Solution: Problem C

Given Problem: Determine the numerical value of the following expression without the use of a calculator:

$$\left(\sqrt{2} + (3^2)^{\frac{1}{4}} + \sum_{m=1}^{3}\left(\frac{1}{m!} - \sqrt{m}\right)\right) \cdot \left(2^{\log_2(8)} + \frac{1}{2^3} - \prod_{k=1}^{8}\left(1 + \frac{1}{k}\right)\right).$$

We are informed the final answer is $-\frac{7}{12}$. We will verify this step-by-step, justifying each simplification thoroughly.

Detailed solution for Problem C...

1. **We first rewrite the main expression:**

Let

$$A = \left(\sqrt{2} + (3^2)^{1/4} + \sum_{m=1}^{3}\left(\frac{1}{m!} - \sqrt{m}\right)\right)$$

and

$$B = \left(2^{\log_2(8)} + \frac{1}{2^3} - \prod_{k=1}^{8}\left(1 + \frac{1}{k}\right)\right).$$

Our target is to compute:

$$A \times B.$$

2. **We then simplify $(3^2)^{1/4}$:**
We have:

$$(3^2)^{1/4} = 9^{1/4}.$$

Since $9 = 3^2$,

$$9^{1/4} = (3^2)^{1/4} = 3^{2/4} = 3^{1/2} = \sqrt{3}.$$

Therefore:

$$(3^2)^{1/4} = \sqrt{3}.$$

3. **Now, we substitute this into A:**
Now:

$$A = \left(\sqrt{2} + \sqrt{3}\right) + \sum_{m=1}^{3}\left(\frac{1}{m!} - \sqrt{m}\right).$$

4. **We evaluate the summation $\sum_{m=1}^{3} \frac{1}{m!}$:**

Computing each factorial:

$$1! = 1, \quad 2! = 2, \quad 3! = 6.$$

Thus:

$$\frac{1}{1!} = 1, \quad \frac{1}{2!} = \frac{1}{2}, \quad \frac{1}{3!} = \frac{1}{6}.$$

Summing these:

$$1 + \frac{1}{2} + \frac{1}{6}.$$

Finding a common denominator (6):

$$1 = \frac{6}{6}, \quad \frac{1}{2} = \frac{3}{6}, \quad \frac{1}{6} = \frac{1}{6}.$$

Adding them:

$$\frac{6}{6} + \frac{3}{6} + \frac{1}{6} = \frac{6+3+1}{6} = \frac{10}{6} = \frac{5}{3}.$$

Therefore:

$$\sum_{m=1}^{3} \frac{1}{m!} = \frac{5}{3}.$$

5. **We now evaluate $\sum_{m=1}^{3} \sqrt{m}$:**

$$\sqrt{1} = 1, \quad \sqrt{2} = \sqrt{2}, \quad \sqrt{3} = \sqrt{3}.$$

Thus:

$$\sum_{m=1}^{3} \sqrt{m} = 1 + \sqrt{2} + \sqrt{3}.$$

6. **We combine the terms in the summation:**
We have:

$$\sum_{m=1}^{3} \left(\frac{1}{m!} - \sqrt{m} \right) = \sum_{m=1}^{3} \frac{1}{m!} - \sum_{m=1}^{3} \sqrt{m}.$$

Substitute the values:

$$= \frac{5}{3} - (1 + \sqrt{2} + \sqrt{3}).$$

7. **The substitute back into A:**
Recall:

$$A = (\sqrt{2} + \sqrt{3}) + \left(\frac{5}{3} - (1 + \sqrt{2} + \sqrt{3}) \right).$$

Distribute the subtraction:

$$A = (\sqrt{2} + \sqrt{3}) + \frac{5}{3} - 1 - \sqrt{2} - \sqrt{3}.$$

Combine like terms:

$$\sqrt{2} - \sqrt{2} = 0, \quad \sqrt{3} - \sqrt{3} = 0.$$

All the radicals cancel out. Hence, we are left with:

$$A = \frac{5}{3} - 1.$$

Convert 1 to $\frac{3}{3}$:

$$A = \frac{5}{3} - \frac{3}{3} = \frac{2}{3}.$$

Hence:

$$A = \frac{2}{3}.$$

8. **We now evaluate the second bracket B:**
Recall:

$$B = 2^{\log_2(8)} + \frac{1}{2^3} - \prod_{k=1}^{8} \left(1 + \frac{1}{k}\right).$$

First, simplify $2^{\log_2(8)}$: Since $8 = 2^3$, $\log_2(8) = 3$. Thus:

$$2^{\log_2(8)} = 2^3 = 8.$$

Next, $\frac{1}{2^3} = \frac{1}{8}$.

Now, we consider the product:

$$\prod_{k=1}^{8} \left(1 + \frac{1}{k}\right) = \prod_{k=1}^{8} \frac{k+1}{k}.$$

Write out a few terms:

$$= \frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \times \frac{7}{6} \times \frac{8}{7} \times \frac{9}{8}.$$

Most terms cancel out in a telescoping manner:

$$= \frac{9}{1} = 9.$$

Therefore:

$$B = 8 + \frac{1}{8} - 9.$$

Combine $8 - 9 = -1$:

$$B = -1 + \frac{1}{8}.$$

Convert $-1 = -\frac{8}{8}$:

$$B = -\frac{8}{8} + \frac{1}{8} = -\frac{7}{8}.$$

Thus:

$$B = -\frac{7}{8}.$$

9. **We now multiply A and B:**

We had found:

$$A = \frac{2}{3}, \quad B = -\frac{7}{8}.$$

Multiplying:

$$A \times B = \frac{2}{3} \times \left(-\frac{7}{8}\right) = \frac{2 \times (-7)}{3 \times 8} = \frac{-14}{24}.$$

We then simplify by dividing numerator and denominator by 2:

$$= \frac{-7}{12}.$$

Therefore:

$$\boxed{-\frac{7}{12}}.$$

Mathematical concepts involved in Problem C...

- **Manipulation of fractional expressions and simplification of factorial-based fractions.**
- **Use of logarithm properties, such as $2^{\log_2(8)} = 8$.**
- **Telescoping products to simplify complex products into simpler fractions.**
- **Careful handling of sums and products involving factorials and radicals.**

*************** END OF SOLUTION: C ***************
