RV Educational Institutions
# RV College of Engineering

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi | Approved by AICTE, New Delhi

| | | |
|---|---|---|
| Course: Fundamentals of Linear Algebra, Calculus and Statistics | DEPARTMENT OF MATHEMATICS<br>TEST-1 | Maximum marks: 50 |
| Course code: MAT211CT | First semester 2023-2024<br>Chemistry Cycle<br>Branch: AI, BT, CD, CS, CY, IS | Time: 2.00PM<br>Date: 20/11/2023 |

| Q.No | Answer all questions | M | BT | CO |
|---|---|---|---|---|
| 1(a) | Find the values of $k$ such that the rank of the matrix $A$ is 2 where $A = \begin{bmatrix} 2 & 1 & k \\ 3 & 2 & k^2 \\ 1 & 1 & 2 \end{bmatrix}$ | 4 | 2 | 1 |
| (b) | The temperature $u_1, u_2, u_3$ of a metal plate under some circumstances is given by: $10u_1 + u_2 - u_3 - 11.2 = 0,$ $u_1 + 10u_2 + u_3 = 20.1,$ $-u_1 + u_2 + 10u_3 = 35.6$ Solve for the temperatures using Gauss- Seidel iterative method. Carry out 3 iterations. | 6 | 3 | 3 |
| 2 | Test the following system of linear equations for consistency and solve if it is consistent $2x_1 + x_2 + 2x_3 - x_4 = 6$ $6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$ $4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$ $2x_1 + 2x_2 - x_3 + x_4 = 10.$ | 10 | 2 | 2 |
| 3(a) | Solve the system of linear equations given by $2x - 3y + z = -1$ $x + 4y + 5z = 25$ $3x - 4y + z = 2$ Using Gauss-Jordan method. | 6 | 3 | 2 |
| 3(b) | Find the eigenvalues and the corresponding eigen vectors of the Matrix $B = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$ | 4 | 1 | 2 |
| 4(a) | Google page rank to find the most powerful page for a particular query is performed using largest eigenvalue and eigen vector for the transition matrix $A = \begin{bmatrix} 4 & 3 & 0 \\ 0 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Identify the dominant eigenvalue and corresponding eigen vector of the matrix $A$ by Rayleigh's power method with the initial approximation $[1 \quad 1 \quad 1]^T$. Perform 4 iterations. | 5 | 3 | 4 |
| 4(b) | Find the radius of curvature of the curve $y = xe^{-x}$ at the point where $y$ is maximum. | 5 | 2 | 3 |
| 5 | Show that the curves $r = a(1 + \cos\theta)$ and $r^2 = a^2\cos 2\theta$ intersect at an angle given by $3\sin^{-1}\left[\left(\frac{3}{4}\right)^{\frac{1}{4}}\right]$. | 10 | 2 | 3 |

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

| Marks Distribution | Particulars | | CO1 | CO2 | CO3 | CO4 | L1 | L2 | L3 | L4 | L5 | L6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Test | Max Marks | 4 | 20 | 21 | 5 | 4 | 29 | 17 | -- | - | - |

***********ALL THE BEST***********

RV Educational Institutions
RV College of Engineering
Go, change the world

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi
Approved by AICTE, New Delhi, Accredited By NAAC, Bengaluru And NBA, New Delhi

# DEPARTMENT OF MATHEMATICS

| Course: Fundamentals of Linear Algebra, Calculus and Statistics | CIE-II | Maximum marks: 50 |
|---|---|---|
| Course code: MAT211CT | First semester 2023-2024<br>Chemistry Cycle<br>Branch: AI, BT, CS, CD, CY, IS, SPARK C | Time: 2:00PM-3:30PM<br>Date: 27-12-2023 |

