Blast Off

SUMMER PROJECT, ASTRONOMY CLUB, IITK

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Contents

1 History

1.1 The Fire Dragon

Medieval and early modern rockets were used militarily as incendiary weapons in sieges. The first gunpowder-powered rockets evolved in medieval China under the song dynasty by the 13th century. There are mentions of the first known multi-stage rocket, the 'Fire-dragon issuing from the water',thought to have been used by the Chinese navy.

1.2 Robert H.Goddard's Rocket

William Leitch first proposed the concept of using rockets to enable human spaceflight in 1861. In 1898 , Tsiolkovsky proposed the idea of liquid propelled rockets.He is called the father of the modern astronautics. Early in the 20th century ,an American,Robert H.Goddard, conducted practical experiments in rocketry. While working on solid-propellant rockets,Goddard became convinced that a rocket could be propelled better by liquid fuel.

He achieved the first successful flight on March 1926,with oxygen and gasoline as fuel. Gasoline was used as a fuel and oxygen was used as an oxidiser. He figured out a way to keep the combustion chamber from exploding by making a revolutionary modification, which is still used in modern rocketry!! He used extremely cold liquid oxygen via a network of pipes to keep the combustion chamber cool and making rocket more efficient since less energy is lost as heat.

Figure 1: Robert H. Goddard's Rocket

The rocket's combustion chamber is the small cylinder at the top ,the nozzle is visible beneath it. The fuel tank is directly beneath the nozzle and is protected from the motor's exhaust by an asbestos cone. Asbestos-wrapped aluminium tubes connect the motor to the tanks providing support and fuel transport. But the design was not stable and the combustion chamber and the nozzle was later put at the bottom.

As Goddard put the payload at the bottom of the rocket,there are some shortcomings in it. The bottom line is, putting the satellite at the top is convenient to dampen vibrations ,reduce aerodynamic turbulence ,improve performance,reduce heating of the spacecraft ,etc.

1.3 Wernher von Braun's Rocket

He was the pioneer of the famous V-2 rocket. The V-2 became the first artificial object travel into space by crossing the Karman line on 20th June 1944. It used 75% Ethanol / 25% water(as coolant) mixture for fuel and liquid oxygen as an oxidiser. The pump was driven by a steam engine which carried fuel and oxygen to the combustion chamber. Sodium permanganate was used as a catalyst to make steam from hydrogen peroxide.

Figure 2: V2 Rocket

1.4 The Space Race

The Space Race began with the 1957 launch of the Soviet satellite Sputnik 1. **Vostok** was a family of rockets derived from the Soviet R-7 Semyorka ICBM and was designed for the human spaceflight programme . This family of rockets launched the first artificial satellite (Sputnik 1) and the first crewed spacecraft (Vostok) in human history. The Soviet Union put the first human ,cosmonaut **Yuri Gagarin**,into a single orbit aboard Vostok 1 on April 12,1961. With the space race heating up,the US also launched programme like Project Mercury and Project Gemini,but they were a little behind in the race to space. But everything changed with **The Apollo Program**. It was the third human spaceflight program which succeeded in landing the first humans on moon from 1969-1972. Several planned missions of the Apollo crewed Moon landing program of the 1960s and 1970s were canceled for a variety of reasons , including changes in technical direction,the Apollo 1 fire ,hardware delays ,and the budget limitations. It is highly recommended to watch Apollo 11 documentary!

Figure 3: Apollo 11

Figure 4: Falcon 9

1.5 SpaceX

After a long long hibernation , finally the world is walking up to space exploration again.And this time it's not any cold war fueling the ignition ,it's a burning desire to level up humanity as a whole .

Figure 5: Starship

Space Exploration Technologies Corp popularly known as **SpaceX** is an American space manufacturer,founded in 2002 by **Elon Musk** with the goal of reducing space transportation costs to enable the colonization of Mars and make humanity a space fairing civilization. SpaceX manufacturer the falcon Heavy launch vehicles , several rocket engines, Cargo Dragon, crew spacecraft, and Starlink communication satellites. SpaceX is the first organisation to produce reusable rockets and boosters!

Currently they are working on **Starship**, a fully reusable super heavy-lift launch vehicle. It is the tallest, heaviest, and most powerful rocket ever built. Both stages combust liquid oxygen and methane with variants of Raptor engines. Starship may deploy satellites and space probes ,serving space tourists, and exploring the Moon via the Artemis program. Further into the future, the rocket may travel between locations on Earth and aid SpaceX's ambition of colonizing Mars. Such operation level is only possible due to reduced launch cost.

2 Anatomy of a rocket

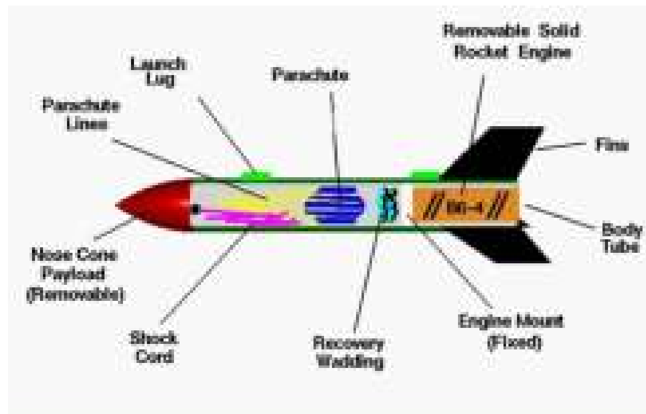


Figure 6: Structure of a rocket

A rocket has four major parts.

1. Faring / Nose Cone
2. Body
3. Engine
4. Fins

2.1 Structural system

The structural system of a rocket includes all the parts which make up the frame of the rocket ; the cylindrical body ,the fairings and the fins.The frame is made up of strong but lightweight materials such as aluminium or titanium .It is coated with thermal protection system to keep out heat of air friction during the flight.The fins are attached at the bottom for stability during the flight.the materials used in the construction of the rocket's boosters and space vehicles range from special high density material for heat absorption to high strength ,lightweight materials to carry flight loads.Vehicle skin thickness must be as low as possible to attain an optimal weight for the rocket.The structure can be internally pressurised either by combustion of gases like helium or using autogenous pressurization to keep the walls from buckling.

2.2 Fairing

The payload present on a spacecraft is delicate and at the initial stages of a launch a large aerodynamic forces act on the aircraft due to earth's atmospheric pressure and contaminants in the atmosphere.Thus fairings are used as a protective covering to safeguard the rocket's body. It also improves the aerodynamics of the launch vehicle by providing a smooth and aerodynamic front cross section to the launch vehicle.

They are generally made of various composite material. Aluminium honeycomb core structure effectively utilises space and provides exceptional strength to the fairings.Ceramic provides thermal insulation and acoustic panel dampens the large vibrations caused due to shock waves.

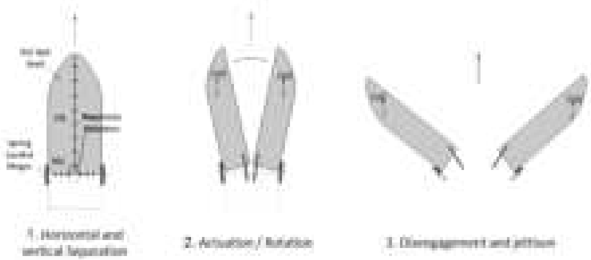


Figure 7: Separation of fairings

The mechanism for the separation of the fairings is to separate the fairings into two halves and jettisoning them.Usually explosive bolts, which are bolts with a designed weak point and a small amount of explosive around it are used for this process.Frangible bolts were simple and strong ways of keeping your rocket as one until a stage had to be jettisoned. When the charges are detonated, the bolts are broken and the stages separate.Generally the fairings fall down in the ocean or get burnt in the way down.

SpaceX with the goal of re-usability of fairings don't use explosive bolts as it would damage the area around them, requiring repair work that they don't want to do. They use hydraulic mechanism to hold the stages together and some springs to gently separate them when the hydraulic catches are opened this process preserve the fairings to a larger extent. SpaceX has successfully recovered the fairings of Falcon 9 creating history.

2.3 Body

The body tube is the airframe of the model rocket.A rocket may have multiple body sections connected with transition sections .It is made from very strong but light weight materials, like titanium or aluminum .It holds the propellant and the rocket engine . Fins are attached to its lower end.

2.4 Engine

A rocket engine uses stored rocket propellants(fuel and the oxidizer) as the reaction mass to form the high-speed propulsive jet of fluid, usually high temperature gases. Rocket engines are reaction engines, producing thrust by ejecting mass rearward in accordance with Newton's Third Law.
There are two main categories of rocket engines ;

1. Solid rocket engine
2. Liquid rocket engine

In a solid rocket engine, the propellants are mixed together and packed into a solid cylinder.They will start burning when exposed to a source of heat provided by an igniter and this proceeds until all the propellant is exhausted.
In a liquid rocket engine, the fuel and the oxidizer are stored separately as liquids and are pumped into the combustion chamber of the nozzle when burning occurs. With a liquid rocket, one can control the thrust by varying the flow of propellants.

2.5 Fins

The fins of the rocket provide aerodynamic stability in flight. It helps the rocket to maintain its actual orientation and intended flight path.Generally a rocket has three fins made up of aluminum and copper; of an optimal size to reduce the drag force. For a rocket to be stable, the centre of pressure needs to be below the centre of gravity.Placing fins at the bottom of a rocket moves the centre of pressure closer towards the bottom end and thus increases stability. However, with the advancement in technology the direction of flight is controlled by onboard computer controlled guidance system.This is typically accomplished by pointing the rocket engine(s) off-center, a technique called gimballing the engine.

3 Aerodynamics

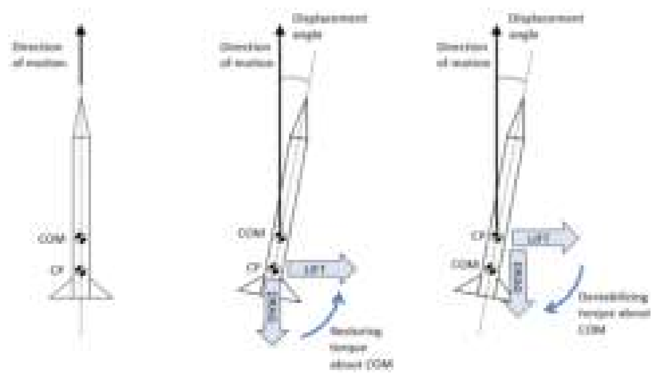


Figure 8: Stability of a rocket model

The stability of a rocket model depends broadly on two major factors

1. Center of Pressure (C_p)
2. Center of Gravity (C_g)

3.1 Location of C_p

Rocket experiences a different velocity and pressure of the fluid at different places along its body.This difference results in a force moving the body from a high pressure area to a low pressure area.Thus we calculate the average location of the pressure called the Center Of Pressure.

$$C_p = \frac{\int xp(x)dx}{\int p(x)dx} \tag{1}$$

where $p(x)$ is the function of pressure dependent on the distance from reference line. The location is highly dependent on the Angle of Attack (α).

3.2 Location of C_g

The center of gravity is the average location of the weight of the rocket.The center of gravity is the mass-weighted average of the component locations. Thus

$$WC_g = [w \cdot c_g]_s + [w \cdot c_g]_{pay} + [w \cdot c_g]_e + [w \cdot c_g]_{prop} \quad (2)$$

where W is the total weight of the rocket and w is the weight of the respective components.

For a rocket to be stable the center of pressure must lie below the center of gravity. The rocket flies stable if the distance between gravity and pressure point is between one caliber and less than 5 caliber. Center of pressure needs to be below center of gravity in order for the rocket to fly straight. Mathematically, the rocket will tilt around the center of gravity but appear to be pushed from the center of pressure, hence the need for the center of pressure to be below the center of gravity, otherwise the rocket will just corkscrew off the pad. The fins move the center of pressure down.

Most of the model rockets follow this rule but with the responses of computers and autopilot systems have become fast enough to steer the rockets even when the launch vehicle becomes aerodynamically unstable for a moment.

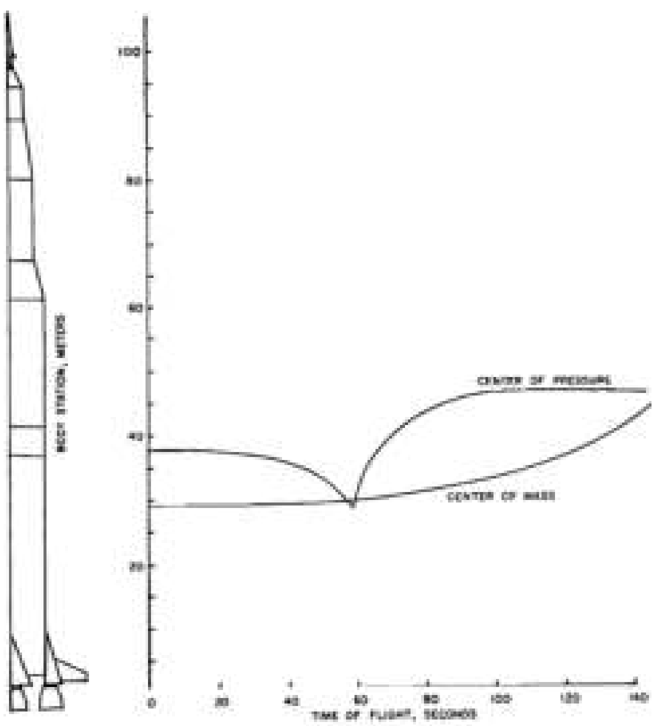


Figure 9: Pressure vs time graph for a rocket

This trend can be seen with the case of mighty Saturn V. In the figure it is clearly visible that C_p is below C_g at one point of time but still this doesn't affected its stability because of its advance autopilot system. Moreover, there are very small fins at the bottom of the rocket to decrease the instability in case of engine-gimbal failure.

Both C_p and C_g follows a certain characteristic nature across the timeline of a rocket. In the initial stage of launch, when the shock waves are created, the pressure at the front part of the rocket increases as shock wave cause a sudden and sharp increment in pressure. Since center of pressure represents the point where the whole pressure can be assumed to be acting upon, C_p moves above. During the course of its flight, location of C_p rises as the ratio of velocity of rocket/speed of sound increases while the location of C_g falls with the burning of propellants.

3.3 Shock Dynamics

Rockets tend to break the sound barrier very fast and often when they haven't left the lower atmosphere creating the area around the rocket with high pressure. Across a shock wave, the static pressure, temperature, and gas density increases almost instantaneously which may destroy the delicate structure of the rocket.

Thus a strong but lightweight fairings is required to protect rocket's body. The performance of rocket depends directly on the weight of the structure.

3.4 Dynamic Pressure and Max Q

The pressure on the rocket due to earth's atmosphere is given by the equation:

$$Q = \frac{1}{2} C_D \rho A v^2 \quad (3)$$

where

- C_D = Coefficient of drag
- ρ = Density of atmosphere
- A = Cross sectional area of the rocket
- v = Velocity of the rocket

The density of the rocket increases with altitude and velocity of the rocket increases with altitude which results into the maxima in the pressure vs time graph of a rocket of the **Max Q** faced by the rocket.This point is generally reached after one minute of the flight .

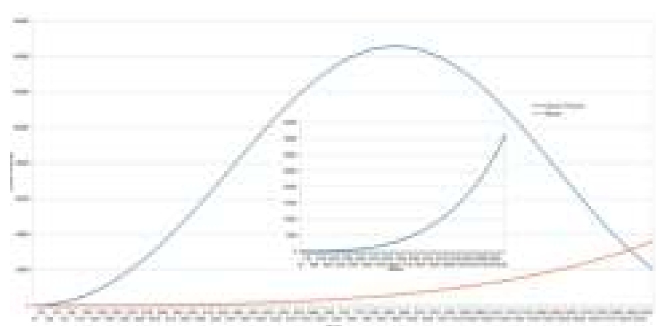


Figure 10: Pressure vs time graph for a rocket

Max Q is very crucial for the structural design of the rocket as high aerodynamic load can cause catastrophic destruction of our rocket body.Thus the speed of the vehicle is generally reduced slightly below this point by throttling the engines to about 60-70%.Engines are throttled down before approaching max q and back up afterwards to reduce the speed and hence the maximum dynamic pressure encountered along the flight.

4 Primitive Mathematical Model

We derived a bare primitive set of rocket equations by assuming gravity and drag force to be zero and by taking the rate of ejection of exhaust gases to be constant. These equations gave a basic idea of rocket motion. The derivation of those equations is written below.

