(378-361)(378-367)(378-387)(378-399)}191+\frac{(x-361)(x-367)(x-378)(x-399)}{(387-361)(387-367)(387-378)(387-399)}212.5+\frac{(x-361)(x-367)(x-378)(x-387)}{(399-361)(399-367)(399-378)(399-387)}244.2$$

$y(x)=-0.00002x^4+0.0314x^3-17.9744x^2+4568.21x-435168=178.1827$
</td><td>2
4</td></tr>
</table>

<table>
<tr><td>2(b)</td><td>

$$x=\frac{(y-24)(y-54)(y-129)}{(10-24)(10-54)(10-129)}+\frac{(y-10)(y-54)(y-129)}{(24-10)(24-54)(24-129)}3+\frac{(y-10)(y-24)(y-129)}{(54-10)(54-24)(54-129)}5+\frac{(y-10)(y-24)(y-54)}{(129-10)(129-24)(129-54)}8$$

When $y=100, x=5.9199$</td><td>2
2</td></tr>
<tr><td>3(a)</td><td>Difference table

<table>
<tr><td>v</td><td>p</td><td>Δp</td><td>$\Delta^2 p$</td><td>$\Delta^3 p$</td><td>$\Delta^4 p$</td></tr>
<tr><td>2</td><td>105</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>-62.3</td><td></td><td></td><td></td></tr>
<tr><td>4</td><td>42.7</td><td></td><td>44.9</td><td></td><td></td></tr>
<tr><td></td><td></td><td>-17.4</td><td></td><td>-36.1</td><td></td></tr>
<tr><td>6</td><td>25.3</td><td></td><td>8.8</td><td></td><td>32.2</td></tr>
<tr><td></td><td></td><td>-8.6</td><td></td><td>-3.9</td><td></td></tr>
<tr><td>8</td><td>16.7</td><td></td><td>4.9</td><td></td><td></td></tr>
<tr><td></td><td></td><td>-3.7</td><td></td><td></td><td></td></tr>
<tr><td>10</td><td>13</td><td></td><td></td><td></td><td></td></tr>
</table>

$$\left(\frac{dp}{dv}\right)_{v=4}=\frac{1}{h}\left[\Delta p_0-\frac{1}{2}\Delta^2 p_0+\frac{1}{3}\Delta^3 p_0-\frac{1}{4}\Delta^4 p_0+\cdots\right]=\frac{1}{2}\left[-17.4-\frac{8.8}{2}-\frac{3.9}{3}\right]=-11.55$$

$$\left(\frac{dp}{dv}\right)_{v=8}=\frac{1}{h}\left[\nabla p_n+\frac{1}{2}\nabla^2 p_n+\frac{1}{3}\nabla^3 p_n+\frac{1}{4}\nabla^4 p_n+\cdots\right]=\frac{1}{2}\left[-8.6+\frac{8.8}{2}-\frac{36.1}{3}\right]=-8.11$$
</td><td>2
2
2</td></tr>
<tr><td>3(b)</td><td>Auxiliary equations $m^3-8m^2+5m+50=0$
Roots $m=-2,5,5$
$x=c_1e^{-2t}+(c_2+c_3t)e^{5t}$</td><td>1
1
2</td></tr>
<tr><td>4</td><td>Auxiliary equations $2m^2+m-1=0$, Roots $m=\frac{1}{2},-1$
$C.F=c_1e^{x/2}+c_2e^{-x}$
$PI=\frac{xe^{x/2}}{3}-\frac{1}{2}+\frac{33\cos(4x)-4\sin(4x)}{2210}-(x^2+2x+6)$
$y=CF+PI$</td><td>1
1
2+3+2
1</td></tr>
<tr><td>5(a)</td><td>Auxiliary equations $m^2-4m+13=0$, Roots $m=2\pm 3i$
$C.F=e^{2x}(c_1\cos(3x)+c_2\sin(3x))$
$PI=\frac{1}{D^2-4D+13}e^{2x}\sin(3x)=e^{2x}\frac{1}{D^2+9}\sin(3x)=-\frac{xe^{2x}\cos(3x)}{6}$
$y=CF+PI=e^{2x}(c_1\cos(3x)+c_2\sin(3x))-\frac{xe^{2x}\cos(3x)}{6}$</td><td>1
1
2
1</td></tr>
<tr><td>5(b)</td><td>Auxiliary equations $m^2+\mu=0$, Roots $m=\pm\sqrt{\mu}i$
$x=c_1\cos(\sqrt{\mu}t)+c_2\sin(\sqrt{\mu}t)$
$x'=-c_1\sqrt{\mu}\sin(\sqrt{\mu}t)+c_2\sqrt{\mu}\cos(\sqrt{\mu}t)$
$c_1=-a$ and $c_2=0$
$x=-a\cos(\sqrt{\mu}t)$</td><td>1
1
2
1</td></tr>
</table>

