AIP ASSIGNMENT-3 REPORT

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MMSE estimation for Laplacian source

Model: Given Noise Model $Y = X + Z$.

X is distributed according to a Laplace distribution:

$$P_X(x) = \frac{1}{2\sigma_X} \exp\left(-\frac{|x|}{\sigma_X}\right)$$

with $\sigma_X = 1$.

Z is distributed according to a Normal Distribution:

$$P_Z(z) = \mathcal{N}(0, \sigma_Z^2) \quad \text{with} \quad \sigma_Z^2 = 0.1$$

The mean of the posterior distribution provides an unbiased least-squares estimate of the variable x, given measurement y. Bayes' rule allows us to write this in terms of the probability densities of the noise and signal:

$$\hat{x}(y) = \frac{\int_{-\infty}^{\infty} P_Z(y-x) P_X(x) x \, dx}{\int_{-\infty}^{\infty} P_Z(y-x) P_X(x) \, dx}$$

where P_Z is the pdf of noise Z, and P_X is the pdf of X.

Since the MMSE estimate could not be written as a closed-form expression of y, I have used the Trapezoidal numerical integration technique to solve the integration.

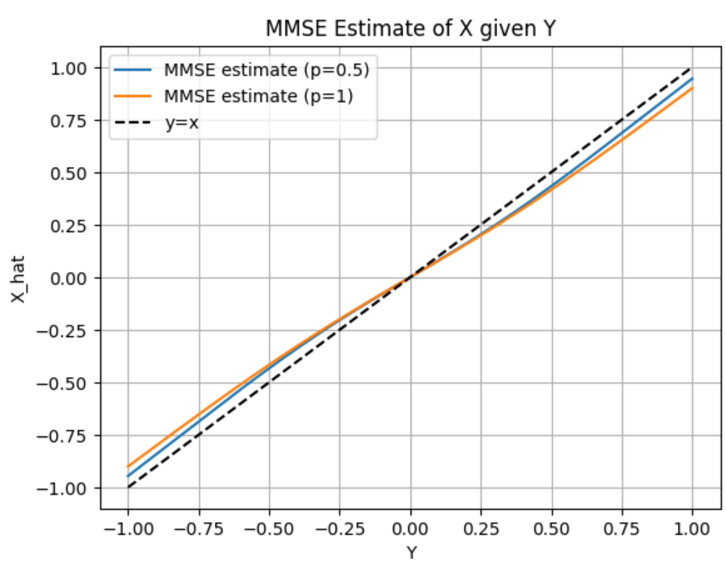

Figure 1: MMSE Estimation of X given

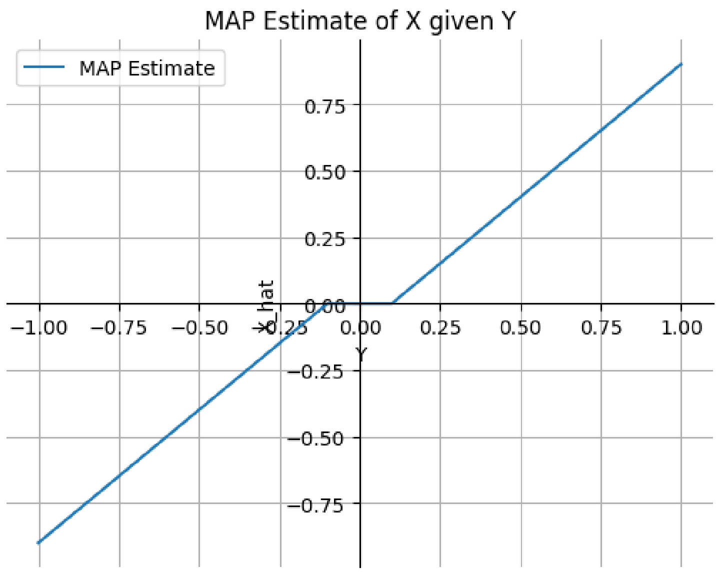

Figure 2: MAP Estimate of X given Y

Problem 2: Image Denoising

2.1. Low Pass Gaussian Filter: The first part of this question involves applying a low pass Gaussian filter to denoise an image. We will use a Gaussian filter to smooth the image and reduce the effect of noise.

The following Python code demonstrates the application of the low pass Gaussian filter for image denoising:

Introduction

In this report, we explore denoising techniques applied to the lighthouse image. The image is initially converted to grayscale, and then white Gaussian noise is added to simulate noise in the grayscale domain. The objective is to compare different denoising methods, specifically focusing on the Low pass Gaussian filter. We vary the filter length and standard deviations to identify the filter that yields the best Mean Squared Error (MSE) compared to the original image.

Methodology

We follow the following steps in our denoising methodology:

1. Load the lighthouse image and convert it to grayscale.
2. Add white Gaussian noise with variance $\sigma_Z^2 = 100$ to the grayscale image.

