ES 631 Mathematical Foundations for Computer Vision and Graphics

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Results

Green points represent points in P cloud, Blue represents points in Q cloud and red points are transformed points.

P_1 and Q_1

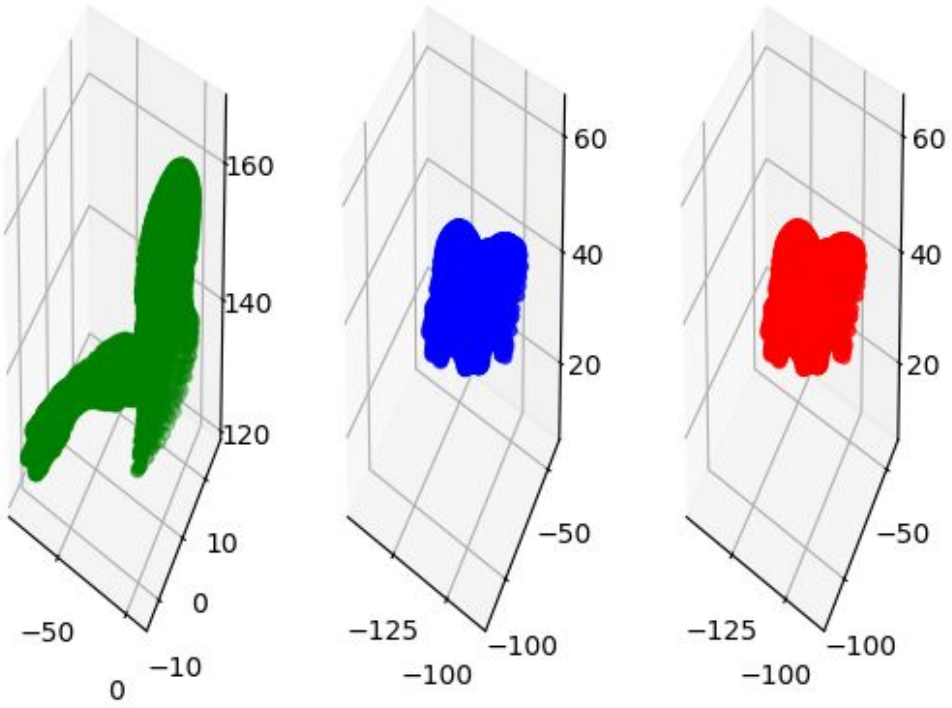
```
Rotation Matrix:
[[ 7.66044443e-01 -2.87019658e-10 -6.42787610e-01]
 [-5.82563416e-01  4.22618262e-01 -6.94272044e-01]
 [ 2.71653783e-01  9.06307787e-01  3.23744371e-01]]

Translation Vector:
[[1.        ]
 [1.00000001]
 [1.00000002]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[ -87.02132408  -88.92669117   56.38706898]
 [ -88.13931699  -90.43370947   56.30827946]
 [ -90.72974625  -94.19743731   55.54262028]
 ...
 [-139.18426503  -41.39007533   20.49891548]
 [-140.15059147  -41.61962581   19.9048074 ]
 [-138.05407819  -43.11358465   20.98970627]]

 Points in Q:
 [[ -87.021324  -88.926691   56.387069]
 [ -88.139317  -90.433709   56.308279]
 [ -90.729746  -94.197437   55.54262 ]
 ...
 [-139.184265  -41.390075   20.498915]
 [-140.150591  -41.619626   19.904807]
 [-138.054078  -43.113585   20.989706]]
```