DEPARTMENT OF MATHEMATICS

Course: NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS	**IMPROVEMENT CIE**	**Maximum marks: 50**
Course code: 22MA21C	**Second semester 2022-2023 Physics Cycle Branch: AI, BT, CD, CS, CY, IS, SPARK-C**	**Time: 02:00PM-3:30PM Date: 06-09-2023**

Sl. No.	**Questions**	Marks
1. (a)	$\frac{d\vec{r}}{dt} = 3t^2\hat{\imath} + 2t\hat{\jmath} + 2\hat{k} \Rightarrow \left(\frac{d\vec{r}}{dt}\right)_{t=1} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ $\frac{d^2\vec{r}}{dt^2} = 6t\hat{\imath} + 2t\hat{\jmath} + 0\hat{k} \Rightarrow \left(\frac{d^2\vec{r}}{dt^2}\right)_{t=1} = 6\hat{\imath} + 2\hat{\jmath}$ Component of velocity $= \frac{11}{\sqrt{11}}$ Component of acceleration $= \frac{8}{\sqrt{11}}$	2 1 1 1
1. (b)	$\nabla\phi = 6x^2y^2z^4\hat{\imath} + 4x^3yz^4\hat{\jmath} + 8x^3y^2z^3\hat{k}$ $div(\vec{f}) = 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2$ $div(\vec{f})_{(1,2,-1)} = 48 + 4 + 96 = 148$	2 2 1
2. (a)	$\nabla\phi = (ay^2 + 3cz^2x^2)\hat{\imath} + (2axy + bz)\hat{\jmath} + (by + 2czx^3)\hat{k}$ $\nabla\phi_{(1,2,-1)} = (4a + 3c)\hat{\imath} + (4a - b)\hat{\jmath} + (2b - 2c)\hat{k}$ Directional derivative of ϕ at $(1,2,-1)$ in the direction parallel to z axis is given by $\nabla\phi.\hat{k} = 64$ $\Rightarrow 2b - 2c = 64$ and $4a + 3c = 0, \quad 4a - b = 0$ Solving $a = 6,\ b = 24, c = -8$.	2 1 1+1+1
2. (b)	$r^2 = x^2 + y^2 + z^2$ and $\frac{\partial r}{\partial x} = \frac{x}{r}, \frac{\partial r}{\partial y} = \frac{y}{r}, \frac{\partial r}{\partial z} = \frac{z}{r}$ $\nabla r^n = nr^{n-1}\frac{x}{r}\hat{\imath} + nr^{n-1}\frac{y}{r}\hat{\jmath} + nr^{n-1}\frac{z}{r}\hat{k} = nr^{n-2}\vec{r}$	1 1+2
3	$curl\,\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x + 2y + az & bx - 3y - z & 4x + cy + 2z \end{vmatrix} = (c+1)\hat{\imath} - (4-a)\hat{\jmath} + (b-2)\hat{k} = \vec{0}$ $a = 4\,, b = 2$ and $c = -1$ $\vec{F} = (x + 2y + 4z)\hat{\imath} + (2x - 3y - z)\hat{\jmath} + (4x - y + 2z)\hat{k}$ $\frac{\partial\phi}{\partial x} = x + 2y + 4z,\ \frac{\partial\phi}{\partial y} = 2x - 3y - z,\ \frac{\partial\phi}{\partial z} = 4x - y + 2z$	1 3 1

