

# Panel Data 3: Difference-in-differences

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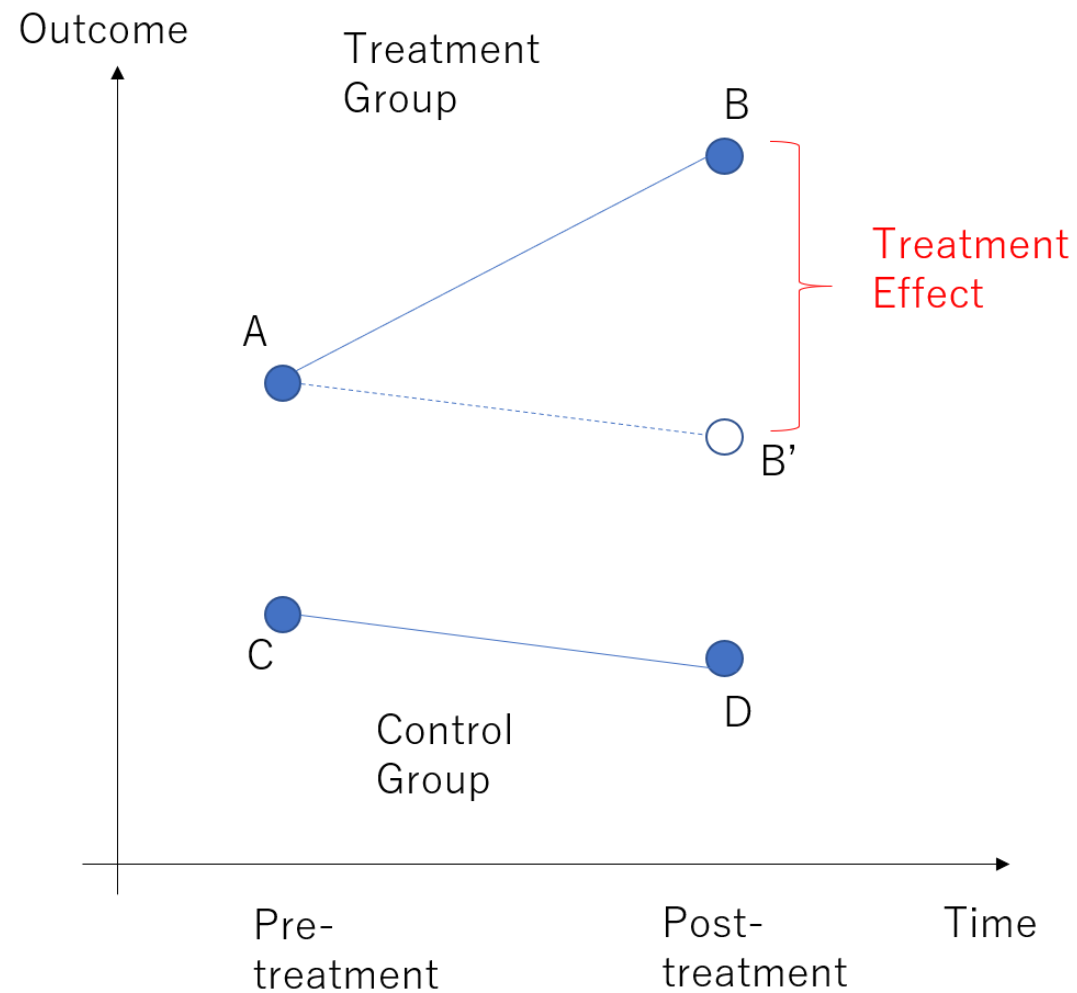
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# Introduction

# Introduction

- **Difference-in-differences (DID, 差の差分法)** exploits the panel structure to estimate the causal effect.
- Two types of simple comparisons
  - Treatment and control group comparison: selection bias
  - Before v.s. after comparison: time trend
- DID combines those two comparisons to draw causal conclusion.