| Sl. No. | Questions | M | BT | CO |
|---|---|---|---|---|
| 1 | Determine the circle of curvature of the cycloid $$x = a(\theta - \sin\theta), \quad y = a(1-\cos\theta) \text{ at } \theta = \pi.$$ | 10 | 3 | 3 |
| 2 | Obtain Maclaurin series for the function $f(x) = e^{\tan^{-1}x}$ upto 5th degree term. | 10 | 2 | 2 |
| 3. (a) | For the function $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, find $xu_x + yu_y$. | 5 | 1 | 1 |
| 3. (b) | If $z = f(x,y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that $$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$ | 5 | 2 | 2 |
| 4. (a) | Obtain local maximum and local minimum values and saddle point of the function $$f(x,y) = y^2 - 2y\cos x, \quad -1 \le x \le 3.$$ | 5 | 2 | 3 |
| 4. (b) | The two legs of a right triangle are measured as 5 m and 12 m with a possible error in measurement of at most 0.2 cm in each. Use differentials to estimate the maximum error in the calculated value of (i) the area of the triangle and (ii) the length of the hypotenuse. | 5 | 2 | 3 |
| 5. (a) | A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters Earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point $(x,y,z)$ on the probe's surface is $T(x,y,z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface. | 6 | 3 | 4 |
| 5. (b) | Let $u = 3x + 2y - z$, $v = x - 2y + z$ and $w = x(x + 2y - z)$. Use Jacobians to prove that $u, v, w$ are functionally related. | 4 | 2 | 3 |

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

| Marks Distribution | Particulars | CO1 | CO2 | CO3 | CO4 | L1 | L2 | L3 | L4 | L5 | L6 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Max Marks | 5 | 15 | 24 | 6 | 5 | 29 | 16 | -- | -- | -- |

# DEPARTMENT OF MATHEMATICS

| | | |
|---|---|---|
| **Course: Fundamentals of Linear Algebra, Calculus and Statistics** | **Improvement CIE** | **Maximum marks: 50** |
| **Course code: MAT211CT** | **First semester 2023-2024**<br>**Chemistry Cycle**<br>**Branch: AI, BT, CS, CD, CY, IS, SPARK** | **Time: 2:00PM-3:30PM**<br>**Date: 22-01-2024** |

*Instructions to candidates: Answer all questions.*

| Q.No | QUESTIONS | M | BT | CO |
|---|---|---|---|---|
| 1(a) | Test for consistency and solve the system:<br>$x+y+z=4\,, 2x+y-z=1\,, x-y+2z=2.$ | 5 | L2 | 1 |
| 1(b) | Apply Rayleigh's power method to find the largest eigenvalue of the matrix<br>$A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Take the initial vector as $X_0=[1\ 1\ 1]^T$. Perform 4 iterations. | 5 | L2 | 2 |
| 2(a) | Apply Gauss-Seidel iteration method to solve the following system of equations:<br>$20x+y-2z=17\,, 3x+20y-z=-18\,, 2x-3y+20z=25.$<br>Carry out 4 iterations. | 5 | L2 | 2 |
| 2(b) | Evaluate $\int_0^1 \int_x^1 e^{\frac{x}{y}}\, dy\, dx$ by changing the order of integration. | 5 | L2 | 2 |
| 3(a) | Determine the area enclosed by the parabola $y=x^2$ and the line $y=x+2$. | 5 | L3 | 3 |
| 3(b) | Transform to polar coordinates and hence evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$. | 5 | L2 | 2 |
| 4 | Evaluate $\int_0^{\log_e 2} \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$. | 10 | L2 | 3 |
| 5 | Obtain the center of gravity of a triangular lamina with vertices $(0,0), (0,3)$ and $(3,0)$ if the density function is $\rho(x,y)=xy$. | 10 | L3 | 4 |

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

| Marks Distribution | Particulars | | CO1 | CO2 | CO3 | CO4 | L1 | L2 | L3 | L4 | L5 | L6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Test** | **Max Marks** | 5 | 20 | 15 | 10 | -- | 35 | 15 | -- | - | - |

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USN 1RV23CD003

# RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

I Semester B. E. Regular / Supplementary Examinations Feb-2024

## (Common to AI & ML, BT, CS, CY, CD and IS)

## FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND STATISTICS

*Time: 03 Hours* *Maximum Marks: 100*

***Instructions to candidates:***

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of mathematics Handbook is permitted. Do not write anything on handbook.