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz$$ 1

$$d\phi = d\left(\frac{x^2}{2}\right) + d(2xy) + d(4xz) - d\left(\frac{3}{2}y^2\right) - d(yz) + d(z^2)$$ 3

$$\phi = \frac{x^2}{2} + 2xy + 4xz - \frac{3}{2}y^2 - yz + z^2 + c$$ 1

4

Difference table

t	θ	$\Delta\theta$	$\Delta^2\theta$	$\Delta^3\theta$	$\Delta^4\theta$
1	85.3				
		-10.8			
3	74.5		**3.3**		
		-7.5		**-2.3**	
5	67		1		**1.6**
		-6.5		**-0.7**	
7	60.5		**0.3**		
		-6.2			
9	54.3				

3

$p = \frac{t-t_0}{h} = \frac{2-1}{2} = 0.5$ and $p = \frac{t-t_n}{h} = \frac{8-9}{2} = -0.5$ 1

Using Newton's forward interpolation formula, we get

$$y = \theta_0 + p\Delta\theta_0 + \frac{p(p-1)}{2!}\Delta^2\theta_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3\theta_0 + \cdots$$

$\theta(2) = 79.28125$ 2

Using Newton's backward interpolation formula, we get

$$y = \theta_n + p\nabla\theta_n + \frac{p(p+1)}{2!}\nabla^2\theta_n + \frac{p(p+1)(p+2)}{3!}\nabla^3\theta_n + \cdots$$

$\theta(8) = 57.34375$ 2

$$\frac{d\theta}{dt} = \frac{1}{h}\left[\nabla\theta_n + \frac{1}{2}\nabla^2\theta_n + \frac{1}{3}\nabla^3\theta_n + \frac{1}{4}\nabla^4\theta_n + \cdots\right] = \frac{1}{2}\left[-6.2 + \frac{0.3}{2} - \frac{0.7}{3} + \frac{1.6}{4}\right] = -2.9416$$ 2

5. (a)

$$= \frac{(x-7)(x-10)(x-12)(x-15)}{(6-7)(6-10)(6-12)(6-15)}3 + \frac{(x-6)(x-10)(x-12)(x-15)}{(7-6)(7-10)(7-12)(7-15)}10 + \frac{(x-6)(x-7)(x-12)(x-15)}{(10-6)(10-7)(10-12)(10-15)}43 + \frac{(x-6)(x-7)(x-10)(x-15)}{(12-6)(12-7)(12-10)(12-15)}75 + \frac{(x-6)(x-7)(x-10)(x-12)}{(15-6)(15-7)(15-10)(15-12)}138$$ 3

$y(x) = x^2 - 6x + 3$ 2

$y(11) = 58$ 1

5. (b)

$$x = \frac{(y-24)(y-54)(y-129)}{(10-24)(10-54)(10-129)} + \frac{(y-10)(y-54)(y-129)}{(24-10)(24-54)(24-129)}3 + \frac{(y-10)(y-24)(y-129)}{(54-10)(54-24)(54-129)}5 + \frac{(y-10)(y-24)(y-54)}{(129-10)(129-24)(129-54)}8$$ 2

When $y = 100, x = 5.9199$ 2

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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

II Semester B. E. Examinations Oct-2023

(Common to AI, BT, CS, CY, CD & IS)

NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Formula book to be provided.

PART-A

1	1.1	General solution of $4x \equiv 7 \bmod 5$ is ________.	01
	1.2	The number of integers less than 181 that are relatively prime to 181 is ________.	01
	1.3	If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\lvert\vec{r}\rvert = r$, then $\nabla r =$ ________.	01
	1.4	If $\vec{F}(t)$ has a constant magnitude then $\vec{F}(t).\frac{d\vec{F}(t)}{dt}$ is ________.	01
	1.5	If $\vec{F}$ represent the velocity of fluid, the $\oint_c \vec{F}.d\vec{r}$ represents ________.	01
	1.6	If R is the projection of surface in XY –plane, then $ds =$______.	01
	1.7	Particular integral of $\frac{d^2x}{dt^2} + \frac{6}{7}\left(x - \sqrt{2}\right) = 0$ is ________.	01
	1.8	If the roots of the auxiliary equation are $\pm i$ and 2 then the corresponding differential equation is ______.	01
	1.9	The 3^{rd} order difference of 3^{rd} degree polynomial is ______.	01
	1.10	The value of $\Delta^3[(1-x)(1-3x)(1-5x)]$ taking the interval of differencing $h = 1$ is ________.	01
	1.11	The sum of all positive divisors of 8620 is ________.	02
	1.12	The velocity and acceleration of a particle along a curve $x = t^2$, $y = 3t^2$, $z = e^t$ at $t = 1$ is ________.	02
	1.13	If $\vec{F} = (x+y)\hat{i} + (2x-y)\hat{j}$ evaluate $\int_c \vec{F}.d\vec{r}$ along the straight line from $A(0,0)$ to $B(0,1)$.	02
	1.14	If $y = e^{-t}$ is the solution of the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + py = 0$, then the value of p is ________.	02
	1.15	Using suitable interpolation, fit a polynomial for the data. x: −1, 2, 4 y: −5, 4, 0	02