P_2 and Q_2

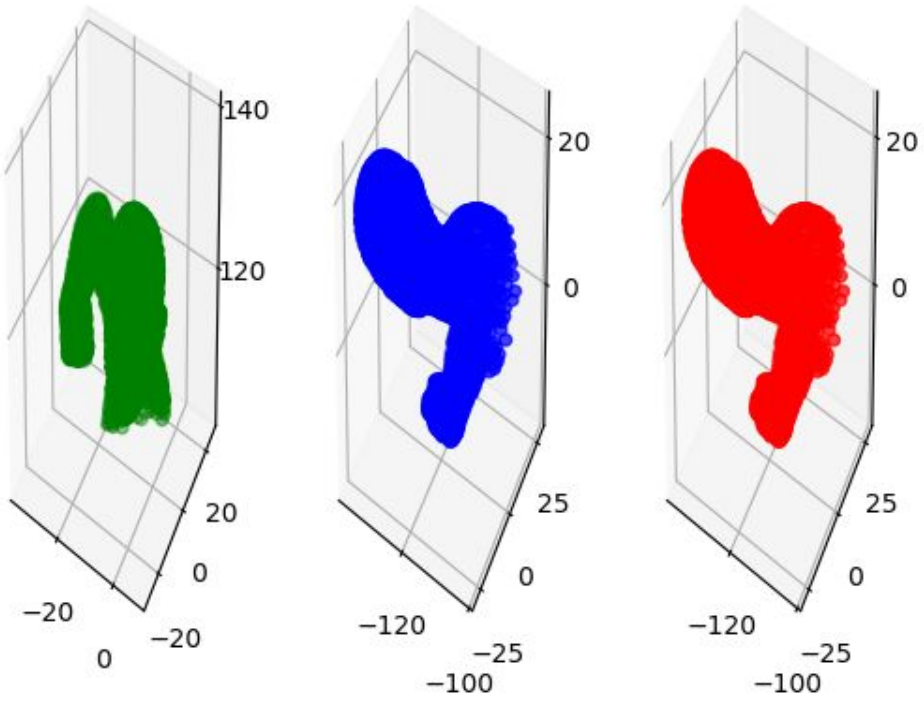
```
Rotation Matrix:
[[ 1.21869343e-01  1.37704151e-10 -9.92546152e-01]
 [-8.65060973e-02  9.96194698e-01 -1.06216130e-02]
 [ 9.88769214e-01  8.71557430e-02  1.21405593e-01]]

Translation Vector:
[[1.        ]
 [0.99999998]
 [1.00000007]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[-111.28287108   -5.45858926   17.97672843]
 [-112.96243932   -6.84405139   17.24242255]
 [-114.81050234   -7.97243845   15.99107884]
 ...
 [-117.25427757   -4.74499866  -10.83813886]
 [-114.99183468   -4.85692904  -10.01271479]
 [-115.39591479   -4.35301086   -9.22725448]]

 Points in Q:
 [[-111.282871   -5.458589   17.976728]
 [-112.962439   -6.844051   17.242423]
 [-114.810502   -7.972438   15.991079]
 ...
 [-117.254278   -4.744999  -10.838139]
 [-114.991835   -4.856929  -10.012715]
 [-115.395915   -4.353011   -9.227254]]
```



P_3 and Q_3

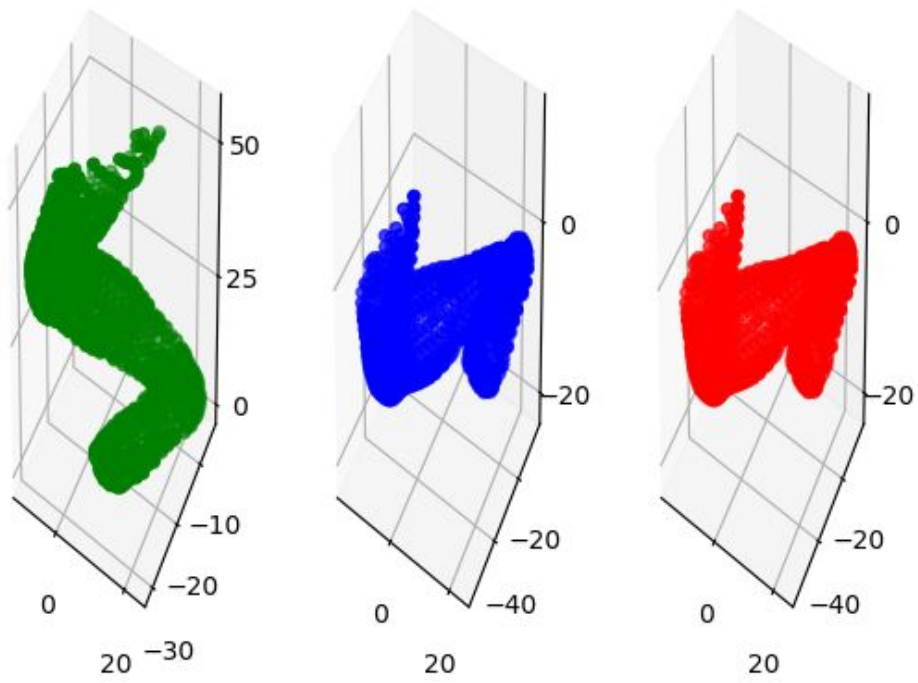
```
Rotation Matrix:
[[ 9.84807753e-01 -4.59655011e-10 -1.73648178e-01]
 [-1.63175911e-01  3.42020144e-01 -9.25416578e-01]
 [ 5.93911753e-02  9.39692621e-01  3.36824089e-01]]

Translation Vector:
[[0.99999999]
 [1.        ]
 [0.99999997]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[  1.76392119 -54.93610242   7.9590807 ]
 [  0.95412896 -56.08701102   6.741309  ]
 [  1.3254964  -54.25178987   6.15398865]
 ...
 [ 14.47360748  -9.94117156 -18.64212291]
 [ 13.18342611  -9.35291116 -19.47329779]
 [  4.47876452  -9.44680723 -19.30521481]]

 Points in Q:
 [[  1.763921 -54.936102   7.959081]
 [  0.954129 -56.087011   6.741309]
 [  1.325496 -54.25179    6.153989]
 ...
 [ 14.473607  -9.941172 -18.642123]
 [ 13.183426  -9.352911 -19.473298]
 [  4.478765  -9.446807 -19.305215]]
```



