Tutorial - 6

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Q1: What do you mean by minimum spanning tree? What are the applications of MST?

Ans:- Minimum spanning tree (MST) or minimum weight spanning tree is a subset of the edges of a connected edge-weighted undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight.

Applications:-

(1) Consider 'n' station are to be linked using a communication network and lying of communication link between any two station involve a coat. The ideal solution would be to exact a subgraph termed as minimum cost spanning tree.

(2) Suppose you meant to construct highways or railroads spanning several cities then we can use the concepts of minimum spanning tree.

(3) Design LAN

(4) Laying pipelines connecting offshore drilling sites, refineries and consume markets.

Q2: Please analyse the time and space complexity of prim, Krushkal, Dijkstra and Bellman ford algorithm.

Ans:- Prim's Algorithm -

TC :- $O((V+E)\log)$

SC :- $O(V)$

Krushkal's Algorithm

TC :- $O(E(\log V))$

SC :- $O(|V|)$

Dijkstra Algorithm -

TC :- $O(V^2)$

SC :- $O(V^2)$

Bellman ford Algorithm

TC :- $O(VE)$

SC :- $O(E)$

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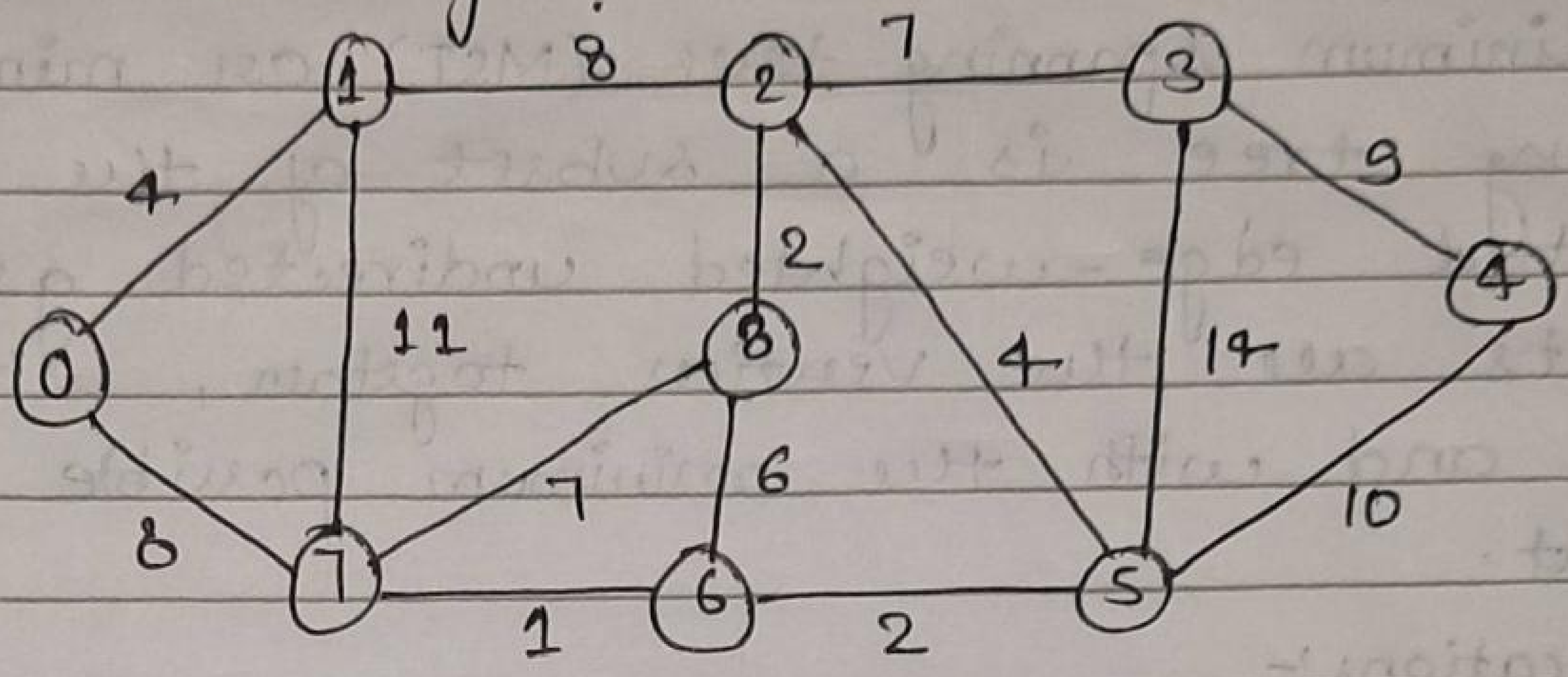
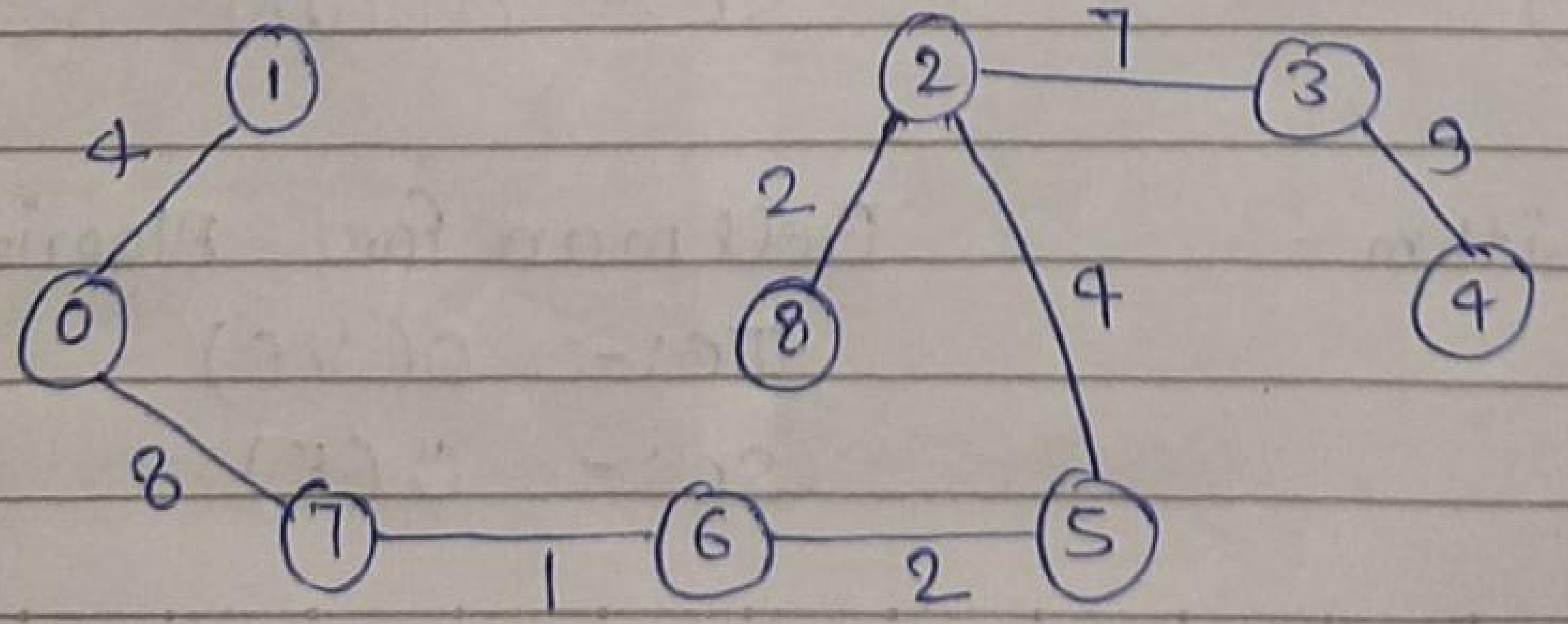
Q3:- Apply Krushkal and Prim's algorithm on graph given on right side to compute MST and its weight?



