MEAM 523.

1. $\tau = \begin{bmatrix} m_1 a_1^2 \ddot\theta_1 + m_2 \ddot\theta_1 [a_1^2 + a_2^2 + 2a_1a_2\cos\theta_2] + m_2 \ddot\theta_2 [a_2^2 + a_1a_2\cos\theta_2] \\ -m_2 [2a_1a_2\sin\theta_2 \; \dot\theta_1\dot\theta_2 + a_1a_2\sin\theta_2 \dot\theta_2^2 \\ -ga_1\cos\theta_1 - ga_2\cos(\theta_1+\theta_2)) + m_1 g a \cos\theta_1 \end{bmatrix}$

$W = \begin{bmatrix} a_1^2\ddot\theta_1 + g a_1\cos\theta_1 & \ddot\theta_1[a_1^2+a_2^2+2a_1a_2\cos\theta_2] + \ddot\theta_2[a_2^2 + a_1a_2\cos\theta_2) - a_1a_2\sin\theta_2\dot\theta_2(2\dot\theta_1 + \dot\theta_2] \\ & + ga_1\cos(\theta_1) + ga_2\cos(\theta_1+\theta_2) \\ 0 & \ddot\theta_1[a_2^2 + a_1a_2\cos\theta_2] + \ddot\theta_2[a_2^2] + a_1a_2\sin\theta_2\dot\theta_2^2 + ga_2\cos(\theta_1+\theta_2) \end{bmatrix}$

$\Delta\theta = \hat\theta(t) - \theta$ is bounded.

$\Delta q = q - q_{des} \to 0.$

b. Error States $\Delta\dot q = \dot q - \dot q_{des} \to 0$

$$\begin{bmatrix} \Delta\dot q_1 \\ \Delta\dot q_2 \\ \Delta\ddot q_1 \\ \Delta\ddot q_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -Kp_1 & 0 & -Kd_1 & 0 \\ 0 & -Kp_1 & 0 & -Kd_1 \end{bmatrix}}_{A} \begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta\dot q_1 \\ \Delta\dot q_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \hat M^{-1} W \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \end{bmatrix}$$

→ Gain matrix

$A^T P + PA = -Q$

Eigenvalue of A lies in left half plane. For the Kp & Kd value we construct a reasonable. choosing $Kp_1 = 2$ $Kd_1 = 5$ $Kp_2 = 2$ $Kd_2 = 5$. We select Q & P such that they are the definite gain matrix.

~~Taking $q =$~~ $\cancel{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$.

Controller Choice

By using $\hat{\theta}$ which is the predicted mass here.

we get $\hat{M}$ $\hat{C}$ $\hat{N}$

By using computed torque method we get

$$\tau = \hat{C}\dot{q} + \hat{N} + \hat{M}\left[\ddot{q}_{des} - K_p \Delta q - K_d \Delta \dot{q}\right]$$

$$W\hat{\theta} = \hat{M}\ddot{q} + \hat{C}\dot{q} + \hat{N}.$$

↳ dynamic matrix

$$\tau = W\hat{\theta} + \hat{M}\left[-\Delta\ddot{q} - K_p \Delta q - K_d \Delta \dot{q}\right] \quad (1)$$

$W\theta \rightarrow$ Actual response of the system

$$W\theta = \tau. \quad \ldots (2)$$

from 1 & 2 we get:

$$\Delta\ddot{q} + K_d \Delta\dot{q} + K_p \Delta q = \hat{M}^{-1} W \Delta\theta.$$

which we represented in the error state space matrix.

Now we have our Gain matrices. we take a lyapunov function.

$$V = \Delta x^T P \Delta x + \frac{1}{2}\Delta\theta^T\Delta\theta.$$

we know $P$ is +ve definite

So $V > 0$ for all $\Delta x$ & $\Delta\theta$ except for $\Delta x = 0$ & $\Delta\theta = 0$.

Now we ~~[struck]~~ want $\dot V$ to be -ve.

$$\dot V = \Delta \dot x^T P \Delta x + \Delta \dot x^T P \Delta x + \Delta\theta^T \Delta\dot\theta$$

$$= \Delta x^T [PA + A^T P] \Delta x + 2 \Delta\theta^T W^T \hat M^{-T} B^T P A \Delta x + \Delta\theta^T \Delta\dot\theta$$

To make $\dot V = -\Delta x^T Q \Delta x$

we choose

$$\Delta\dot\theta = -2 W^T \hat M^{-T} B^T P \Delta x .$$

we can say $\dot V = 0$ if $\Delta x^T Q \Delta x \to 0$ as $t \to \infty$.

making error states to converge.

Thus by using barbalat lemma we show that system is stable.

We say its locally stable because we can have $\theta \in R$ where $\hat M$ becomes singular making system unstable.

d. from last Assignment

$|M| = a_1^2 a_2^2 (m_1 m_2 + m_2^2 (1 - \cos^2 q_2))$ to make this matrix singular $m_2$ can be 0 ~~or~~ $m_2$.

or $m_1 = 0$ with $q_2 = \pi/2, \frac{3\pi}{2}$ as initial conditions. we cannot design a adaptive controller that can stabilize the system.

With $m_1 = 0$ the dynamics like of $\dot q$ $\ddot q$ become $\infty$ at a small $\Delta t$. Same case occurs with $m_1 = 0$ $q_2 = \pi/2$. Physically when $q_2 = \pi/2$ & $\frac{3\pi}{2}$ if $m_1$ is massless we cannot control $m_2$. This means system will never converge.