P_4 and Q_4

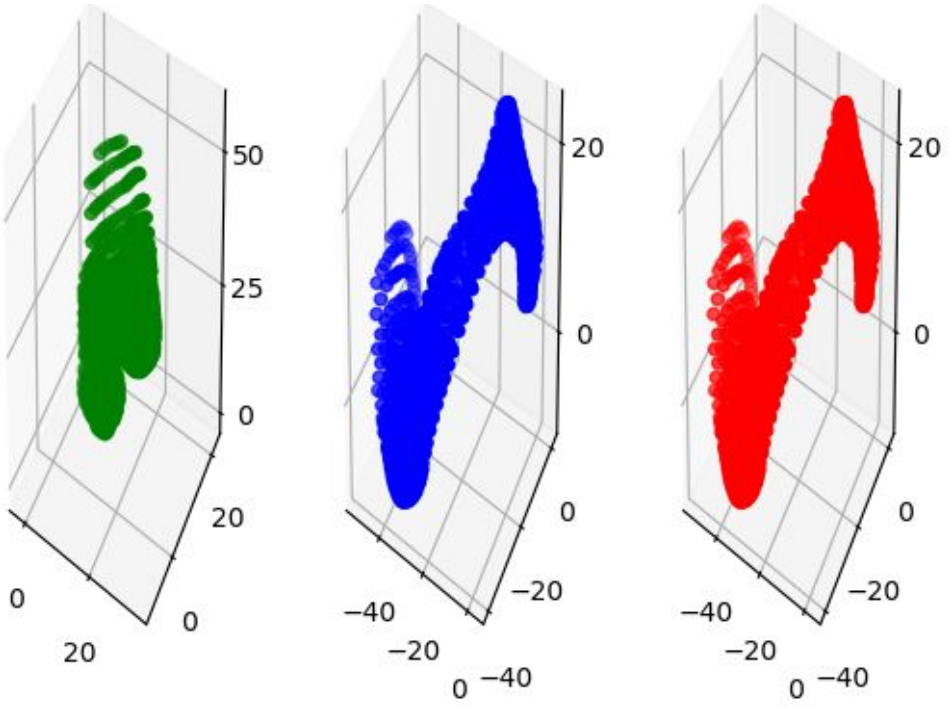
```
Rotation Matrix:
[[ 1.73648177e-01 -9.39373101e-10 -9.84807753e-01]
 [-9.84807753e-01  6.64230337e-10 -1.73648177e-01]
 [ 8.17259588e-10  1.00000000e+00 -8.09759659e-10]]

Translation Vector:
[[1.00000002]
 [1.        ]
 [1.00000001]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[-50.0102507  -23.16049324  11.04764097]
 [-48.11707609 -18.16159847  10.39504897]
 [-49.52563059 -22.09818804  11.07248497]
 ...
 [  1.5599498  -11.31158843   8.15722202]
 [  0.33914572  -7.32428879  17.38109501]
 [  0.73588811  -7.20096462  17.07796501]]

 Points in Q:
 [[-50.010251 -23.160493  11.047641]
 [-48.117076 -18.161598  10.395049]
 [-49.525631 -22.098188  11.072485]
 ...
 [  1.55995  -11.311588   8.157222]
 [  0.339146  -7.324289  17.381095]
 [  0.735888  -7.200965  17.077965]]
```



