Practical - 1

Q1 $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} \quad 2\sqrt{x}} \right]$ 2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3) $\lim_{x \to \pi/6} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$ 4) $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

5) Examine the continuity of the following function at given points.

i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \leq \frac{\pi}{2}$ } at $x = \frac{\pi}{2}$

$= \frac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$

ii) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \leq x < 6$ } at $x=3$ & $x=6$

$= \frac{x^2-9}{x+3}$ $6 \leq x < 9$

Q6 Find value of k, so that the fn f(x) is cts. at the indicated point.

i) $f(x) = \frac{1-\cos 4x}{x^2}$ $x < 0$ } at $x = 0$

$= k$ $x = 0$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$ } at $x = 0$

$= k$ $x = 0$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \frac{\pi}{3}$

$= k$ $x = \frac{\pi}{3}$ } at $x = \frac{\pi}{3}$

Q4 Discuss the continuity of the following fn. which of these fn. have a removable discontinuity? Redefine the fn. so as to remove the discontinuity

i) $f(x) = \dfrac{1 - \cos 3x}{x \tan x}$ $x \neq 0$

$= 9$ $x = 0$ } at $x = 0$

ii) $f(x) = \dfrac{(e^{3x} - 1) \sin x}{x^2}$ $x \neq 0$

$= \dfrac{\pi}{60}$, $x = 0$ } at $x = 0$

8) If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is cts at $x = 0$, find $f(0)$

9) If $f(x) = \dfrac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^3 x}$ for $x \neq \frac{\pi}{2}$ is cts at $x = \frac{\pi}{2}$ find $f\left(\frac{\pi}{2}\right)$

1 $$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right.$$

$$\lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \quad \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{2\sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\overset{2}{\cancel{4}}\sqrt{a}}{\cancel{2}\sqrt{3a}}$$

$$= \frac{1}{3} \times \cancel{2} \quad \frac{2}{3\sqrt{3}}$$

2 $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right)$

$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$\lim_{y \to 0} \frac{y}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

li $\frac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a})}$

$\frac{1}{\sqrt{a}(\sqrt{a} + \sqrt{a})}$

$= \frac{1}{\sqrt{a}(2\sqrt{a})}$

$= \frac{1}{2a}$

3. $\lim_{x \to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \frac{\pi}{6} = h$

$$x = h + \frac{\pi}{6}$$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{6}\right) - \sqrt{3}\sin\left(h + \frac{\pi}{6}\right)}{\pi - 6\left(h + \frac{\pi}{6}\right)}$$

using $\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

$$= \lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h + \pi}{6}\right)}$$

$\cos\frac{\pi}{6} = \cos 30^\circ$

$$= \lim_{h \to 0} \frac{\left[\left(\cos h \frac{\sqrt{3}}{2} - \sin h \frac{1}{2}\right]\right. - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \frac{1}{2}\right)}{-6h}$$

$$= \lim_{h \to 0} \frac{\left(\cos\frac{\sqrt{3}h}{2} - \sin\frac{h}{2}\right) - \left(\sin\frac{3h}{2} + \cos\frac{\sqrt{3}}{2}h\right)}{-6h}$$

$$= \lim_{h \to 0} \frac{-\frac{\sin 4h}{2}}{-6h}$$

$$\lim_{h \to 0} \frac{\sin 2h}{6h} = \frac{1}{3}$$

$$\frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h}$$

4] $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

Sol^n

By rationalizing

$$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2-1}}{\sqrt{x^2+3} \sqrt{x^2+1}} \right]$$

$$\lim_{x \to \infty} \left[\frac{(x^2+5-x^2+3)}{(x^2+3-x^2+1)} \cdot \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})} \right]$$

$$\lim_{x \to \infty} \frac{8(\sqrt{x^2+3} + \sqrt{x^2+1})}{2\sqrt{x^2+5} + \sqrt{x^2-3}}$$

$$4 \lim_{x \to \infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit we get

$= 4$

5) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 \le x \le \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, for $\dfrac{\pi}{2} \le x \le \pi$ } at $x = \dfrac{\pi}{2}$

Sol^n

a) $f\left(\dfrac{\pi}{2}\right) = \dfrac{\sin^2(\pi/2)}{\sqrt{1-\cos(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \pi/2$ define.