4.1 Equation Derivation

Without loss of generality we have assumed the direction of rocket's motion as positive. Let us assume that a rocket of initial mass m_0 is launched with initial velocity v_0 and the relative speed of the exhaust gases w.r.t. the rocket is equal to v_e at any time t. So, the speed of gases w.r.t. ground frame is $v_e - v$ where v is the speed of rocket at that instant. Let m be the constant rate of ejection of gases.
At time t,

- Mass = m
- Speed = v

At time $t + dt$,

- Mass = $m - dm$
- Speed = $v + dv$

Since there is no external force (by hypothesis) we can conserve the total momentum of the system at two instants.

$$\begin{aligned} mv &= (m - dm)(v + dv) - dm(v_e - v) \\ 0 &= mdv - v_e dm \\ \frac{dm}{m} &= \frac{dv}{v_e} \end{aligned} \tag{4}$$

Integrating both sides from the limit M_i to M_f and v_i to v_f, we get

$$\begin{aligned} \int_{M_i}^{M_f} \frac{dm}{m} &= \int_{v_i}^{v_f} \frac{dv}{v_e} \\ \log(\frac{M_f}{M_i}) &= \frac{v_f - v_i}{v_e} \end{aligned} \tag{5}$$

Substituting

- $\Delta M = M_f - M_i$
- $\Delta v = v_f - v_i$

We get,

$$\frac{\Delta M}{M_i} = 1 - e^{\frac{\Delta v}{v_e}} \tag{6}$$

4.2 Specific Impulse

Specific impulse (usually abbreviated I_{sp}) is a measure of how efficiently a reaction mass engine (a rocket using propellant or a jet engine using fuel) creates thrust. For engines whose reaction mass is only the fuel they carry, specific impulse is exactly proportional to the effective exhaust gas velocity.

A propulsion system with a higher specific impulse uses the mass of the propellant more efficiently. In the case of a rocket, this means less propellant needed for a given Δv, so that the vehicle attached to the engine can more efficiently gain altitude and velocity.

In an atmospheric context, specific impulse can include the contribution to impulse provided by the mass of external air that is accelerated by the engine in some way, such as by an internal turbofan or heating by fuel combustion participation then thrust expansion or by external propeller. Jet engines breathe external air for both combustion and by-pass, and therefore have a much higher specific impulse than rocket engines. The specific impulse in terms of propellant mass spent has units of distance per time, which is a notional velocity called the effective exhaust velocity. This is higher than the actual exhaust velocity because the mass of the combustion air is not being accounted for. Actual and effective exhaust velocity are the same in rocket engines operating in a vacuum. The most common unit for specific impulse is the second. Mathematically,

$$I_{sp} = \frac{F_{thrust}}{m \times g_0} \tag{7}$$

where

- m = Mass rate of exhaust ejection
- g_0 = Gravity at surface of the earth
- F_{thrust} = Thrust force acting on the rocket

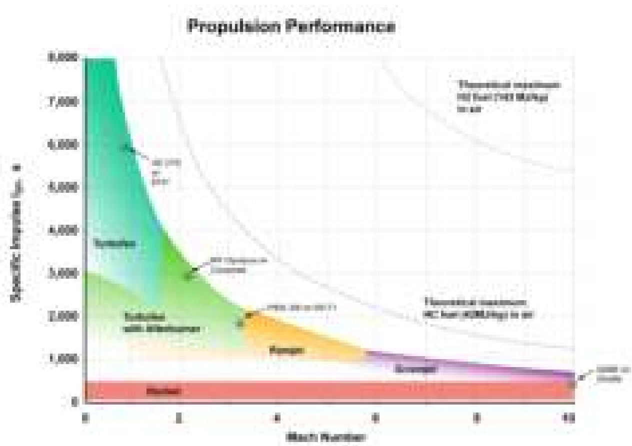


Figure 11: Specific impulse of various jet engines

5 Staging

Staging is the combination of several rocket sections, or stages, that fire in a specific order and then detach, so a ship can penetrate Earth's atmosphere and reach space.

The operative principle behind rocket stages is that you need a certain amount of thrust to get above the atmosphere, and then further thrust to accelerate to a speed fast enough to stay in orbit around Earth (orbital speed, about five miles per second).

It's easier for a rocket to get to that orbital speed without having to carry the excess weight of empty propellant tanks and early-stage rockets. So when the fuel/oxygen for each stage of a rocket is used up, the ship jettisons that stage, and it falls back to Earth. This becomes part of the rocket's mass fraction—the portion of its fully fueled pre-launch mass that does reach orbit. We have studied two types of rocket staging:

- Serial Staging
- Parallel Staging

5.1 Serial Staging

Stages are attached, one on top of the other, or stacked. The first stage ignites at launch and burns through its fuel until its propellants are spent. Now useless dead weight, in a staging maneuver the first stage breaks free from the previous stage, then begins burning through the next stage in straight succession. Depending on the rocket, the second stage may get the payload into orbit or require a third or fourth stage to ultimately deliver it to space. It depends on the individual rocket and mission.

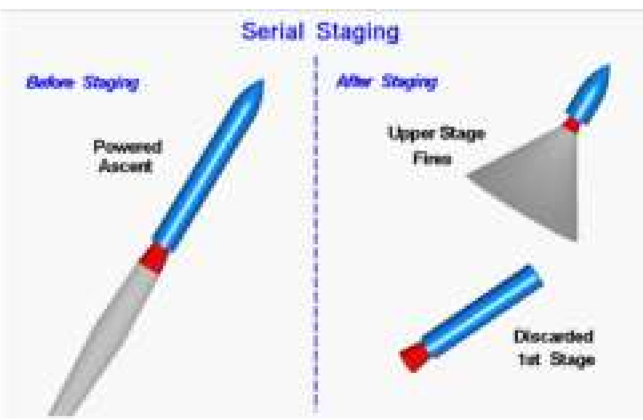


Figure 12: Serial Staging

5.2 Parallel Staging

Serial staging involves stacked stages, parallel staging features one or multiple booster stages strapped to a central sustainer, as on the space shuttle. At launch, all the engines ignite. When their propellant runs out, the strapped-on boosters fall away. The sustainer engine keeps burning to put the payload into orbit. With the shuttle, solid rocket boosters are the stages that fall away from the main sustainer, the external tank that fed the main engines. The Titan III is an example of a rocket that uses both serial and parallel staging; it used a two-stage Titan II as the sustainer and added two solid rocket stages as boosters that fell away once they were done, much like the Solid Rocket Boosters on the Space Shuttles.

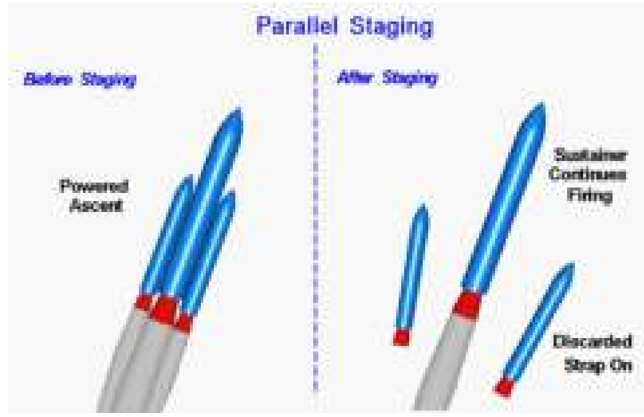


Figure 13: Parallel Staging

6 Single Stage To Orbit

6.1 Introduction

A single-stage-to-orbit (or SSTO) vehicle reaches orbit from the surface of a body using only propellants and fluids and without expending tanks, engines, or other major hardware. The term usually, but not exclusively, refers to reusable vehicles.

6.2 Advantages

- The main projected advantage of the SSTO concept is elimination of the hardware replacement inherent in expendable launch systems.
- These are kind of reusable hence less costly body.

6.3 Disadvantages

- The non-recurring costs associated with design, development, research and engineering (DDRE) of reusable SSTO systems are much higher than expendable systems due to the substantial technical challenges of SSTO, assuming that those technical issues can in fact be solved.
- SSTO vehicles may also require a significantly higher degree of regular maintenance.

6.4 Challenges with SSTO vehicles

- A high orbital velocity of 7400m/s needs to be achieved for successful launch.
- Earth atmosphere provides a great resistance to rocket hence creating great strain on rocket's body.
- For greater efficiency lower structural fraction needs to be achieved.
- Using same structural coefficient a two-stage-to-orbit vehicle will always have a better payload-to-weight ratio than a single stage designed for the same mission.

6.5 A Few Calculations based on Staged Rockets

The Tsiolkovsky rocket equation expresses the maximum change in velocity any single rocket stage can achieve:

$$\Delta v = I_{sp}.g_0.ln(M_0/M_f)$$

$$M_0 = M_{prop} + M_{pay} + M_{rocket}$$

$$M_f = M_{pay} + M_{rocket}$$

$$\implies \Delta v = I_{sp}.g_0.ln(1 + \frac{M_{prop}}{M_{pay} + M_{rocket}})$$

Let

$$\zeta = \frac{M_{prop}}{M_0}$$

$$\lambda = \frac{M_{rocket}}{M_f}$$

where

ζ= propellent mass fraction

λ=structural coefficient

$$\implies M_0 = \frac{M_{prop}}{1 - \frac{\zeta}{1-\lambda}}$$

- Maximum value of structural coefficient is equals to 1
- For extending this model to DSTO(Double Stage To Rocket) we can extend it by putting similar expressions for λ_1 ,ζ_1 for stage 1 and λ_2 ,ζ_2 for stage 2 and so on for higher order stage rockets.

7 Re-entry

Atmospheric entry is the movement of an object from outer space into and through the gases of an atmosphere of a planet, dwarf planet, or natural satellite. There are two types of atmospheric entry:

- Uncontrolled entry, such as the entry of astronomical objects, space debris, or bolides;
- Controlled entry (or re-entry) of a spacecraft capable of being navigated or following a predetermined course.

7.1 Challenges and it's solutions while re-entry

Objects entering an atmosphere experience atmospheric drag, which puts mechanical stress on the object, and aerodynamic heating—caused mostly by compression of the air in front of the object, but also by drag. These forces can cause loss of mass (ablation) or even complete disintegration of smaller objects, and objects with lower compressive strength can explode.

Space vehicles must be slowed to subsonic speeds before parachutes or air brakes may be deployed. Such vehicles have kinetic energies typically between 50 and 1,800 megajoules per kilogram, and atmospheric dissipation is the only way of expending the kinetic energy.

7.2 Re entry Corridor

The re-entry corridor is a narrow region in space that a re-entering vehicle must fly through.

If the vehicle strays above the corridor, it may skip out. If it strays below the corridor, it may burn up.

7.3 Re-entry Maneuver

As a spacecraft re-enters the earth's atmosphere, it is traveling very much faster than the speed of sound. The aircraft is said to be hypersonic. Typical low earth orbit re-entry speeds are near 17,500 mph. The chief characteristic of re-entry aerodynamics is that the temperature of the flow is so great that the chemical bonds of the diatomic molecules of the air are broken.Strong shock waves are generated on the lower surface of the spacecraft.

The Shuttle flies at a high angle of attack during re-entry to generate drag to dissipate speed. It executes hypersonic "S-turn" maneuvers to kill off speed during re-entry. The lift of the wings is only important in the final flare maneuver at touchdown.

7.4 Heating of Rocket during Re-entry

The Shuttle uses a rocket propulsion system to get into orbit, but during re-entry the aircraft is actually an un-powered glider. Small steering rockets are used for maneuvering early in the re-entry because the low density of the air at altitudes above 50 miles makes aerodynamic surfaces ineffective. The heat is so great during re-entry that a special thermal protection system is used to keep the spacecraft intact. This gives rise to need of something called heat shield.

8 Heat Shield

A heat shield is designed to protect an object from overheating by dissipating, reflecting, absorbing heat, or simply gradually burn and fall away from the aircraft, pulling the excess heat with it.Heat shields protect structures from extreme temperatures and thermal gradients by two primary mechanisms.

Thermal insulation and radiative cooling, which respectively isolate the underlying structure from high external surface temperatures, while emitting heat outwards through thermal radiation. To achieve good functionality the three attributes required of a heat shield are low thermal conductivity (high thermal resistance), high emissivity and good thermal stability (refractoriness).

Many types of heat shield are available in the market but mainly they can be categorised into five types:

- Insulation Blankets
- Insulation Tiles
- Reinforced Carbon carbon
- Ablative Heat Shield
- Regenerative Cooling

8.1 Insulation Blanket

Insulation blanket is an especially low-weight, low-bulk blanket made of heat-reflective, thin, plastic sheeting. They are used on the exterior surfaces of spacecraft for thermal control.

- These are used where maximum temperature a rocket is exposed is less than 649°C
- It holds the aircraft firmly and is easier to maintain. It looks like a blanket.

8.2 Insulation Tiles

Insulation tiles are used all over the orbiter; there are nearly 20,000 of these tiles on the orbiter. They do not encounter the most extreme temperatures but must withstand some heat. The tiles protect areas where temperatures are below 2,300°F.

- These tiles can withstand temperature upto 2300°F.
- These tiles are made up of ceramic material which is bad conductor of heat and electricity
- These are extremely fragile and there is always a possibility of dropping off the tiles

Even if one tile get loosened,it could result in a big disaster. Such a disaster has happened before with Kalpana Chawla in Columbia space shuttle. On February 1, 2003, the Space Shuttle Columbia was destroyed on reentry due to a failure of the Thermal Protection System. The investigation team found and reported that the probable cause of the accident was that during launch, a piece of foam debris punctured an Reinforced carbon carbon panel on the left wing's leading edge and allowed hot gases from the reentry to enter the wing and disintegrate the wing from within, leading to eventual loss of control and breakup of the shuttle.

8.3 Reinforced Carbon-Carbon

Reinforced Carbon–Carbon (RCC) is a composite material consisting of carbon fiber reinforcement in a matrix of graphite. It was developed for the reentry vehicles of intercontinental ballistic missiles, and is most widely known as the material for the nose cone and wing leading edges of the Space Shuttle orbiter.

- Carbon–carbon is well-suited to structural applications at high temperatures, or where thermal shock resistance and/or a low coefficient of thermal expansion is needed
- I can be heavy but is very strong and can bear high thermal and mechanical stresses.
- While it is less brittle than many other ceramics, it lacks impact resistance; Space Shuttle Columbia was destroyed during atmospheric re-entry after one of its RCC panels was broken by the impact of a piece of polyurethane foam insulation

8.4 Ablative Heat Shield

An ablative heat shield consists of a layer of plastic resin, the outer surface of which is heated to a gas, which then carries the heat away by convection. Such shields were used on the Mercury, Gemini, and Apollo spacecraft, and are currently used by the SpaceX Dragon 2 spacecraft, and will be used on the Orion spacecraft.

They are of three types:

1. Super Lightweight Ablator :-SLA stands for super lightweight ablator. SLA is a proprietary ablative made by Lockheed Martin that has been used as the primary TPS material.SLA is applied by packing the ablative material into a honeycomb core that is pre-bonded to the aeroshell's structure thus enabling construction of a large heat shield.
2. Phenol Ablative Carbon Ablator:-Phenolic-impregnated carbon ablator (PICA), a carbon fiber preform impregnated in phenolic resin,[27] is a modern TPS material and has the advantages of low density (much lighter than carbon phenolic) coupled with efficient ablative ability at high heat flux. It is a good choice for ablative applications such as high-peak-heating conditions found on sample-return missions or lunar-return missions. PICA's thermal conductivity is lower than other high-heat-flux-ablative materials, such as conventional carbon phenolics.
3. AVCOAT :-AVCOAT is an epoxy novolac resin with special additives in a fiberglass honeycomb matrix. In fabrication, the empty honeycomb is bonded to the primary structure and the resin is gunned into each cell individually.The Avcoat to be used on Orion is reformulated to meet environmental legislation that has been passed since the end of Apollo.

8.5 Regenerative Cooling

Regenerative cooling, in the context of rocket engine design, is a configuration in which some or all of the propellant is passed through tubes, channels, or in a jacket around the combustion chamber or nozzle to cool the engine. This is effective because the propellants are often cryogenic. The heated propellant is then fed into a special gas-generator or injected directly into the main combustion chamber.

Figure 14: The NASA Apollo Missions

9 Rocket Propulsion

Rocket propulsion is the system that powers a rocket, lifts it off the ground, and propels it through the air. Unlike jets, rockets carry their own propellant. Their invention dates back thousands of years, but modern rocketry did not begin until the past few hundred years. Today, solid or liquid propellants are used to blast rockets to immense heights. They have sent robots to Mars and have even carried people to the moon.

Rocket propulsion is simply based on Sir Issac Newton's third law of motion which states that "for every action, there is an equal and opposite reaction." It is upon this principle that a rocket operates. Propellants are combined in a combustion chamber where they chemically react to form hot gases which are then accelerated and ejected at high velocity through a nozzle, thereby imparting momentum to the engine. The thrust force of a rocket motor is the reaction experienced by the motor structure due to ejection of the high velocity matter. This is the same phenomenon which pushes a garden hose backward as water flows from the nozzle, or makes a gun recoil when fired.