P_5 and Q_5

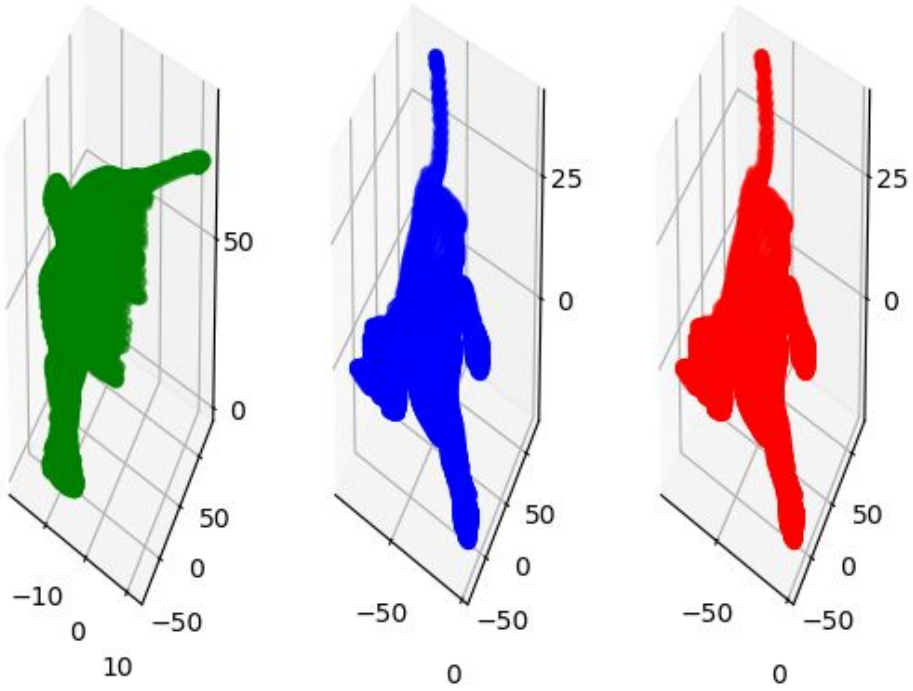
```
Rotation Matrix:
[[ 1.73654244e-01  1.27712587e-06 -9.84806683e-01]
 [-1.71002515e-01  9.84809112e-01 -3.01521662e-02]
 [ 9.69846557e-01  1.73640472e-01  1.71016500e-01]]

Translation Vector:
[[1.04999246]
 [1.04999274]
 [1.04982608]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[-59.11419237  39.54189977  18.58050332]
 [-59.23786481  39.21487485  19.41787306]
 [-38.10578037  21.32495152  13.27583441]
 ...
 [-67.48808866  70.79817916  34.39677542]
 [-65.86700725  69.3996221   33.47694183]
 [-66.90186288  70.34556135  35.03540183]]

 Points in Q:
 [[-59.086668  39.511183  18.593695]
 [-59.223168  39.192221  19.436265]
 [-38.121226  21.362833  13.32531 ]
 ...
 [-67.534227  70.801241  34.365869]
 [-65.856193  69.370245  33.473824]
 [-66.855385  70.327497  35.071552]]
```