$\lim\limits_{x \to \pi/2^+} f(x) = \lim\limits_{x \to \pi/2^+} \dfrac{\cos x}{\pi - 2x}$

By substituting method

$x - \dfrac{\pi}{2} = h$

$x = h + \dfrac{\pi}{2}$

where $h \to 0$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - 2\left(\dfrac{2h + \pi}{2}\right)}$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{-2h}$

Using $\cos(A+B) = \cos A \cos B - \sin A \sin$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h\to 0} \frac{-\sin h}{-2h}$$

$$\frac{1}{2} \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2}$$

6 For $x = 3$

$f(x) = f(3) = 3+3$

$= 6$

f is defined at $x=3$

$$\lim_{x\to 3^+} f(x)$$

$$= \lim_{x\to 3^+} (x+3)$$

$= 6$

$$\lim_{x \to 3^-} f(x) - \lim_{x \to 3^-} \frac{x^2-9}{x-3}$$

$$= \lim_{x \to 3^-} \frac{(x-3)(x+3)}{x-3}$$

$$= 6$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$

from (i) & 2

F is continuous at $x=3$

for $x = 6$

i) $f(x) = \frac{x^2+9}{x+3}$

$$= \frac{36-9}{6+3}$$

$$= 3$$

ii) $\lim_{x \to 6^+} f(x)$

$$\lim_{x \to 6^+} \frac{x^2-9}{x+3}$$

3

$$\lim_{x \to 6^-} f(x)$$

$$= x+3$$

$$= 6+3$$

$$= 9$$

6) 1) $F(x) = \dfrac{1-\cos 4x}{x^2}$ $x \neq 0$

$= k$ $x = 0$ } at $x=0$

F is continuous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$\therefore \lim_{x \to 0} \dfrac{1-\cos 4x}{x^2} = k$

$\lim_{x \to 0} \dfrac{2\sin^2 x}{x^2} = k$

$\therefore 2\lim_{x \to 0} \left(\dfrac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$\boxed{k = 8}$

2) $F(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$

$= k$ $x = 0$ } at $x=0$

F is continuous at $x=0$

$\therefore \lim_{x \to 0} f(x) = k$

$\lim_{x \to 0} (\sec^2 x)^{\cot^2 x} = k$

$$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x} = k$$

$$\lim_{x\to 0} (1+x)^{1/x} = e$$

$$e = k$$
$$k = e$$

3) $F(x) = \dfrac{\sqrt{3}-\tan x}{\pi - 3x}$ $\quad x \neq \pi/3$

$= k \quad x = \pi/3$ $\quad$ at $x = \pi/3$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$x - \pi/3 = h$
$x = h + \pi/3$

$$(\pi/3 + h) = \frac{\sqrt{3} \cdot \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{x\to 0} \frac{(\sqrt{3}\cdot\sqrt{3} \times \sqrt{3}\tan h) - (\sqrt{3} + \tan h)/1 - \sqrt{3}\cdot\tan h}{-3h}$$

$$\lim_{x\to 0} \frac{(\sqrt{3} - 3\tan h) - (\sqrt{3} + \tan h)/1 - \sqrt{3}\cdot\tan h}{-3h}$$

$$\lim_{x\to 0} \frac{-4\tan h}{\sqrt{3}\, h(1 - \sqrt{3}\tan h)}$$

$$4/3 \lim_{x\to 0} \frac{\tan h}{h} \lim_{x\to 0} \left\{ 1/1 - \sqrt{3}\tan h \right.$$

Practical 2

1) Show that following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

2) C

1) $\cot x$

$f(x) = \cot x$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)(\tan x \tan x)}$$

put $x - a = h$ $\quad x = a + h$ $\quad$ as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to a} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

Formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B) - (1 + \tan A \cdot \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan a + \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} -\frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$= -\operatorname{cosec}^2 a$

$Df(a) = -\cos^2 a$

$\therefore$ F is differentiable $\forall a \in R$

2) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\sin x - 1/\sin a}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x - a = h$ $\quad x = a + h$ as $x \to a, h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$Df(h) = \lim_{h \to a} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times 1/2 \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2\cos(2a/2)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

3) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec(x) - \sec(a)}{x - a}$$

$$= \lim_{x \to a} \frac{1/\cos(x) - 1/\cos(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cos(a) - \cos(x)}{(x-a)\cos a \cos x}$$ put $x - a = h$, $x = h + a$

as $x \to a$ $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \cos a \cos(a+h)}$$

formula :- $-2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{-2 \sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{\cos a \cos(a+h) \times -h/2}$$

$$= -1/2 \times -2 \frac{\sin a}{\cos a \cdot \cos a}$$

$$= \tan a \sec a$$

Sol^n

LHD: ~~Df~~

$$Df(2-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$Df(2-) = 4$

RHD:

$$Df(2+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$$= 2+2 = 4$$

$Df(2+) = 4$ $\therefore$ RHD = LHD

f is diffrentiable at $x=2$

Q3 Soln: RHD:

$$Df(3+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3-3+1)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9$$

$Df(3^+) = 9$

$LHD = Df(3-)$

$$= \lim_{x \to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3} = \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4(x-3)}{x-3}$$

$Df(3^+) = 4$

RHD = LHD

f is not differentiable at $x=3$.

Qu soln

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD : $Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3 \times 2 + 2 = 8$

$Df(2^+) = 8$

LHD:

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$ $Df(2^-) = 8$

LHD = RHD f is diffrentiable at $x = 3$

Practical- 3

Topic: Application of derivativ

1) Find the intervals in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upwards.

