Topic 2: Method of Moments

Kashish Khangar

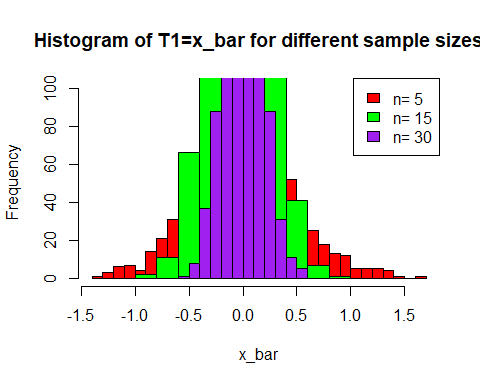
2025-01-21

*Topic 2*

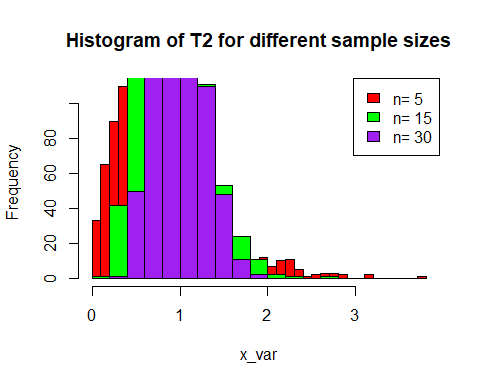
1. To conduct the simulation study from N(=0,=1) distribution. Demonstrate the distribution and it’s descriptive statistics for different sample sizes of following statistic/estimators.

1)T1=x\_bar 2) T2=(1/n)\*sum(Xi-x\_bar)^2

n=c(5,15,30)  
sam\_mean\_T1=sam\_var\_T1=sam\_var\_mean=sam\_var\_var=numeric(0)  
T1\_list=list()  
T2\_list=list()  
  
for (i in 1:length(n)){  
 x=matrix(rnorm(1000\*n[i],0,1),nrow=1000,ncol=n[i])  
   
 T1=apply(x,1,mean)  
 x\_var=apply(x,1,var)  
 T2=((n[i]-1)/n[i])\*x\_var  
   
 sam\_mean\_T1[i]=mean(T1)  
 sam\_var\_mean[i]=mean(T2)  
   
 sam\_var\_T1[i]=var(T1)  
 sam\_var\_var[i]=var(T2)  
   
 T1\_list[[i]]=T1  
 T2\_list[[i]]=T2  
}  
  
hist(T1\_list[[1]],breaks="FD",col="red",main="Histogram of T1=x\_bar for different sample sizes",xlab="x\_bar")  
hist(T1\_list[[2]],col="green",add=T)  
hist(T1\_list[[3]],col="purple",add=T)  
legend("topright",legend=paste("n=",n),  
 fill=c("red","green","purple"))



hist(T2\_list[[1]],breaks="FD",col="red",main="Histogram of T2 for different sample sizes",xlab="x\_var")  
hist(T2\_list[[2]],col="green",add=T)  
hist(T2\_list[[3]],col="purple",add=T)  
legend("topright",legend=paste("n=",n),  
 fill=c("red","green","purple"))



cbind(n,sam\_mean\_T1,sam\_var\_T1)

## n sam\_mean\_T1 sam\_var\_T1  
## [1,] 5 0.002489765 0.21051017  
## [2,] 15 -0.014859756 0.07012379  
## [3,] 30 0.002177539 0.03192162

cbind(n,sam\_var\_mean,sam\_var\_var)

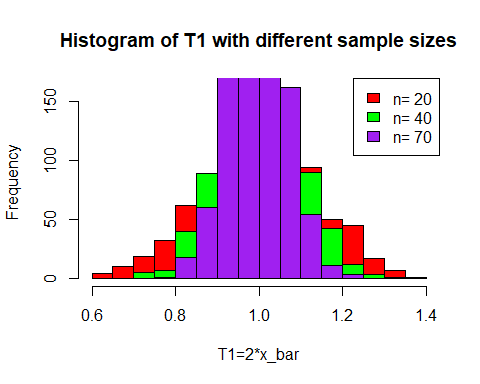
## n sam\_var\_mean sam\_var\_var  
## [1,] 5 0.7699123 0.31709677  
## [2,] 15 0.9326848 0.11718472  
## [3,] 30 0.9673025 0.06396639

From this output, we conclude that as sample size increases, variability in the 1st moment estimator (T1=x\_bar) is decreases.

1. To conduct the simulation study from U(0,),>0 distribution. Demonstrate the distribution and it’ s descriptive statistics for different sample sizes of the moment estimator.