9.1 How Rocket Engines Work?

Modern rockets work by igniting a propellant. As the propellant is combusted, it is converted from a solid or liquid form into a gas. The gases produced have such a high volume, and are under such intense pressure from the heat of combustion, that they are pushed downward, through the nozzle, and out of the open end of the rocket at high velocities. From there, Sir Isaac Newton takes over. Newton's third law of motion explains that, for every action, there is an equal and opposite reaction. This means that simply stated, forces come in pairs. As the exhaust gas is pushed downward out of the bottom of the rockets, the gases also push the rocket upward with an equal and opposite force. The amount of force depends upon both the mass of the gases and the acceleration of the gases. And because the rocket itself is so much heavier than the exhaust, large rockets will usually move slowly at first. But as the combustion process continues, and more and more gas is ejected, the rocket receives a constant push. From there acceleration can take it to truly astronomical heights.

Figure 15: NASA RS-25 engines

9.2 Rocket Propulsion Types

Based on the types of propellants used there can be two types of rocket engines- liquid rockets and solid rockets. In a liquid rocket, the propellants, the fuel, and the oxidizer are stored separately as liquids and are pumped into the combustion chamber of the nozzle where burning occurs. In a solid rocket, the propellants are mixed together and packed into a solid cylinder. Under normal temperature conditions, the propellants do not burn; but they will burn when exposed to a source of heat provided by an igniter. Once the burning starts, it proceeds until all the propellant is exhausted. With a liquid rocket, you can stop the thrust by turning off the flow of propellants; but with a solid rocket, you have to destroy the casing to stop the engine. Liquid rockets tend to be heavier and more complex because of the pumps and storage tanks. The propellants are loaded into the rocket just before launch. A solid rocket is much easier to handle and can sit for years before firing. Liquid-fuel rockets carry a fuel and an oxidizer (often liquid hydrogen and liquid oxygen) in separate tanks. During flight, the two components are pumped into the combustion chamber and burned as needed. Liquid rockets are handy because you can adjust the thrust as needed, simply by adjusting the flow of fuel. However, they do tend to be heavier, more complex, and more costly than solid-fuel rockets because their design must include separate tanks and pumps. Solid-fuel rockets, on the other hand, have their fuel and oxidizer (often aluminium and ammonium perchlorate) pre-mixed in the appropriate ratios. When the fuel is heated to high enough temperatures, combustion begins. The thrust from solid rockets cannot be adjusted during combustion. But they are lighter, simpler, and less expensive than liquid-fuel rockets. The end of the story with both types of rockets is the same. The fuel and oxidizer are combusted at high temperatures. Exhaust gases are violently pushed out the bottom of the nozzle, and the rocket begins to take flight.

Figure 16: The Space Shuttle used both solid and liquid propellants.

Liquid Rocket Propellants

Rockets that need higher energy propellants and controllability (throttling or restart capability) often choose liquid propellants. Generally speaking, liquid propellant combinations has higher energy levels (specific impulse) than solid propellant mixtures. A high energy propellant combination often used in launch vehicles is the mixture between oxygen and hydrogen. Depending on the mixture ratio, the specific impulse level can reach above 4300 m/s (438 s). This is 1.7 times more efficient than a modern solid propellant. When we are talking about liquid propellants, we are often dividing them into the following parts:

- Petroleum-based propellants
- Cryogenic propellants
- Hypergolic Propellants
- Monopropellants

Petroleum-based Petroleum fuels are those refined from crude oil and are a mixture of complex hydrocarbons, i.e. organic compounds containing only carbon and hydrogen. The petroleum used as rocket fuel is a type of highly refined kerosene, called RP-1 in the United States. Petroleum fuels are usually used in combination with liquid oxygen as the oxidizer. Kerosene delivers a specific impulse considerably less than cryogenic fuels, but it is generally better than hypergolic propellants.

Figure 17: Atlas I rocket burning RP-1 fuel

Figure 18: Apollo 11 Saturn V lifting off in July 16, 1969

During the early 1950s, the chemical industry in the US was assigned the task of formulating an improved petroleum-based rocket propellant which would not leave residue behind and also ensure that the engines would remain cool. The result was RP-1. A highly refined form of jet fuel, RP-1 burned much more cleanly than conventional petroleum fuels and also posed less of a danger to ground personnel from explosive vapours.

Liquid oxygen and RP-1 are used as the propellant in the first-stage boosters of the Atlas and Delta II launch vehicles. It also powered the first stages of the Saturn 1B and Saturn V rockets.

Cryogenic Liquid propellants in the cryogenic state means that the temperature required for liquid state is well below the room temperature. Typically, a gas like hydrogen or oxygen becomes a liquid when chilled down to a low enough temperature. In liquid rocket engines the reactants are stored in their respective tanks. Then they are either pushed by pressure or pumped into the combustion chamber where they are mixed and burned. By having the reactants separated makes the liquid rocket engine very safe compare to the solid propellant rocket motor where the propellant is premixed and stored inside a single compartment. The downside is increased complexity since several tanks/pressure vessels are needed alongside tubing and valves. On top of that we also need electronic sensors and control devices. That said, an important benefit with liquid rocket engines is that they are much more controllable in respect to thrust modulation and the ability turn the engine off and even on again. Features which are extremely difficult in solid propellant motors. Liquid rocket engines often are used in launch vehicles, spacecrafts and landers.

Hypergolic Hypergolic propellants are fuels and oxidizers that ignite spontaneously on contact with each other and require no ignition source. The easy start and restart capability of hypergols make them ideal for spacecraft manoeuvring systems. Also, since hypergols remain liquid at normal temperatures, they do not pose the storage problems of cryogenic propellants. Hypergols are highly toxic and must be handled with extreme care.

Monopropellants The fourth type of liquid propellants is named monopropellants. This means propellants that alone can perform an exothermal decomposition process releasing energy that can be utilized for creating thrust. Monopropellants just needs one tank. However they need a feed system, often in form of a gaseous feed system using ether helium or nitrogen gas. In order to convert the liquid into a thermally hot gas suitable for expansion, the liquid need to come into contact with a material can lower the molecules activation energy to enter a self-sustained decomposition process. Hydrogen peroxide decompose into oxygen and steam (water vapor). There is no combustion going on, therefor no burn, however the amount of gas generated and its temperature is more than sufficient to produce thrust through expansion through an ordinary nozzle. Hydrogen peroxide as monopropellant are seeing renewed interest due to its lower cost and non-toxicity compare to the state-of-the-art Hydrazine, in particular so for launchers attitude control systems. Typical thrust levels for such applications are 10 to 250 N.

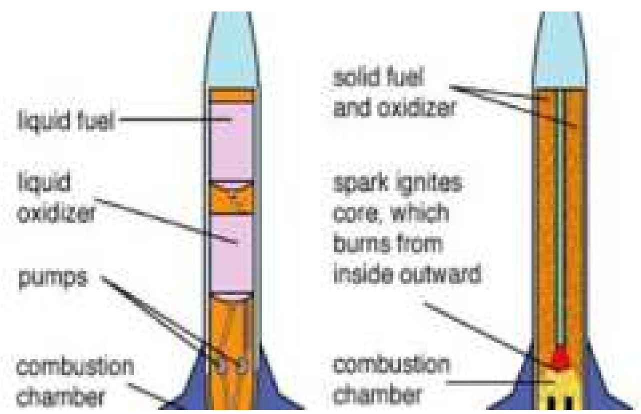


Figure 19: Cross section views of liquid- and solid -propellant rockets

Solid Rocket Propellants

A solid-propellant rocket or solid rocket is a rocket with a rocket engine that uses solid propellants (fuel/oxidizer). The earliest rockets were solid-fuel rockets powered by gunpowder; they were used in warfare by the Chinese, Persians, Mongols, and Indians as early as the 13th century.

All rockets used some form of solid or powdered propellant up until the 20th century when liquid-propellant rockets offered more efficient and controllable alternatives. Solid rockets are still used today in military armaments worldwide, model rockets, solid rocket boosters, and on larger applications for their simplicity and reliability.

Since solid-fuel rockets can remain in storage for a long time without much propellant degradation and because they almost always launch reliably, they have been frequently used in military applications such as missiles. The lower performance of solid propellants (as compared to liquids) does not favor their use as primary propulsion in modern medium-to-large launch vehicles customarily used to orbit commercial satellites and launch major space probes. Solids are, however, frequently used as strap-on boosters to increase payload capacity or as spin-stabilized add-on upper stages when higher-than-normal velocities are required. Solid rockets are used as light launch vehicles for low Earth orbit (LEO) payloads under 2 tons or escape payloads up to 500 kilograms.

Figure 20: NASA Image of a solid rocket booster (right) being mated to a Delta II rocket (teal)

Solid Rocket Booster(SRB) A Solid rocket booster (SRB) is a large solid propellant motor used to provide thrust in spacecraft launches from initial launch through the first ascent. Many launch vehicles, including the Ariane 5, Atlas V, and the space shuttle, have used SRBs to give launch vehicles much of the thrust required to place the vehicle into orbit. The space shuttle used two space shuttle SRBs, which were the largest solid propellant motors ever built and the first designed for recovery and reuse. Compared to liquid propellant rockets, the solid-propellant SRBs have been capable of providing large amounts of thrust with a relatively simple design. They provide greater thrust without significant refrigeration and insulation requirements and produce large amounts of thrust for their size. Adding detachable SRBs to a vehicle also powered by liquid-propelled rockets known as staging reduces the amount of liquid propellant needed and lowers the launch rig mass. Solid boosters are cheaper to design, test, and produce in the long run compared to the equivalent liquid propellant boosters. Reusability of components across multiple flights, as in the Shuttle assembly, also has decreased hardware costs.

10 Rocket Engines

A rocket engine uses stored rocket propellants as the reaction mass for forming a high-speed propulsive jet of fluid, usually high-temperature gas. Rocket engines are reaction engines, producing thrust by ejecting mass rearward, in accordance with Newton's third law.

Compared to other types of jet engines, rocket engines are the lightest and have the highest thrust, but are the least propellant-efficient (they have the lowest specific impulse). The ideal exhaust is hydrogen, the lightest of all elements, but chemical rockets produce a mix of heavier species, reducing the exhaust velocity.

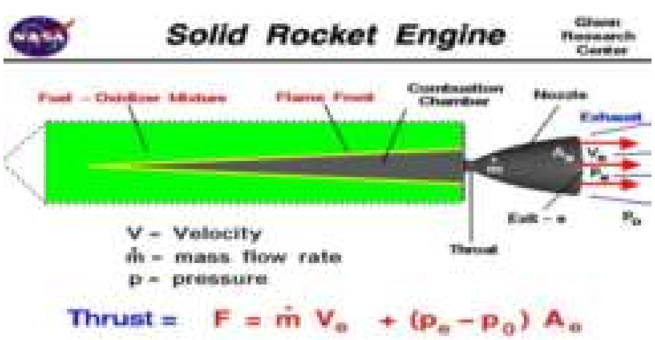


Figure 21

10.1 Injection

Liquid-fuelled rockets force separate fuel and oxidizer components into the combustion chamber, where they mix and burn. Hybrid rocket engines use a combination of solid and liquid or gaseous propellants. Both liquid and hybrid rockets use injectors to introduce the propellant into the chamber. These are often an array of simple jets – holes through which the propellant escapes under pressure; but sometimes may be more complex spray nozzles. When two or more propellants are injected, the jets usually deliberately cause the propellants to collide as this breaks up the flow into smaller droplets that burn more easily.

10.2 Combustion Chamber

For chemical rockets the combustion chamber is typically cylindrical, and flame holders, used to hold a part of the combustion in a slower-flowing portion of the combustion chamber. The dimensions of the cylinder are such that the propellant is able to combust thoroughly; different rocket propellants require different combustion chamber sizes for this to occur.

In order for fuel and oxidizer to flow into the chamber, the pressure of the propellants entering the combustion chamber must exceed the pressure inside the combustion chamber itself. This may be accomplished by a variety of design approaches including turbopumps or, in simpler engines, via sufficient tank pressure to advance fluid flow. Tank pressure may be maintained by several means, including a high-pressure helium pressurization system common to many large rocket engines or, in some newer rocket systems, by a bleed-off of high-pressure gas from the engine cycle to autogenously pressurize the propellant tanks. For example, the self-pressurization gas system of the SpaceX Starship is a critical part of SpaceX's strategy to reduce launch vehicle fluids from five in their legacy Falcon 9 vehicle family to just two in Star ship, eliminating not only the helium tank pressurant but all hypergolic propellants as well as nitrogen for cold-gas reaction-control thrusters.

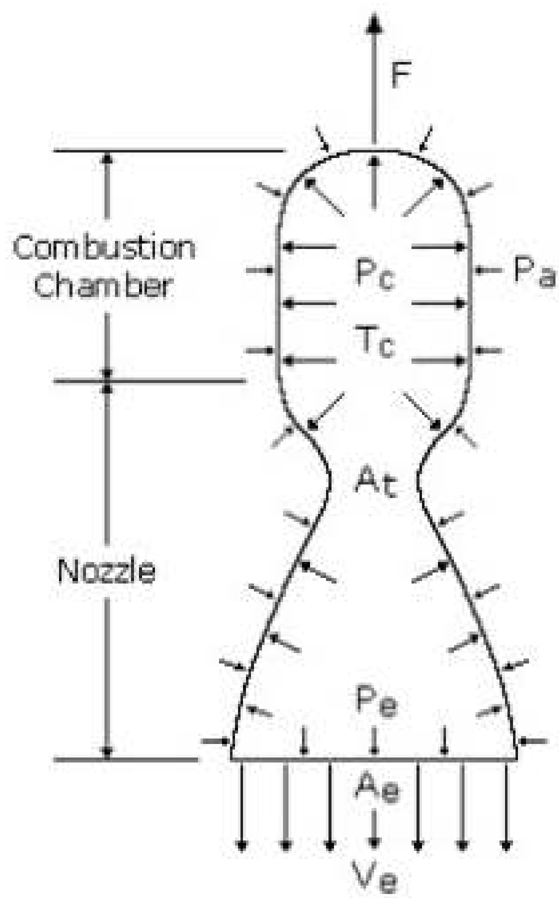


Figure 22: Detailed Rocket engine

10.3 Nozzle

Rocket thrust is caused by pressures acting in the combustion chamber and nozzle. From Newton's third law, equal and opposite pressures act on the exhaust, and this accelerates it to high speeds.

The hot gas produced in the combustion chamber is permitted to escape through an opening (the ”throat”), and then through a diverging expansion section. When sufficient pressure is provided to the nozzle (about 2.5–3 times ambient pressure), the nozzle chokes, and a supersonic jet is formed, dramatically accelerating the gas, converting most of the thermal energy into kinetic energy. Exhaust speeds vary, depending on the expansion ratio the nozzle is designed for, but exhaust speeds as high as ten times the speed of sound in air at sea level are not uncommon.

The most commonly used nozzle is the de Laval nozzle, a fixed geometry nozzle with a high expansion ratio. The large bell- or cone-shaped nozzle extension beyond the throat gives the rocket engine its characteristic shape.

The exit static pressure of the exhaust jet depends on the chamber pressure and the ratio of exit to throat area of the nozzle. As exit pressure varies from the ambient (atmospheric) pressure, a choked nozzle is said to be under-expanded (exit pressure greater than ambient), perfectly expanded (exit pressure equals ambient), over-expanded (exit pressure less than ambient; shock diamonds form outside the nozzle), or grossly over-expanded (a shock wave forms inside the nozzle extension).

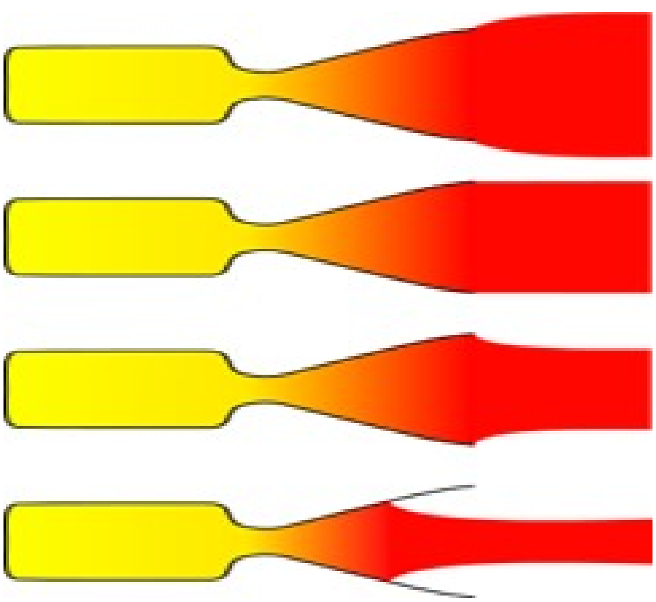


Figure 23: The four expansion regimes of a Laval nozzle: • under-expanded • perfectly expanded • over-expanded • grossly over-expanded

11 Rocket Plumbing

Key to utilising liquid fuel's high energy density is an engine's ability to consistently supply the combustion chamber with propellants at high pressures and high specific mass flow rates. Rocket engines supply themselves the energy required to do this via feed systems which can be thought of as an engines internal plumbing. “Liquid propellant feed systems consist of piping, a series of valves, provisions for filling and usually also for removing (draining and flushing) the liquid propellants, filters, and control devices to initiate, stop, and regulate their flow and operation” — George P. Sutton, Rocket Propulsion Elements These systems are predominantly powered by either high-pressure gas or turbopumps but importantly there is no ‘one size fits all’. Almost every engine has a different feed system, each of which provides its own design benefits and drawbacks, similar to the design pattern analogy drawn on earlier. So let's explore some of the most common feed systems and do a general comparison between them.