P_6 and Q_6

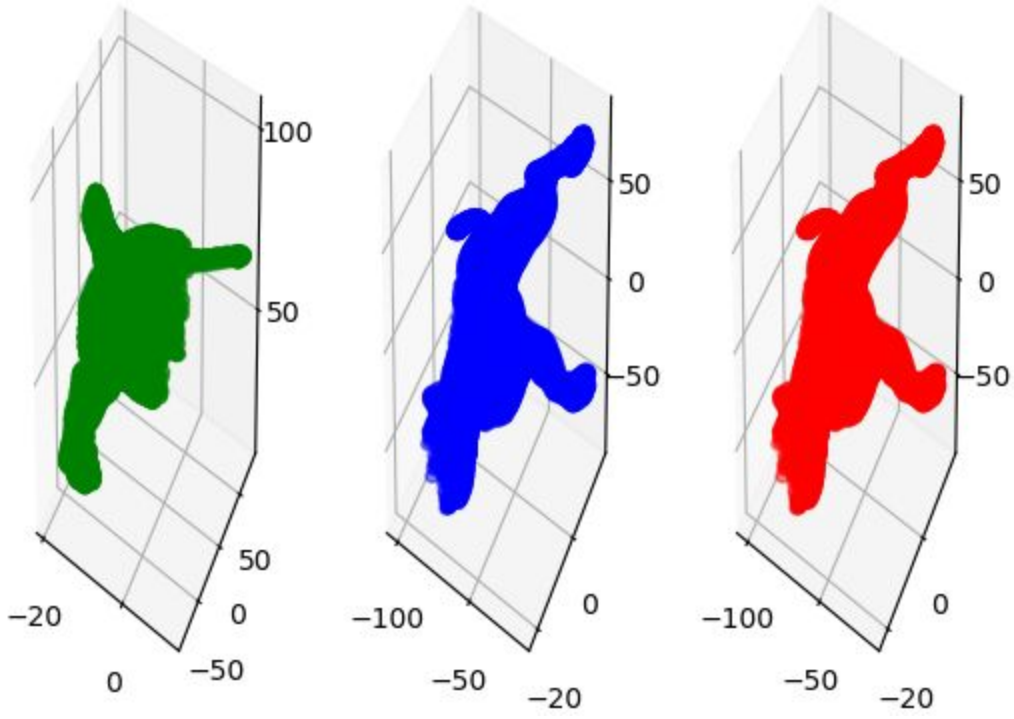
```
Rotation Matrix:
[[ 1.73657802e-01  7.17131275e-06 -9.84806056e-01]
 [-9.84806056e-01 -4.39512442e-06 -1.73657802e-01]
 [-5.57369955e-06  1.00000000e+00  6.29910458e-06]]

Translation Vector:
[[1.04963805]
 [1.05009564]
 [1.04961709]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[-60.90953429  -9.87623659  43.12212739]
 [-61.06091026 -10.78799893  43.02515647]
 [-45.00770437  -8.68403756  19.7976256 ]
 ...
 [-61.0063398  -18.53478383  78.05944003]
 [-59.80125039 -17.94504558  76.15371698]
 [-60.51305606 -19.25884316  77.62451531]]

 Points in Q:
 [[-60.921269  -9.917696  43.076782]
 [-61.08786  -10.760061  43.05403 ]
 [-44.992722  -8.658787  19.818118]
 ...
 [-60.995396 -18.507985  78.07236 ]
 [-59.760371 -17.990852  76.18898 ]
 [-60.562614 -19.303621  77.594549]]
```