→ Krushkal's Algorithm

0	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	X
2	3	7	✓
7	8	7	X
0	7	8	✓
1	2	8	X
4	3	9	✓
4	5	10	X
1	7	11	X
3	5	14	X



Total weight =
1+2+2+4+4+7+
8+9 = 37

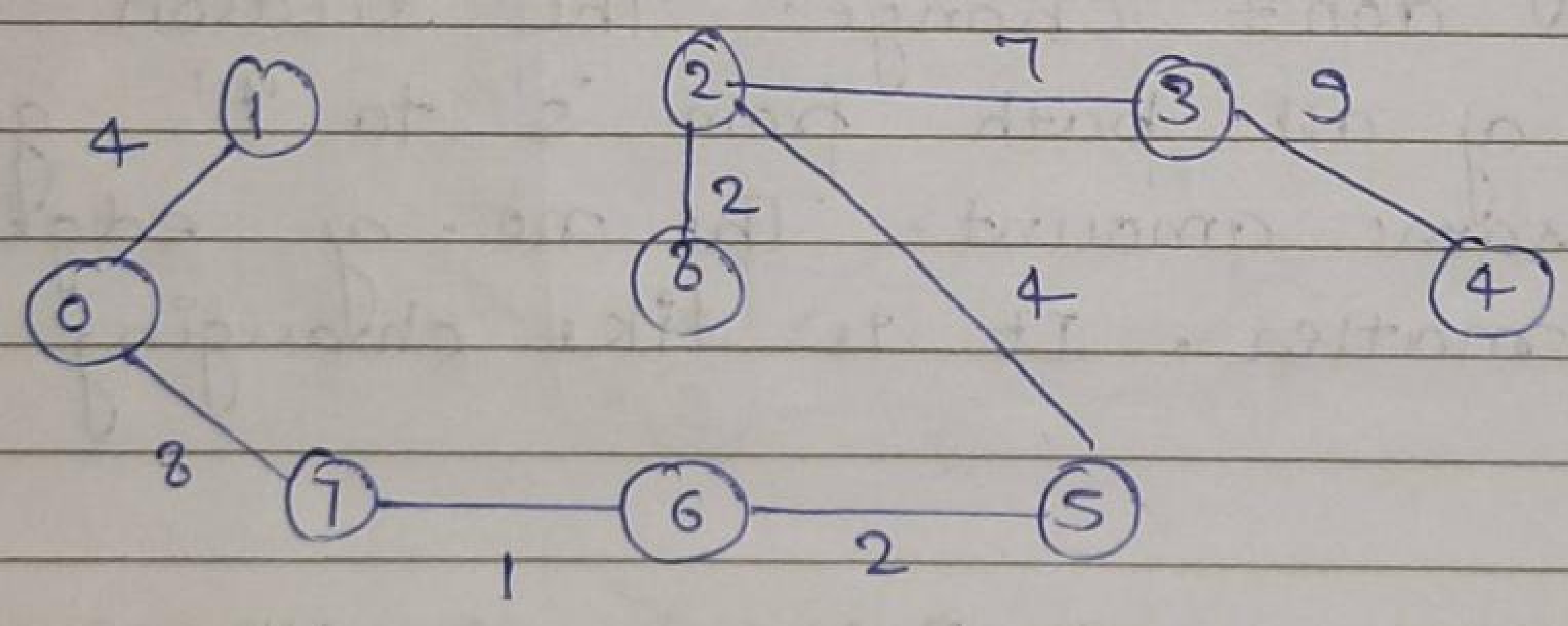
→ Prim's Algorithm

Weights:-

0	1	2	3	4	5	6	7	8
$\boxed{0\ \cancel{\infty}}$	∞	∞	∞	∞	∞	∞	∞	∞
	4						$\boxed{8}$	
		8				$\boxed{1}$		7
	11		7		4	1		$\boxed{2}$
			7		2			6
	4	14	1	10				
		$\boxed{7}$						
				$\boxed{9}$				

