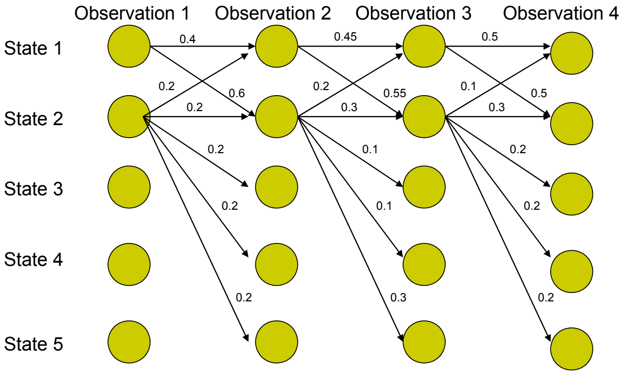
MEMM: the Label bias problem

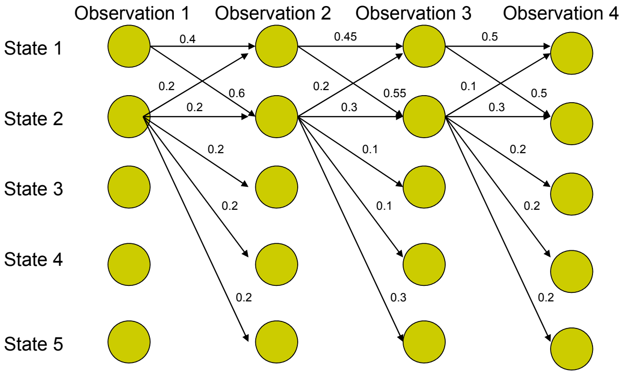


Probability of path 2->2->2->2 :

- 0.2 X 0.3 X 0.3 = 0.018

Other paths:
1-> 1-> 1-> 1: 0.09

MEMM: the Label bias problem

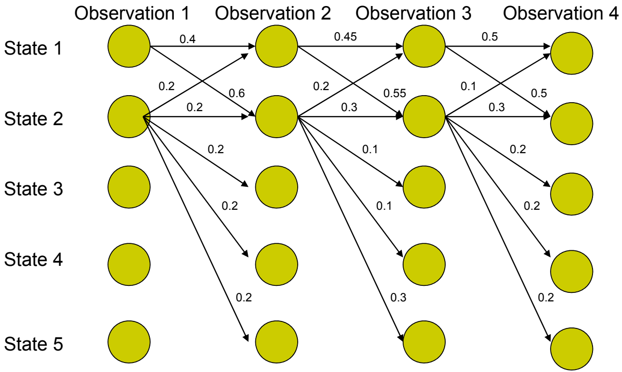


Probability of path 1->2->1->2:

- 0.6 X 0.2 X 0.5 = 0.06

Other paths:
1->1->1->1: 0.09
2->2->2->2: 0.018

MEMM: the Label bias problem

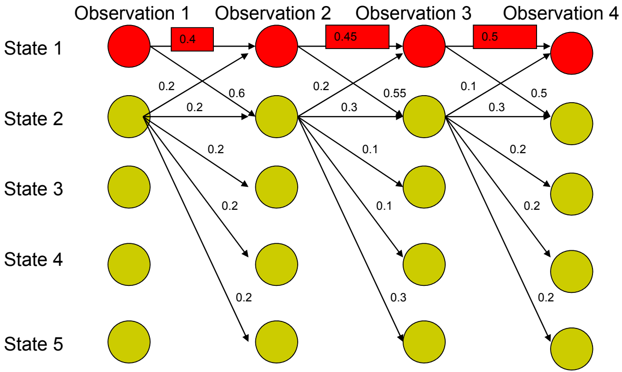


Probability of path 1->1->2->2:

