Solution: Problem A

The Problem Statement:
We are given two sequences of numbers:

n	1	2	3	4	5	6
a_n	2	5	10	17	26	37
b_n	1	2	8	48	384	3840

The problem states to analyze each sequence, continue the pattern, and find a closed-form (an explicit formula) for the n-th term of each sequence, a_n and b_n.

Detailed solution for Problem A...

Part 1: Deriving the Formula for a_n

1. **We inspect the given terms for a_n:**
The sequence is:

$$a_1 = 2, \quad a_2 = 5, \quad a_3 = 10, \quad a_4 = 17, \quad a_5 = 26, \quad a_6 = 37.$$

We have six terms. To find a pattern, a common method and my first instinct was to look at the differences between consecutive terms.

2. **First Differences Δa_n:**
We compute $\Delta a_n = a_{n+1} - a_n$ for each pair of consecutive terms:

$$a_2-a_1 = 5-2 = 3, \quad a_3-a_2 = 10-5 = 5, \quad a_4-a_3 = 17-10 = 7, \quad a_5-a_4 = 26-17 = 9, \quad a_6-a_5 = 37-$$

So the first differences are:

$$\Delta a_n : 3, 5, 7, 9, 11, \ldots$$

We observe that these differences form an arithmetic progression with a common difference of 2.

3. **Second Differences $\Delta^2 a_n$:**
Now, the second differences are checked which are the differences of the first differences:

$$5 - 3 = 2, \quad 7 - 5 = 2, \quad 9 - 7 = 2, \quad 11 - 9 = 2.$$

All second differences are constant and equal to 2. A sequence with a constant second difference indicates that a_n can be expressed as a quadratic polynomial in n.

4. **We will assume a Quadratic Form:**
Since we have constant second differences, we assume:

$$a_n = An^2 + Bn + C,$$

where A, B, C are constants we need to determine.

5. **Plugging in the known terms:**
Using the first three terms of the sequence (a_1, a_2, a_3) to form three equations in terms of A, B, C.

- For $n = 1$, $a_1 = 2$:

$$A(1)^2 + B(1) + C = A + B + C = 2.$$

So:

$$A + B + C = 2. \quad (1)$$

- For $n = 2$, $a_2 = 5$:

$$A(2^2) + B(2) + C = 4A + 2B + C = 5.$$

So:

$$4A + 2B + C = 5. \quad (2)$$

- For $n = 3$, $a_3 = 10$:

$$A(3^2) + B(3) + C = 9A + 3B + C = 10.$$

So:

$$9A + 3B + C = 10. \quad (3)$$

Now we have three linear equations:

$$(1)\ A + B + C = 2$$

$$(2)\ 4A + 2B + C = 5$$

$$(3)\ 9A + 3B + C = 10$$

6. **We now solve the system for A, B, C:**
To solve, we eliminate variables step-by-step:

Step A: Subtract Equation (1) from Equation (2):
$(2) - (1)$:

$$(4A + 2B + C) - (A + B + C) = 5 - 2.$$

On the left: - The C cancels. - $4A - A = 3A$. - $2B - B = B$. On the right: $5 - 2 = 3$.
Thus:

$$3A + B = 3. \quad (4)$$

Step B: Subtract Equation (1) from Equation (3):
$(3) - (1)$:

$$(9A + 3B + C) - (A + B + C) = 10 - 2.$$

On the left: - C cancels. - $9A - A = 8A$. - $3B - B = 2B$. On the right: $10 - 2 = 8$.
We have:

$$8A + 2B = 8. \quad (5)$$

Step C: Simplify Equation (5):
Divide (5) by 2:

$$4A + B = 4. \quad (6)$$

Now compare (4) and (6): From (4): $3A + B = 3$. From (6): $4A + B = 4$.