P_7 and Q_7

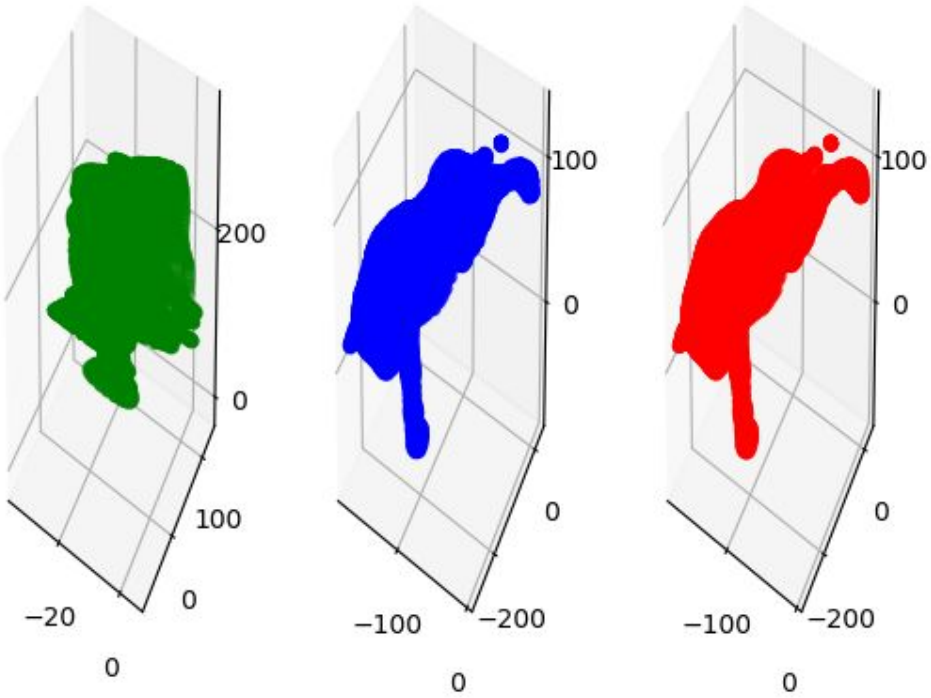
```
Rotation Matrix:
[[ 8.65964264e-01  3.38601941e-05 -5.00105881e-01]
 [-5.00105878e-01 -7.80213568e-05 -8.65964263e-01]
 [-6.83406574e-05  9.99999996e-01 -5.06300247e-05]]

Translation Vector:
[[1.65750456]
 [1.629593  ]
 [1.64122068]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[ -95.90887127 -167.28506647  103.45781371]
 [ -60.96941653 -109.50393172   60.13215666]
 [ -81.53674743 -142.42209003  116.20261296]
 ...
 [ -31.65382815  -54.34837312  128.43502859]
 [ -31.34450641  -54.25277293  127.21117257]
 [ -31.24805252  -54.67799912  128.86078813]]

 Points in Q:
 [[ -95.987732 -167.152829  103.163571]
 [ -61.604522 -109.151756   60.493962]
 [ -81.004722 -142.405538  116.289986]
 ...
 [ -31.482744  -53.902356  128.451564]
 [ -31.211192  -54.264797  127.373724]
 [ -30.63529   -54.73426   128.983834]]
```



P_8 and Q_8

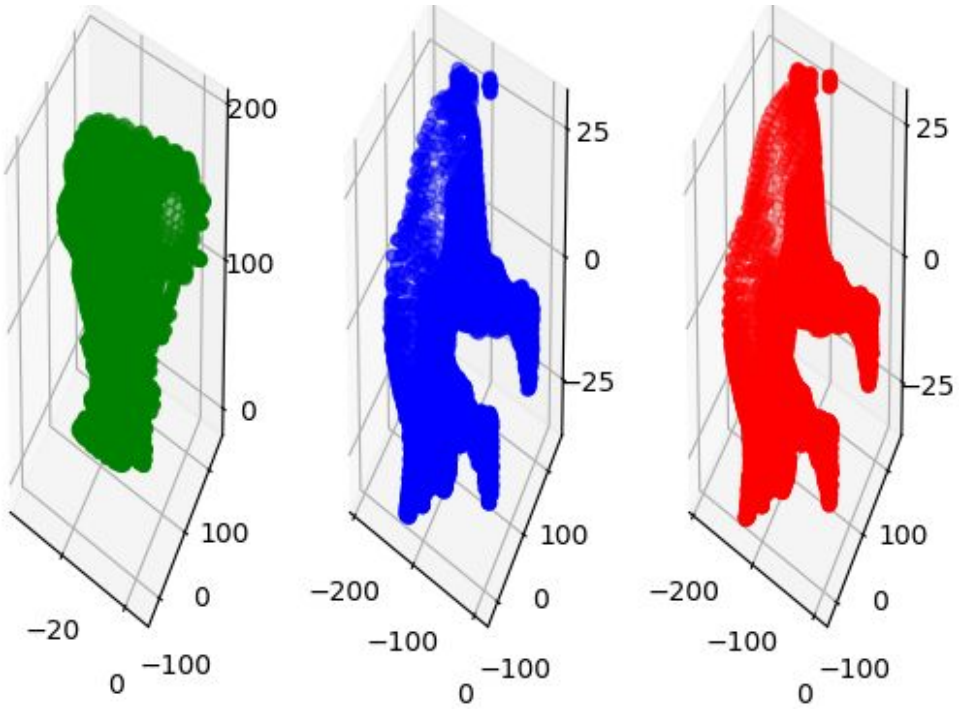
```
Rotation Matrix:
[[-1.72820077e-04 -1.99637138e-04 -9.99999965e-01]
 [-1.73595801e-01  9.84816973e-01 -1.66605208e-04]
 [ 9.84816972e-01  1.73595767e-01 -2.04852314e-04]]

Translation Vector:
[[2.58789001]
 [2.53798862]
 [2.33417868]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[-181.84801015   41.91198793    9.23095943]
 [-110.11418223   75.42931541   16.52117028]
 [-179.87170673   79.70339156   15.89135698]
 ...
 [ -95.36780826  137.24096228   26.05684047]
 [ -95.92366409  138.0900949    27.1909972 ]
 [ -95.94104662  139.08524114   26.37936233]]

 Points in Q:
 [[-181.992521   42.572559   10.342477]
 [-109.41337    75.328134   15.472105]
 [-179.578404   80.533482   14.637877]
 ...
 [ -94.703953  137.727602   26.352163]
 [ -95.306897  137.925132   26.39587 ]
 [ -96.20512   138.181523   26.5767  ]]
```