Figure 24: Falcons Heavy roaring Merlin Engines(SpaceX)

11.1 Pressure-fed Engines

Pressure-fed engines are the simplest and most reliable feed systems in use. Fundamentally, these systems use a separate supply of very high-pressure gas, usually inert such as helium, to force propellant from its tanks into the combustion chamber. As a result, the plumbing required is simple typically comprising of a gas starting valve, a gas pressure regulator, propellant tanks, propellant valves, feed lines and a high-pressure gas tank.

In general pressure-fed engines are the best choice for missions where both the required Delta-v and rocket payload is low. This is due to their best-in-class efficiency, cost and reliability at such mission specifications. During SpaceX's infancy, they opted for a ‘regulated’ pressure-fed system in their Kestrel engine, used in the upper stage of the Falcon 1 launch vehicle, crucial for achieving system simplicity and driving down costs.

For missions with heavier payloads, higher Delta-v's or both, the engine must generate more thrust which demands an increase in combustion chamber pressure. To produce higher pressures in a pressure-fed system, larger and more reinforced tanks are required which greatly increases the mass of the rocket. This makes pressure-fed systems hard-sells and is why many engine designs turn to turbopump feeds.

Advantages:

• Simplistic and low cost design

• Engines typically produce a high specific impulse (thrust per unit mass of propellant used)

Disadvantages:

• Generally low Delta-v's due to pressure limits imposed by feed gas

• Necessitates use of heavy gas tanks

Figure 25: Armadillo Aerospace's Pixel I

11.2 Gas-generator cycle

By far the most common form of turbopump engine cycle used is the gas-generator cycle. A separate gas-generator, also referred to as the pre-burner, is used to power the turbine of the turbopump. The propellant used to generate inlet gas for the turbine is usually drawn from the main propellant pumps via control valves.

The gas is then discharged to the atmosphere separately from the combustion chamber. This has the benefit of allowing the turbine flow-path to run in parallel with the combustion chamber flow-path, separating the two functions. As a result, the complexity of the engine is greatly reduced as there is no need to deal with the counter pressure of the exhaust discharging into the combustion chamber.

Given a gas-generator expands to atmospheric pressure there is a large pressure ratio across the turbine and thus a high available energy per unit mass of flow across the turbine. Combined with an overall efficient turbopump and high thermal operating temperature, a low turbine flow rate can produce a high specific impulse.

The caveat to this is gas-generator cycles are by definition open cycles, meaning not all propellant that can be burnt in the combustion chamber is. This results in losses in overall specific impulse.

Advantages:

• Although more complex than pressure-fed engines, the two-system design remains simplistic

• Reliable choice for ensuring a high thrust engine

Disadvantages:

• Generally low specific impulse versus counterparts

• Recycling secondary exhaust through the nozzle adds the extra complexity of back-pressure

Figure 26: RD-107 gas-generator engine powers Russia's famous Soyuz rockets (NPO Energomash)

11.3 Expander Cycle

The expander cycle is in contrast a closed engine cycle meaning all fuel is burnt in the combustion chamber. Cryogenic fuel is pumped around the combustion chamber and nozzle acting as a coolant picking up heat. The fuel once hot, changes phase to a gas and powers the turbine of the turbopump. As well as closing the cycle this design has the added benefit of removing the gas-generator and its accompanying plumbing making the cycle much cheaper and more reliable.

Several expander cycle engines have been successfully flown in the past with the RL10 engine being the most notable, having flown multiple missions with a very reliable track record. Currently ESA is developing a state-of-the art expander cycle engine called the VINCI which uses two separate turbopumps and a long combustion chamber to capitalise on a larger surface area.

Advantages:

• Simplest design of all engines mentioned

• Engines typically produce a high specific impulse

Disadvantages:

• Necessitates use of cryogenic fuel, not suitable for all missions

• Square-cube law imposes a thrust limit on these designs

Figure 27: VINCI is a fresh attempt at a high performance Expander Cycle engine (ESA)

11.4 Staged combustion cycle

The final notable turbopump engine cycle is the staged combustion cycle, a closed cycle, high performance and high thrust engine cycle that comes with the trade-off of greatly increased complexity. Opposite to gas-generator cycles, the pre-burner and turbine flow-path is in series with the combustion chamber flow-path and thus experiences counter pressures of the exhaust being discharged into the combustion chamber.

As a result, staged combustion engines have more complex plumbing and more resilient turbines to withstand the higher-pressure engine environment. Moreover, the newly introduced feedback between the pre-burner and combustion chamber makes the engine significantly harder to start-up correctly.

Full flow staged combustion cycles are most complex staged combustion cycles but boast the most impressive performance and thrust characteristics of all cycles. SpaceX's next generation Raptor rocket engine, which will power the famous Starship to Mars, is a full flow staged combustion cycle methalox engine.

Figure 28: Starship fitted with SpaceX's powerful full-flow staged combustion Raptor Engines (SpaceX)

Advantages:

• Highest specific impulse currently possible

• Engines are typically high thrust

Disadvantages:

• Performance benefits are at the expense of a high complexity and high cost design

• The additional plumbing involved makes stages engines hard to start-up

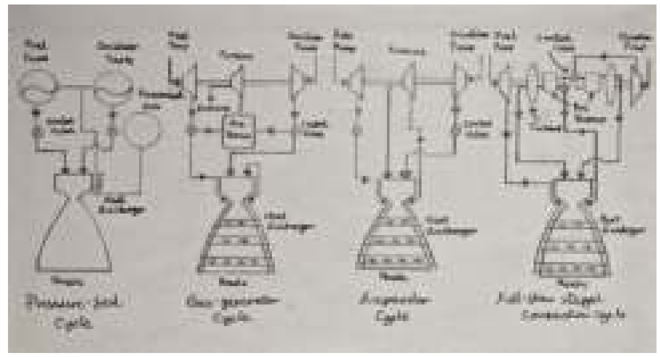


Figure 29: All Plumbing Cycles Compared

12 Coordinate Systems

Coordinate systems are organised arrangements for specifying positions of satellites, planets, stars, galaxies, and other celestial objects relative to physical reference points available to a situated observer. Coordinate systems in astronomy can specify an object's position in three-dimensional space or plot merely its direction on a celestial sphere, if the object's distance is unknown or trivial. We have studied mainly two coordinate systems:

- Alt-Azimuth Coordinate System
- RADec Coordinate System

12.1 The Alt-Az Coordinate System

This coordinate system uses the observer's local horizon as the fundamental plane to define two angles: altitude and azimuth. This system is used to define the exact position of objects in the sky, such as planets, the Sun, or the Moon. This system is also called the Horizontal Coordinate System.

Imagine the sky as a dome towering above you, its edges resting on the horizon. This is the backdrop the horizontal coordinate system uses to map the sky and describe the positions of its objects. To compare, the geographic coordinate system uses the Earth's surface as a backdrop to determine a position.

In effect, the system also includes the invisible half of the sky that is below the horizon. The dome above you is called the upper hemisphere and the invisible part of the sky below you is the lower hemisphere. Together, they form the celestial sphere, an imaginary globe surrounding you, with you at its center.

Figure 14: Upper hemisphere of the celestial sphere

Elucidation of the terms involved:

Celestial Horizon The horizontal line separating the two hemispheres is called the celestial horizon. It is a continuation into space of the imaginary plane created between you and the horizon around you. If the Earth were flat, the celestial horizon would follow the terrestrial plane. However, since we are living on a globe, it is defined as the imaginary plane perpendicular to the direction of gravity at the observer's location.

Altitude Altitude or elevation is the angle the object makes with the horizon. Objects that seem to touch the horizon have an altitude of 0°, while those straight above you are at 90° (see Figure 2). Anything below the horizon has a negative angle, with -90° describing a location straight down. In this and other celestial coordinate systems, the location straight above you is called zenith while the point exactly below you is referred to as nadir.

Figure 15: Altitude: Angle made by object with horizon

Azimuth Azimuth is the object's cardinal direction, such as north, east, south, or west. It is specified as the horizontal angle the object makes with a reference direction, such as true north (see Figure 3). Imagine a vertical line connecting the object with the horizon. The azimuth is the angle between the spot where that line crosses the horizon and the reference direction. If true north is used as reference, it is represented by an azimuth of 0°, and angle values increase towards the east. This means, for example, that an azimuth of 180° means due south.

Figure 15: Azimuth: representing the object's cardinal direction

Limitations of the System:

Depends on Location and Time The horizontal coordinate system owes its name to the fact that it is based upon the observer's horizon. As the horizon's limits – and, therefore, the portion of the sky you see – depends on your location, an object's altitude and azimuth angles shift as you move to a different spot on the Earth's surface. What's more, most celestial objects move across the sky, so their coordinates change as time goes by, even if you stay put.

This means that the angles provided by the horizontal coordinate system apply only to a specific location at a specific time.

Doesn't Work at the Poles While the horizontal coordinate system provides an easy way to define a location in the sky at almost any location on Earth, it is not possible to define an azimuth at the North Pole or the South Pole, rendering the system useless there.

At the North Pole, for example, it is easy to find the Polaris, the North Star. It is very close to the zenith position, so you have to look straight up to see it. However, you will not be able to describe the location of any other star using an azimuth angle because all of them are south of Polaris.

12.2 The RADec Coordinate System

It is a celestial coordinate system widely used to specify the positions of celestial objects. It may be implemented in spherical or rectangular coordinates, both defined by an origin at the centre of Earth, a fundamental plane consisting of the projection of Earth's equator onto the celestial sphere (forming the celestial equator), a primary direction towards the vernal equinox, and a right-handed convention.

The origin at the centre of Earth means the coordinates are geocentric, that is, as seen from the centre of Earth as if it were transparent. The fundamental plane and the primary direction mean that the coordinate system, while aligned with Earth's equator and pole, does not rotate with the Earth, but remains relatively fixed against the background stars. A right-handed convention means that coordinates increase northward from and eastward around the fundamental plane.

The coordinates of a body are expressed in terms of two angles, hence the name of the system:

- Right Ascension (RA)

- Declination (Dec)

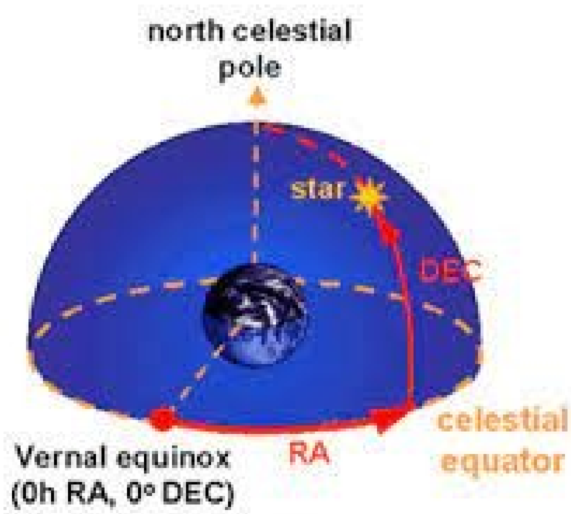


Figure 16: Outline of the RADec Coordinate System

Elucidation of the terms involved:

Vernal Equinox A solar equinox is a moment in time when the Sun crosses the Earth's equator, which is to say, appears directly above the equator (rather than north or south of the equator). On the day of the equinox, the Sun appears to rise "due east" and set "due west". This occurs twice each year, around 20 March and 23 September.

More precisely, an equinox is traditionally defined as the time when the plane of Earth's equator passes through the geometric center of the Sun's disk. Equivalently, this is the moment when Earth's rotation axis is directly perpendicular to the Sun-Earth line, tilting neither toward nor away from the Sun.

In the Northern Hemisphere, the March equinox is called the vernal or spring equinox while the September equinox is called the autumnal or fall equinox. In the Southern Hemisphere, the reverse is true. During the year, equinoxes alternate with solstices.

The March equinox occurs about when the Sun appears to cross the celestial equator northward. In the Northern Hemisphere, the term vernal point is used for the time of this occurrence and for the precise direction in space where the Sun exists at that time. In the equatorial coordinate system, the vernal point is the origin of the Right Ascension (demonstrated in Figure 2).

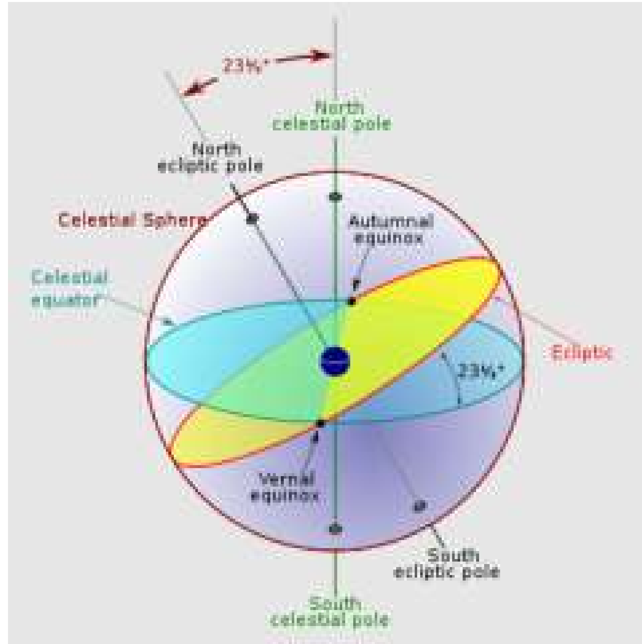


Figure 17: Celestial Sphere

Right Ascension Right ascension (abbreviated RA; symbol alpha) is the angular distance of a particular point measured eastward along the celestial equator from the Sun at the March equinox to the (hour circle of the) point in question above the earth.

Right ascension is the celestial equivalent of terrestrial longitude. Both right ascension and longitude measure an angle from a primary direction (a zero point) on an equator. Right ascension is measured from the Sun at the March equinox i.e. the First Point of Aries, which is the place on the celestial sphere where the Sun crosses the celestial equator from south to north at the March equinox and is currently located in the constellation Pisces. Right ascension is measured continuously in a full circle from that alignment of Earth and Sun in space, that equinox, the measurement increasing towards the east.

Declination Declination (abbreviated Dec; symbol) is the other angle that locates a point on the celestial sphere in the equatorial coordinate system, first being the Right Ascension. Declination's angle is measured north or south of the celestial equator, along the hour circle passing through the point in question.

Declination in astronomy is comparable to geographic latitude, projected onto the celestial sphere, and hour angle is likewise comparable to longitude. Points north of the celestial equator have positive declinations, while those south have negative declinations. Any units of angular measure can be used for declination, but it is customarily measured in the degrees (°), minutes (), and seconds () of sexagesimal measure, with 90° equivalent to a quarter circle. Declinations with magnitudes greater than 90° do not occur, because the poles are the northernmost and southernmost points of the celestial sphere. An object at the

- celestial equator has a declination of 0°

- north celestial pole has a declination of +90°
- south celestial pole has a declination of 90°

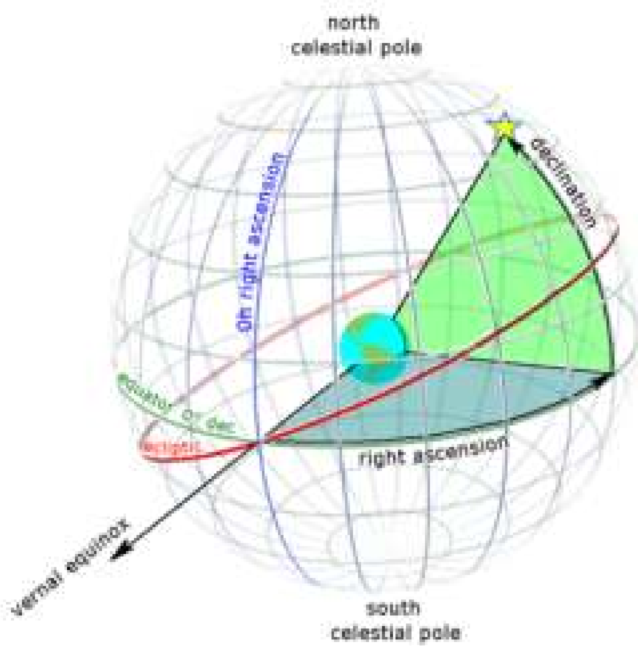


Figure 18: A nice demonstration of RA and Dec

Advantage of RADec over Alt-Azimuth
Unlike the Alt-Az system, where the location of a body depends upon the location of the observer and time of observation, the RADec system is nearly independent of the location and time of the observation. I have used the word "nearly" because of the Earth's precessional motion and the proper motion of the Star.