- 0.4 X 0.55 X 0.3 = 0.066

Other paths:
1->1->1->1: 0.09
2->2->2->2: 0.018
1->2->1->2: 0.06

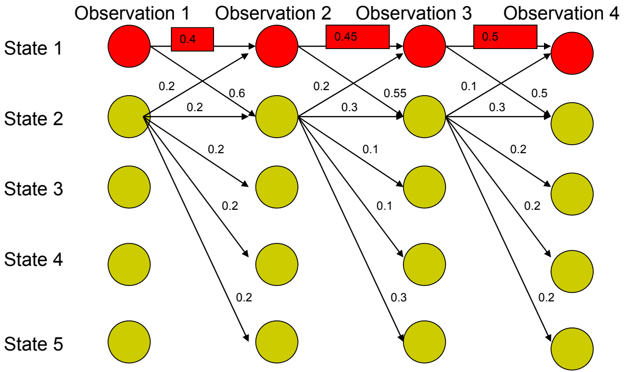
MEMM: the Label bias problem



Most Likely Path: 1-> 1-> 1-> 1

- Although **locally** it seems state 1 wants to go to state 2 and state 2 wants to remain in state 2.
- **why?**

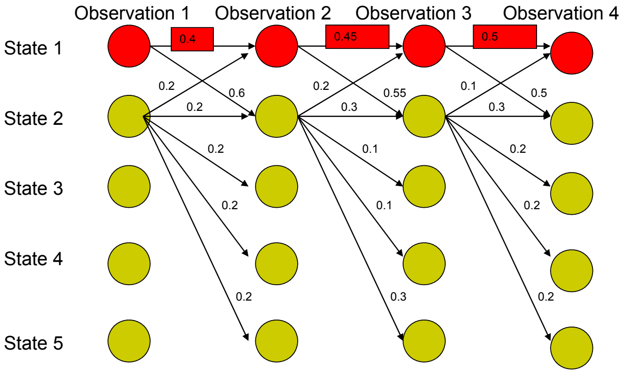
MEMM: the Label bias problem



Most Likely Path: 1-> 1-> 1-> 1

- State 1 has only two transitions but state 2 has 5:
  - Average transition probability from state 2 is lower

MEMM: the Label bias problem



Label bias problem in MEMM:

- Preference of states with lower number of transitions over others

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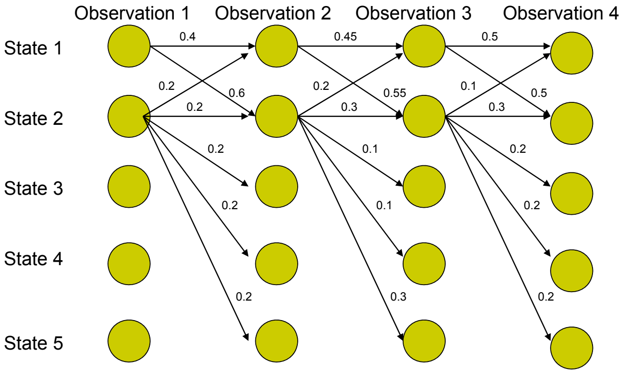
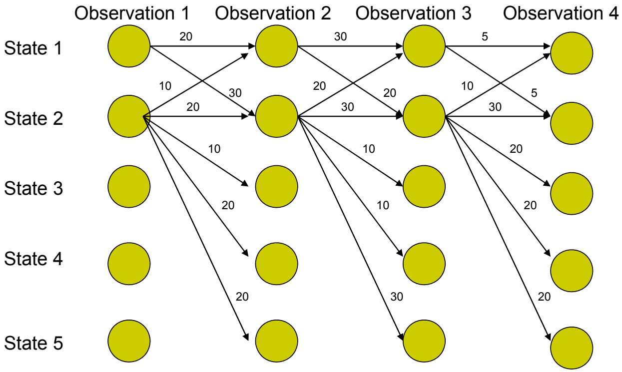
Solution: Do not normalize probabilities locally



From local probabilities .

Solution: Do not normalize probabilities locally



From local probabilities to local potentials

- States with lower transitions do not have an unfair advantage!

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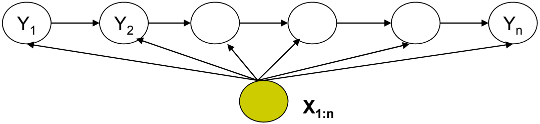
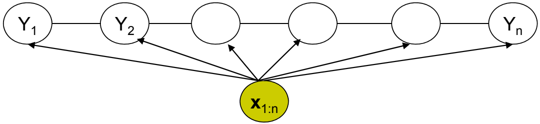
From MEMM .



$$P(\mathbf{y}_{1:n}|\mathbf{x}_{1:n}) = \prod_{i=1}^{n} P(y_i|y_{i-1},\mathbf{x}_{1:n}) = \prod_{i=1}^{n} \frac{\exp(\mathbf{w}^T\mathbf{f}(y_i,y_{i-1},\mathbf{x}_{1:n}))}{Z(y_{i-1},\mathbf{x}_{1:n})}$$