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$

Sol^n

Q1

1) $f(x) = x^3 - 5x - 11$

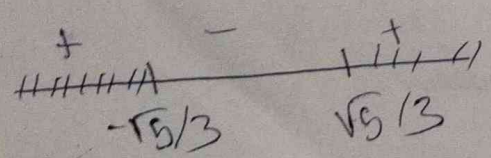
$\therefore f'(x) = 3x^2 - 5$

$\because$ f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$



$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \quad \sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f(x)$ is increasing

$2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x < 2$

$\therefore x \in (-\infty, 2)$

iii) $f(x) = \cancel{6x}\ 2x^3 + x^2 - 20x + 4$

$f(x) = 6x^2 + 2x - 60$

$\therefore f$ is increasing iff $f'(x) > 0$

$3x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x + 2) > 0$

$+ \quad - \quad +$

$-2 \quad 10/5$

$\therefore x \in (-\infty, -2) \cup (10/6, \infty)$

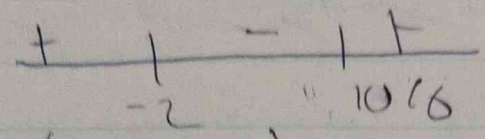
f is decreasing iff $f(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x^2 + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$(6x-10)(x+2) < 0$



$\therefore x \in \left(-2, \frac{10}{6}\right)$

iv) $f(x) = x^3 - 27x + 5$

$= 3x^2 - 27$

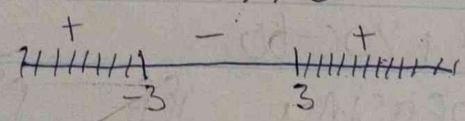
$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$



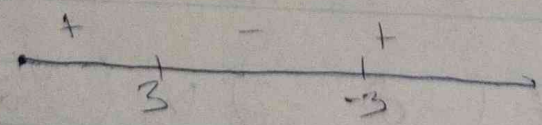
$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$



$\therefore x \in (-3, 3)$

v) $f(x) = 6a - 24x - 9x^2 + 2x^3$

$f(x) = -24 - 18x - 24$

i.e $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

f is increasing iff $f'(x) > 0$

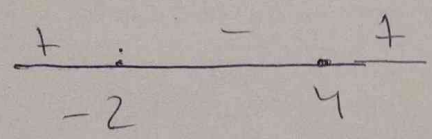
$\therefore 6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x+1)(x-4) > 0$



$\therefore x \in (-\infty, -1) \cup (4, \infty)$

f is decreasing iff $f'(x) < 0$

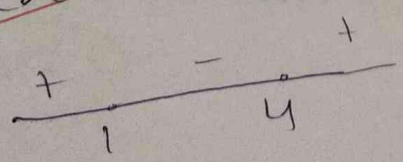
$6(x^2 - 3x - 4) < 0$

$x^2 - 3x - 4 < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x+1)(x-4) < 0$



$x \in (1, 4)$

Q2

i) $y = 3x^2 - 2x^3$

let

$f(x) = y = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$= 6(1-2x)$

$f''(x)$ is concave upward iff,

$f''(x)$ is concave upward iff,

$f''(x) > 0$

$6(1-2x) > 0$

$1-2x > 0$

$-2x > -1$

$2x < 1$

$x < 1/2$

$\therefore x \in \left(-\infty, \frac{1}{2}\right)$

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$6(1-2x) < 0$

$1-2x < 0$

$-2x < -1$

$2x > 1$

$x > \frac{1}{2}$

$x \in \left(\frac{1}{2}, \infty\right)$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

Let

$f(x) = y = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$= 6(1-2x)$

$f''(x)$ is concave upward iff,

$f''(x) > 0$

$6(1-2x) > 0$

$1-2x > 0$

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$2x < 1$

$x < 1/2$

$\therefore x \in \left(-\infty, \frac{1}{2}\right)$

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$6(1-2x) < 0$

$1-2x < 0$

$-2x < -1$

$2x > 1$

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$x < 1/2$

$\therefore x \in \left(-\infty, \frac{1}{2}\right)$

$f''(x)$ is concave downwards iff,

$f''(x) < 0$

$6(1-2x) < 0$

$1 - 2x < 0$

$-2x < -1$

$2x > 1$

$x > \frac{1}{2}$

$\therefore x \in \left(\frac{1}{2}, \infty\right)$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

let

$f(x) = y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$f''(x)$ is concave upwards iff

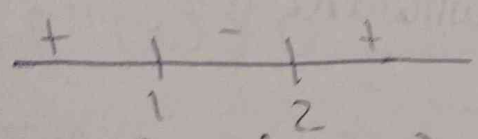
$f''(x) > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$x(x-1) - 2(x-1) > 0$

