USN | | | | | | | | |

RV COLLEGE OF ENGINEERING®
Autonomous Institution affiliated to VTU
II Semester B. E. Examinations July - Aug 2024

NUMBER THEORY, VECTOR CALCULUS AND COMPUTATIONAL METHODS
MODEL QUESTION PAPER
Branch: AI, BT, CD, CS, CY, IS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.

PART – A

1	1.1	For what value of b, the vector $\vec{F} = y(bx^2 + z)\hat{\imath} + x(y^2 - z^2)\hat{\jmath} + 2xy(z - xy)\hat{k}$ is solenoidal.	01
	1.2	If $\phi = x^2y + y^2z + 4$, then $\nabla\phi =$______________.	01
	1.3	Particular integral of $2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2y = e^{3x}$ is__________.	01
	1.4	If $x = e^{-3t}$ is the solution of the equation $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + kx = 0$ then value of k is __________.	01
	1.5	The value of $\Delta^3[(1 + 3x)(1 - 5x)(1 - 4x)]$ taking interval of differencing $h = 1$ is ______.	01
	1.6	Construct a forward difference table for the data $(0,2), (2,6), (4,7)$.	01
	1.7	If $\vec{F}$ represents force acting on a particle then $\int_c \vec{F}.d\vec{r}$ represents __________.	01
	1.8	If $\vec{F}$ is velocity of fluid then $\iint_S \vec{F}.\hat{n}ds$ represents _________.	01
	1.9	Calculate the sum of positive divisors of the integer 882.	02
	1.10	Find the remainder when $135 \times 74 \times 48$ is divided by 7.	02
	1.11	Find the directional derivative of $\phi = 3x^2 + 2y - 3z$ at $(1,1,1)$ in the direction of $2\hat{\imath} + 2\hat{\jmath} - \hat{k}$.	02
	1.12	If $\vec{F} = (x^2 + y)\hat{\imath} + (3y - 5x)\hat{\jmath}$, evaluate $\int_c \vec{F}.d\vec{r}$ along the straight line from $A(0,0)$ to $B(2,0)$.	02
	1.13	The complementary function of the differential equation is $c_1cos3x + c_2sin3x$, then Wronskian is __________.	02
	1.14	Given	02

		x: 1, 3, 5, 7; $f(x)$: 2, 7, 16, 29 The value of $f'(7) =$____________.	

x	1	3	5	7
$f(x)$	2	7	16	29

The value of $f'(7) =$____________.

PART – B

2	a	Find the greatest common divisor d of the numbers 1819 and 3587 using Euclid's algorithm and then obtain the integers x and y to satisfy $1819\,x + 3587\,y = d$.	6
	b	Solve the linear congruence $6x \equiv 15\ (mod\ 21)$.	4
	c	Given the public key $(e, n) = (7,51)$, encrypt plain text $L\ I\ V$, where the alphabets $A, B, C, \cdots X, Y, Z$ are assigned the numbers $3,4,5, \cdots ,26,27,28$. Give the cipher text and find the private key d.	6

3	a	Find the unit tangent vector s to the curve $\hat{r} = 4\sin t\,\hat{\imath} + 4\cos t\,\hat{\jmath} + 3t\,\hat{k}$ at the points $t = \frac{\pi}{6}$ and $t = \frac{\pi}{4}$. Obtain the angle between these tangent vectors.	5
	b	If $\hat{F} = \nabla(x^2y + y^2z + z^2x - xyz)$, then compute $div\,\hat{F}$ and $curl\,\hat{F}$ at $(1,2,1)$.	5
	c	Find the constants a and b such that the surfaces $ax^2 - byz = ax + 2x$ and $4x^2y + z^3 = 4$ are orthogonal at the point $(1, -1,2)$.	6
		OR	
4	a	A particle moves along the curve C: $x = (2t^2 + 1), y = (5t - 3)$ and $z = (t^2 - 5t)$; where t is the time. Find the components of velocity and acceleration at $t = 2$ in the direction of $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.	5
	b	Show that the vector $r^n\hat{r}$ is irrotational for any constant n and solenoidal only for $n = -3$.	5
	c	Compute the values of the constants a, b, c such that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is curl free vector. Determine the scalar potential function ϕ such that $\vec{F} = \nabla\phi$.	6

5	a	Verify Green's theorem in the plane for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where c is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$.	8
	b	Evaluate $\iint_S \vec{F}.\hat{n}\,ds$, if $\vec{F} = 4y\hat{\imath} + 18z\hat{\jmath} - x\hat{k}$ and S is the surface of the plane $3x + 2y + 6z = 6$ contained in the first octant.	8
		OR	
6	a	If $\vec{F} = (2x + y^2)\hat{\imath} + (3y - 4x)\hat{\jmath}$, evaluate $\oint_C \vec{F}.d\vec{r}$ around the triangle in the xy plane with vertices $(0,0), (2,0)$ and $(2,1)$.	8
	b	Use Stokes theorem to evaluate $\int_c \sin z\,dx - \cos x\,dy + \sin y\,dz$ where c is the boundary of the rectangle $0 \le x \le \pi, 0 \le y \le 1, z = 3$.	8

7	a	Solve $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log_e x)$.	8
	b	Apply the method of variation of parameters to solve the ordinary differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{1}{1+e^x}$	8
		OR	

8	a	Obtain the general solution of the differential equation: $2\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 6\frac{dy}{dt} - 9y = \cos^2\left(\frac{\sqrt{3}}{2}t\right) + t.$	8
	b	An electric circuit consists of an inductance of 0.1 henry, a resistance of 20 ohms and a condenser of capacitance 25 micro-farads connected in series. Find the charge q and the current i at any time t. Given that $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$ at $t = 0$, $q = 0.05$ C, $i = \frac{dq}{dt} = 0$ when $t = 0$.	8

9	a	The following table gives the relation between steam pressure and temperature.	8

T°C	361	367	378	387	399
P	154.9	167.9	191	212.5	244.2

Using suitable interpolation formula find the pressure at the temperature 372 and 404.

	b	The following table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface:	8

x = height	100	150	200	250	300	350	400
y = distance	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Estimate the value of y when $x = 180$ and $x = 410$.

OR

10	a	Given that	8

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Compute the velocity and acceleration at $x = 1.1$.

	b	From the following data, estimate the number of students who obtained marks between 40 and 45 using Newton's interpolation method	8

Marks:	30-40	40-50	50-60	60-70	70-80
Number of Students:	31	42	51	35	31

******