# DID in Figure



# Plan of the Lecture

- Framework
- Implementation in a regression framework
- Discussion on Parallel Trend Assumption

# Reference

- Angrist and Pischke "Mostly Harmless Econometrics" Chapter 5
- Bertrand, Duflo, and Mullainathan (2004, QJE)
  - Discuss issues of calculating standard errors in the DID method.
- Ishise, Kitamura, Kudamatsu, Matsubayashi, and Murooka (2019)
  - Guidance on how to conduct policy evaluation with DID
  - Slide
  - Paper

# Framework

# Framework

- Two periods:  $t = 1, 2$ . Treatment implemented at  $t = 2$ .
- $Y_{it}$ : observed outcome for person  $i$  in period  $t$
- $G_i$ : dummy for treatment group
- $D_{it}$ : treatment status
  - $D_{it} = 1$  if  $t = 2$  and  $G_i = 1$
- **Potential outcomes**
  - $Y_{it}(1)$ : outcome for  $i$  when she is treated
  - $Y_{it}(0)$ : outcome for  $i$  when she is not treated
- Then

$$Y_{it} = D_{it}Y_{it}(1) + (1 - D_{it})Y_{it}(0)$$



# Identification

- Goal: ATT at  $t = 2$

$$E[Y_{i2}(1) - Y_{i2}(0)|G_i = 1] = E[Y_{i2}(1)|G_i = 1] - E[Y_{i2}(0)|G_i = 1]$$

- What we observe

|                         | Pre-period ( $t = 1$ )   | Post ( $t = 2$ )         |
|-------------------------|--------------------------|--------------------------|
| Treatment ( $G_i = 1$ ) | $E[Y_{i1}(0)   G_i = 1]$ | $E[Y_{i2}(1)   G_i = 1]$ |
| Control ( $G_i = 0$ )   | $E[Y_{i1}(0)   G_i = 0]$ | $E[Y_{i2}(0)   G_i = 0]$ |

- Under what assumptions can we the ATT?

# Simple Comparisons

- Within period comparison

- If  $E[Y_{i2}(0)|G_i = 1] = E[Y_{i2}(0)|G_i = 0]$ , ATT is identified by

$$ATT = E[Y_{i2}(1)|G_i = 1] - E[Y_{i2}(0)|G_i = 0]$$

- Selection bias?

- Before-after comparison

- If  $E[Y_{i2}(0)|G_i = 1] = E[Y_{i1}(0)|G_i = 1]$ , ATT is identified by

$$ATT = E[Y_{i2}(1)|G_i = 1] - E[Y_{i1}(0)|G_i = 1]$$

- Time trend?

- More reasonable assumption?

# Parallel Trend (並行トレンド) Assumption

- Assumption:

$$E[Y_{i2}(0) - Y_{i1}(0)|G_i = 0] = E[Y_{i2}(0) - Y_{i1}(0)|G_i = 1]$$

- Interpretation: Change in the outcome *without treatment* is the same across two groups.
- This puts an assumption on **counterfactual** trend in treatment group if it had not been treated.

# Difference-in-differences

- Note that

$$\begin{aligned}\underbrace{E[Y_{i2}(1) - Y_{i2}(0)|G_i = 1]}_{ATT} &= E[Y_{i2}(1)|G_i = 1] - E[Y_{i2}(0)|G_i = 1] \\ &= E[Y_{i2}(1)|G_i = 1] - E[Y_{i1}(0)|G_i = 1] \\ &\quad - \underbrace{(E[Y_{i2}(0)|G_i = 1] - E[Y_{i1}(0)|G_i = 1])}_{=E[Y_{i2}(0)-Y_{i1}(0)|G_i=0] \text{ (parallel trend)}}\end{aligned}$$

- Thus,

$$ATT = E[Y_{i2}(1) - Y_{i1}(0)|G_i = 1] - E[Y_{i2}(0) - Y_{i1}(0)|G_i = 0]$$

- ATT is identified by **the difference of two differences**

# Estimation

# Estimation Approach 1: Plug-in Estimator

- Remember that the ATT is identified by

$$ATT = E[Y_{i2}(1) - Y_{i1}(0) | G_i = 1] - E[Y_{i2}(0) - Y_{i1}(0) | G_i = 0]$$

- Replace them with the sample average.

$$\begin{aligned} \hat{ATT} = & \{ \bar{y}(t = 2, G = 1) - \bar{y}(t = 1, G = 1) \} \\ & - \{ \bar{y}(t = 2, G = 0) - \bar{y}(t = 1, G = 0) \} \end{aligned}$$

where  $\bar{y}(t, G)$  is the sample average for group  $G$  in period  $t$ .

## Example: Card and Kruger (1994, AER)

- Question: Effects of the minimum wage increase on employment.
- On April 1, 1992, New Jersey's minimum wage rose from 4.25 to 5.05 USD.
- Compare fast-food restaurants in New Jersey (treatment group) and eastern Pennsylvania (control group) before and after the rise.