T1=

n=c(20,40,70)  
sam\_mean\_T1=sam\_var\_T1=numeric(length(n))  
T1\_list=list()  
  
for (i in 1:length(n)){  
 x=matrix(runif(1000\*n[i],0,1),nrow = 1000,ncol = n[i])  
 x\_bar=apply(x,1,mean)  
 T1=2\*x\_bar  
 sam\_mean\_T1[i]=mean(T1)   
 sam\_var\_T1[i]=var(T1)  
 T1\_list[[i]]=T1  
}  
  
hist(T1\_list[[1]],breaks="FD",col="red",main="Histogram of T1 with different sample sizes",xlab="T1=2\*x\_bar")  
hist(T1\_list[[2]],col="green",add=T)  
hist(T1\_list[[3]],col="purple",add=T)  
  
  
legend("topright",legend=paste("n=",n),  
 fill=c("red","green","purple"))



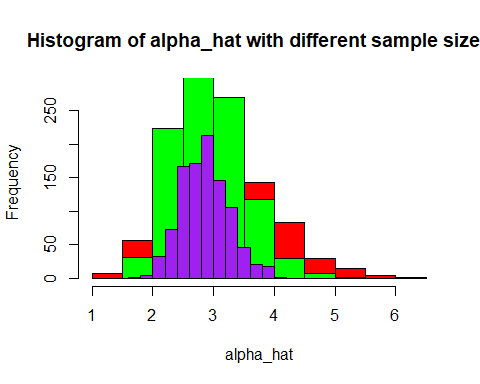
cbind(n,sam\_mean\_T1,sam\_var\_T1)

## n sam\_mean\_T1 sam\_var\_T1  
## [1,] 20 1.0006734 0.016785320  
## [2,] 40 1.0024291 0.008565162  
## [3,] 70 0.9975787 0.004820991

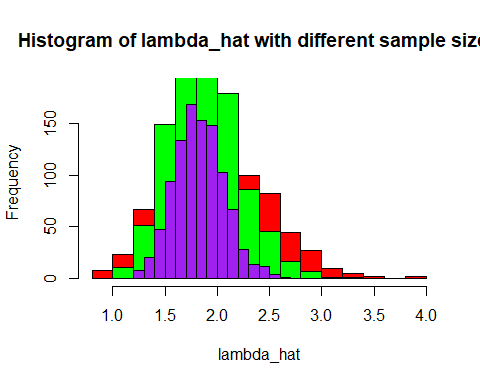
From this output, we conclude that as sample size increases variability in T1 and T2 decreases.

1. To conduct the simulation study from Gamma distribution with density given by . Demonstrate the distribution and it’s descriptive statistics for different sample sizes of following statistic/estimators.

rm(list=ls())  
n=c(50,100,200) #To generate 5 r. s. from G(alpha=2.8, lambda(shape)=1.8) dist  
set.seed(120)  
sam\_mean\_alpha\_hat=sam\_mean\_lambda\_hat=sam\_var\_alpha\_hat=sam\_var\_lambda\_hat=numeric(length(n))  
  
alpha\_hat\_list=list()  
lambda\_hat\_list=list()  
  
for (i in 1:length(n)){  
 x=matrix(rgamma(1000\*n[i],1.8,2.8),nrow=1000,ncol=n[i]) #shape=1.8  
   
 m1\_=apply(x,1,mean) #m1'  
 m2\_=apply(x^2,1,mean) #m2'  
 m2=m2\_-m1\_^2 #m2=(1/n\*sum(x-x\_bar)^2)  
   
 alpha\_hat=m1\_/m2 #moment est of alpha  
 lambda\_hat=m1\_^2/m2 #moment estimate of lambda  
   
 sam\_mean\_alpha\_hat[i]=mean(alpha\_hat)  
 sam\_mean\_lambda\_hat[i]=mean(lambda\_hat)  
   
 sam\_var\_alpha\_hat[i]=var(alpha\_hat)  
 sam\_var\_lambda\_hat[i]=var(lambda\_hat)  
   
 alpha\_hat\_list[i]=list(alpha\_hat)  
 lambda\_hat\_list[i]=list(lambda\_hat)  
}  
  
hist(alpha\_hat\_list[[1]],col="red",main="Histogram of alpha\_hat with different sample sizes",xlab="alpha\_hat")  
hist(alpha\_hat\_list[[2]],col="green",add=T)  
hist(alpha\_hat\_list[[3]],col="purple",add=T)



hist(lambda\_hat\_list[[1]],col="red",main="Histogram of lambda\_hat with different sample sizes",xlab="lambda\_hat")  
hist(lambda\_hat\_list[[2]],col="green",add=T)  
hist(lambda\_hat\_list[[3]],col="purple",add=T)



cbind(n,sam\_mean\_alpha\_hat, sam\_mean\_lambda\_hat)

## n sam\_mean\_alpha\_hat sam\_mean\_lambda\_hat  
## [1,] 50 3.077406 1.955498  
## [2,] 100 2.926502 1.873718  
## [3,] 200 2.859333 1.830444