$(x-2)(x-1) > 0$



$\therefore x \in (-\infty, 1) \cup (2, \infty)$

$f''(x)$ is concave downward iff

$f''(x) < 0$

$12(x^2 - 3x + 2) < 0$

$x^2 - 3x + 2 < 0$

$x^2 - x - 2x + 2 < 0$

$x(x-1) - 2(x-1) < 0$

$(x-2)(x-1) < 0$

$\therefore x \in (1, 2)$

iii) $y = x^3 - 27x + 5$

let,

$f(x) = y = x^2 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is concave upwards iff

$f''(x) > 0$

$6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

$f''(x)$ is concave downward iff

$f''(x) < 0$

$6x < 0$

$x < 0$

$\therefore x \in (-\infty, 0)$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

let

$f(x) = 2x^3 - 9x^2 - 24x + 4$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

3) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12x > -2$

$x > -1/6$

$\therefore f''(x) > 0$

$\therefore$ There exist an interval $(-1/6, \infty)$

Practical 4

TOPIC :- Application of derivative & Newton's method

Q1) Find max^m & min^m value of following

i) $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$\therefore f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/16$

$= 2 + 6$

$= 8 \quad > 0$

$\therefore$ f has minimum value at $x = 8$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8$

$\therefore$ f has minimum value at $x = -2, 2$

~~fn reache mi~~

$f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$\cancel{15}x^2 = \cancel{15}x^2$

$x^2 = 1$

$\therefore x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$ $\therefore$ f has minimum value at $x=1$.

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 3 - 5 + 3$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= -3 < 0$ $\therefore$ f has max^m value at $x=-1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ f has max^m value 5 at $x=-1$ and has the min^m value 1 at $x=1$

iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has max^m value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has max^m value at $x = 2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 12 + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has max^m value 1 at $x = 0$ and f has min^m value -3 at $x = 2$.

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

consider

$f'(x) = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1$ & $x = 2$

$f''(x) = 12x - 6$

$= 6(2x - 1)$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore$ f has max^m value of $x=-1$

$f(-1) = 2(-1)^3 - 3\cdot(-1)^2 - 12(-1)+1$

$= -2-3+12+1$

$= 18$

$f''(2) = 12\cdot(2) - 6$

$= 24-6$

$= 18 > 0$

$\therefore$ f has min^m value at $x=2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) - 1$

$= 16-12-24-1$

$= 16-37$

$= -27$

$\therefore$ f has min^m value as -27 at $x=2$

$\therefore$ f has max^m value as 8 at $x=-1$

Q2 Find the root of the eq^n correct upto 4 decimal place

1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0824$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$
$= -55.9467$

$\therefore x_2 = x_1 \; f(x_1)/f'(x_1)$
$= 0.1727 - 0.829/55.9467$
$= 0.1712$

$f(x_2) = 3(0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 4.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 5$
$= -55.9343$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$
$= 0.1712 + 0.0011/55.9343$
$= 0.1712$

$\therefore$ The root of the eqn is 0.1712.

Practical No-5
Integration

1) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$I = \int \frac{dx}{\sqrt{x^2+2x-3}} = \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

Comparing with $\int \frac{dx}{\sqrt{x^2-a^2}}$ $= x^2 = (x+1)^2$

$I = \log |x + \sqrt{x^2-a^2}| + C$

$= \log |x+1+\sqrt{(x+1)^2-(2)^2}| + C$

ii) $\int (4e^{3x}+1)\, dx$

$\int 4e^{3x} dx + \int 1\, dx$

$= 4\int e^{3x} dx + \int 1 dx$ $\int e^{ax} dx = \frac{1}{2a} \times e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + C$

3. $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$

$= \int 2x^2 dx - \int 3\sin(x) dx + \int 5x^{1/2} dx$

$\frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4. $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$I = \int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$= \int \left(\frac{x^2}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}} \right) dx$

$= \int (x^3 \cdot x^{-1/2} + 3x \cdot x^{-1/2} + 4x^{-1/2}) dx$

$= \frac{2}{7} x^{7/2} + 3 \cdot \frac{2}{3} x^{3/2} + 4 \cdot x^{1/2} \cdot 2 + C$

$= \frac{2}{7} x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

5) $\int t^7 \sin(2t^4)\, dt$

$I = \int t^7 \sin(2t^4) dt$

let, $t^4 = x$

$4t^3 dt = dx$

$I = \frac{1}{4}\int 4t^3 \cdot t \sin(2t^4)\,dt$

$= \frac{1}{4}\int x \cdot \sin(2x)\,dx$

$= \frac{1}{4}\left[x\int \sin 2x - \int\left[\int \sin 2x \frac{d}{dx}(x)\right]\right]$

$= \frac{1}{4}\left[\frac{-x\cos^2 x}{2} + \frac{1}{2}\int \cos 2x \cdot 1\right]$