# Estimation Approach 2: Linear Regression

- Run the following regression

$$y_{it} = \alpha_0 + \alpha_1 G_i + \alpha_2 T_t + \alpha_3 D_{it} + \beta X_{it} + \epsilon_{it}$$

- $G_i$ : dummy for treatment group
  - $T_t$ : dummy for treatment period
  - $D_{it} = G_i \times T_t$ .  $\alpha_3$  captures the ATT.
- Regression framework can incorporate covariates  $X_{it}$ , which is important to control for observed confounding factors.



# Two-way Fixed Effects Estimator

- With panel data

$$y_{it} = \alpha D_{it} + \beta X_{it} + \epsilon_i + \epsilon_t + \epsilon_{it}$$

- $\epsilon_i$  is individual FE
  - $\epsilon_t$  is time FE.
- Use the cluster-robust standard errors for inference! (Bertrand, Duflo, and Mullainathan 2004, QJE)

# Event Study (イベントスタディ) Specification

- With multiple periods, we can estimate **the treatment effect in the post-treatment periods**
- The specification with  $k$ -periods lags and leads

$$y_{it} = \epsilon_i + \epsilon_t + \sum_{\tau=-k}^k \gamma^{\tau} D_{it}^{\tau} + \beta X_{it} + \epsilon_{it}$$

- $D_{it}^{\tau}$  takes 1 if unit  $i$  is in treatment group and period  $t$  is  $\tau$  period far from the treatment timing.
  - Normalize  $\gamma^{-1} = 0$ .
- Why include lags (i.e.,  $\tau = -k, \dots, -1$ )? See this later.

# Discussion on Parallel Trend Assumption

# Discussions on Parallel Trend

- Parallel trend assumption can be violated in various situations.
- If treatment status depends on **time-varying factors**, treatment and control group may have differential time trend in their outcome.
  - Note: DID can only deal with **time-invariant factors**.
- Example: Self-selection into treatment based on time-varying factor
  - People participate in worker training program because they expected a future decrease in earnings before they enter the program.

# Diagnostics for Parallel Trends: Pre-treatment trends

- Check if the trends are parallel in the pre-treatment periods (プレトレンド)
- Requires data on multiple pre-treatment periods.
- Should do this if you have multiple pre-treatment periods.
- Note: this is only **diagnostics**. NOT a direct test of the assumption!
  - Remember: Parallel trend assumption is not testable.
  - You **should never say** "the key assumption for DID is satisfied if the pre-treatment trends are parallel."

# Other Diagnostics: Placebo Effects in Pre-Treatment

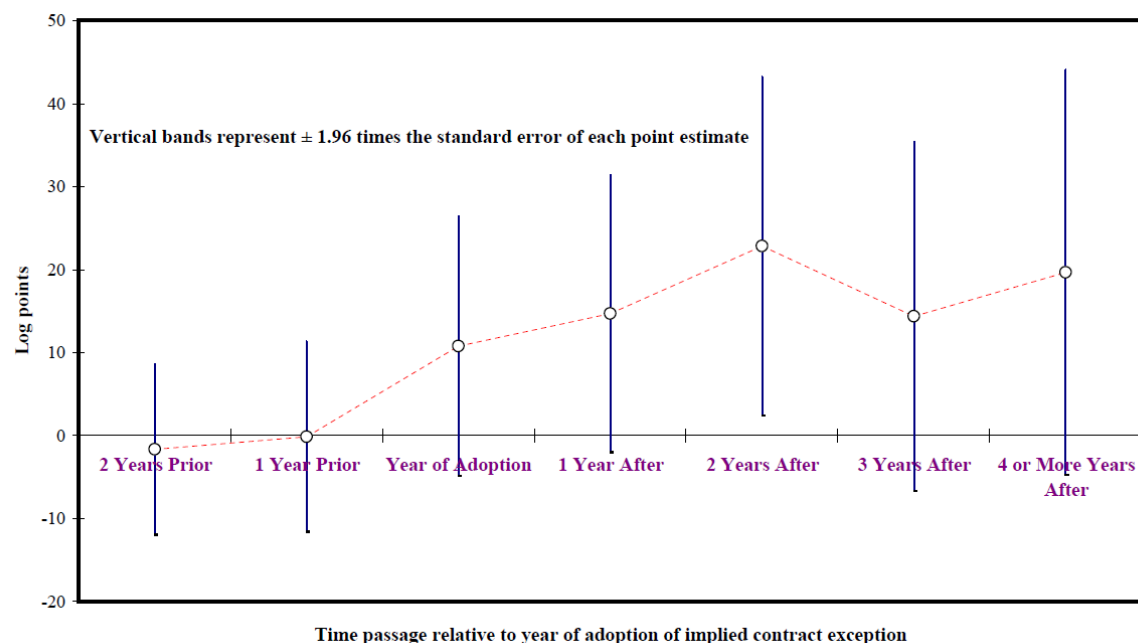
- Idea: If both groups are indeed similar, there should be **no treatment effect in the pre-treatment period**. (placebo outcome)
- We run the following event-study specification