From MEMM to CRF

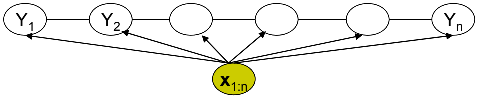


$$P(\mathbf{y}_{1:n}|\mathbf{x}_{1:n}) = \frac{1}{Z(\mathbf{x}_{1:n})}\prod_{i=1}^{n}\phi(y_i, y_{i-1}, \mathbf{x}_{1:n}) = \frac{1}{Z(\mathbf{x}_{1:n}, \mathbf{w})}\prod_{i=1}^{n}\exp(\mathbf{w}^T\mathbf{f}(y_i, y_{i-1}, \mathbf{x}_{1:n}))$$

- CRF is a partially directed model
  - Discriminative model like MEMM
  - Usage of global normalizer Z(**x**) overcomes the label bias problem of MEMM
  - Models the dependence between each state and the entire observation sequence (like MEMM)

Conditional Random Fields

- General parametric form:



$$
\begin{aligned}
P(\mathbf{y}|\mathbf{x}) &= \frac{1}{Z(\mathbf{x},\lambda,\mu)} \exp\Big(\sum_{i=1}^{n}\big(\sum_{k}\lambda_k f_k(y_i, y_{i-1}, \mathbf{x}) + \sum_{l}\mu_l g_l(y_i, \mathbf{x})\big)\Big) \\
&= \frac{1}{Z(\mathbf{x},\lambda,\mu)} \exp\Big(\sum_{i=1}^{n}\big(\lambda^T \mathbf{f}(y_i, y_{i-1}, \mathbf{x}) + \mu^T \mathbf{g}(y_i, \mathbf{x})\big)\Big)
\end{aligned}
$$

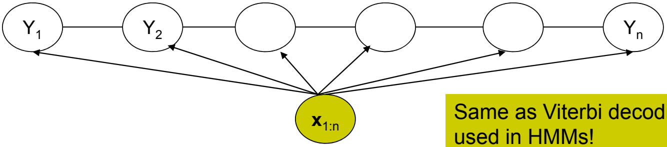
$$
\text{where } Z(\mathbf{x},\lambda,\mu) = \sum_{\mathbf{y}} \exp\Big(\sum_{i=1}^{n}\big(\lambda^T \mathbf{f}(y_i, y_{i-1}, \mathbf{x}) + \mu^T \mathbf{g}(y_i, \mathbf{x})\big)\Big)
$$

CRFs: Inference

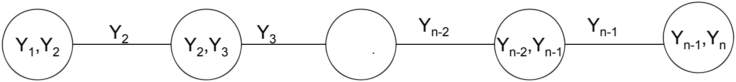
- Given CRF parameters λ and μ, find the **y*** that maximizes P(**y**|**x**)

$$\mathbf{y}^* = \arg\max_{\mathbf{y}} \exp\left(\sum_{i=1}^{n}(\lambda^T \mathbf{f}(y_i, y_{i-1}, \mathbf{x}) + \mu^T \mathbf{g}(y_i, \mathbf{x}))\right)$$

  - Can ignore Z(**x**) because it is not a function of **y**
- Run the max-product algorithm on the junction-tree of CRF:



Same as Viterbi decoding used in HMMs!



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CRF learning

- Given $\{(\mathbf{x}_d, \mathbf{y}_d)\}_{d=1}^N$, find λ^*, μ^* such that

$$\begin{aligned}
\lambda*, \mu* &= \arg\max_{\lambda,\mu} L(\lambda, \mu) = \arg\max_{\lambda,\mu} \prod_{d=1}^{N} P(\mathbf{y}_d | \mathbf{x}_d, \lambda, \mu) \\
&= \arg\max_{\lambda,\mu} \prod_{d=1}^{N} \frac{1}{Z(\mathbf{x}_d, \lambda, \mu)} \exp(\sum_{i=1}^{n} (\lambda^T \mathbf{f}(y_{d,i}, y_{d,i-1}, \mathbf{x}_d) + \mu^T \mathbf{g}(y_{d,i}, \mathbf{x}_d))) \\
&= \arg\max_{\lambda,\mu} \sum_{d=1}^{N} (\sum_{i=1}^{n} (\lambda^T \mathbf{f}(y_{d,i}, y_{d,i-1}, \mathbf{x}_d) + \mu^T \mathbf{g}(y_{d,i}, \mathbf{x}_d)) - \log Z(\mathbf{x}_d, \lambda, \mu))
\end{aligned}$$

