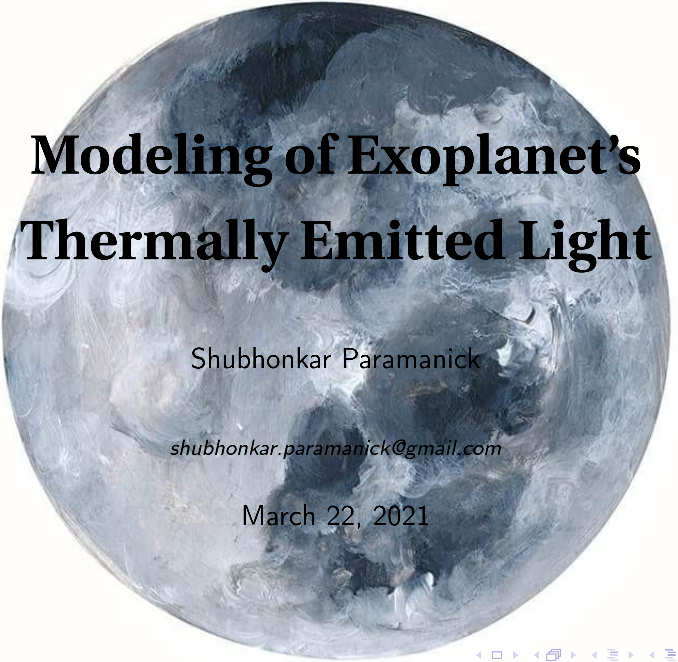

Modeling of Exoplanet's Thermally Emitted Light

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Overview

1. Description of the Geometry
2. Plane Parallel Stellar Radiation
3. Obtaining the Phase Variations of the Thermal Light

General Problem

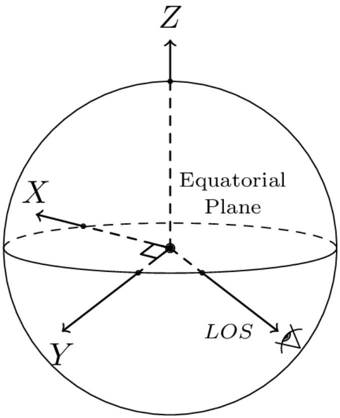
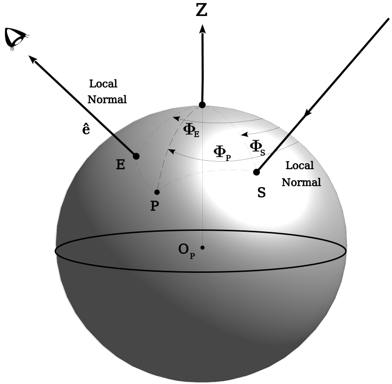
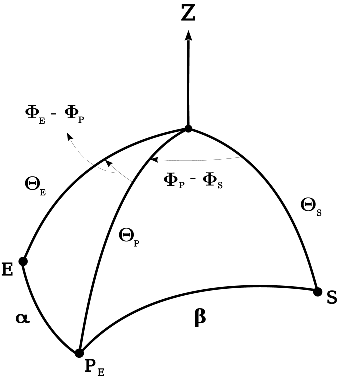


Figure: Planeto-centric Frame



(a) Sphere irradiated at S.



(b) Arcs on the Geodesics.

$$\cos(\alpha) = \cos(\Theta_E)\cos(\Theta_P) + \sin(\Theta_E)\sin(\Theta_P)\cos(\Phi_E - \Phi_P)$$

$$\cos(\beta) = \cos(\Theta_S)\cos(\Theta_P) + \sin(\Theta_S)\sin(\Theta_P)\cos(\Phi_P - \Phi_S)$$