$$y_{it} = \epsilon_i + \epsilon_t + \sum_{\tau=-k}^k \gamma^{\tau} D_{it}^{\tau} + \beta X_{it} + \epsilon_{it}$$

- The estimates of  $\gamma^{\tau}$  should be close to zero up to the beginning of treatment

# Example: Autor (2003) (from Angrist and Pischke MHE)

- Figure 5.2.4: Estimated impact of state courts' adoption of an implied-contract exception to the employment-at-will doctrine on use of temporary workers (from Autor 2003). The dependent variable is the log of state temporary help employment in 1979 - 1995. Estimates are from a model that allows for effects before, during, and after adoption.



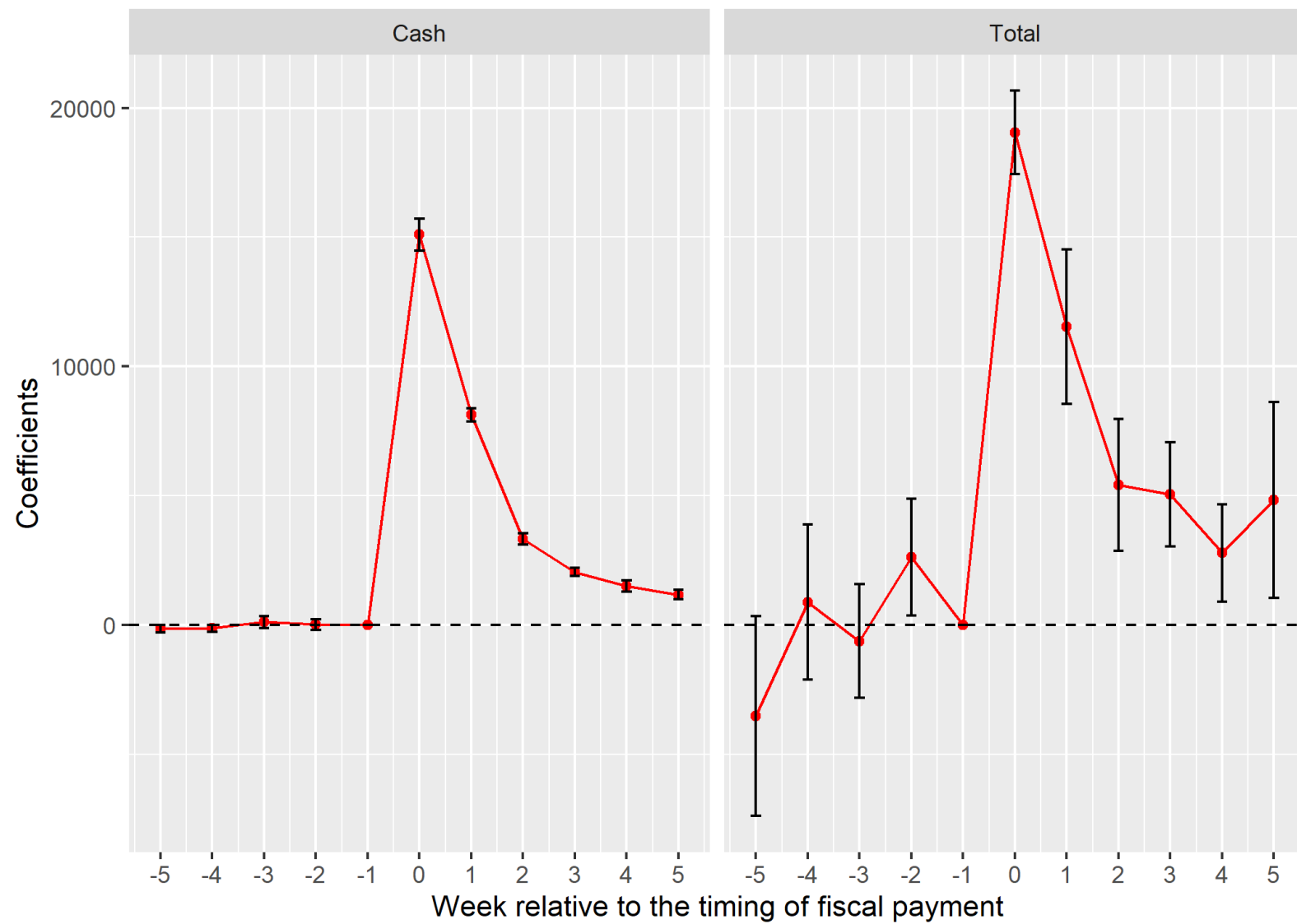
# Example: Kubota, Onishi, and Toyama (2021)