P_9 and Q_9

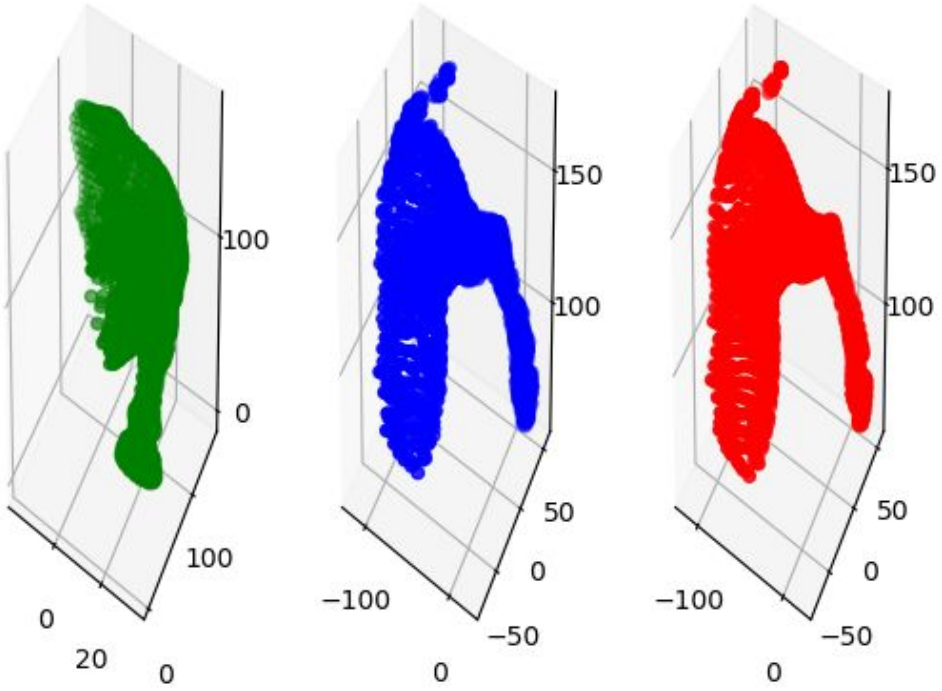
```
Rotation Matrix:
[[ 0.70770908  0.00119533 -0.70650296]
 [-0.50035401  0.70684763 -0.50001229]
 [ 0.49879226  0.70736483  0.50084058]]

Translation Vector:
[[2.77566664]
 [2.68317239]
 [2.53451462]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[-67.19792302 -25.37892631 141.60157483]
 [-66.7045097  -22.81016148 143.74687216]
 [-67.33324975 -21.59709675 145.48132739]
 ...
 [-89.18472651  36.74195388 166.22894796]
 [-87.27590508  35.95402357 164.55904162]
 [-88.72609474  36.7854117  166.73801444]]

 Points in Q:
 [[-67.39478  -24.955578 142.084774]
 [-65.844384 -23.234071 145.259384]
 [-67.516819 -22.75273  144.444008]
 ...
 [-90.976093  36.126668 166.672837]
 [-87.691789  35.163292 165.729485]
 [-88.799086  37.068874 165.642041]]
```