Emitted Luminosity

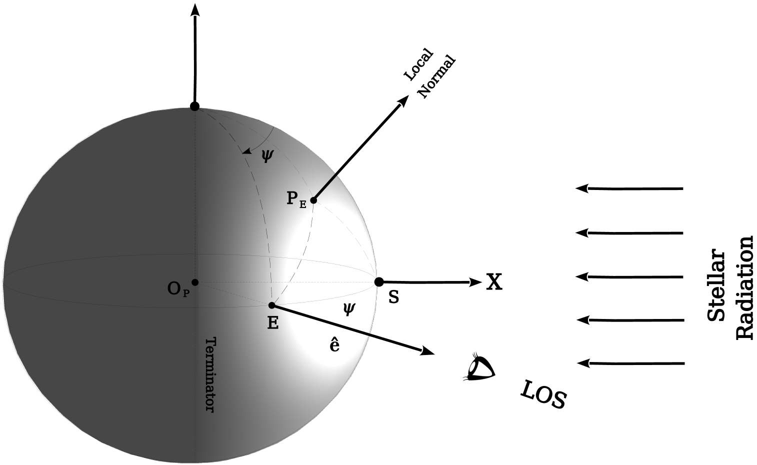
The luminosity ($\mathscr{L}_{\text{P, Emit.}}(\alpha, \nu, t)$) of the planet at point P_E as observed by an observer on Earth is given by

$$\mathscr{L}_{\text{P, Emit.}}(\alpha, \nu, t) = \int\limits_{\substack{A = \text{Planet area} \\ \text{visible to Earth}}} \overrightarrow{\mathscr{F}_{\text{P, Emit.}}(\Theta_P, \Phi_P, \nu, t)} \cdot \hat{e} \; dA_E$$

$$= \int\limits_{\substack{A = \text{Planet area} \\ \text{visible to Earth}}} \mathscr{F}_{\text{P, Emit.}}(\Theta_P, \Phi_P, \nu, t) \, \widehat{p_e} \cdot \hat{e} \; dA_E$$

$$= \int\limits_{\substack{A = \text{Planet area} \\ \text{visible to Earth}}} \mathscr{F}_{\text{P, Emit.}}(\Theta_P, \Phi_P, \nu, t) \, \cos\alpha \, {R_P}^2 \, \sin\Theta_P \, d\Theta_P \, d\Phi_P \quad (1)$$

Approximations



Emitted Thermal Light

When the observer is along the planet's equator and the incident stellar radiation is parallel to the equatorial plane,

$$\Phi_{\mathrm{S}} = 0, \Theta_{\mathrm{S}} = \Theta_{\mathrm{E}} = \frac{\pi}{2}, \text{and } \Phi_{\mathrm{E}} = \psi$$

$$\begin{aligned}\cos(\alpha) &= \cos(\frac{\pi}{2})\cos(\Theta_{\mathrm{P}}) + \sin(\frac{\pi}{2})\sin(\Theta_{\mathrm{P}})\cos(\psi - \Phi_{\mathrm{P}}) \\ &= \sin(\Theta_{\mathrm{P}})\ \cos(\psi - \Phi_{\mathrm{P}})\end{aligned}$$

Then,

$$\mathscr{L}_{\mathrm{P,\ Emit.}}(\alpha, \nu, \mathrm{t}) = \int\limits_{\substack{\mathrm{A\ =\ Planet\ \ area} \\ \mathrm{visible\ to\ Earth}}} \mathscr{F}_{\mathrm{P,\ Emit.}}(\Theta_{\mathrm{P}}, \Phi_{\mathrm{P}}, \nu, \mathrm{t})\ \cos\alpha\ \mathrm{dA_E}$$

Two Zone Black-body Model

For plane parallel rays illuminating the exoplanet, it can be assumed that there are two temperature zones for the exoplanet: the day-side ($\mathrm{T}_{\mathrm{Day}}$) and night-side ($\mathrm{T}_{\mathrm{Night}}$) temperatures.