### PART-A

| | | | |
|---|---|---|---|
| 1 | 1.1 | If '$a$' is an irrational number, then rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & a \\ 0 & 2 & a \end{bmatrix}$ is ________. | 02 |
| | 1.2 | If $A = \begin{bmatrix} 3 & -1 \\ 1 & -3 \end{bmatrix}$, then the eigenvalues of $A^4$ are ________. | 02 |
| | 1.3 | The equation of circle passing through origin centered at $(a, 0)$ in polar coordinates is ________. | 02 |
| | 1.4 | The coefficient of $x^2$ in Maclaurin's series expansion of $\tan^{-1}x$ is ________. | 02 |
| | 1.5 | If $y^x = \sin x$, determine $\frac{dy}{dx}$ using partial differentiation. | 02 |
| | 1.6 | Determine $\frac{\partial^2 u}{\partial x\, \partial y}$, if $u(x, y) = x \sin y + x\, e^x$ | 02 |
| | 1.7 | Sketch the region of integration for the integral $\int_1^4 \int_1^{\sqrt{y}} f(x, y)\, dx\, dy$. | 02 |
| | 1.8 | Transform the integral $\int_0^2 \int_1^{\sqrt{4-x^2}} dy\, dx$ into polar coordinates. | 02 |
| | 1.9 | Give the relation between mean, mode and median in a negatively skewed distribution, and in a normal distribution. | 02 |
| | 1.10 | Determine mean of x and y if two regression equations of the variables x and y are $x = 19.13 - 0.87y$, $y = 11.6 - 0.5x$. | 02 |

### PART-B

| | | | |
|---|---|---|---|
| 2 | a | Check for the consistency of the following system and solve if consistent $5x + 3y + 7z = 4$; $3x + 26y + 2z = 9$; $7x + 2y + 10z = 5$ | 05 |
| | b | Determine an approximate solution of the following system of linear equation by Gauss-Siedel method with initial approximation $[1 \quad 0 \quad 0]^T$. Apply Four iterations. $8x + y - z = 8$; $2x + y + 9z = 12$; $x - 7y + 2z = -4$ | 06 |
| | c | Determine the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$. | 05 |
| | | | |
| 3 | a | Show that the angle of intersection of the lemniscate $r^2 = a^2 \cos 2\theta$ and the cardioid $r = a(1 + \cos\theta)$ is $\frac{3}{2}\sec^{-1}\left(\frac{1}{1-\sqrt{3}}\right)$. | 08 |
| | b | Find the coordinates of the centre of curvature at $(at^2, 2at)$ on the parabola $y^2 = 4ax$. | 08 |

**OR**

4 a For the curve $x = b(\cos\theta + \log(\tan\frac{\theta}{2}))$, $y = b\sin\theta$, show that the radius of curvature at any point $\theta$ is $b\cot\theta$. 08

b Obtain the Maclaurin's series expansion for the function $f(x) = \ln(1+x)$ up to fourth degree term. 08

5 a Jacobian represents transformation function to traverse between coordinate system. If $x = u\cos v$ and $y = u\sin v$, determine $\frac{\partial(x,y)}{\partial(u,v)}$. 08

b The temperature T at any point $(x, y, z)$ in space is $T = xyz^2$. Determine the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = a^2$, using Lagrange Method. 08

**OR**

6 a If $u = g(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $6\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} + 3\frac{\partial u}{\partial z} = 0$. 08

b The lengths x, y and z of edges of a rectangular box are changing with time. At the instant in question $x = 1$ m, $y = 2$ m, $z = 3$ m $dx/dt = dy/dt = 1$ m/sec and $dz/dt = -3$ m/sec. At what rates the box's volume, surface area and length of diagonal are changing at that instant? 08

7 a Determine the area of the region inside the cardioid $r = 4(1 + \cos\theta)$ and outside the circle $r = 4$, using double integrals. 08

b A metal plate is in the form of a triangle with vertices (0,0), (1,0) and (1,1). The density at any point on the metal plate is given by $\rho(x, y) = xy$. Determine the center of gravity of the metal plate. 08

**OR**

8 a Using triple integral obtain the volume of the solid bounded between the cone $z^2 = x^2 + y^2$ and the plane $z = 2$. 08

b Evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ by changing the order of integration. 08

9 a Compute the correlation coefficient for this data. Also find the regression lines of y on x and x on y.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | 5 | 7 | 8 | 10 | 11 | 13 | 16 |
| y | 33 | 30 | 28 | 20 | 18 | 16 | 9 |

08

b Fit a second-degree polynomial $y = a + bx + cx^2$ for the following data using least square method.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
| y | 4.63 | 2.11 | 0.67 | 0.09 | 0.63 | 2.15 | 4.58 |

08

**OR**

10 a From the following frequency distribution compute first four central moments.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Size x | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| Frequency f | 4 | 10 | 20 | 36 | 16 | 12 | 2 |

08

b Fit a curve of the form $y = ab^x$ to the data by the method of least squares.

| | | | | | |
|---|---|---|---|---|---|
| x | 2 | 4 | 6 | 8 | 10 |
| y | 1 | 3 | 6 | 12 | 24 |

08