Question 1

When a measurement is performed on the composite system, which is a measurement of one of the enclosing system, this is called a partial measurement.

$|\psi\rangle$ be the composite system

According to the measurement postulate, any measurement on the system, the state becomes

$$|\psi'\rangle = \frac{\{M \otimes I\}|\psi\rangle}{p'}$$

Where

$$\begin{aligned} p' &= \langle \psi \mid M^\dagger M \mid \psi \rangle \\ &= \| M \mid \psi \rangle \|^2 \end{aligned}$$

Let a state with 2 qubits be
$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

Let the measurement operator measuring 0 as the 1st bit be

$$\begin{aligned} M_0^{(1)} &= M_{00} + M_{01} \\ &= |00\rangle\langle 00| + |01\rangle\langle 01| \\ &= |0\rangle\langle 0| \otimes |0\rangle\langle 0| + |0\rangle\langle 0| \otimes |1\rangle\langle 1| \\ &= |0\rangle\langle 0| \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|) \\ &= |0\rangle\langle 0| \otimes I \end{aligned}$$

Similarly the measurement operator measuring 1 as the 1st bit be

$$\begin{aligned} M_1^{(1)} &= M_{10} + M_{11} \\ &= |10\rangle\langle 10| + |11\rangle\langle 11| \\ &= |1\rangle\langle 1| \otimes |0\rangle\langle 0| + |1\rangle\langle 1| \otimes |1\rangle\langle 1| \\ &= |1\rangle\langle 1| \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|) \\ &= |1\rangle\langle 1| \otimes I \end{aligned}$$

The probability of measuring the 1st qubit in state 0 is

$$\begin{aligned} Pr_0^{(1)} &= \langle \psi | M_0^{\dagger} M_0^{(1)} | \psi \rangle \\ &= (\alpha^*\langle 00| + \beta^*\langle 01| + \gamma^*\langle 10| + \delta^*\langle 11|)(|0\rangle\langle 0| \otimes I)(\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle) \\ &= (\alpha^*\langle 00| + \beta^*\langle 01|)(\alpha|00\rangle + \beta|01\rangle) \\ &= |\alpha|^2 + |\beta|^2 \end{aligned}$$

The state then get updated to

$$
\begin{aligned}
|\psi'\rangle &= \frac{1}{\sqrt{Pr_0^{(1)}}} M_0^{(1)} |\psi\rangle \\
&= \frac{1}{|\alpha|^2 + |\beta|^2} \ (|0\rangle\langle 0| \otimes I) \ \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \\
&= \frac{1}{|\alpha|^2 + |\beta|^2} \ (\alpha|00\rangle + \beta|01\rangle)
\end{aligned}
$$

The relative probability of the states $|00\rangle$ and $|01\rangle$ remains same. We eliminate the probability of the state to be in $|10\rangle$ or $|11\rangle$.

Question 2

Measure the state
$|\psi\rangle = 1/\sqrt{2}\ [|01\rangle - |10\rangle]$
Along the axis $\overrightarrow{v} = (v_x, v_y, v_z)$ with the observable $A = \overrightarrow{v}\ .\ \overrightarrow{\sigma}$

Let $|a\rangle$ and $|b\rangle$ be the eigenstates of A

Let the basis be

$$|0\rangle = \alpha|a\rangle + \beta|b\rangle$$
$$|1\rangle = \gamma|a\rangle + \delta|b\rangle$$

So the state becomes

$$|\psi\rangle = (\alpha\delta - \beta\gamma)\frac{|ab\rangle - |ba\rangle}{\sqrt{2}}$$

The $(\alpha\delta - \beta\gamma)$ is equal to the global phase factor, thus

$$|\psi\rangle = \frac{|ab\rangle - |ba\rangle}{\sqrt{2}}$$

Thus the qubits are always anti-correlated, irrespective of the observable (spin vector).