Parents:-

0	1	2	3	4	5	6	7	8
-1	~~-1~~	~~-1~~	-1	-1	-1	~~-1~~	~~-1~~	-1
	6	1				1	1	



Total weight = 4 + 8 + 1 + 2 + 4 + 2 + 7 + 9 = 37 Ans

Q4:- Given a directed weighted graph. You are also given the shortest path from a source vertex 's' to a destination vertex 't'. Does shortest path remain same in the modified graph in following cases?

→ If weight of every edge is increased by 10 units.

→ If weight of every edge is multiplied by 10 units.

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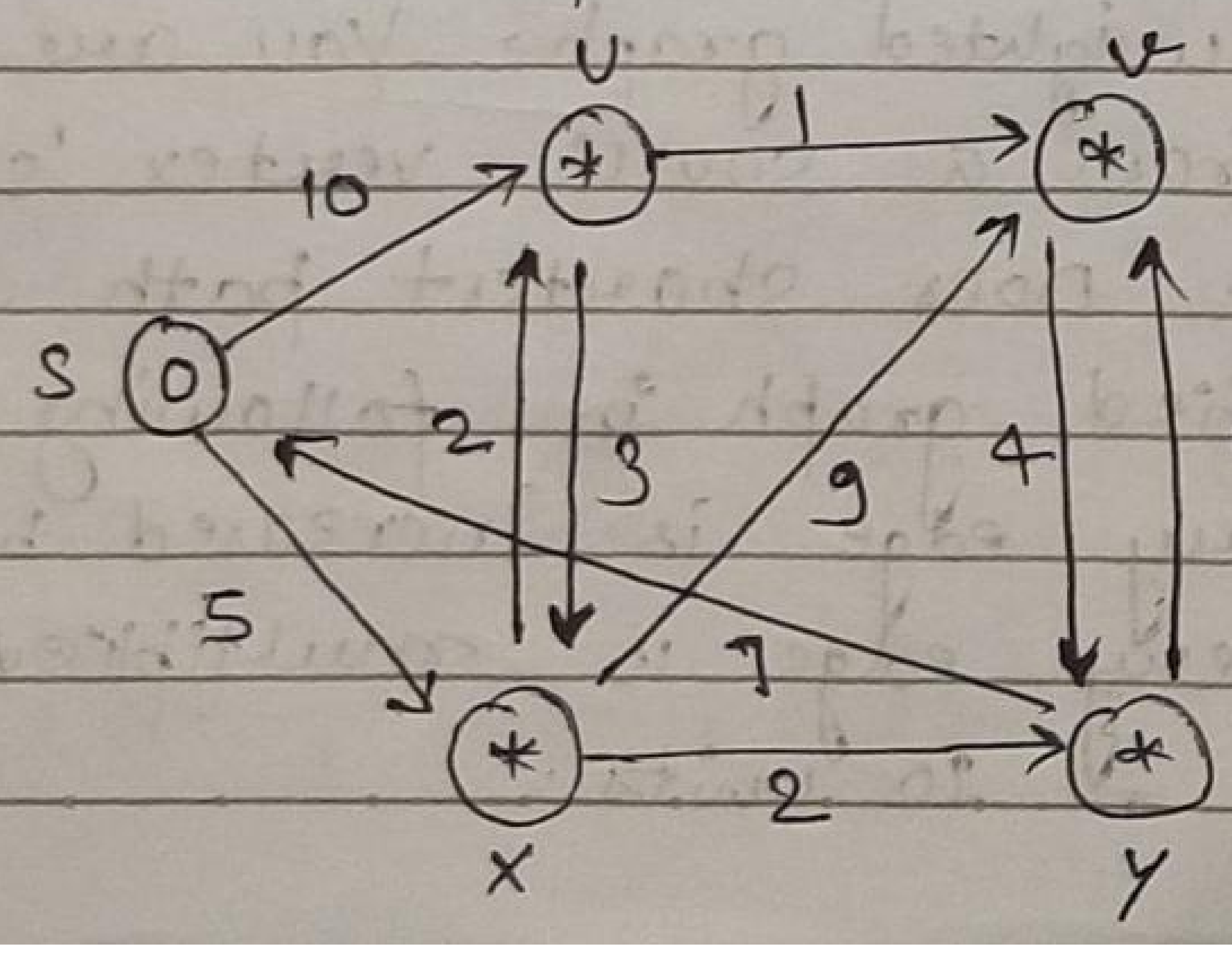
Ans:- (i) If weight of every edge is increased by 10 units.