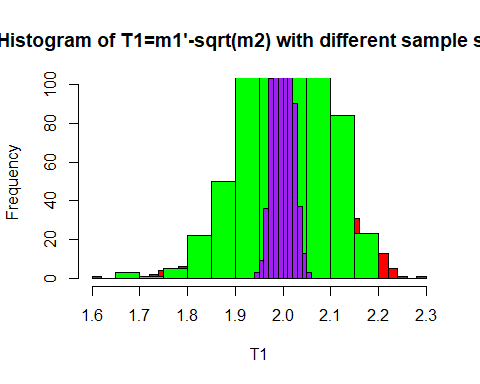
cbind(n,sam\_var\_alpha\_hat,sam\_var\_lambda\_hat)

## n sam\_var\_alpha\_hat sam\_var\_lambda\_hat  
## [1,] 50 0.6235670 0.20971140  
## [2,] 100 0.3280680 0.11227818  
## [3,] 200 0.1572297 0.05594251

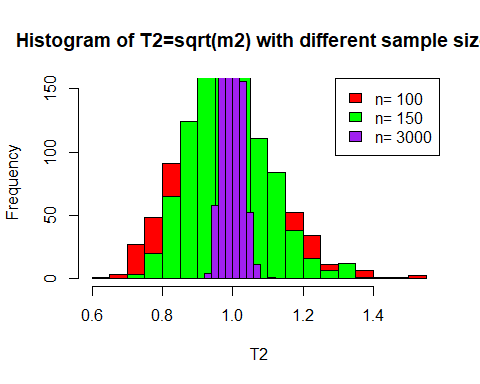
4.Example To conduct the simulation study from Exponential distribution with location parameter and scale parameter . Demonstrate the distribution and it’s descriptive statistics for different sample sizes of following statistic/estimators for and respectively.

1. - 2)

n=c(100,150,3000)  
sam\_mean\_T1=sam\_var\_T1=numeric(length(n))  
sam\_mean\_T2=sam\_var\_T2=numeric(length(n))  
mu=2  
sigma=1  
  
T1\_list=list()  
T2\_list=list()  
  
for (i in 1:length(n)){  
 y=matrix(runif(1000\*n[i],0,1),nrow = 1000,ncol = n[i])  
 x=mu-(sigma\*log(1-y))  
   
 m1\_=apply(x,1,mean) #m1'  
 m2\_=apply(x^2,1,mean) #m2'  
 m2=m2\_-((m1\_)^2) #m2=(1/n\*sum(x-x\_bar)^2)  
   
 T1=m1\_-sqrt(m2)   
 T2=sqrt(m2)   
   
 sam\_mean\_T1[i]=mean(T1)   
 sam\_mean\_T2[i]=mean(T2)  
   
 sam\_var\_T1[i]=var(T1)  
 sam\_var\_T2[i]=var(T2)  
   
 T1\_list[[i]]=T1  
 T2\_list[[i]]=T2  
   
}  
  
hist(T1\_list[[1]],breaks="FD",col="red",main="Histogram of T1=m1'-sqrt(m2) with different sample sizes",xlab="T1")  
hist(T1\_list[[2]],col="green",add=T)  
hist(T1\_list[[3]],col="purple",add=T)



hist(T2\_list[[1]],breaks="FD",col="red",main="Histogram of T2=sqrt(m2) with different sample sizes",xlab="T2")  
hist(T2\_list[[2]],col="green",add=T)  
hist(T2\_list[[3]],col="purple",add=T)  
  
legend("topright",legend=paste("n=",n),  
 fill=c("red","green","purple"))



cbind(n,sam\_mean\_T1,sam\_var\_T1)

## n sam\_mean\_T1 sam\_var\_T1  
## [1,] 100 2.019090 0.0086199415  
## [2,] 150 2.010864 0.0059385257  
## [3,] 3000 2.000307 0.0003435028

cbind(n,sam\_mean\_T2,sam\_var\_T2)

## n sam\_mean\_T2 sam\_var\_T2  
## [1,] 100 0.9815215 0.0185036824  
## [2,] 150 0.9892342 0.0120541253  
## [3,] 3000 0.9999965 0.0006765653

1. Example Consider a random sample of size 20 from given below from distribution. Obtain moment estimates of and based on given sample.

x=c(0.133904,0.224011,0.062282,0.122690,0.051213,0.620577,0.000650,0.271087,0.827109,0.929479,0.561905,0.160865,  
 0.362565,0.001039,0.883135,0.393477,0.990179,0.045737,0.243155,0.080471)  
n=length(x)  
x\_bar=mean(x)  
var\_x=var(x)\*(n-1)/n  
alpha\_hat=x\_bar\*(x\_bar-mean(x^2))/var\_x  
beta\_hat=(1-x\_bar)\*alpha\_hat/x\_bar  
cbind(alpha\_hat,beta\_hat)

## alpha\_hat beta\_hat  
## [1,] 0.3952689 0.7396595