3. Implement the Low pass Gaussian filter with varying filter lengths (3, 7, 11) and standard deviations (0.1, 1, 2, 4, 8).

4. Compute the denoised images for each combination of filter length and standard deviation.

5. Assess the denoising performance using both subjective evaluation and MSE comparison with the original image.

Results

Best Denoising Method

The best denoising method is determined by evaluating the Mean Squared Error (MSE) between the denoised images and the original image.

Mean Squared Error (MSE)

We compute the MSE for each denoised image and compare them to identify the method with the lowest MSE.

Visual Comparison

We provide visual comparisons between the original image, noisy image, and the best denoised image obtained from the Low pass Gaussian filter with the optimal parameters.

Plots

Result Details

- Best Gaussian Filter: Size=3, Sigma=1
- Mean Squared Error (MSE) with the best filter: 67.59274492665308

Conclusion

From the analysis, we observe that the Low pass Gaussian filter with specific filter lengths and standard deviations effectively reduces noise in the image. The choice of parameters significantly influences the denoising performance, with certain combinations yielding lower MSE values. Overall, the denoising methods help preserve important details in the image while reducing noise, demonstrating the effectiveness of the Low pass Gaussian filter for denoising grayscale images.

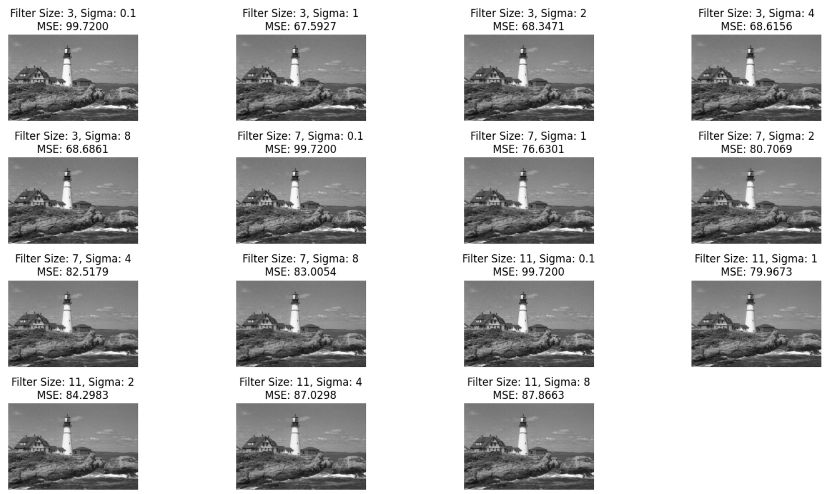

Figure 3: Denoised Images with Different Filter Sizes and Sigma along with MSE

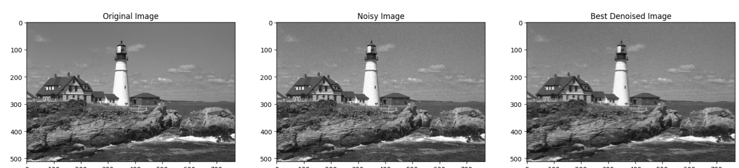

Figure 4: Best Denoised Image along with Original and Noisy Image

2.2 Adaptive MMSE Introduction

In this report, we explore the Adaptive Minimum Mean Squared Error (MMSE) denoising method applied to the lighthouse image. The objective is to develop an adaptive version of the MMSE filter, where estimates are computed for patches of size 32×32 with an overlap of 16 in the high pass domain. We calculate the variance of the high pass coefficients of the original image and the variance of noise in the high pass image. The noise variance is given as $\sigma_Z^2 = 100$ in the pixel domain. The adaptive MMSE filter aims to reduce noise while preserving important image details.

Methodology

We follow these steps in our denoising methodology:

1. Calculate the variance of the high pass coefficients of the original image and the variance of noise in the high pass image.

2. Implement the adaptive MMSE filter, where estimates are computed for

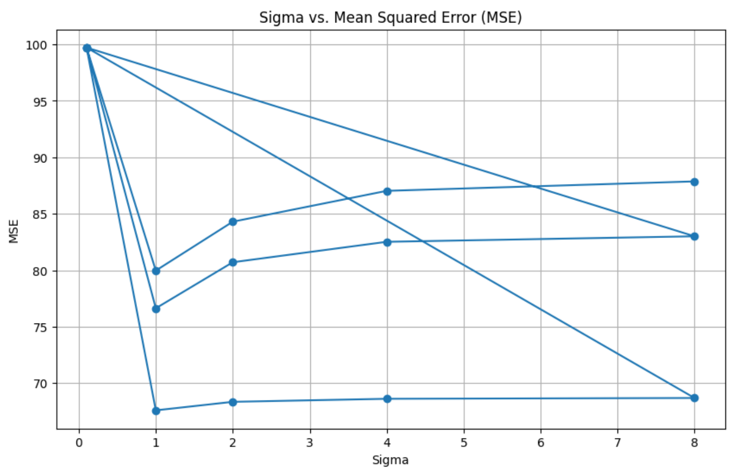

Figure 5: Sigma vs. MSE Plot

patches of size 32×32 with an overlap of 16 in the high pass domain.