The shortest path may changes. The reason is there may be different number of edges in different paths from 's' to 't'. for example:- let shortest path be of weight 15 and has edge 5. Let there be another path with 2 edge and total weight 25. The weight of the shortest path is increased by 5*10 and becomes 15 + 50. Weight of the other path is increased by 2*10 and becomes 25+20 so the shortest path changes to the other path with weight 20 45.

(ii) if weight of every edge is multipled by 10 units.

If we multiply all edges weight by 10, the shorest path don't change. The reason is simple, weight of all path from 's' to 't' get multiplied by same amount. The no. of edges on a path don't matter. It is like changing limits of weight.

Q5:- Apply Dijkshtra and Bellman algorithm on graph given on right side to compute shortest path to all nodes from node S.



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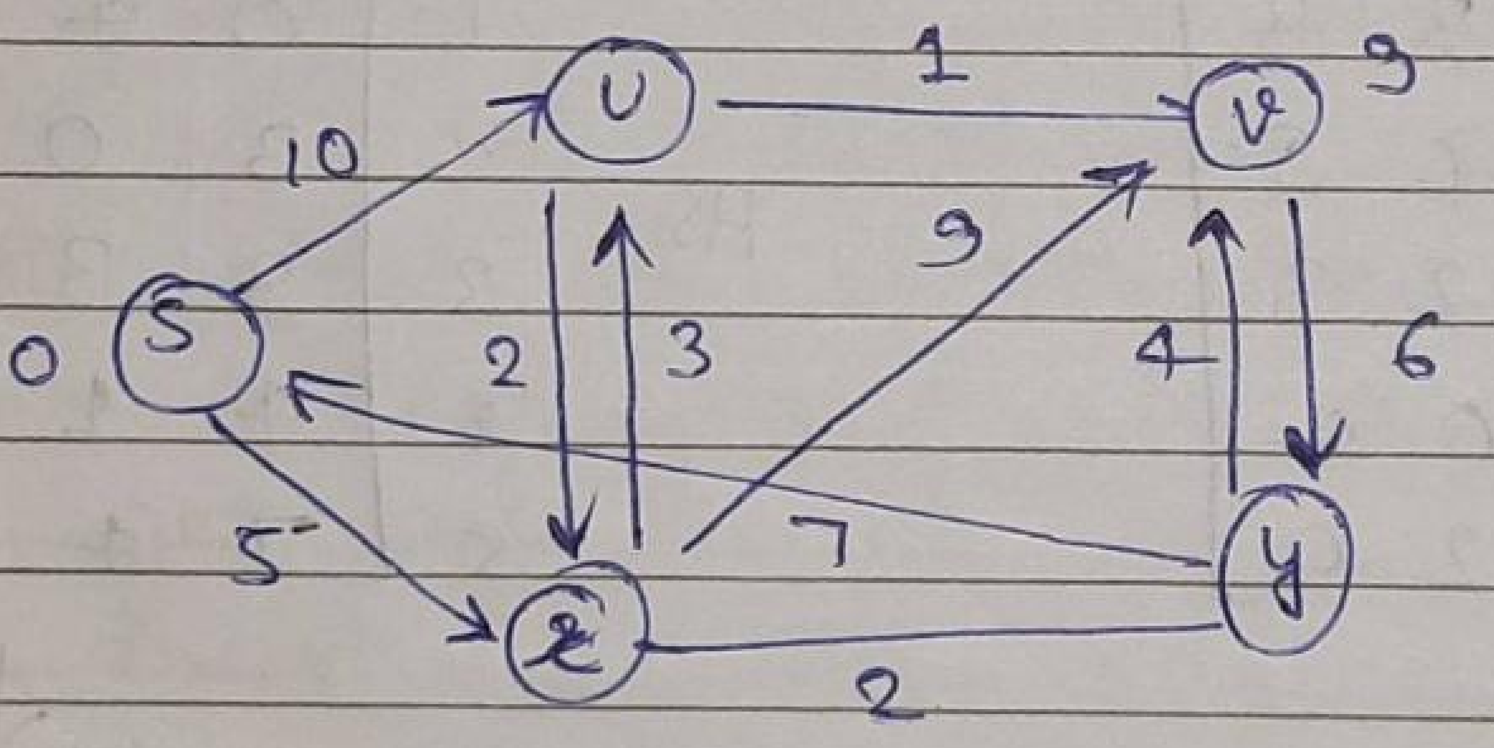
node	shortest distance from source node
u	8
x	5
v	9
y	7