Assuming that both the star and the exoplanet behave like black bodies with radiance

$$\mathrm{B}_\nu(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

The temperature dependent flux is then given by

$$\mathscr{F}(\mathrm{T}) = \int_\nu \mathrm{B}_\nu(\nu, T)\,\mathrm{K}(\nu)\,\mathrm{d}\nu$$

where $\mathrm{K}(\nu)$ is the filter response function.

Day Side

Integrating Eq. 1 over the day-side of the exoplanet that is visible along the LOS,

$$= \mathrm{R_P}^2 \times \mathscr{F}_{\mathrm{P,\ Emit.}}(\mathrm{T_{Day}}) \times \int\limits_{\psi-\frac{\pi}{2}}^{\frac{\pi}{2}} \int\limits_{0}^{\pi} \sin^2\Theta_\mathrm{P}\ \cos(\Phi_\mathrm{P}-\psi)\, d\Theta_\mathrm{P}\, d\Phi_\mathrm{P}$$

$$\begin{aligned}\mathscr{L}_{\mathrm{Thermal,\ Day}}(\psi, \mathrm{T_{Day}}) &= \pi \times \mathrm{R_P}^2 \times \mathscr{F}_{\mathrm{P,\ Emit.}}(\mathrm{T_{Day}}) \times \left\{\frac{1+\cos\psi}{2}\right\} \\ &= \pi \times \mathrm{R_P}^2 \times \mathscr{F}_{\mathrm{P,\ Emit.}}(\mathrm{T_{Day}}) \times \cos^2\left(\frac{\psi}{2}\right)\end{aligned}$$

Night Side

Integrating Eq. 1 over the night-side of the exoplanet,

$$= {R_P}^2 \times \mathscr{F}_{\text{P, Emit.}}(T_{\text{Night}}) \times \int\limits_{\frac{\pi}{2}}^{\psi+\frac{\pi}{2}} \int\limits_{0}^{\pi} \sin^2\Theta_P \ \cos\left(\Phi_P - \psi\right) d\Theta_P \, d\Phi_P$$

$$\begin{aligned}
\mathscr{L}_{\text{Thermal, Night}}(\psi, T_{\text{Day}}) &= \pi \times {R_P}^2 \times \mathscr{F}_{\text{P, Emit.}}(T_{\text{Night}}) \times \left\{\frac{1-\cos\psi}{2}\right\} \\
&= \pi \times {R_P}^2 \times \mathscr{F}_{\text{P, Emit.}}(T_{\text{Night}}) \times \left\{\frac{1+\cos(\pi-\psi)}{2}\right\} \\
&= \pi \times {R_P}^2 \times \mathscr{F}_{\text{P, Emit.}}(T_{\text{Night}}) \times \sin^2\left(\frac{\psi}{2}\right)
\end{aligned}$$

The night-side luminosity of an exoplanet is exactly out of phase of that of the day-side luminosity.

Disc-Integrated Day-Side & Night-Side Flux

Emitted Day-Side flux of the Planet received at Earth

$$
\begin{aligned}
&= \frac{\mathscr{L}_{\text{Thermal, Day}}(\psi, \mathsf{T}_{\text{Day}})}{4\pi\mathscr{R}^2} \\
&= \mathscr{F}_{\text{P, Emit.}}(\mathsf{T}_{\text{Day}}) \times \left(\frac{\mathsf{R}_{\mathsf{P}}}{\mathscr{R}}\right)^2 \times \left\{\frac{1+\cos\psi}{8}\right\}
\end{aligned}
$$

Emitted Night-Side flux of the Planet received at Earth

$$
\begin{aligned}
&= \frac{\mathscr{L}_{\text{Thermal, Night}}(\psi, \mathsf{T}_{\text{Night}})}{4\pi\mathscr{R}^2} \\
&= \mathscr{F}_{\text{P, Emit.}}(\mathsf{T}_{\text{Night}}) \times \left(\frac{\mathsf{R}_{\mathsf{P}}}{\mathscr{R}}\right)^2 \times \left\{\frac{1-\cos\psi}{8}\right\}
\end{aligned}
$$

Disc-Integrated Flux Continued...