$= \frac{1}{4}\left[\frac{-x\cos 2x}{2} + \frac{1}{4}\sin 2x\right] + C$

$= -\frac{1}{8}x\cos^2 x + \frac{1}{16}\sin 2x + C$

Resubstituting $x = t^4$

$\therefore I = -\frac{1}{8}t^4\cos(2t^4) + \frac{1}{6}\sin(2t^4) + C$

5) $\int \sqrt{x}(x^2-1)\,dx$

$I = \int \sqrt{x}(x^2-1)\,dx$

$= \int (\sqrt{x}\cdot x^2 - \sqrt{x})\,dx$

$= \int x^{5/2}\,dx - \int \sqrt{x}\,dx$

$= \frac{2}{7}x^{7/2} - \frac{2}{3}x^{3/2} + C$

7) $\frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$I = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

let,

$\frac{1}{x^2} = t$

$\therefore x^{-2} = t$

$\therefore \frac{-2}{x^3}\,dx = dt$

$$I = \frac{1}{-2}\int \frac{-2}{x^3}\sin\left(\frac{1}{x^2}\right)dx$$

$$= \frac{-1}{2}\int \sin t \, dt$$

$$= \frac{-1}{2}\left[-\cos t\right] + C$$

$$= \frac{1}{2}\cos t + C$$

Resubstituting $t = \frac{1}{x^2}$

$$I = \frac{1}{2}\cos\left(\frac{1}{x^2}\right) + C$$

8) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

$$I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$$

Let $\sin x = t$

$\cos x \, dx = dt$

$$I = \int \frac{dt}{\sqrt[3]{t^2}}$$

$$I = \int \frac{dt}{t^{2/3}}$$

$$= \int t^{-2/3}\,dt$$

$$= 3t^{1/3} + C$$

$$= 3t^{1/3} + C$$

$$= 3(\sin x)^{1/3} + C$$

$$= 3\sqrt[3]{\sin x} + C$$

9) $\int e^{\cos^2 x} \sin 2x \, dx$

$I = \int e^{\cos^2 x} \sin^2 2x \, dx$

let $\cos^2 x = t$

$-2\cos x \sin x \, dx = dt$

$-2\sin 2x \, dx = dt$

$I = -\int -\sin 2x \, e^{\cos^2 x} \, dx$

$= -\int e^t \, dt$

$= -e^t + c$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

10) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

$I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

Let

$x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x) \, dx = dt$

$3(x^2 - 2x) \, dx = dt$

$(x^2 - 2x) \, dx = dt/3$

$$\therefore I = \int \frac{1}{t} \frac{dt}{3}$$

$$\frac{1}{3} \int \frac{dt}{t}$$

$$\frac{1}{3} \log t + C$$

Resubstituting $t = x^3 - 3x^2 + 1$

$$I = \frac{1}{3} \log (x^3 - 3x^2 + 1) + C$$

Practical 6

Application of integration & numerical integration

Q. Find the length of the following curve

$x = t\sin t,\ y = 1-\cos t,\ t \in [0, 2\pi]$

Sol^n

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x = t - \sin t$ $\quad\therefore \frac{dx}{dt} = 1-\cos t$

$y = 1-\cos t$ $\quad\therefore \frac{dy}{dt} = 0-(-\sin t) = \sin t$

$$L = \int_0^{2\pi} \sqrt{(1-\cos t + \sin t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1-2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2-2\cos t}\, dt = \int_0^{2\pi} \sqrt{2(1-\cos t)}$$

$$= \int_0^{2\pi} \sqrt{2\times 2\sin^2 t/2} = \sqrt{4\sin^2 t/2}$$

$$= \int_0^{2\pi} 2\left|\sin\frac{t}{2}\right| dt$$

$$\therefore \sin^2\frac{t}{2} = \frac{1-\cos t}{2}$$

$$= \int_0^{2\pi} 2 \sin \frac{t}{2} \, dt$$

$$= \left(-4\cos(t/2)\right)^{2\pi} = (-4\cos\pi) - (4\cos$$

$$= 4+4$$

$$= 8$$

2. $y = \sqrt{4-x^2} \quad x \in [-2,2]$

Sol^n.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$$y = \sqrt{4-x^2} \quad \therefore \frac{dy}{dx} = 2 \int_0^2 1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2 dt$$

$$= 2 \int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}} \, dx$$

$$= 4 \left(\sin^{-1} x/2\right)^2$$

$$= 2\pi$$

3) $y = x^{3/2}$ in $[0,4]$

sol^n:-

$$f'(x) = \frac{3}{2} x^{1/2}$$

$$[f'(x)]^2 = \frac{9}{4} x$$

$$L = \int_a^b \sqrt{1+[f'(x)]^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$$

$$= 1/2 \int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2} \left[\frac{(4+9x)^{1/2+1}}{1/2+1} \right]_0^4 dx$$

$$= \frac{1}{2} \left[(4+9x)^{3/2} \right]_0^4$$

$$= \frac{1}{2} \left[(4+0)^{3/2} - (4+36)^{3/2} \right]$$

$$= \frac{1}{2} \left[(4)^{3/2} - (40)^{3/2} \right.$$

4) $x = 3\sin t \quad y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{4(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{4(1)}\, dt$$

$$= \int_0^{2\pi} 3\, dt$$

$$= 3\int_0^{2\pi} dt$$

$$= 3\,[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

02) Using simpson rule solve the following $\int_0^2 e^{x^2}\, dx$ with $n=4$

Sol^n :- $a=0,\ b=2,\ n=4$

$$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.2840	2.718244877	54.54581	
	y_0	y_1	y_2	y_3	y_4