PART-B

2	a	Find the greatest common divisor d of the number 4076 and 1024 using Euclid's algorithm and then obtain the integers x and y to satisfy $4076x + 1024y = d$.	06
	b	Given the public key $(e, n) = (7, 55)$, encrypt plain text $M\,I\,T$, where the alphabets $\{A, B, C, \ldots.X, Y, Z\}$ are assigned the numbers $\{5, 6, \ldots.29, 30\}$. Find the cipher text and the private key d.	06
	c	Compute the remainder when 3^{247} is divided by 17.	04
3	a	Find the angle between the tangents to the curve $x = \left(t - \frac{t^2}{2}\right)$, $y = t^2$ and $z = \left(t + \frac{t^2}{2}\right)$ at $t = \pm 1$.	08
	b	Prove that $div(r^n\vec{r}) = (n+3)r^n$. Hence show that $\vec{r}/r^3$ is solenoidal.	08

OR

4 a Show that an electromagnetic field $\vec{f} = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative and find the scalar potential ϕ such that $\vec{f} = \nabla\phi$. Also show that $div\,\vec{f} = \nabla^2\phi$. **10**

b Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(1,-2,-1)$ in the direction of the normal to the surface $x\,log_e z - y^2 = -4$ at $(-1,2,1)$. **06**

5 a Using line integral, compute the work done by a force $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ when it moves a particle from the point $(0,0,0)$ to $(2,1,3)$ along the curve $x = 2t^2,\ y = t,\ z = 4t^2 - t$ from $t = 0$ to 1. **08**

b Evaluate using divergence theorem:
$\iint_s \left((x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}\right).\hat{n}\,ds$ where S is the surface of the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. **08**

OR

6 a Verify Green's theorem in the plane for $\oint\{(x^2 + y)dx - xy^2dy\}$ taken around the boundary of the rectangle whose vertices are $(0,0)$, $(a,0)$, (a,b) and $(0,b)$. **08**

b Evaluate by Stokes theorem $\oint_c ((x + y)dx + (2x - z)dy + (y + z)dz)$, C is the boundary of the triangular surface with vertices $(0,0,0)$, $(1,0,0)$ and $(1,1,0)$. **08**

7 a Obtain the radial displacement x in a rotating disc at a distance s from the axis, given by the differential equation $\frac{d^2x}{ds^2} + \frac{1}{s}\frac{dx}{ds} + \frac{x}{s^2} = \frac{\log_e s}{s^2}\sin(\log_e s)$ where $s > 0$. **08**

b Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ using method of variation of parameters. **08**

OR

8 a Obtain the solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \cos^2 x$ given that $y(0) = \frac{1}{9}$, $y'(0) = \frac{2}{9}$ **08**

b Solve $(D^2 - 1)y = \sin x + (1 + x^2)e^x$ **08**

9 a The following table gives the values of pressure P and specific volume V of saturated steam:

V	40	50	60	70	80
P	304	276	250	204	184

Find the rate of change of pressure with respect to volume at $V = 50$ and $\frac{d^2V}{dP^2}$ at $V = 70$. **08**

b The velocity of a rocket as a function of time is given as follows: $v(1) = 2,\ v(3) = 10,\ v(4) = 17$. Obtain the functional representation of velocity as a function of time using Lagrange's interpolation formula. Also find the velocity at time 3.8 *units*. **08**

OR

10 a The population of a town is given by the table

Years	1961	1971	1981	1991	2001
Population in thousands	19.96	39.65	58.81	77.21	94.61

Using appropriate interpolation formula, calculate the increase in population from the year 1955 to 1985. **08**

b Apply Lagrange's formula inversely to find a root of the equation $f(x) = 0$ given that $f(30) = -30$, $f(34) = -13$, $f(38) = 3$, $f(42) = 18$. **08**