13 Orbital Dynamics

Orbital mechanics or astrodynamics is the application of ballistics and celestial mechanics to the practical problems concerning the motion of rockets and other spacecraft. The motion of these objects is usually calculated from Newton's laws of motion and law of universal gravitation. Orbital mechanics is a core discipline within space-mission design and control.

13.1 Keplerian elements

There are 6 parameters required for describing a orbit. And these parameters are as follows:

The semi major axis (a)
The semi-major axis determines the size of the conic section. For a circle, it is the radius, while for an ellipse, it describes the width of the ellipse. For a hyperbola, the semi-major axis describes the distance from the origin of the Cartesian coordinate system.

The ecentricity (e)
The eccentricity describes the deviation of the trajectory from a circle. When e=0, the orbit is circular; for values of e¡1, the orbit is elliptical. When e=1, the trajectory is parabolic and for e¿1, the trajectory is hyperbolic.

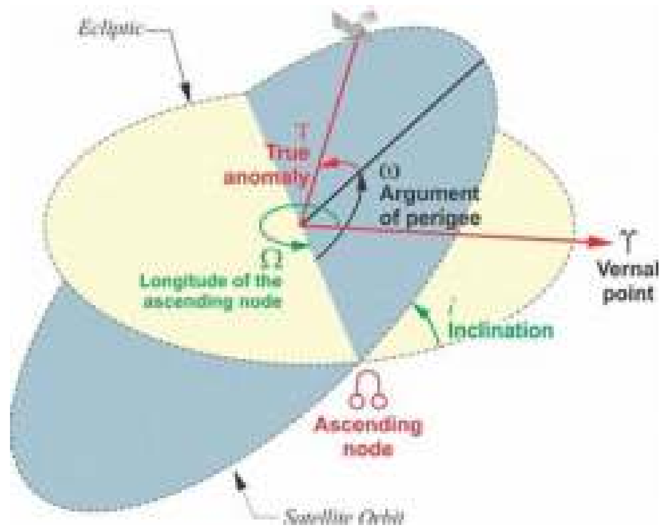


Figure 1: Keplerian elements

The inclination (i)
The angle between the orbital and equatorial plane. The inclination ranges from 0° to 180°. An inclination of 0° is an equatorial orbit. Orbits with inclinations from 0° to 90° are called prograde orbits because they rotate counterclockwise when viewed from above the north pole. This is the same direction as the surface of the earth rotates and the same direction that planets orbit around the sun. An orbit with an inclination of 90° is called a polar orbit because it passes directly over the north and south poles of the primary object. Orbits from 90° to 180° are called retrograde orbits because they rotate clockwise when viewed from above the north pole. This is the opposite direction of the surface of the earth or the planets.

The Right ascension of the ascending node (RAAN)(Ω)
The angle from the Vernal Equinox vector to the ascending node on the equatorial plane.The spacecraft spends part of its time above the reference plane and part of the time below the reference plane. The descending node: The point when the spacecraft goes from above to below the reference plane. The ascending node: The point when the spacecraft goes from below to above the reference plane. The right ascension of the ascending node (abbreviated RAAN) can range from 0° to 360°, inclusive.

Argument of perigee (ω)
Argument of perigee is the angle between ascending node and perigee. If both perigee and ascending node are existing at same point, then the argument of perigee will be zero degrees Argument of perigee is measured in the orbital plane at earth's center in the direction of satellite motion.

True anomaly (v)
It is an angular parameter that defines the position of a body in a Keplerian orbit. The angle between the perigee and the satellite in the orbital plane at a specified time.

13.2 Lagrange points

Lagrange points or Lagrangian points are the points in space at which the force that is exerted between any two given objects becomes equal. The objects at the Lagrangian point experience a neutral kind of force. The two-body systems such as Earth and the sun produce some enhanced regions of attraction as well as repulsion. It plays a very important role in astronomy and can be used by spacecraft to remain in their position without consuming much fuel. There are basically five Lagrange points wherein a body of a small mass can orbit in a pattern with any two larger masses.

What are Lagrange points
Lagrangian Point or Lagrange Point can be understood as a point that is near two large celestial bodies moving in an orbit in a way that the smaller object is able to maintain its position in relation to the large bodies. Lagrangian points can also be called Lagrange points or Liberation Points, and are denoted by L.

]

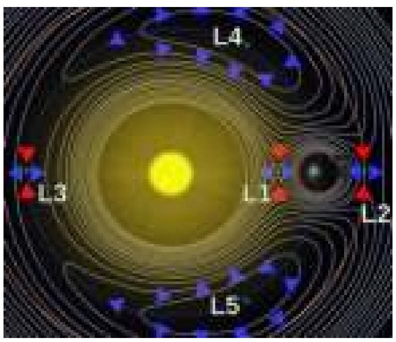


Figure 2: Lagrange points

For every known combination of 2 large bodies, there exist up to 5 Lagrangian points from L1 to L5. Leonhard Euler discovered the first three Lagrangian points i.e. L1, L2, and L3. The remaining two points, L4 and L5, were later discovered by Joseph Louis.

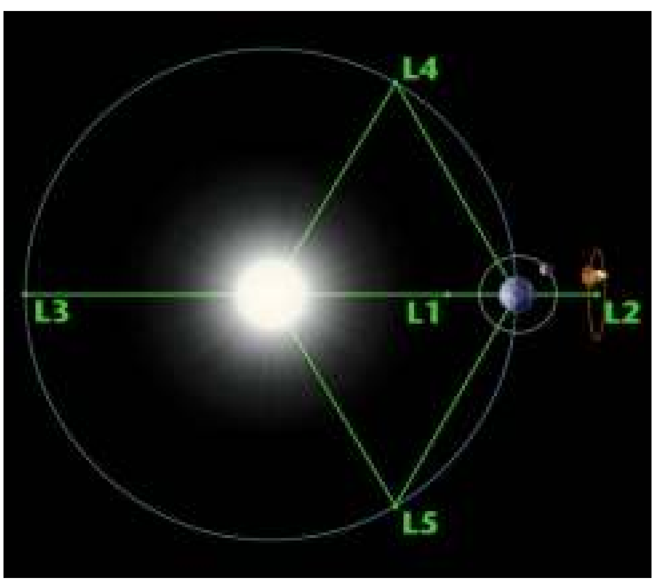


Figure 3: Lagrangian point of a two-body system

Stability of Lagrange points
Of all the five Lagrangian points, three of them are unstable while two are stable.

- L1, L2, and L3 are the three unstable Lagrange points that lie along the line that connects two of the large masses.
- L4 and L5 Lagrangian points however are stable points that form the apex of 2 equilateral triangles having large masses on their vertices.

In the system of Earth and sun, L1 and L2, the first two Lagrangian points exist at about 1,500,000 km or 900,000 miles from the Earth away and towards the sun. Satellites are located at the Lagrangian of the sun earth system.

Locations of Lagrange points
Location of L1
The point existing on the line between two large masses, known as M1 and M2 is the Lagrange Point 1. The gravitational attraction of a mass is partially canceled by the gravitational force of the other one. The mathematical representation of this point is:

$$L1 : (R[1 - (\alpha/3)^{1/3}], 0) \tag{8}$$

where,

- R = Distance between Sun and Earth
- M_2 = Mass of Earth
- M_1 = Mass of Sun
- $\alpha = M_2/M_1$

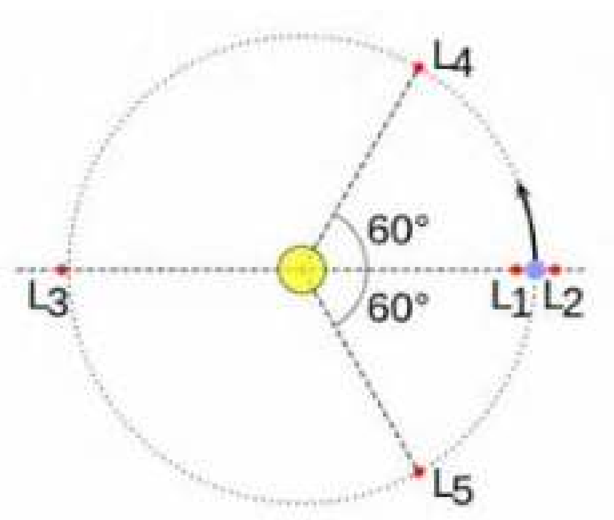


Figure 4: Lagrange points

Location of L2
Lagrange Point 2 is the point that exists beyond the smaller of the two masses and on the line defined by them. The centrifugal effect on a body present at the L2 point is balanced by the gravitational force of both the large masses. The mathematical representation of this point is:

$$L2 : (R[1 + (\alpha/3)^{1/3}], 0) \tag{9}$$

Location of L3
The point that lies beyond the larger of the two masses and on the line defined by them is known as the Lagrange Point 3. The mathematical representation of this point is:

$$L3 : (-R[1 + 5/12(\alpha)], 0) \tag{10}$$

Location of L4 and L5
L4 and L5 points exist on the line defined by the centers of both the masses in a way that they lie at the third corner of both the equilateral triangles.

$$L4 : (R/2(\frac{M_1 - M_2}{M_1 + M_2}), \sqrt{3}R/2) \tag{11}$$

$$L5 : (R/2(\frac{M_1 - M_2}{M_1 + M_2}), -\sqrt{3}R/2) \tag{12}$$

13.3 Non Spherical Earth

The earth is not a sphere. In fact, the Earth is neither a homogeneous mass nor a sphere. Consequently, the several attributes of the Earth's shape and composition have noticeable affects on a satellite's orbit. The bulge at the equator, the flattening at the poles, and the slight pear shape of the Earth are important contributors to the perturbation of an orbit due to a non-spherical Earth.

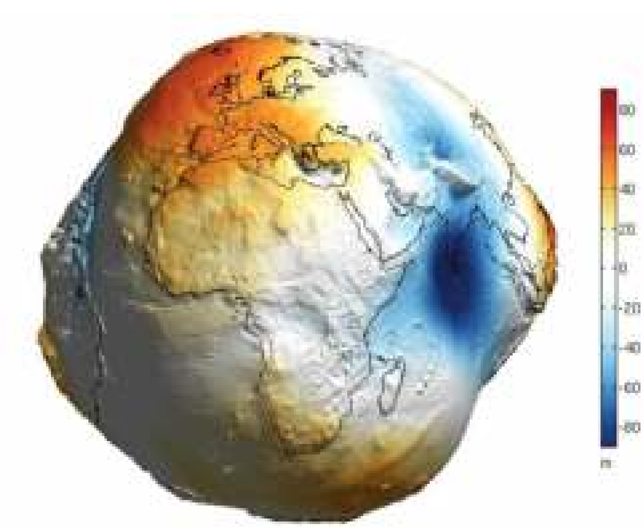


Figure 5: Non Spherical Earth

13.4 J2 Perturbation

Actually, the Earth really isn't a sphere - it is an oblate spheroid. Because of the rotation of the Earth on its axis, centrifugal force bulges the equator. In fact, the radius at the Earth's equator is about 21 km larger than the radius at the poles.

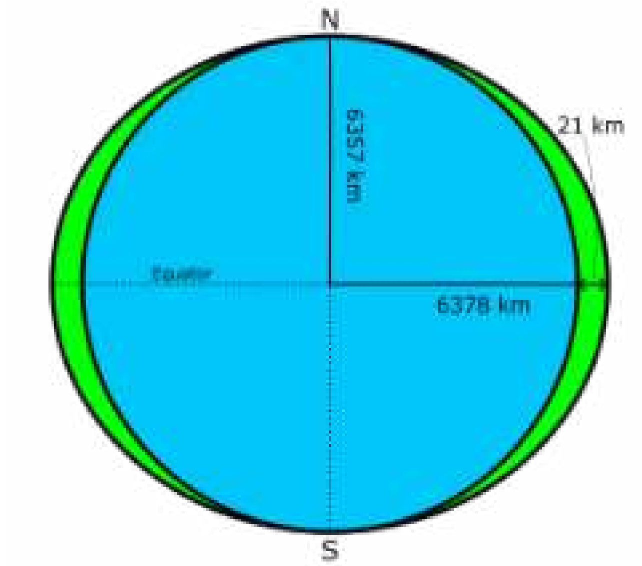


Figure 6: Oblate Earth

Now that we understand that our Earth is really more oblate than spherical, we need to ask ourselves some questions. How does this affect our orbits? There is a perturbing force based on this oblate Earth called "J2 Perturbations." But where does the term "J2" come from? The term J2 comes from an infinite series mathematical equation that describes the perturbational effects of oblation on the gravity of a planet. The coefficients of each term in this series is described as Jk, of which J2, J3, and J4 are called "zonal coefficients." However, J2 is over 1000 times larger than the rest and has the strongest perturbing factor on orbits. J2 Nodal Regression: The ascending node migrates opposite the direction of flight. The equatorial bulge produces an extra pull in the equatorial plane which creates an averaged torque on the angular momentum vector. Like gravity, the torque causes h vector to precess as shown in the figure below.

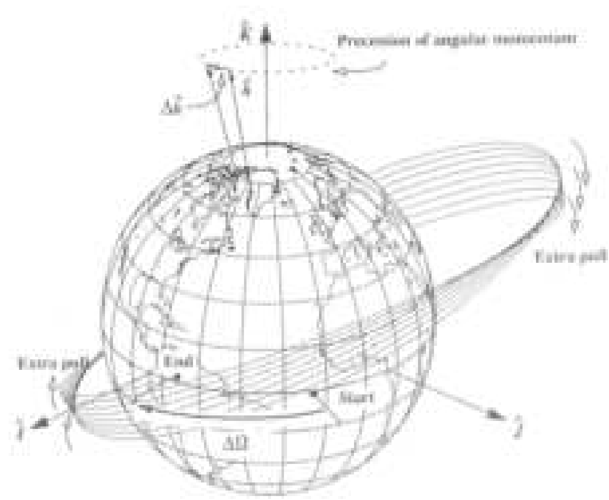


Figure 7: J2 Nodal Regression

J2 Apsidal Rotation: The Apsidal precession represents major axis shift, respectively the argument of perigee deviation.

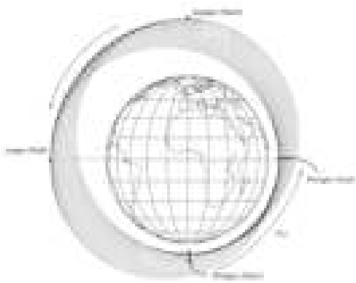


Figure 8: J2 Apsidal Rotation

13.5 Hohmann Transfer

The Hohmann transfer orbit is an orbital maneuver used to transfer a spacecraft between two circular orbits of different altitudes around a central body.It is accomplished by placing the craft into an elliptical orbit that is tangential to both the initial and target orbits in the same plane. The maneuver uses two engine impulses: the first prograde impulse places it on the transfer orbit by raising the craft's apoapsis to the target orbit's altitude; and the second raises the craft's periapsis to match the target orbit. Thrusters are fired at apogee and perigee of the elliptical orbit to provide the required Δv.

$$\Delta v_1 = \sqrt{\mu/r_1}(\sqrt{2r_2/(r_1+r_2)} - 1) \tag{13}$$

$$\Delta v_2 = \sqrt{\mu/r_2}(1 - \sqrt{2r_1/(r_1+r_2)}) \tag{14}$$

- r_1 = radius of smaller orbit
- r_2 = radius of larger orbit
- μ = GM (M = Mass of Earth)

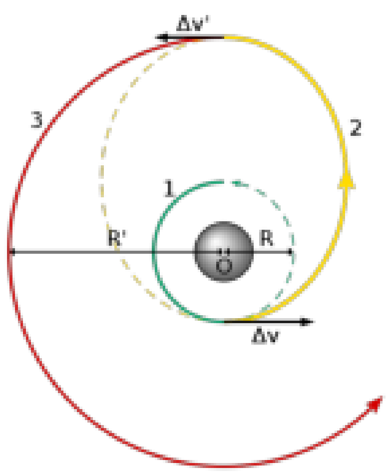


Figure 9: Hohmann Orbit Transfer

13.6 Graveyard Orbits

The energy required for the re-entry of a satellite into earth's atmosphere is approx.ΔV = 1500 m/s which requires a ton of fuel. Hence it is impractical to do so. A much easier and less energy intensive way is to park it into one of the super synchronous orbit, for example, the Graveyard Orbit, the maneuver from Geo-stationary orbit requires only a ΔV of about 11 m/s. This can be achieved via small perturbations to the orbit within a few months.

13.7 Gravitational assist

In orbital mechanics and aerospace engineering, gravity assist maneuver, or swing-by is the use of the relative movement and gravity of a planet or other astronomical object to alter the path and speed of a spacecraft, typically to save propellant and reduce expense. Gravity assistance can be used to accelerate a spacecraft, that is, to increase or decrease its speed or redirect its path. The "assist" is provided by the motion of the gravitating body as it pulls on the spacecraft. Any gain or loss of kinetic energy and velocity by a passing spacecraft is correspondingly lost or gained by the gravitational body, in accordance with Newton's Third Law.