By simpson's Rule

$$\int_0^2 e^{x^2}\,dx = \frac{0.5}{3}\left(1+54.5981\right)+4\left(1.2840+9.4877\right)+2\left(2.71828+54.5981\right)$$

$$= 1.779$$

2) $\int_0^4 x^2\,dx$

$$L = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2\,dx = \frac{1}{3}\left[16+4(10)+8\right]$$

$$= 64/3$$

$$\int_0^4 x^2\,dx = 21.533$$

3. $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n=6$

Sol^n :-

$$L = \frac{\pi/3 - 0}{6} = \pi/18$$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$8\pi/18$
y	0	0.4166	0.58	0.70	0.8068	0.8726	0.948

$$\int_0^{\pi/3} \sqrt{\sin x} = \pi/54 \times 12.1163$$

$$= \int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7044$$

Practical 7

Differential Equation

$$x\frac{dy}{dx} + y = e^x$$

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$$

$$x = \quad = \frac{1}{x}$$

$$IF = e^{\int(x)\,dx}$$
$$= e^{1/x\,dx}$$
$$= e^{\ln x}$$
$$IF = x$$

$$y = (IF) = \int Q(x)\,(IF)\,dx + C$$
$$= \int \frac{e^x}{x}\,x\,dx + C$$
$$= \int e^x\,dx + C$$

$$xy = e^x + C$$

$$e^x\,dy/dx + 2e^x y = 1$$

$$\frac{dy}{dx} + \frac{2e^x}{e^x}y = \frac{1}{e^x} \quad (\div \text{ by } e^x)$$

$$\frac{dy}{dx} + 2y = \frac{1}{e^x}$$

$P(x) = 2 \quad Q(x) = e^{-x}$

$\int P(x)\,dx$

$IF = e^{\int 2\,dx}$

$= e^{2x}$

$y\,(IF) = \int Q(x)\,(IF)\,dx + C$

$y \cdot e^{2x} \int e^{x} + 2x\,dx + C$

$= \int e^{x}\,dx + C$

$y \cdot e^{2x} = e^{x} + C$

3. $x\dfrac{dy}{dx} = \dfrac{\cos x}{x^{2}} \cdot 2y$

$x\dfrac{dy}{dx} = \cos\dfrac{x}{x} \cdot 2y$

$\dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{\cos x}{x^{2}}$

$p(x) = 2(x) \quad Q(x) = \dfrac{\cos x}{x^{2}}$

$IF = e^{\int p(x)\,dx}$

$= e^{\int 2/x\,dx}$

$= \ln x^{2}$

$y \cdot (IF) = \int Q(x)\,(IF)\,dx + C$

$= \int \cos x + C$

$x^2 y = \sin x + C$

$\cdot x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$p(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e \int p(x)\, dx$

$= e \int 3/x \, dx$

$= e^{3/x} \, dx$

$= e^{\ln x^3}$

$IF = x^3$

$y(IF) = \int Q(x)(IF)\, dx + C$

$= \int \frac{\sin x}{x^3} \cdot x^3 \, dx + C$

$= \int \sin x \, dx + C$

$x^3 y = -\cos x + C$

5. $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x / e^{2x} = 2x^{-2x}$

$IF = e \int P(x)\, dx$

$= e \int 2 dx = e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + C$

$= \int 2x e^{-2x} e^{2x}\,dx + C$

$ye^{2x} = x^2 + C$

6. $\sec^2 x \cdot \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$

$\frac{\sec^2 x}{\tan x}\,dx = \frac{-\sec^2 y}{\tan y}\,dy$

$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y}{\tan y}\,dy$

$\log|\tan x| = -\log|\tan y| + C$

$\log|\tan x - \tan y| + C$

$\tan x \cdot \tan y = e^C$

7. $\frac{dy}{dx} = \sin^2(x - y + 1)$

put $x - y + 1 = v$

Differentiating BTS

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{1 - dv}{dx} = \frac{dv}{dx}$

$\frac{1 - dv}{dx} = dy/dx$

$1 - \frac{dv}{dx} = \sin^2 v$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x + y - 1) = x + c$$

8. $$\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$$

Put $2x + 3\frac{dy}{dx} = \frac{dv}{dx}$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$$

$$\frac{dv}{dx} = \frac{v - 1 + 2v + 4}{v + 2}$$

$$= \frac{2v+3}{v+2}$$

$$\frac{3(v+1)}{v+1}$$

$$\int \left(\frac{v+2}{v+1} \right) dv = 3dx$$

$$\int \frac{v+1}{v} \, dx + \int \frac{1}{v+1} \, dv = 3x$$

$$v + \log |x| = 3x + C$$

$$2x + 3y + \log |2x+3y+1| = 3x + C$$

$$3y = x - \log |2x+3y+1| + C$$

Practical No. 8

$$\frac{dy}{dx} = y + e^x - 2$$

$f(x, y) = y + e^x - 2$, $y_e = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.14	3.57435
2	1	3.5	4.24	5.3615

