**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

From above data we have *μ* = 45, *σ* = 8

Total time expected to be taken by manager is 50 mins

So the success :p(x<=50)= np.round(stats.norm.cdf(50,loc=45,scale=8),4) = 0.7340=73.4%

Unsuccess:p(x>50)= 100-73.4=26.6% or 0.2666 or 0.2676

1. 0.3875
2. **0.2676 #** **manager cannot meet his commitment**
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

p(38<=age<=44)

stats.norm.cdf(44,loc=38,scale=6) - stats.norm.cdf(38,loc=38,scale=6)=

0.3413447460685429=34.13%

P(x>44)=1- stats.norm.cdf(44,loc=38,scale=6)= 0.15865525393145707= 15.86%

: So above statement is **false**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

: P(age<30)= stats.norm.cdf(30,loc=38,scale=6)=0.0912

No number of people being under 30 = p(age<30)\*400

0.0912\*400=36.48 or 36

So this statement is **true**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

As we know that X1 and X2 are two random variable and have same mean and variance

Since X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are two independent variable so, X1+X2 ~ N(μ+ μ, σ2+σ2 )

So we can say : 2X1~ N(2μ, 4σ2)

: X1+X2 ~ N(μ+ μ, σ2+σ2 )=> N(2 μ,2σ2)

: 2X1-(X1+X2) = N(2μ - 2μ , 4σ2 + 2σ2 )

= N(0 , 6σ2 )

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

We have to find a, b symmetric about mean whose probability of random variable between them is 0.99

So probability of not getting is 0.01

So probability to the left side of a=0.01/2 = -0.005

probability to the right side of b = 0.01/2 = 0.005

z=(X- μ) / σ

Z=stats.norm.ppf(0.005)= -2.576

So Z\* σ + μ = -(-2.576)\*20 +100=151.52

Z\* σ + μ = (-2.576)\*20 +100=48.48

**So the interval is [151.5,48.5]**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Profit1 ~ N(5, 32)+ Profit2 ~ N(7, 42)= N(5+7, 42+32)=N(12, 52)

For 95% Confidence interval Z=1.96

So range=[ 12-1.96\*5,12+1.96\*5]=[$2.2,$21.8]=(INR)[99,981]

Mean(INR)=12\*45=540

Std(INR)=5\*45=225

stats.norm.interval(0.95,540,225)

(99.00810347848784, 980.9918965215122)

1. Specify the 5th percentile of profit (in Rupees) for the company

So to compute 5th percentile of profit=

X=μ + Zσ; wherein from z table, 5 percentile = -1.645

=225\*(-1.645)+540=170

1. Which of the two divisions has a larger probability of making a loss in a given year?

#div1

round(stats.norm.cdf(0,loc=5,scale=3),4)\*100 = 4.78

#div2

round(stats.norm.cdf(0,loc=7,scale=4),4)\*100 = 4.01

so division 1 has higher probability of making loss.