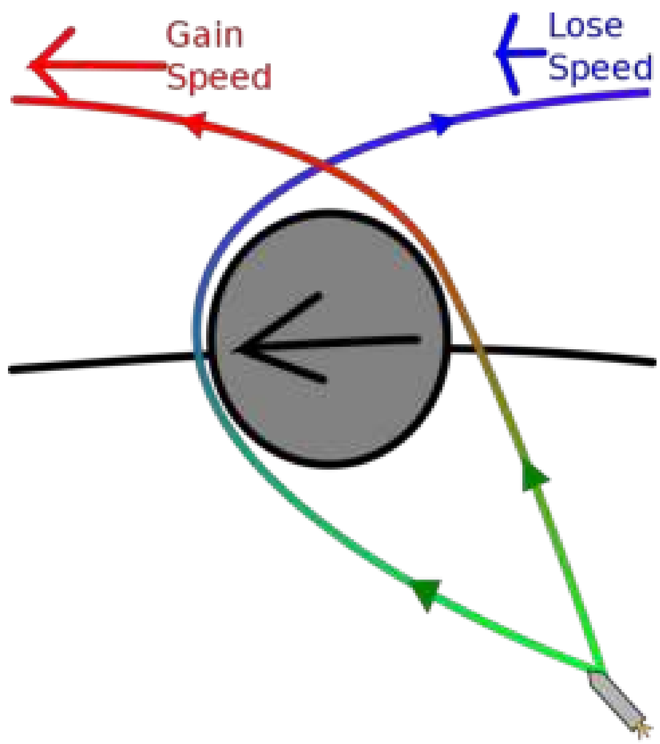


Figure 10: Gravitational Assist

' 1 A gravity assist around a planet changes a spacecraft's velocity (relative to the Sun) by entering and leaving the gravitational sphere of influence of a planet. The spacecraft's speed increases as it approaches the planet and decreases as it leaves the planet. The speed gained from approaching and the speed lost from leaving is nearly identical, but the spacecraft is affected by the planet's motion around the sun during the maneuver. To increase speed, the spacecraft approaches the planet from the direction of the planet's orbital velocity, and departs in the opposite direction. To decrease speed, the spacecraft approaches the planet from a direction away from the planet's orbital velocity – in both types of maneuver the energy transfer compared to the planet's total orbital energy is negligible. The sum of the kinetic energies of both bodies remains constant (see elastic collision). A gravitational assist can therefore be used to change the spaceship's trajectory and speed relative to the Sun.

Voyager 1 and 2

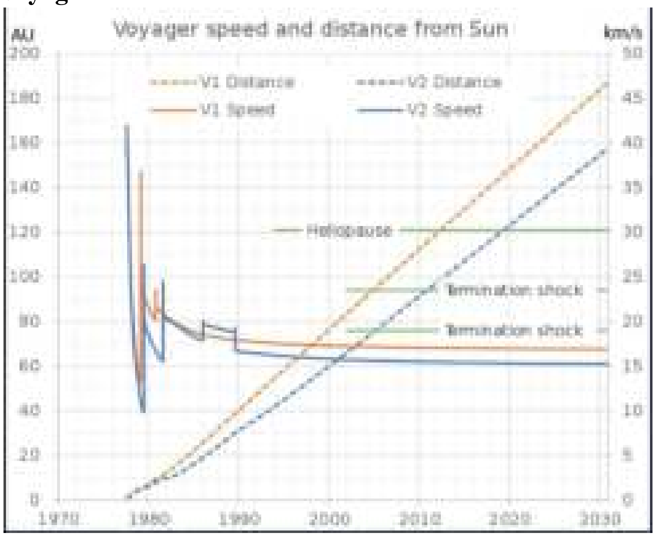


Figure 11: Voyager 1 and 2 speed

The graph plotted below shows as how velocity of Voyager 2 changes as it passes through nearby of different planets. We can observe certain peaks in the velocity when it passes through nearby of planets. The reason behind this peaks is the gravitational assist. So when it passes through nearby of Jupiter, a gravitational assist from the Jupiter increases it's velocity and thus the peak is observed. Similarly other peaks are observed when it passes through nearby of Saturn, Uranus and Neptune respectively.

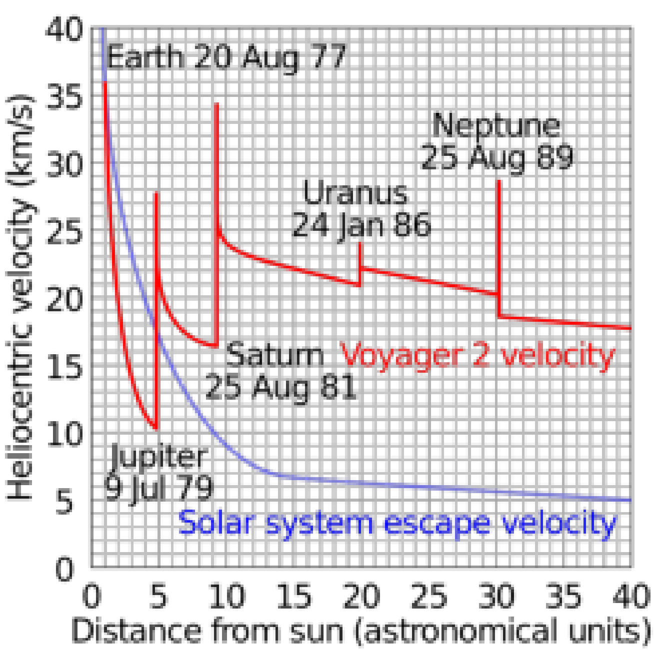


Figure 12: Voyager 2 speed

14 Trajectory planning

14.1 Parameters affecting Rocket's trajectory

There are 5 parameters which are used to determine a rocket's trajectory in space. These are-

- Thrust force by engine at an altitude of 0,50,100,200,400 km from the earth's surface.$[T_1, T_2, T_3, T_4, T_5]$
- Thrust force angle with respect to the normal from the earth's surface (α).
- Half angle of the nose cone (θ_c).
- Rocket radius (R).
- Initial wet mass ($m_{initial}$).

14.2 Equation of Thrust and Drag

Thrust

We have been given the magnitude of the thrust force at 5 different altitudes.The thrust force at any other altitude is obtained by using linear interpolation.Take $a_1 = 0, a_2 = 50, a_3 = 100, a_4 = 200, a_5 = 400$ (in km) and the corresponding thrust at these altitudes as T_1, T_2, T_3, T_4, T_5

For our rocket model,

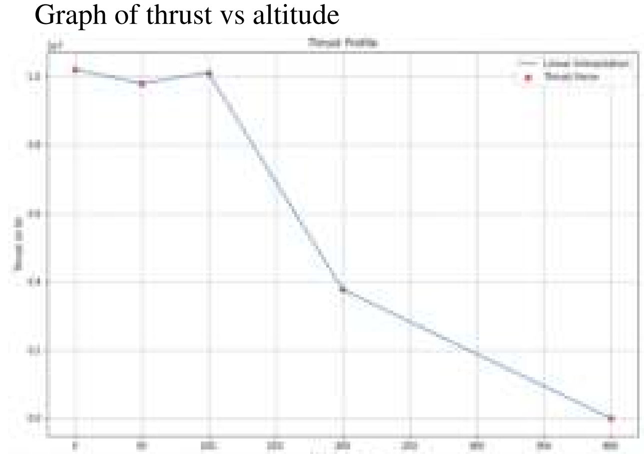
- $T_1 = 10.2 \times 10^6$
- $T_2 = 9.79 \times 10^6$
- $T_3 = 10.1 \times 10^6$
- $T_4 = 3.77 \times 10^6$
- $T_5 = 3.0 \times 10^3$

We first plot these 5 points on a graph and then join them by using straight lines. Now using this graph we can easily find the thrust at any altitude(a) between 0 and 400.

Eg- take any arbitrary a lying between a_i and a_{i+1} then the thrust t at this a is given by the formula-

$$T = T_i + \frac{a - a_i}{a_{i+1} - a_i} \cdot (T_{i+1} - T_i)$$

Graph of thrust vs altitude



The thrust initially decreases slightly such that the velocity of rocket doesn't increase too much so as to avoid excess drag .Once max Q is attained(for our rocket model it is attained near 50 km altitude) thrust starts increasing again till a particular point after which it again decreases.

Angle of thrust

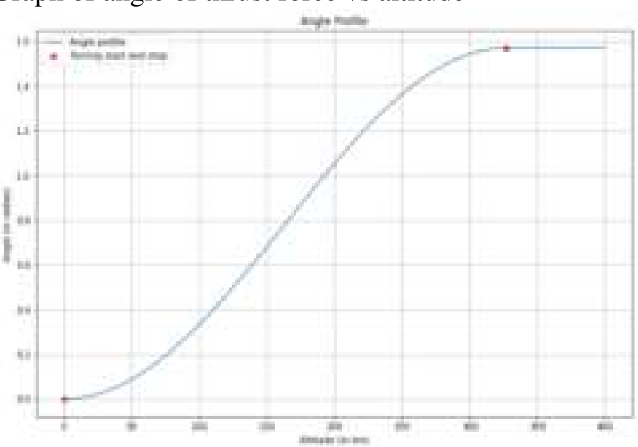
The angle of thrust with respect to the normal from the earth's surface is determined by gravity turn altitudes of start (α_1) and end (α_2).
The angle of force is represented by-

- if altitude $< \alpha_1$ then $\theta = 0$
- if altitude $> \alpha_1 + \alpha_2$ then $\theta = \frac{\pi}{2}$
- else $\theta = (1 - \cos\frac{\pi(alt - \alpha_1)}{\alpha_2}) \cdot \frac{\pi}{4}$

For our rocket model-

- $\alpha_1 = 400km$
- $\alpha_2 = 327.1 \times 10^3$

Graph of angle of thrust force vs altitude -



Drag

The drag force acting on a rocket is a dependent on it's radial distance from center of the earth(r) and it's radial velocity($\dot{r}$).It is given by the formula-

$$D = \frac{1}{2} \cdot C_D \cdot A \cdot \rho \cdot \dot{r}^2$$

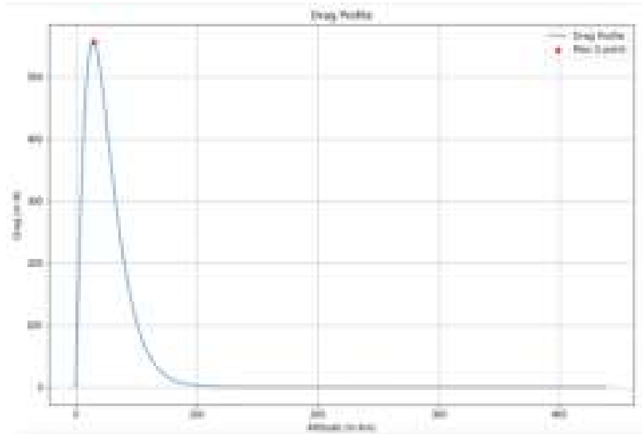
where,

- A(area of cross- section of rocket)=πR^2
- C_D(coefficient of drag)= $2\sin^2(\theta_c)$
- ρ(density of air)= $1.2 \cdot e^{-(\frac{r - R_E}{10.4})}$
- R_E(radius of the earth)=6400 km

For our rocket model-

- $\theta_c = 0.108\ radians$
- $R = 3.98\ m$

Graph of drag force vs altitude -



The drag force first increases due to increasing radial velocity($\dot{r}$). After reaching a maxima it starts decreasing as the density decreases too rapidly at higher altitudes. The maxima of the graph(known as max Q) is the point where the rocket experiences maximum drag force.

14.3 Equations of motion of a rocket going to LEO

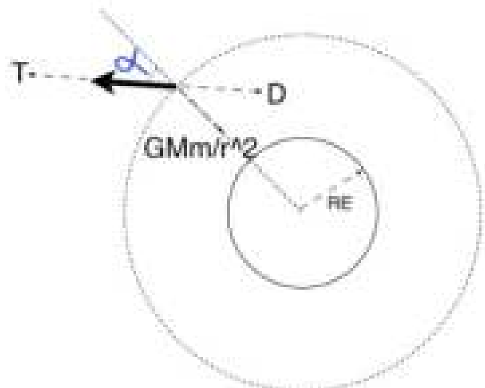
There are 3 equations which are used to determine a rocket's trajectory in space

$$\ddot{r} = \frac{-GM}{r^2} + r\dot{\theta}^2 + \frac{T(r) - D(r, \dot{r})}{m} \cdot \cos\alpha$$
$$\ddot{\theta} = \frac{T(r) - D(r, \dot{r})}{r \cdot m} \cdot \sin\alpha$$
$$\dot{m} = -\frac{T(r)}{I_{sp} \cdot g_0}$$

where,

- r and θ are coordinates of rocket in plane polar coordinate system attached with the earth.
- G(gravitational constant)= 6.674×10^{-11} in SI
- M(mass of earth)= 5.97219×10^{24} in SI
- T it the thrust
- D is the drag force
- m is mass of the rocket
- α is the angle between thrust and normal from the earth's surface.
- I_{sp} is the specific impulse
- g_0(gravity at earth's surface)= 9.8 in SI

Derivation



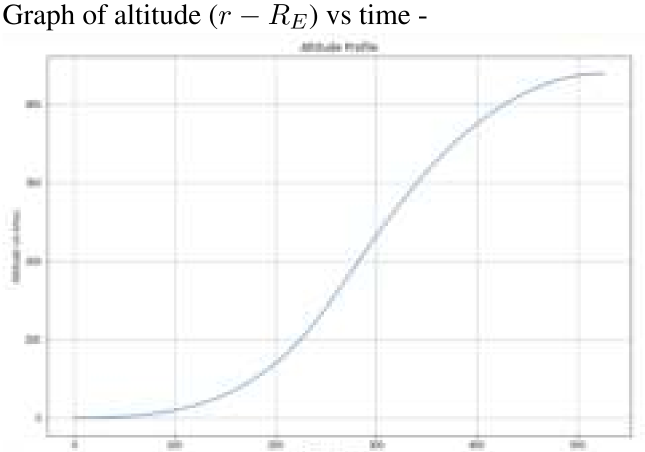
Here the dark arrow represents the rocket, directions of the thrust, drag and gravitational force are shown. angle α between normal from the earth and thrust is also shown.Resolving the forces along $\hat{r}$ and $\hat{\theta}$, we get the following set of equations-

$$\begin{aligned} F_r &= (T-D)\cos\alpha - \frac{GMm}{r^2} \\ m(\ddot{r} - r\dot{\theta}^2) &= -\frac{GMm}{r^2} + (T-D)\cos\alpha \\ \ddot{r} &= r\dot{\theta}^2 - -\frac{GM}{r^2} + \frac{T-D}{m}\cos\alpha \qquad (15) \\ F_\theta &= (T-D)\sin\alpha \\ m(2\dot{r}\dot{\theta} + r\ddot{\theta}) &= (T-D)\sin\alpha \\ \ddot{\theta} &= \frac{T-D}{m}\sin\alpha \qquad (16) \end{aligned}$$

We have neglected the $\dot{r}\dot{\theta}$ term while deriving the expression for $\ddot{\theta}$.For deriving the last equation we simply use the definition of specific impulse,

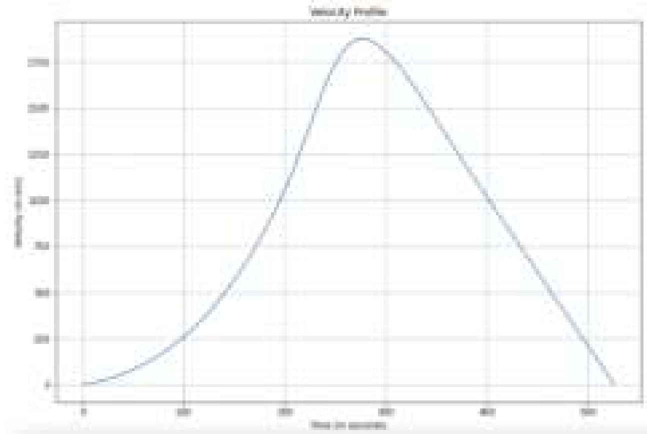
$$\begin{aligned} I_{sp} &= \frac{T}{-\dot{m} \cdot g_0} \\ \dot{m} &= \frac{T}{I_{sp} \cdot g_0} \qquad (17) \end{aligned}$$

Graph of altitude $(r - R_E)$ vs time -



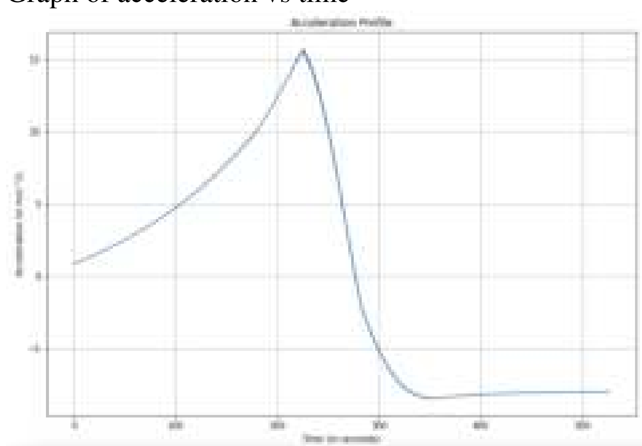
The altitude of the rocket increases with time. the graph is concave up till acceleration $\ddot{r}$ is positive and then becomes concave downwards when acceleration is negative. At the point of inflection of this graph the rocket attains maximum velocity which further decreases with time.