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3	7.84	9.28305
4	2	9.2		

$\therefore$ By Euler's formula

$y(2) = 9.2831$

2. $\frac{dy}{dx} = 1+y^2$

$f(x, y) = 1+y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h \; f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8630	1.2944
5	1	0.2942		

$\therefore$ By Euler's Formula

$y(1) = 1.2942.$

3. $\frac{dy}{dx} \sqrt{\frac{x}{y}}$ $y(0)=1$ $x_0=0$ $h=0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

4. $\frac{dy}{dx} = 3x^2+1$ $\quad y_0=2,\ x_0=1,\ h=1$

For $h=0.5$

$y_{n+1} = y_n + hF(x_n, y_n)$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	44	28.5
2	2	28.5		

By Euler's formula

$y(2) = 28.5$

For $h = 0.25$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4214
2	1.5	4.4214	7.75	3.3544
3	1.75	6.3544	10.1815	8.9048
4	2	8.9048		

By Euler's Formula

$y(2) = 8.9048.$

4. $\frac{dy}{dx} = 3x^2+1$ $\quad y_0=2,\ x_0=1,\ h=1$

For $h=0.5$

$y_{n+1} = y_n + hF(x_n, y_n)$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	44	28.5
2	2	28.5		

By Euler's formula

$y(2) = 28.5$

For $h = 0.25$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4214
2	1.5	4.4214	7.75	3.3544
3	1.75	6.3544	10.1815	8.9048
4	2	8.9048		

By Euler's Formula

$y(2) = 8.9048.$

$\frac{dy}{dx} = \sqrt{xy} + 2 \qquad y_0 = 1, \; x_0 = 1, \; h = 0.2$

using Euler's iteration formula.

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

$\therefore$ By euler's formula

$y(1.2) = 1.6$

Practical 9
Limit & Partial order derivatives

Q1
i) $$\lim_{(x,y)\to(-4,1)} \frac{x^3-3y+y^2-1}{xy+5}$$

At $(-4,-1)$ Denominator $\neq 0$
$\therefore$ By applying limit

$$= \frac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$$

$$= \frac{-64+3+1-1}{4+5}$$

$$= \frac{-61}{9}$$

i) $$\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

At $(2,0)$ Denominator $\neq 0$
$\therefore$ By applying limit,

$$= \frac{(0+1)((2)^2+0-4(2))}{2+0}$$

$$= \frac{1(4+0-8)}{2}$$

$$= \frac{-4}{2}$$

$$= -2$$

1) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2 \cdot z}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^2-x^2yz}$

Apply limit

$= \frac{(1)^2+(1)^2(1)}{(1)^2}$

$= \frac{1+1(1)}{(1)^2} = 2$

2 $f(x,y) = xye^{x^2+y^2}$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\therefore f(x) = 2xye^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$f_y = 2yxe^{x^2+y^2}$

ii) $f(x) = e^x \cos y$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$f_y = -e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_x = 3x^2y^2 - 6x$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(x^2y^2 - 3x^2y^2 + y^3 + 1)$

$2x^3y - 3x^2 + 3y^2$

$$f(x,y) = \frac{2x}{1+y^2}$$

$$f(x) = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y^2-0}{(1+y^2)^2}$$

$$= \frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y)^2}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \times \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0$$

Q4

1. $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{\partial}{\partial x} y^2 - xy \quad \frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{x^2(xy) - 2x\,(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x\,(2y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x} - x^2y - 2xy^2 + 2x^2y\right) - x^2y - 2xy + 2x^2y)\frac{\partial(x^4)}{\partial x}}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6} \rightarrow (1)$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2} \rightarrow (2)$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \rightarrow (3)$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2 \frac{d}{dx}(2y - x) - (2y - x)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2 - 4xy + 2x^2}{x^4} \rightarrow (4)$$

From (3) & (4)

$f_{xy} = f_{x^4}$

ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{\partial}{\partial x}\left(x^3 + 3x^2y - \log(x^2+1)\right)$

$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f_y = \frac{\partial}{\partial y}\left(x^2 + 3x^2y^2 - \log(x^2+1)\right)$

$= 0 + 6x^2y - 0$

$= 6x^2y$

$f_{xx} = 6x + 6y^2 - \left(x^2+1 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(x^2+1)\right)$