Step D: Subtract (4) from (6):

$$(4A + B) - (3A + B) = 4 - 3.$$

Left side: - B cancels. - $4A - 3A = A$. Right side:

$$4 - 3 = 1.$$

So:

$$A = 1.$$

With $A = 1$, substitute into (4) $3A + B = 3$:

$$3(1) + B = 3 \implies 3 + B = 3 \implies B = 0.$$

Now $A = 1, B = 0$. From (1): $A + B + C = 2$:

$$1 + 0 + C = 2 \implies C = 1.$$

Thus:

$$A = 1, \; B = 0, \; C = 1.$$

7. **Final Quadratic Formula for a_n:**

$$a_n = n^2 + 1.$$

$$\boxed{a_n = n^2 + 1.}$$

Part 2: Deriving the Formula for b_n

1. **We first inspecting the given terms for b_n:**
The sequence is:

$$b_1 = 1, \quad b_2 = 2, \quad b_3 = 8, \quad b_4 = 48, \quad b_5 = 384, \quad b_6 = 3840.$$

The terms grow very fast which is key hint. We will hendforth look at the ratio of consecutive terms.

2. **Check Ratios $\frac{b_{n+1}}{b_n}$:**
Compute:

$$\frac{b_2}{b_1} = \frac{2}{1} = 2, \quad \frac{b_3}{b_2} = \frac{8}{2} = 4, \quad \frac{b_4}{b_3} = \frac{48}{8} = 6, \quad \frac{b_5}{b_4} = \frac{384}{48} = 8, \quad \frac{b_6}{b_5} = \frac{3840}{384} = 10.$$

The ratios are:

$$2, 4, 6, 8, 10, \ldots$$

These ratios form an arithmetic sequence starting at 2 and increasing by 2 each time. The k-th ratio is $2k$.

3. **Constructing b_n using ratios:**
Start with $b_1 = 1$:
- To get b_2: multiply b_1 by 2:

$$b_2 = 1 \times 2.$$

- To get b_3: multiply b_2 by 4:

$$b_3 = (1 \times 2) \times 4 = 1 \times 2 \times 4.$$

- To get b_4: multiply b_3 by 6:

$$b_4 = (1 \times 2 \times 4) \times 6 = 1 \times 2 \times 4 \times 6.$$

- To get b_5: multiply b_4 by 8:

$$b_5 = (1 \times 2 \times 4 \times 6) \times 8 = 1 \times 2 \times 4 \times 6 \times 8.$$

4. **Visualizing the Pattern ("b Triangle"):**
We arrange factors in a triangular form:

$$\begin{array}{c} b_1: \ 1 \\ \\ b_2: \ 1 \ \times 2 \\ \\ b_3: \ 1 \ \times 2 \ \times 4 \\ \\ b_4: \ 1 \ \times 2 \ \times 4 \ \times 6 \\ \\ b_5: \ 1 \ \times 2 \ \times 4 \ \times 6 \ \times 8 \end{array}$$

Each b_n (for $n > 1$) is formed by multiplying together even numbers starting from 2.

5. **Expressing b_n in General Form:**
For $n > 1$:

$$b_n = 1 \times 2 \times 4 \times 6 \times \cdots \times [2(n-1)].$$

There are $(n-1)$ even factors. Write them as $2k$:

$$b_n = \prod_{k=1}^{n-1} (2k).$$

Factor out 2 from each term:

$$b_n = 2^{n-1}(1 \cdot 2 \cdot 3 \cdots (n-1)) = 2^{n-1}(n-1)!.$$

Check for a few terms matches perfectly.

$$\boxed{b_n = 2^{n-1}(n-1)!}.$$

Mathematical concepts involved in Problem A

- **Arithmetic Progressions (AP):** Used to identify patterns in the first differences of a_n and the sequence of ratios for b_n.
- **Constant Second Differences & Quadratic Sequences:** For a_n, the constant second difference confirmed it can be represented by a quadratic polynomial.
- **Solving Linear Equations:** Found A, B, C for a_n by forming and solving a system of linear equations.
- **Factorials:** To recognize the product pattern for b_n led to a factorial expression $(n-1)!$ and a power of 2.
- **Decomposition into Basic Factors:** For b_n, factoring out 2 from every even factor gave a neat closed form.

Final Answers

$$\boxed{a_n = n^2 + 1, \quad b_n = 2^{n-1}(n-1)!.}$$

**************** END OF SOLUTION: A ****************