Graph of velocity of rocket vs time -



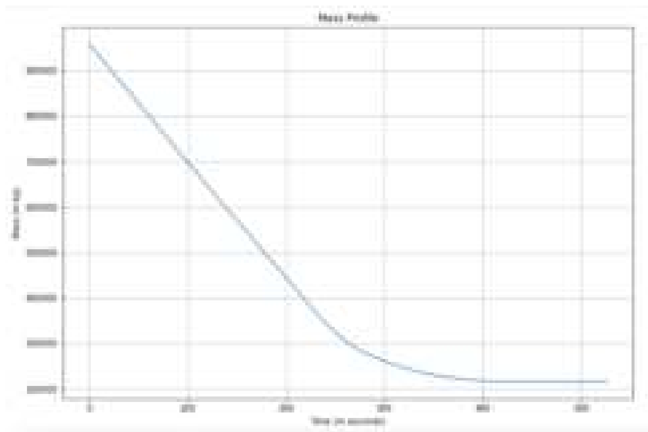
The velocity of the rocket first increases with time and reaches a maxima. At this point the acceleration of the rocket becomes 0. as the acceleration becomes negative after this point the velocity starts decreasing until it reaches 0.

Graph of acceleration vs time -



The acceleration first increases with time as the thrust dominates over the drag and gravitational force. it reaches a maxima after which drag becomes too much(due to high velocity) and thrust decreases which results in acceleration decreasing with time .when the acceleration becomes 0 we get the maximum velocity of the rocket. the acceleration decreases continuously after that till we get a slight minima (due to drag and gravity). It then increases till g ($9.8m/s^2$) as the rocket attains topmost point in it's flight.

Graph of mass vs time-



The mass of the rocket decreases with time as amount of propellant decreases.We can formulate the following set of equations for mass of various components of a rocket-

$$m_{initial} = m_{engine} + m_{payload} + m_{propellant} + m_{structure}$$

$$m_{final} = m_{engine} + m_{payload} + m_{structure}$$

Subtracting final mass from initial mass we get,

$$m_{propellant} = m_{initial} - m_{final}$$

The mass of engine is calculated using space shuttle engine mass. max thrust produced by a rocket is directly proportional to the engine mass which implies that the engine mass by max thrust ratio will be same for both rocket and space shuttle.

$$\frac{m_{engine}}{max\ thrust} = \frac{m_{space\ shuttle\ engine}}{space\ shuttle\ max\ thrust}$$

$$m_{engine} = \frac{max\ thrust}{space\ shuttle\ max\ thrust} \cdot (m_{space\ shuttle\ engine})$$

For the space shuttle-

- $max\ thrust = 2.15 \times 10^6\ N$
- $m_{engine} = 3500\ kg$

The structural mass of the rocket is usually fixed and accounts for 15% of the initial mass of rocket.

$$m_{structure} = 0.15 \cdot m_{initial}$$

Now we know $m_{engine}, m_{structure}$ and $m_{propellant}$. To get $m_{payload}$ simply subtract mass of these three components from the initial mass of rocket.

$$m_{payload} = m_{initial} - m_{structure} - m_{engine} - m_{propellant}$$

For our rocket model we got the following data (using $I_{sp} = 400 sec$)-

- $m_{initial}$ = 958800.0 kg
- m_{final} = 215525.7548288 kg
- m_{engine} = 16604.65116279 kg
- $m_{structure}$ = 143820.0 kg
- $m_{propellant}$ = 743274.2451711 kg
- $m_{payload}$ = 55101.103666 kg

15 Rocket Modelling using Python

15.1 Genetic Algorithm

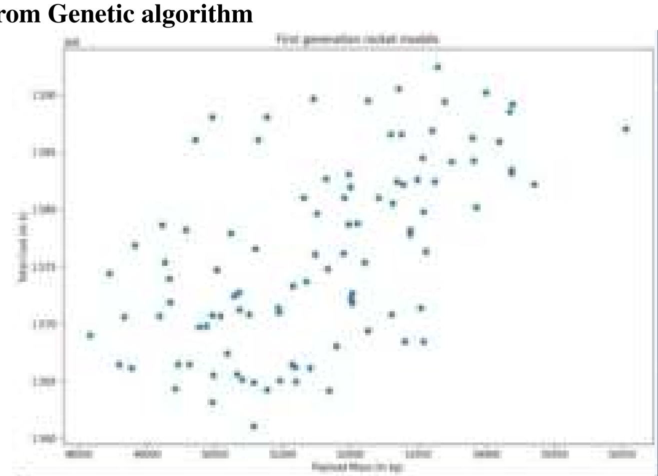
Genetic Algorithm is a probabilistic search algorithm that randomly transforms a set (population) of mathematical objects each with an associated fitness value into a new population of offspring objects using Darwinian principle of natural selection using patterns such as mutation and crossover .

1. Individuals in a population compete for resources
2. Those that are successful (fittest) mate to create more offspring
3. Each inherent quality of the parent can be thought of as its gene.
4. Genes from the “fittest” parents are passed on to the offspring, which is better than the parent
5. Hence, the offspring is better suited for the environment (more optimised)

Pseudo Code **Initialise** population with random candidates
Evaluate all individuals
While termination criteria not met
Select parents
Apply crossover
Mutate offspring
Replace current generation
end while

Fitness Function Since GAs operate on survival of the fittest, we need to give a score based on how good an individual is at surviving. Fitness can be a function of the initial parameters and used in successive genetic operations. Individuals with higher fitness have a higher chance to reproduce and pass on their genes. Hence, that’s why the population is usually sorted based on its fitness.

Sample plot of first generation rocket model obtained from Genetic algorithm



15.2 MOGA

So, Genetic Algorithms as we know are brute force optimization algorithms based on the idea of natural selection and genetics. We calculate the fitness for the individuals on basis of some objective to be achieved and try to respectively maximise or minimise it. Earlier we have seen single dimension optimisation, i.e., optimising only one parameter. MOGA as the name suggests is a genetic algorithm to optimise multiple objectives. Here we might try to maximise one parameter and minimise some other at the same time. So we calculate the fitness likewise. It finds application in various designing and engineering problems.

Here in our project we use it to find the optimum rocket design. The parameters we want to optimise are

1. Payload Mass
2. Total Project Cost
3. The Altitude it reaches

The first 2 are interlinked and form a 2D optimization problem as one affects the other and we want to maximise the payload mass and minimise project cost. To solve this problem we use something called pareto dominance.

15.3 Pareto Dominance

To understand this we need to first understand pareto front. So, basically when we plot a graph like this

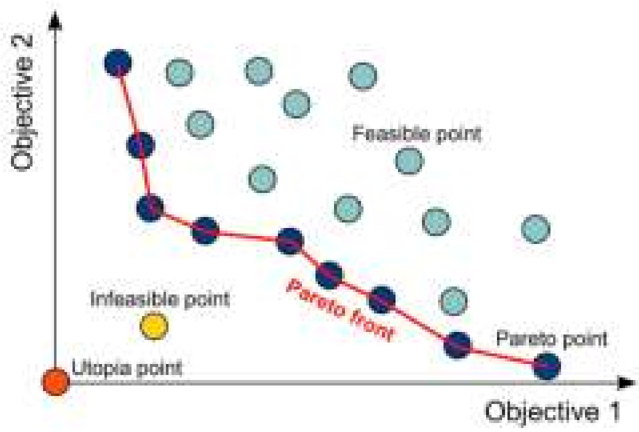


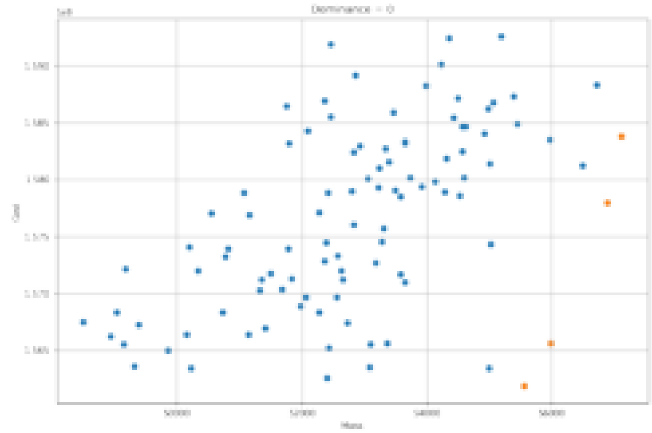
Figure 15: Pareto Front

The points on this type of front are called pareto points and the front is called pareto front. This can be considered as simply a front formed by points which are the most efficient solutions to some problem. They mainly form a boundary of this kind. After finding this front we remove all the pareto points and find the pareto front for the remaining points. Like this we keep finding the pareto points and noting the iteration in which they were pareto points till all points are done. Pareto Dominance is this iteration no. of the pareto points. It represents in which iteration they were the pareto points and optimum solutions. The higher the dominance, the less optimum the solution as compared to the previous pareto points.

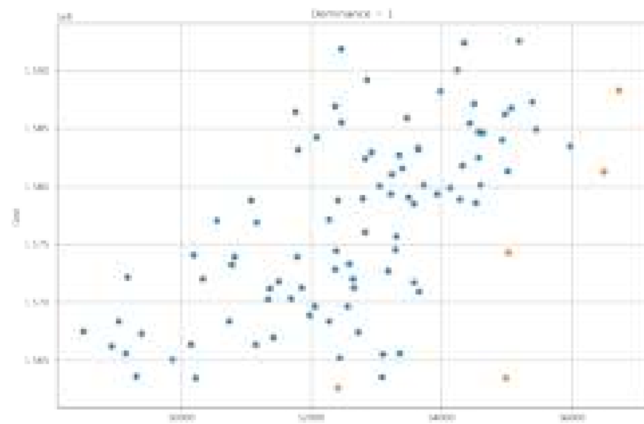
15.4 Finding Optimum Rocket Model

In our model we find optimum points with minimum cost and maximum payload mass. So we plot graph between cost and mass of payload and our objective is to minimise the cost to mass ratio. For this we find pareto front for which the pareto points have least slope. A sample plot is given below with the pareto fronts after each iteration. This was done on python and the pygmo library was used to find the pareto points. By this we calculated the dominance of each individual rocket in the population and used it to calculate their fitness.

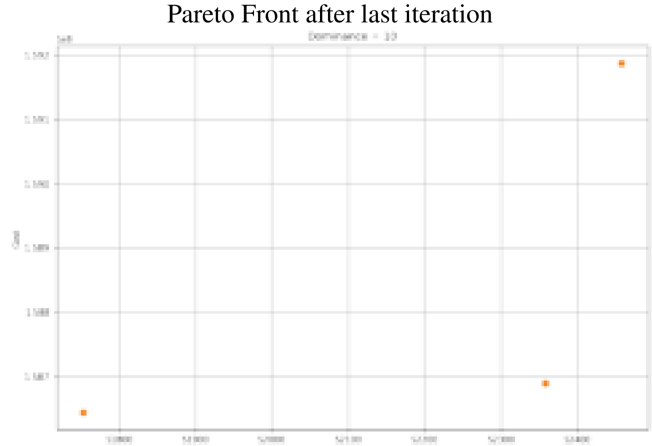
Pareto Front after 1st iteration



Pareto Front after 2nd iteration



Pareto Front after last iteration



15.5 MOGA for Rockets

We need to find the most optimum rocket model. So our individuals in the population are various rocket models. We create a population of 100 of them. Their genes are the 5 basic features of a rocket that define it, i.e., Thrust Vector, Angle Vector, Cone Half Angle, Rocket Radius and Total Mass. We define 3 classes namely rocket, random variable and generation.

1. First Generation

The first Step is to create the first generation. So, we create the first generation by randomly choosing values of the genes from the ranges given below.

$$\begin{aligned}
\text{Thrust Vector Range} &= [10.1MN - 10.3MN, \\
&\quad 9.7MN - 9.9MN, \\
&\quad 10.0MN - 10.2MN, \\
&\quad 3.7MN - 3.9MN, \\
&\quad 0.002MN - 0.004MN] \\
\text{Angle Vector Range} &= [0.35km - 0.45km, \\
&\quad 320km - 330km] \\
\text{Cone Half Angle} &= 0.1 - 0.5 \\
\text{Rocket Radius} &= 3.8m - 4.2m \\
\text{Total Mass} &= 9.5 \times 10^5 kg - 9.8 \times 10^5 kg
\end{aligned}$$

We create 100 such individuals to create the population.

2. Fitness

Then we have to assign the fitness to each individual. For this we use the following formula

$$\text{Fitness } F = (\max(1 - 0.01 \times n_{dom} - p(A_{final}), 0))^2$$

Here, fitness depends on 2 factors:

1. Penalty p(max Altitude)- This accounts for the optimum altitude to be reached to be 400 km.
2. Number of Dominance n_{dom}- This accounts for the maximisation of payload mass and minimisation of total cost.

For both these we first calculate the various profiles and properties of the rocket like altitude profile, max altitude, payload mass, total cost, etc. We calculate the cost as follows

Engine Cost

Cost and Mass of the engine can be taken to be directly proportional. Therefore we use mass and cost of the space shuttle engine to compute the engine cost. The Engine Cost comes out to be as follows

$$C_E = M_E \cdot \left(\frac{C_{Ess}}{M_{Ess}}\right)$$

C_{Ess} and M_{Ess} are given to be 30 million \$ and $3500kg$

Structural Cost

Similar to engine cost a direct proportionality can be assumed for cost and mass here also. So the structural mass is as follows

$$C_S = M_S \cdot \left(\frac{C_{Sss}}{M_{Sss}}\right)$$

C_{Sss} and M_{Sss} are given to be 75 million \$ and $760,000kg$

Propellant Cost

The propellant mixture that is used is $\frac{1}{7}$ liquid H_2 and $\frac{6}{7}$ liquid O_2. So we use the following formula to determine the cost of propellant

$$C_p = \left(M_p \cdot C_{O_2} \cdot \left(\frac{6}{7}\right)\right) + \left(M_p \cdot C_{H_2} \cdot \left(\frac{1}{7}\right)\right)$$

Where M_p is the total propellant mass.

C_{O_2} and C_{H_2} are given to be $0.155\$/kg$ and $9.5\$/kg$ respectively.

Now the total cost is given by the sum of these 3 costs

After that we obtain the max altitude, payload mass and the total cost and use them to compute the penalty and dominance.

Calculation of Penalty

A variable penalty is awarded to the rocket based on the maximum altitude it reaches i.e. A_{final} and the generation in which it achieves that. The following formula is used

$$p(A_{final}) = \left[\frac{(400 - A_{final})}{\max(1, 400 - 4 \times gen)}\right]^2$$

This is considering 10 generations. This ensures that optimum altitude is reached and that too in lesser computation.

Calculation of Number of Dominance

This accounts for the optimization of payload mass and total cost. It is calculated by plotting all the individuals on a cost v/s mass graph and then finding the pareto points in each iteration as explained before. The lesser the dominance, more fit the rocket.

After we have assigned the fitness to each individual, we sort them in decreasing order of fitness. The fittest is at the zeroth index.

3. Mating

Now to create the new offspring we mate the parents, i.e., the individuals of previous generation. Before mating we first select the top 10 fittest individuals and pass them into the next generation as it is like the natural selection. The rest 90 individuals of the new generation are produced by mating the top 50 fittest individuals of the parent generation.

The mating is done as was explained before in genetic algorithm section. We generate a random probability and on the basis of it we take the respective gene from either of the parent or generate a mutation. We took the convention that if probability is less than 0.45 we took the gene from parent 1, if it is between 0.45 and 0.9 we took it from the other parent else we mutate it. This mating ensures selection and survival of the fittest individuals.

We keep on generating the new population until about 15-20 generation so as to get the fittest individuals or rocket. After the last generation the fittest individual is the optimum solution or the optimum rocket model that we desire.

15.6 Conclusion

After we get our fittest rocket we find its properties like the Payload Mass, Total Cost and Mass to Cost ratio. We then compare this with the actual Masses and Cost of some real mission and find the error that we get. The following is the result that we got from our modelling.

