Assignment 3

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$|\psi\rangle = \frac{1}{\sqrt{2}}\, |01\rangle - |10\rangle$ measure in $\vec{v}_\alpha \cdot \vec{\sigma}$ for 1st qubit
$\vec{v}_\beta \cdot \vec{\sigma}$ for 2nd qubit

$A = \vec{v}_\alpha \cdot \vec{\sigma} = v_{\alpha x}\sigma_x + v_{\alpha y}\sigma_y + v_{\alpha z}\sigma_z$;

B $\vec{v}_\beta \cdot \vec{\sigma} = v_{\beta x}\sigma_x + v_{\beta y}\sigma_y + v_{\beta z}\sigma_z$;

+1 for 1st qubit for 2nd qubit

$M_{+1}^{(1)} = \frac{I + A}{2}$ $M_{+1}^{(2)} = \frac{I + B}{2}$

$M_{-1}^{(1)} = \frac{I - A}{2}$ $M_{-1}^{(2)} = \frac{I - B}{2}$

(1)

$M_{+1+1} = \frac{I+A}{2} \otimes \frac{I+B}{2}$ $M_{-1-1} = \frac{I-A}{2} \otimes \frac{I-B}{2}$

$M_{+1-1} = \frac{I+A}{2} \otimes \frac{I-B}{2}$ $M_{-1+1} = \frac{I-A}{2} \otimes \frac{I-B}{2}$

$P_{+1+1} = \langle\psi| M_{+1+1}^{\dagger} M_{+1+1} |\psi\rangle$ $P_{--} = \langle\psi| M_{-1-1}^{\dagger} M_{-1-1} |\psi\rangle$

$P_{+1-1} = \langle\psi| M_{+1-1}^{\dagger} M_{+1-1} |\psi\rangle$ $P_{-+} = \langle\psi| M_{-1+1}^{\dagger} M_{-1+1} |\psi\rangle$

(2) Observable $\rightarrow A \otimes B$

$M_+ = \frac{I + A\otimes B}{2}$ $M_- = \frac{I - A\otimes B}{2}$

$A \otimes B = \vec{v}_\alpha \vec{\sigma} \otimes \vec{v}_B \vec{\sigma}$

$$\begin{bmatrix} v_{\alpha z} & v_{\alpha x} - v_{\alpha y} i \\ v_{\alpha x} + v_{\alpha y} i & -v_{\alpha z} \end{bmatrix} \otimes \begin{bmatrix} v_{\beta z} & v_{\beta x} - v_{\beta y} i \\ v_{\beta x} + v_{\beta y} i & v_{\beta z} \end{bmatrix}$$

$A \otimes B$ has eigen values +1 and -1 so it has results +1 and -1.

$P_{++} = \langle\psi| M_{++}^{\dagger} M_{++} |\psi\rangle$ $\qquad$ $P_{--} = \langle\psi| M_{--}^{\dagger} M_{--} |\psi\rangle$

$M_{++} = \frac{1}{4}(I+A)\otimes(I+B)$ $\qquad$ $M_{--} = \frac{1}{4}\left[(I-A)\otimes(I-B)\right]$

$E_{++} = M_{++}^{\dagger} M_{++} = \frac{1}{16}\left[\left((I+A)^{\dagger}\otimes(I+B)^{\dagger}\right)\otimes\left((I+A)\otimes(I+B)\right)\right]$ $\qquad$ $= \frac{1}{4}\left[I + AB - A - B\right]$

$= \frac{1}{16}\left[(I^{\dagger}+A^{\dagger})\otimes[I+A]\otimes\left(I^{\dagger}+B^{\dagger}\right)\otimes(I+B)\right]$

$= \frac{1}{16}\left[[I+A+A^{\dagger}+I]\otimes[I+B+B^{\dagger}+I]\right]$

$\left[\begin{array}{ll} A^{\dagger}A = I & \text{properties of} \\ AA = A & \text{observable} \end{array}\right]$

$= 1/16\left[2(I+A)\otimes 2[I+B]\right]$

$E_{++} = 1/4\left[I + A + B + AB\right]$

Similarly

$E_{--} = 1/4\left[I - A - B + AB\right]$

$P_{++} + P_{--} = \frac{1}{4}\langle\psi|[I+A+B+AB]|\psi\rangle + \frac{1}{4}\langle\psi|[I-A-B+AB]|\psi\rangle$

$= 1/4\langle\psi| 2I + 2AB|\psi\rangle$

$= 1/2\langle\psi| I + A\otimes B|\psi\rangle \quad \text{———} \quad (1)$

$P_{+} = \langle\psi| M_{+}^{\dagger} M_{+} |\psi\rangle$

$M_{+} = \frac{1}{2}(I + AB)$

$M_{+}^{\dagger} M = \frac{1}{4}\left[(I + (AB)^{\dagger})\otimes(I+AB)\right]$

$= \frac{1}{4}\left[I + AB + (AB)^{\dagger} + (AB)^{\dagger}(AB)\right]$

$= 1/4\left[2I + 2AB\right]$

$= 1/2\left[I + A\otimes B\right]$

$P_{+} = 1/2\langle\psi|[I + A\otimes B]|\psi\rangle \quad \text{———} \quad (2)$

The results (1) and (2) are same so,

$P_{++} + P_{--} = P_{+}$

Similarly

$P_{+-} = \langle \psi | M_{+-}^{\dagger} M_{+-} | \psi \rangle$

$M_{+-} = \frac{1}{4} (I + A) \otimes (I - B)$

$E_{+-} = \frac{1}{4} (I + A - B - AB)$

$P_{-+} = \langle \psi | M_{-+}^{\dagger} M_{-+} | \psi \rangle$

$M_{-+} = \frac{1}{4} (I - A) \otimes (I + B)$

$E_{-+} = \frac{1}{4} [I - A + B - AB]$

$$P_{++} + P_{--} = \frac{1}{4} \langle \psi | [I + A - B - AB] | \psi \rangle + \frac{1}{4} \langle \psi | [I - A + B - AB] | \psi \rangle$$

$$= \frac{1}{4} \langle \psi | 2I - 2AB | \psi \rangle$$

$$= \frac{1}{2} \langle \psi | I - A \otimes B | \psi \rangle \quad \text{—— (3)}$$

$P_{-} = \langle \psi | M_{-}^{\dagger} M_{-} | \psi \rangle$

$M_{-} = \frac{1}{2} (I - AB)$

$E_{-} = M_{-}^{\dagger} M = \frac{1}{2} [I - A \otimes B]$

$P_{-} = \frac{1}{2} \langle \psi | I - A \otimes B | \psi \rangle \quad \text{—— (4)}$

As equation (3) is equal to (4)

$P_{+-} + P_{-+} = P_{-}$

(3)

$$E(A \otimes B) = (-1) P_{+} + (+1) P_{-}$$

$$= -\frac{1}{2} [I + A \otimes B] + \frac{1}{2} [I - A \otimes B]$$

$$= -(A \otimes B)$$

$A \otimes B$ which is equivalent to the dot product between the vectors

$$E(A \otimes B) = -\vec{v}_{\alpha} \cdot \vec{v}_{\beta}$$

$$= -\cos\theta$$

where θ is the angle between the vectors $\vec{v}_{\alpha}$ and $\vec{v}_{\beta}$