- To estimate the effect of receiving the SCP (特別定額給付金) on consumption, we run

$$y_{itw} = \alpha_i + \alpha_{iw} + \alpha_{tpw} + \sum_{k=-5}^5 \gamma^k D_{itw}^k + u_{itw}$$

- Household  $i$  in week  $w$  in year  $t$  ( $= 2019, 2020$ ).
- $D_{itw}^k = 1\{T_i - w = k\}$ ,  $T_i$  is the timing of receiving the SCP (i.e., treatment)
- $y_{itw}$ : consumption measure (total withdrawal, ATM withdrawal)





# Application

# Application: JAL-JAS Merger (Doi and Ohashi 2019)

- Question: What is the effects of JAL-JAS merger on market outcome in airline industry.
- They use both DID and structural estimation (構造推定) approach to answer the question.
- Here, I only discuss their DID analysis.

# Background of JAL-JASmerger

- Around 2000, JAL and JAS tried to create a holding company (持株会社) to merge.
  - Market share: JAL 25%, JAS 24%, ANA 48%
- The JFTC (公正取引委員会) issued an interim report in March 2002 and claimed that it was likely to be **a substantial restraint of competition in the domestic air passenger market.**
- The merging party proposed remedial measures.
  - Set the pre-merger price level as the price ceiling for the post-merger period.
  - Release 9 slots at the most congested airport, and be assigned to new entrants.
- The JFTC approved the merger with the remedies and it was consummated on Oct 2002.

# Theoretical Effects of a Merger

- **Anticompetitive effect (反競争効果)**: The market becomes more concentrated, less competition, higher price, lower welfare.
- **Efficiency effect (効率性効果)**: The merged party may become more efficient through scale effect, synergy, knowledge transfer, etc.
- These two have different implications on economic welfare (Williamson's tradeoff)
- The JFTC (公取委) reviews the merge case and decides whether approve it.

# Descriptive Statistics

**Table 2**  
Monthly relative price and flight frequency.

|                     | Number of Routes<br>[Number of slots<br>available by remedy] | Average<br>shares (%) | 2000           |              | 2002           |              | 2004            |              | (Post-merger averaged<br>outcomes) - (pre-merger<br>averaged outcomes) |           |
|---------------------|--|-----------------------|----------------|--------------|----------------|--------------|-----------------|--------------|--|-----------|
|                     |  |                       | Price          | Frequency    | Price          | Frequency    | Price           | Frequency    | Price  | Frequency |
| All routes          | 274  |                       | 1.3<br>(14.8)  | 2.2<br>(1.8) | 1.8<br>(14.9)  | 2.3<br>(1.9) | 1.9<br>(15.3)   | 2.7<br>(2.6) | 1.2  | 1.2***    |
| By market structure |  |                       |                |              |                |              |                 |              |  |           |
| Merger-to-monopoly  | 6  | 1.5                   | 11.5<br>(7.5)  | 2.0<br>(0.6) | 11.5<br>(4.8)  | 2.3<br>(0.6) | 15.8<br>(4.3)   | 3.3<br>(0.7) | 1.3***   | 1.5***    |
| Merger-to-oligopoly | 27   | [3] 58.7              | -9.8<br>(11.6) | 4.6<br>(2.6) | -9.7<br>(10.7) | 4.8<br>(2.9) | -10.1<br>(10.2) | 6.5<br>(4.2) | 1.0  | 1.3***    |
| Monopoly            | 209  | 20.3                  | 2.8<br>(15.3)  | 1.7<br>(1.2) | 3.4<br>(15.5)  | 1.8<br>(1.3) | 3.9<br>(15.9)   | 1.9<br>(1.6) | 1.3  | 1.1*      |
| Oligopoly           | 32   | [2] 19.4              | 0.2<br>(9.9)   | 3.4<br>(1.9) | 0.9<br>(10.0)  | 3.5<br>(1.9) | -0.7<br>(11.0)  | 3.7<br>(2.2) | -2.6   | 1.1       |