Calculated Payload Mass : $52818.27kg$
Actual Payload Mass : $52100kg$
Error : 1.38%
Calculated Total Cost : 159.04 million \$
Actual Total Cost : 157.3 million \$
Error : 1.11%
Calculated M_p/C : $332.11kg/M\$$
Actual M_p/C : $331.21kg/M\$$
Error : 0.27%

16 Stability and Optimization

16.1 OpenRocket

OpenRocket is a free rocket simulator that enables you to design and simulate your rockets before you build and fly them. There are inbuilt tools with which we can virtually configure the model rocket we desire to create and launch. We can select components like the nose cone, the body tube, fins etc. There are further components (some may not be essential) which fit inside the main ones.

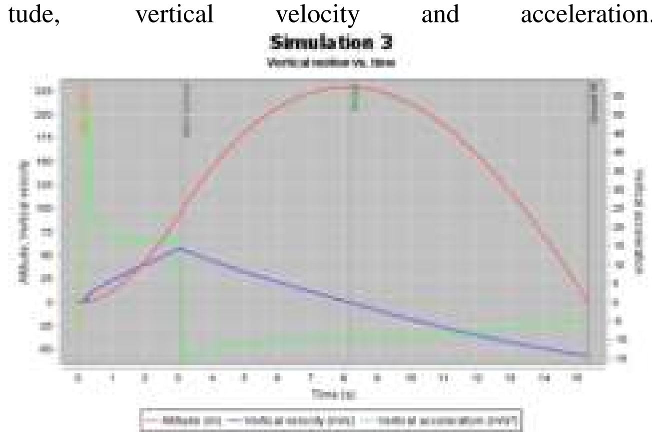
16.2 Simulation of Launch

OpenRocket can simulate conditions at the launch site, so you can estimate how high the rocket will ascend and how far downrange it will drift.

We observed that the forces due to external agents like the wind are minimal as compared to the thrust offered by the rocket motor. Therefore those forces are neglected when we are studying the behaviour of the rocket after motor burnout, parachute deployment etc.

In the screen shown below, you can set parameters (and units) for your Launch site i.e. Latitude, Longitude and Altitude. The solid propellant motor that we used (for simulation as well as the final launch) is Estes E9.

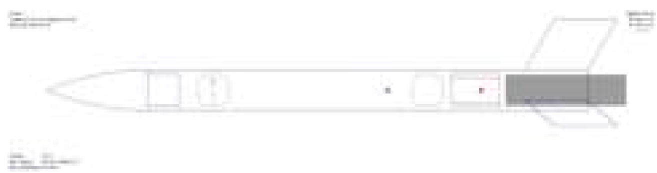
These will be the graphs for altitude, vertical velocity and acceleration.



During the powered ascent phase of the flight, the motor would provide a constant thrust of 9 Newtons. The mass would decrease linearly with time and so would the gravitational force acting on the rocket. Its speed would increase in this phase and the air drag would rise till motor burnout. At this point, the rocket will undergo maximum aerodynamic pressure (also called Max-Q). Observe that the portion of the altitude vs. time graph after motor burnout is roughly parabolic. Once the rocket achieves its maximum altitude (apogee), it descends and its velocity increases, resulting in higher air drag. This causes the magnitude of the acceleration to drop. We might want to add a parachute to protect the instruments from damage due to impact. The rocket would then achieve a terminal velocity much quicker and the altitude vs. time graph would look like a straight line towards the end of the flight. From this we can infer that it would be counter-productive to deploy the parachute before maximum altitude i.e. apogee is reached.

16.3 Optimisation of the Model Rocket

Once we have chosen the components of reasonable dimensions, we will need to simulate the launches for various configurations to determine the maximum possible apogee. The simulator assumes that the rocket travels only vertically. Additionally, our rocket may achieve a high apogee value at the cost of stability. Therefore, we use the stability constraint to set a lower bound (one calibre) or an upper bound (five calibres). A diagram of the optimised (and feasible) rocket is given below.



We did multiple simulations for different values of the lengths of both the nosecone and body tube. The masses of the parachute, wadding and instruments to be kept in the body tube were also taken into account.

We narrowed down on the set of higher altitudes attained to get the optimal version. The blue dot indicates centre of gravity and the red one indicates centre of pressure. A centre of pressure farther away from the nosecone (as compared to the centre of gravity) implies greater stability.

16.4 Testing the Stability of the Rocket

We intend to test the actual model before flight. For this purpose, a rope is tied around the body tube where the centre if gravity is located. Now we swing the rocket body using the rope so that it revolves around a fixed point and observe its motion. If it starts spinning around the center of gravity or appears to revolve with its nosecone pointed backwards, the rocket is unstable. Otherwise, it implies that the centre of pressure is indeed below the center of gravity i.e. the results from our earlier simulation are correct. This is a way of evaluating how the rocket would perform during flight (which is essentially the objective of a wind tunnel).

17 Rocket Building and Launch

17.1 Rocket Parts

Rocket body The body tube(or tubes) are the airframe of the model rocket. The body tube usually contains an engine mount to hold the motor,and space for the recovery system. Body of rocket is made up of PVC pipes having height of about 35cm and diameter of about

Figure: Rocket body

Inner tube It is used to fit motor properly in rocket body.As

motor's diameter is little bit smaller than rocket body's diameter,so to hold motor at its place we use inner tube.
We made inner tube by PVC pipe having diameter somewhat similar to motor.We placed inner tube 5cm from one end and 7cm from other end of rocket body.

Figure: Inner tube

Fins We used cardboard material to make fins.Fins are parallelogram in shape with base 5cm and height 4cm.

Figure: Fins

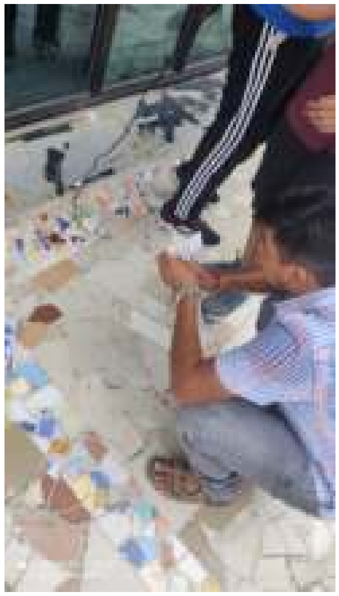


Figure: Attaching fins to the rocket body

Reason behind why we add fins:
The fins of the rocket provide aerodynamic stability in flight so that the rocket will fly straight(in the same way that the feathers of an arrow help it fly straight).Without the stabilization provided by fins in the rear of the rocket, the center of pressure would be too close to the front of the rocket in relation to the motor. This would inevitably cause the rocket to spin out.

Recovery Wadding Recovery wadding is frame-resistant material that protects the parachute(or other recovery system components) from the hot blast of the motor ejection charge.The ejection charge would melt a plastic parachute ,so this protection is necessary.Recovery wadding is installed in between the motor and the parachute and is designed to prevent damage to the parachute.Recovery wadding is typically chemically treated tissue paper or cellulose insertion.
We used sodium bicarbonate and tissue paper to make recovery wadding.To make it first we made a saturated solution of sodium bicarbonate.And after that we soaked tissue paper in solution and finally did the burning test with it.

Figure: Soaking tissue paper in solution

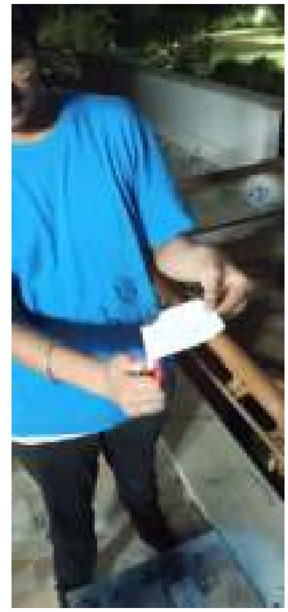


Figure: Burning test

Nose cone The nose cone of the rocket has a shape that causes the air to flow smoothly around the rocket.It could be conical in shape , but at subsonic speeds a rounded shape gives lower aerodynamic drag.Here it is conical in shape and made them by using 3-D printing method in lab.

Figure: Nose cones

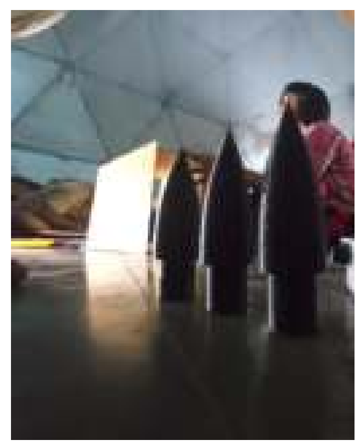


Figure: Nose cones

Launch lug The launch lugs are small tubes(straws) which are attached to the body tube.
The sole purpose of launch lug is to provide stability for a model rocket prior to and during liftoff by forcing the rocket to remain parallel to the launch rod during the first seconds of flight,before significant velocities are reached and enough momentum is built up to maintain stability.
We made launch lugs by body of sketch pens.

Figure: These purple, green and grey tubes on rocket body are launch lugs

Motor

Figure: Estes E9 Motor

We used Estes E9 motor in our rocket.It provides highly efficient performance and is simple, reliable and economical. It is made up of Black Powder propellant and has a mass of 35.8 grams. Black powder motors are easy to ignite and when they burn, they produce a orange/reddish color flame, and light grey smoke.Its casing dimensions are 24mm *95mm. It provides approximately :-

- Total Impulse: 29 N/s
- Peak Thrust: 19.5 N
- Average Thrust: 9 N
- Burn Time: 3 s
- Mass left after firing: 22 g

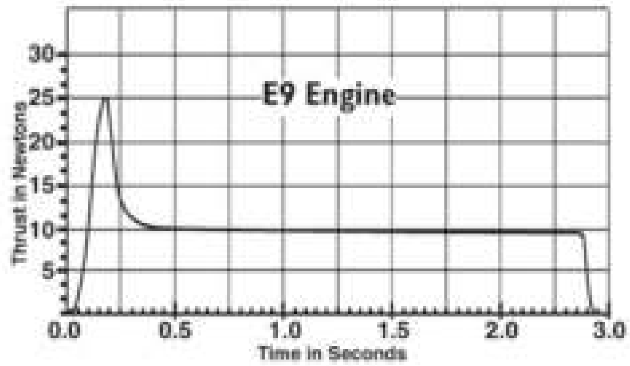


Figure: Thrust vs Time graph

A typical thrust vs time graph of Estes motor is shown above.

Sensors Used in the rocket

1. Altimeter
2. Accelerometer

Altimeter

Figure: Altimeter BMP280

Altimeter is a device used to measure the height of an object above the sea level.Altimeters use barometer sensors which measure atmospheric pressure.The atmospheric pressure varies with the height the rocket attains and the altimeter measures the changes in atmospheric pressure as the rocket climbs or descends.

We used BMP280 altimeter for our rocket. It is economical,small in size so don't take too much space and provides precise temperature, atmospheric pressure values and altitude data.This module uses an environmental sensor manufactured by Bosch with temperature, barometric pressure sensor which is great for all sorts of weather sensing providing an accuracy of ±1 meter when used as altimeter.Its other features are :-

- Operating Voltage: 1.71V to 3.6V – would typically be operated from 3.3V

- Operating Temperature: -40 to +85 deg. Celsius (full accuracy between 0 and +65 deg. C)
- Operating Pressure: 300 hPa to 1100 hPa
- Peak current: 1.12mA

Accelerometer

Figure: Accelerometer ADXL345

An accelerometer is a tool that measures proper acceleration.The accelerometer works on the movement or the vibration of the body.It measures acceleration either by piezoelectric effect or with the capacitance sensor. Piezoelectric effect is more common .It senses the vibration and converts that vibration into the piezoelectric effect. A piezoelectric effect occurs when energy is generated due to pressure and stress.On the other hand ,the capacitance accelerometer sense changes in capacitance between microstructures located next to the device so if an accelerated force moves one of these any given structures, the capacitance will change which will cause the accelerometer to translate that capacitance into voltage for interpretation.

We used ADXL345 accelerometer for our rocket. ADXL345 is a small 3-axis accelerometer with a dynamic range of +/-16g with 13-bit resolution, the maximum bandwidth of 3200Hz, and a maximum data transfer rate of 3200 times a second. It is a digital accelerometer sensor and outputs digital values of acceleration in all the three axes. It is a MEMS accelerometer consisting of a polysilicon surface-micro-machined structure built on the top of a Polysilicon wafer. It is a capacitive accelerometer sensor. The MEMS stands for Micro- Electro- Mechanical- System. MEMS is a fabrication technology. In this type of accelerometer, the changes in capacitance are detected instead of a change in resistance. Other qualities of ADXL345 accelerometer are:-

Measurement range – ADXL345 sensor can measure acceleration in three axes using a user-selectable range of +/-2g, +/-4g, +/-8g, and +/-16g.
Output data rate and bandwidth – The output data rate can range from 0.1 Hz (once in 10 seconds) to 3200 Hz (3200 times a second).
Operating voltage – The sensor requires an operating voltage of 2.5V that can range from 2.0V to 3.6V.
Operating temperature range – ADXL345 has an operating temperature range of -40C to +85C.

Figure: Power source for altimeter and accelerometer

Both altimeter and accelerometer were provided power by the battery connection as shown in the figure.They were placed inside the rocket body below the parachute wrapped inside wadding to protect them from the heat produced during the flight.They are the payload of our rocket .

Parachute All model rockets require a recovery system to slow their descent and return them safely to the ground .The most common type of recovery system is the parachute.The parachute is expelled from the body tube by the ejection charge of the rocket motor after a delay to allow the rocket to reach apogee and be traveling at a relatively slow speed.

Construction of Parachute

- We measured and cut out a square piece of the parachute plastic material with the sides of length 6 inches
- We folded it twice making it a square of half the initial length.We obtained a side containing all the free ends and marked it as 1.
- Then we folded it diagonally obtaining a isosceles triangle.
- We removed a small piece of isosceles triangle from the side marked in step 2 and unfolded our parachute material.We got a square with a small square piece removed from all the four ends.
- We joined the ends of small square pieces at the corners using duct tape.
- After that ,we cut out two sections of string to make the shroud lines . We took one string and connected its two ends at the two adjacent corners of our square piece and repeated the same for the other string.

Figure: Parachute

Attaching the parachute to the rocket

We stretched the two strings and joined both of them from the center using an elastic cording .With the help of this elastic cording we joined our parachute to both rocket body and the nose cone .Then we folded our parachute, put it inside the rocket body with some wadding to protect it from the heat generated.

17.2 Rocket Launch

Launch Stand

We have used a launch stand to launch the rockets. It's purpose was to give a support to the rocket and launch the rocket in a straight alignment. It consisted of a metal base and metal rod welded onto it as shown in the figure.

Figure: Launch Stand With Rocket

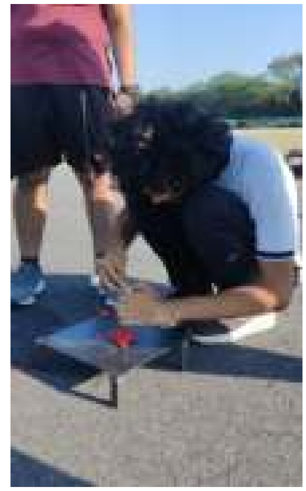


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Figure: Preparing Launch Stand

double sided tape is wound and stuck to the vertical rod at a height of about 10cm from the base. The launch lug is mounted on it so that the rocket is raised at a height. This provide a support so that the rocket can stand.

Launch Mechanism

To launch the rocket, we need to ignite the motor from certain safe distance. To ignite the motor in our rocket we used a resistor to cause the necessary ignition. We fitted a 20Ω resistor into the motor and supplied a very high voltage to it from a 12V battery. This caused the resistor to become red hot and blow up. The heat evolved from the resistor caused the motor to ignite. It took about 10 seconds for the motor to ignite. Due to so much thrust the wires got detached and the rocket went off.

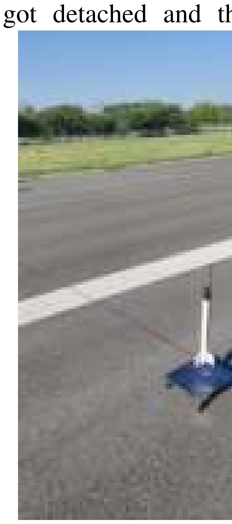

Figure: Launch Setup

The Launch

The launch took place at the Airstrip, IIT Kanpur. The weather conditions were ambient with less wind speeds and sufficient light. Ideal for the launch. On the launch day the necessary parts like motor and waddings were added. The launch took place with proper safety measures:

- Fire Extinguishers were there for any mishappening
- 3 Waddings were added so as to protect the components from heat
- Wires were connected to the battery only 10 seconds before the launch
- All people stayed at a safe distance from the rocket launch site during the launh
- The launching person had saftey helmet and barrier for protection
- Alligator clips were used
- The base was frimly attached to the ground

18 References

- **Multiobjective Optimization of Two-Stage Rockets for Earth-To-Orbit Launch**, *Bairstow et al.*
- **Orbital Perturbations**, *Astronomy Club, IITK*
- **U.S. Standard Atmosphere, 1962**, *U.S. Government Printing Office, Washington, D.C., 1962.*