3. Apply the adaptive MMSE filter to the noisy image to obtain the denoised image.

4. Compute the Mean Squared Error (MSE) between the denoised image and the original image to assess the denoising performance.

Results

Variance Calculation

- Variance of the high pass coefficients of the original image: 93.88473536847465

- Variance of noise in the high pass image: 71.72445320530537

Visual Comparison

We provide visual comparisons between the original image, noisy image, and the denoised image obtained using the Adaptive MMSE filter.

Plots

Mean Squared Error (MSE)

The MSE between the denoised image and the original image is calculated to quantify the denoising performance.

- MSE in Adaptive MMSE denoising: 61.1215894429302

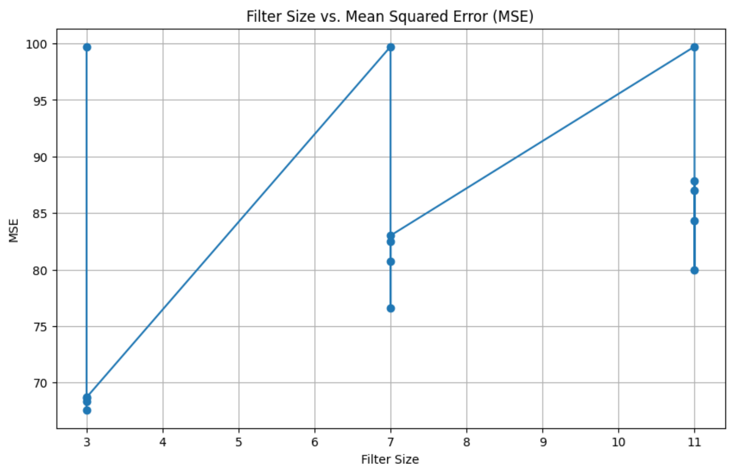

Figure 6: Filter Size vs. MSE Plot

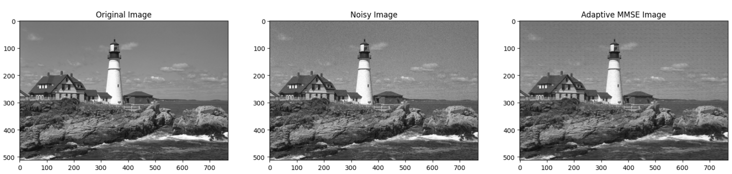

Figure 7: Original Noisy and Adaptive mmse image

Conclusion

The Adaptive MMSE denoising method effectively reduces noise in the lighthouse image while preserving important details. By computing estimates for patches of size 32×32 with an overlap of 16 in the high pass domain, the adaptive MMSE filter adapts to local image characteristics, resulting in improved denoising performance. The calculated variance values provide insights into the characteristics of the original image and the noise present in the high pass domain. Overall, the Adaptive MMSE denoising technique demonstrates its effectiveness in enhancing image quality and reducing noise.

1 2.3. Adaptive Shrinkage:Introduction

This report investigates the application of Adaptive Shrinkage denoising methods to the lighthouse image.

2 Results

2.1 Adaptive Shrinkage Image

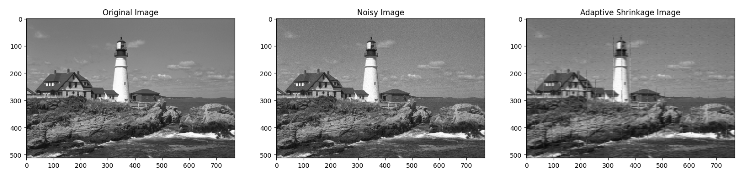

Figure 8: Adaptive Shrinkage Image

2.2 Threshold Values for Each Patch

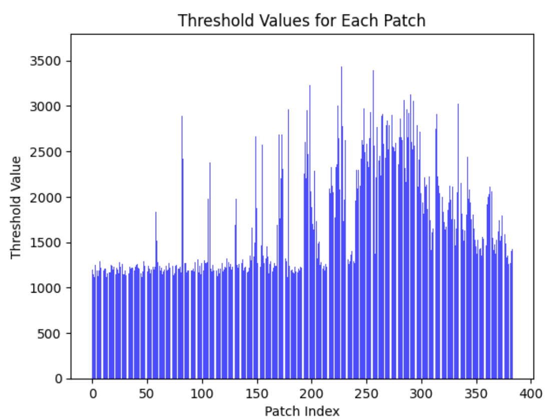

Figure 9: Threshold Values for Each Patch

2.3 All best images together

Figure 10: All best denoised images