P10 and Q10

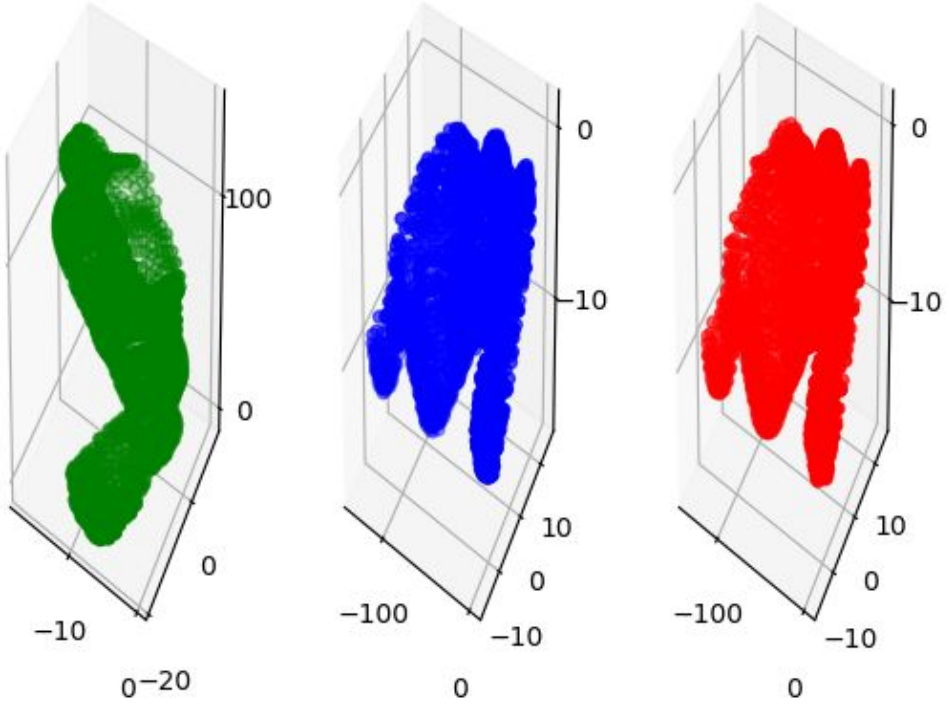
```
Rotation Matrix:
[[-3.51344325e-05 -1.69372019e-04 -9.99999985e-01]
 [-1.73671185e-01  9.84803683e-01 -1.60696352e-04]
 [ 9.84803695e-01  1.73671177e-01 -6.40155578e-05]]

Translation Vector:
[[1.31338681]
 [1.32454898]
 [1.31419297]]
```

```
print('Transformed Points: \n',transformed_points,
      '\n\n','Points in Q: \n', q_points)
```

```
Transformed Points:
 [[-8.77725621e+01 -5.20371443e+00  1.99898592e-01]
 [-8.71327311e+01 -4.31596070e+00  1.76762073e-02]
 [-1.08951844e+02  7.11634864e+00 -8.68793582e+00]
 ...
 [ 1.06405810e+00 -9.80382671e+00 -1.04175227e+01]
 [ 1.24381706e+00 -8.73654784e+00 -1.10027420e+01]
 [ 1.80909640e+00 -1.55419526e+01 -8.59273665e+00]]

 Points in Q:
 [[-8.78049830e+01 -4.99346500e+00 -4.49700000e-02]
 [-8.68828720e+01 -4.32942600e+00  1.67470000e-02]
 [-1.09094544e+02  7.25609000e+00 -8.96784100e+00]
 ...
 [ 1.35217900e+00 -9.56424500e+00 -1.06551750e+01]
 [ 1.24130000e+00 -8.67403000e+00 -1.11085690e+01]
 [ 1.82333200e+00 -1.52829870e+01 -8.78688500e+00]]
```



Algorithm

The initial correspondences are found by matching distance histogram of 128 closest points to a given point. Matching is performed by matching descriptor of one point in the first point cloud to the descriptor of each point in the second point cloud.
Then ICP(Iterative Closest Point) algorithm is then used to find the rotation matrix and translation vector.
First, the centroids are calculated for point clouds P and Q which are named p' and q'. Then two vectors x and y are calculated as x = p-p' and y = q-q'. A covariance matrix is calculated $S = XY^T = U\Sigma V^T$. The rotation matrix is calculated as $R = VU^T$ and translation vector t = q' - Rp'. After calculating both of these, new correspondences are again calculated by minimizing the distance between Rp + t and q and the process is repeated until convergence.