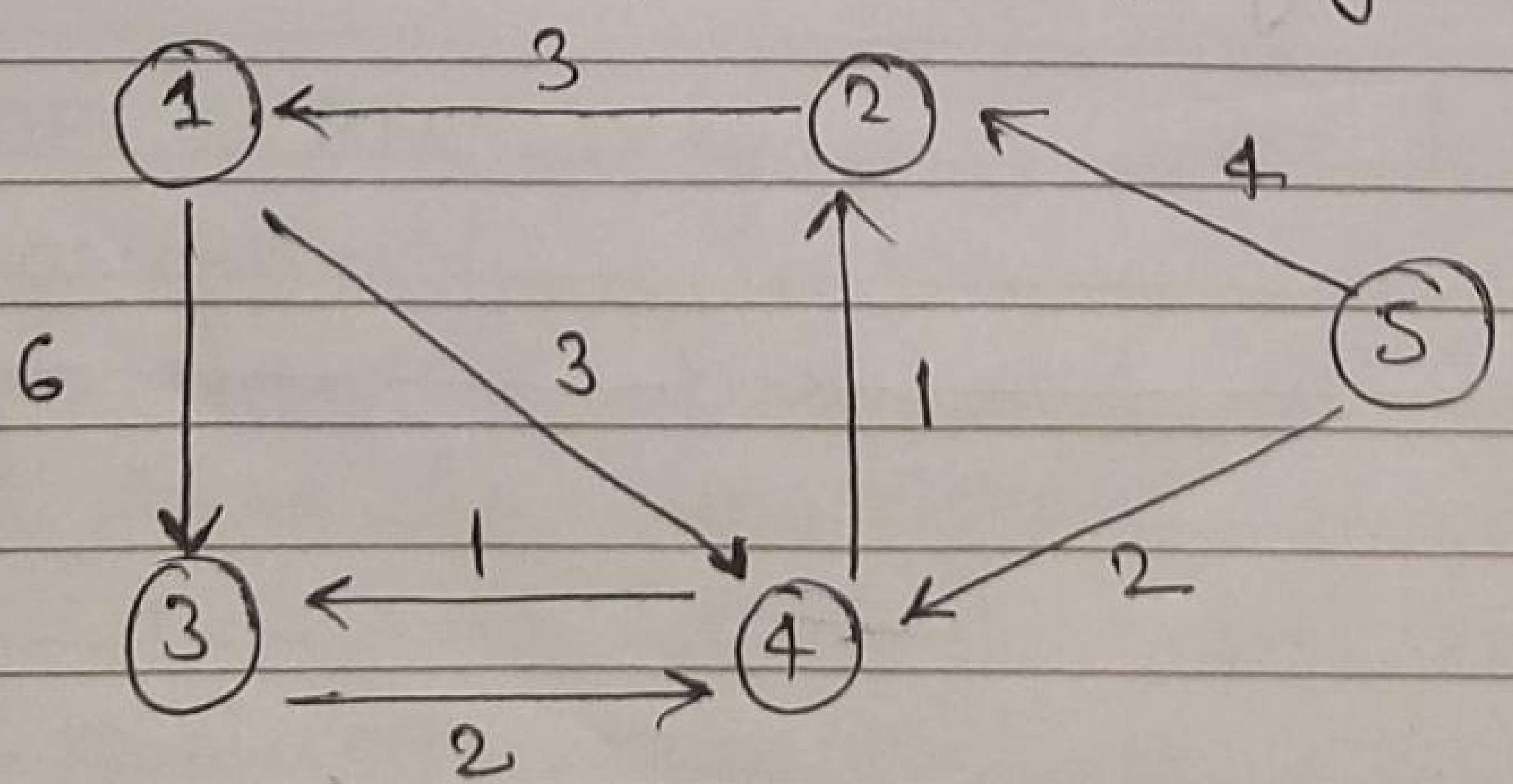
Bellman ford algorithm

	s	u	v	x	y
1st →	0	10	∞	5	∞
2nd →	0	10	11	5	∞
3rd →	0	8	9	5	7
4th →	0	8	9	5	7

→ "graph" does not have cycle



Q6:- Apply all pair shortest path algorithm - Floyd Warshall on below mentioned graph and also analyse the time and space complexity of algorithm.



$$A^0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & \infty & \infty & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$A[a,b] < A[a,1] + A[1,b]$

$$A^{1} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & 3 \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & 6 & 2 & 0 \end{array}$$

$A[a,b] < A[a,2] + A[2,b]$

$$A^2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & 3 \\ 2 & 3 & 0 & 9 & 6 & 3 \\ 3 & 3 & \infty & 0 & 2 & 3 \\ 4 & 4 & 1 & 1 & 0 & 4 \\ 5 & 7 & 4 & 6 & 2 & 0 \end{array}$$

$A[a,b] < A[a,3] + A[3,b]$

$$A^3 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 6 & 6 & 3 & 3 \\ 2 & 3 & 0 & 9 & 6 & 3 \\ 3 & 3 & \infty & 0 & 2 & 3 \\ 4 & 4 & 1 & 1 & 0 & 4 \\ 5 & 7 & 4 & 6 & 2 & 0 \end{array}$$

$A[a,b] < A[a,4] + A[4,b]$

$$A^4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 4 & 3 & 3 \\ 2 & 3 & 0 & 7 & 6 & 3 \\ 3 & 3 & 3 & 0 & 2 & 3 \\ 4 & 4 & 1 & 1 & 0 & 4 \\ 5 & 6 & 3 & 3 & 2 & 0 \end{array}$$

$A[a,b] < A[a,5] + A[5,b]$

$$A^5 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 4 & 3 & 3 \\ 2 & 3 & 0 & 6 & 5 & 3 \\ 3 & 3 & 3 & 0 & 2 & 3 \\ 4 & 4 & 1 & 1 & 0 & 4 \\ 5 & 7 & 4 & 6 & 2 & 0 \end{array}$$

$\downarrow$

All pairs shortest path

Time complexity $= O(n^3)$

Space complexity $= O(n^2)$

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