Flux received from the host star

$$= \frac{\mathscr{L}_{\mathrm{S}}}{4\pi\mathscr{R}^2}$$
$$= \mathscr{F}_{\mathrm{Star}}(\mathrm{T}_{\mathrm{Eff.}}) \times \left(\frac{\mathrm{R}_{\mathrm{S}}}{\mathscr{R}}\right)^2$$

where R_{S} is the stellar radius and $\mathscr{R}$ is the distance between the planetary system and Earth.

Day-Side & Night-Side Emitted Fractional Flux

$$\mathfrak{F}_{\mathfrak{Thermal},\mathfrak{Day}} = \frac{\mathscr{F}_{\text{P, Emit.}}(\text{T}_{\text{Day}})}{\mathscr{F}_{\text{Star}}(\text{T}_{\text{Eff.}})} \times \left(\frac{\text{R}_\text{P}}{\text{R}_\text{S}}\right)^2 \times \left\{\frac{1+\cos\psi}{8}\right\}$$

$$\mathfrak{F}_{\mathfrak{Thermal},\mathfrak{Night}} = \frac{\mathscr{F}_{\text{P, Emit.}}(\text{T}_{\text{Night}})}{\mathscr{F}_{\text{Star}}(\text{T}_{\text{Eff.}})} \times \left(\frac{\text{R}_\text{P}}{\text{R}_\text{S}}\right)^2 \times \left\{\frac{1-\cos\psi}{8}\right\}$$

$$\mathfrak{F}_{\mathfrak{Thermal},\mathfrak{Total}} = \mathfrak{F}_{\mathfrak{Thermal},\mathfrak{Day}} + \mathfrak{F}_{\mathfrak{Thermal},\mathfrak{Night}}$$

Obtaining the Thermal Phase Curve

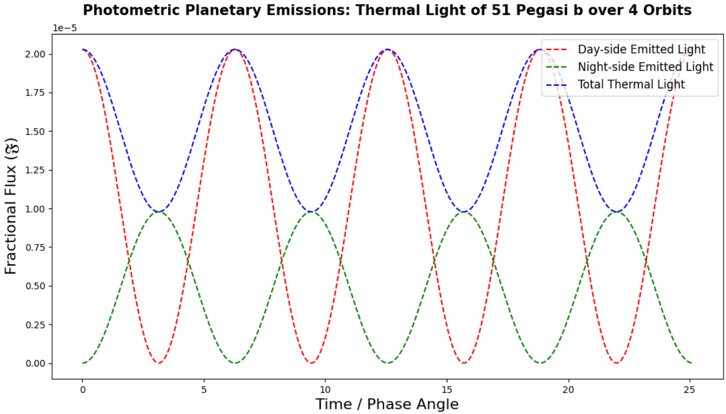


Figure: Fractional Flux due to Thermal Radiation

Plots Continued...

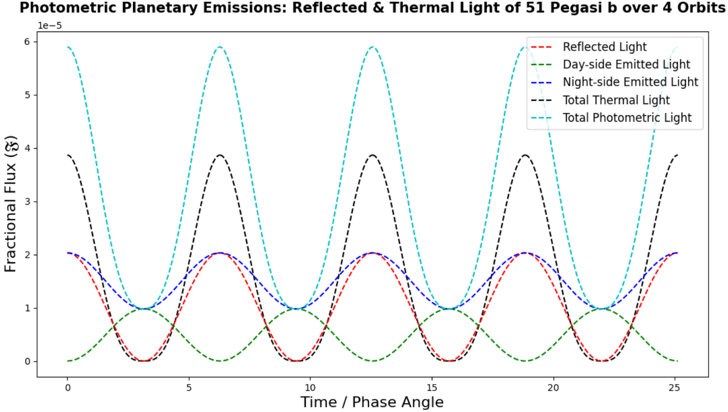


Figure: Total Fractional Flux

The End