*Notes:* The numbers are means for July of each year with standard deviations in the parenthesis. The numbers in the squared brackets are the numbers of slots transferred under the structural remedy. Price is the distance-adjusted price level (in constant 2005 JPY), as a ratio to the industry average. Frequency is the number of round trips per day. The superscripts, \*\*\*, \*\*, and \* indicate significance at the 99-, 95-, and 90% confidence levels, respectively. The pre-merger period ranges from April 2000–July 2002, whereas the post-merger period is from October 2002 to October 2005.

# DID Analysis

- Outcome: Price, flight frequency, # of domestic passengers
- 274 routes in 2000–2005
- Regression equation

$$\ln(y_{jmt}) = \gamma_1^A \cdot JJ_{jmt} + \gamma_2^A \cdot post_t + \gamma_3^A \cdot JJ_{jmt} \cdot post_t + \mathbf{x}'_{jmt} \cdot \lambda^A + \kappa_{jmt}^A$$

- $j$ : firm,  $m$ : market (route),  $t$ : time (monthly)
  - $post_t = 1$  if  $t$  is after October 2002.
  - $JJ_{jmt} = 1$  if a firm  $j$  in market  $m$  is either JAL or JAS (JAL group).
  - $\mathbf{x}'_{jmt}$ : covariates.
- Control group: Non-merged domestic airlines (ANA and others)
    - Either in the same market as JAL–JAL or in a market without JAL-JAS.

- Allow heterogeneous effects of a merger

$$\begin{aligned}\ln(y_{jmt}) = & \gamma_1^B \cdot JJ_{jmt} + \gamma_2^B \cdot post_t + \gamma_3^B \cdot JJ_{jmt} \cdot post_t \\ & + MtM_{jmt} \cdot (\gamma_4^B \cdot JJ_{jmt} + \gamma_5^B \cdot post_t + \gamma_6^B \cdot JJ_{jmt} \cdot post_t) \\ & + MtO_{jmt} \cdot (\gamma_7^B \cdot JJ_{jmt} + \gamma_8^B \cdot post_t + \gamma_9^B \cdot JJ_{jmt} \cdot post_t) \\ & + x'_{jmt} \cdot \lambda^B + \kappa_{jmt}^B\end{aligned}$$

- $MtM_{jmt} = 1$  if the route became a monopoly after the JJ merger
- $MtO_{jmt} = 1$  if it became oligopoly.



# Results

Table 3  
DID estimates.

|                 | Prices             |                     | Flight frequency    |                     | Number of passengers |                     |
|-----------------|--------------------|---------------------|---------------------|---------------------|----------------------|---------------------|
| JJ* post        | -0.01**<br>(0.004) | -0.013**<br>(0.005) | 0.285***<br>(0.016) | 0.082***<br>(0.021) | 0.385***<br>(0.020)  | 0.185***<br>(0.027) |
| JJ * post * MTM |                    | 0.051***<br>(0.012) |                     | 0.578***<br>(0.048) |                      | 0.966***<br>(0.062) |
| JJ * post * MTO |                    | -0.004<br>(0.008)   |                     | 0.287***<br>(0.033) |                      | 0.246***<br>(0.042) |
| R-squared       | 0.97               | 0.97                | 0.93                | 0.94                | 0.96                 | 0.96                |
| Number of obs.  | 5329               |                     | 5329                |                     | 5329                 |                     |

*Notes:* Each dependent variable is a logarithmic form. The numbers in the parentheses are the standard errors. The superscripts, \*\*\*, \*\*, and \* indicate significance at the 99-, 95-, and 90% confidence levels, respectively.

# Robustness Check

- Pretrend: Regress the outcome on route-FE, time FE, and time FE interacted with the dummy for control routes.
- Placebo test: Use the period b.w. April 2000 to July 2002. Consider placebo merger in July 2001 and see its effects using DID.

# Limitations of DID in this context

- Is the choice of control group appropriate?
- How to interpret the results?
- Further analysis using structural estimation approach.