$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \rightarrow (1)$

$f_{yy} = \frac{\partial}{\partial y}(6x^2y)$

$= 6x^2 \rightarrow (2)$

$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$

$= 12xy$

$f_{yx} = \frac{\partial}{\partial x} 6x^2y$

$= 12xy$

From ③ & ④

$\therefore f_{xy} = f_{yx}$

Q 1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}} \quad 2x$ $\qquad f_y = \frac{1}{2\sqrt{x^2+y^2}} \quad 2y$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\qquad = \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\qquad f_y$ at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f(x)(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1$

$$= \sqrt{2} + \frac{1}{\sqrt{2}} (x+y-2)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}} x + \frac{1}{\sqrt{2}} y - \frac{2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

24/01/2020

Practical 10

Q1 i) $f(x,y) = x - 2y + 3$ $\quad a = (1,-1)$, $u = 3i - j$

→ Here,
$u = 3i - j$ is not a unit vector
$\bar{u} = 3\bar{i} - \bar{j}$
$|u| = \sqrt{10}$
∴ Unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3\bar{i} - \bar{j})$

$$= \frac{1}{\sqrt{10}}(3,-1)$$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a+hu) = f((1,-1) + h)\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a) = f(1,-1) = (1) - 2(-1) - 3 = 1 - 2 - 3 = -4$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1, -\frac{h}{\sqrt{10}}\right)$$

$$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(2 - 1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{4} + h/\sqrt{10} + \cancel{4}}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

ii) $f(x) = y^2 - 4x + 1$ $\quad a = (3, 4)$ $\quad u = i + 5j$

Here, $u = i + 5j$ is not a unit vector

$|u| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$f(a) = f(3, 4) = (4)^2 - 4(3) + 1 = 5$

$$f(a+hu) = f(3, 4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$Dvt(a) = \lim_{h \to 0} \frac{25h^2/26 + 36h/\sqrt{26} + 8 - 8}{h}$$

$$= h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)$$

$$Dvt(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

$f(x,y) = x^y + y^x \Rightarrow a(1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + y^x \log y$

$f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^2 \log y,\ x^y \log x + xy^{x-1})$

$f(1,1) = (1+0,\ 1+0)$

$= (1,1)$

$f(x,y) = (\tan^{-1} x) \cdot y^2 \qquad a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \cdot \tan^{-1} x$

$f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2},\ 2y \tan^{-1} x\right)$

$$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2) \right)$$

$$= \left(\frac{1}{2}, \frac{\pi}{\cancel{4}2}(-\cancel{2}) \right)$$

$$= \left(\frac{1}{2}, \frac{-\pi}{2} \right)$$

iii) $f(x,y,z) = xyz - e^{x+y+z}$ $\quad a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz \cdot e^{x+y+z}, xz - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{1+(-1)+0}, (1)(0) - e^{1+(-1)+0}$

Q3

1) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y\, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} x$

$(x_0, y_0) = (1,0)$ $\quad x_0 = 1 \quad y_0 = 0$

eq^n tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0\, 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0)0 + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1)+1(y-0)=0$

$2x-2+y=0$

$2x+y-2=0$ → It is the required eq^n of normal so eq^n of tangent

$= ax+by+c=0$

$bx+ay+d=0$

$1(1)+2(y)+d=0$

$1+2y+d=0$ at $(1,0)$

$\therefore d=-1$

$x^2-2yz+3y+xz=7$ at $(2,1,0)$

$f_x = 2x-0+0+z$

$f_x = 2x+z$

$f_y = 0-2z+3+0$

$= 2z+3$

$f_z = 0-2y+0+x$

$= -2y+x$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore x_0=2, y_0=1, z_0=0$

$f_x(x_0, y_0, z_0) = 2(2)+0=4$

$f_y(x_0, y_0, z_0) = 3$

$f_z(x_0, y_0+z_0) = -2(1)+2=0$

eq^n of tangent

$f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$

$4(x-2)+3(y-1)+0(z-0)=0$

$4x-8+3y-3=0$

$4x+3y-11=0$

This is req. eq^n of tangent.

eq^n of normal at $(4, 3, -1)$

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+1}{0}$$

iii) $3xyz - x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$

$f(x) = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$
$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0 \rightarrow$ This is required eq^n of tangent.

Eq^n of normal at $(-7, 5, -2)$

$$\therefore \frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$

$f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ — ①

$f_y = 0$

$2y - 3x - y = 0$

$2y - 3x = 4 \rightarrow$ ②

Multiply eq^n ① with ②

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

$$12x^2 + 4y = 0$$
$$(-)\quad 12x^2 - 8y = 0$$
$$(+)$$

$$12y = 0$$
$$y = 0$$

Substitute value of y in eq^n ①

$$4x^2 + 3(0) = 0$$
$$x = 0$$

Critical pt is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$
$= 24(0) + 6(0) = 0$
$r = 0$

$f(x,y)$ at $(0,0)$
$2(0)^4 + 3(0)^2(0)$
$= 0$

$rt - s^2 = 0(-2) - (s)^2$
$= 0 - 0$
$= 0$

$r = 0$ & $rt - s^2 = 0$
(nothing to say)

All
03/04/2020