- Computing the gradient w.r.t λ:

Gradient of the log-partition function in an exponential family is the expectation of the sufficient statistics.

$$\nabla_\lambda L(\lambda, \mu) = \sum_{d=1}^{N} (\sum_{i=1}^{n} \mathbf{f}(y_{d,i}, y_{d,i-1}, \mathbf{x}_d) - \sum_{\mathbf{y}} (P(\mathbf{y}|\mathbf{x_d}) \sum_{i=1}^{n} \mathbf{f}(y_{d,i}, y_{d,i-1}, \mathbf{x}_d)))$$

CRF learning

$$\nabla_\lambda L(\lambda,\mu) = \sum_{d=1}^{N}\left(\sum_{i=1}^{n}\mathbf{f}(y_{d,i},y_{d,i-1},\mathbf{x}_d) - \boxed{\sum_{\mathbf{y}}\left(P(\mathbf{y}|\mathbf{x_d})\sum_{i=1}^{n}\mathbf{f}(y_i,y_{i-1},\mathbf{x}_d)\right)}\right)$$

- Computing the model expectations:
  - Requires exponentially large number of summations: Is it intractable?

$$\sum_{\mathbf{y}}\left(P(\mathbf{y}|\mathbf{x}_d)\sum_{i=1}^{n}\mathbf{f}(y_i,y_{i-1},\mathbf{x}_d)\right) = \sum_{i=1}^{n}\left(\sum_{\mathbf{y}}\mathbf{f}(y_i,y_{i-1},\mathbf{x}_d)P(\mathbf{y}|\mathbf{x}_d)\right)$$

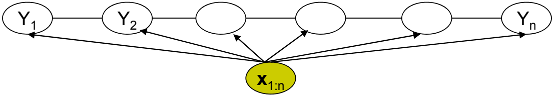
$$= \sum_{i=1}^{n}\sum_{y_i,y_{i-1}}\mathbf{f}(y_i,y_{i-1},\mathbf{x}_d)P(y_i,y_{i-1}|\mathbf{x}_d)$$

Expectation of **f** over the corresponding marginal probability of neighboring nodes!!

- Tractable!
  - Can compute marginals using the sum-product algorithm on the chain

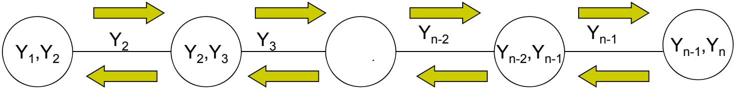
CRF learning

- Computing marginals using junction-tree calibration:



- Junction Tree Initialization:

$$\alpha^0(y_i, y_{i-1}) = \exp(\lambda^T \mathbf{f}(y_i, y_{i-1}, \mathbf{x}_d) + \mu^T \mathbf{g}(y_i, \mathbf{x}_d))$$



- After calibration:

Also called forward-backward algorithm

$$P(y_i, y_{i-1}|\mathbf{x}_d) \propto \alpha(y_i, y_{i-1})$$

$$\Rightarrow P(y_i, y_{i-1}|\mathbf{x}_d) = \frac{\alpha(y_i, y_{i-1})}{\sum_{y_i, y_{i-1}} \alpha(y_i, y_{i-1})} = \alpha'(y_i, y_{i-1})$$

CRF learning

- Computing feature expectations using calibrated potentials:

$$\sum_{y_i, y_{i-1}} \mathbf{f}(y_i, y_{i-1}, \mathbf{x}_d) P(y_i, y_{i-1} | \mathbf{x}_d) = \sum_{y_i, y_{i-1}} \mathbf{f}(y_i, y_{i-1}, \mathbf{x}_d) \alpha'(y_i, y_{i-1})$$

- Now we know how to compute r$_\lambda$L(λ,μ):

$$\begin{aligned} \nabla_\lambda L(\lambda, \mu) &= \sum_{d=1}^{N} \Big(\sum_{i=1}^{n} \mathbf{f}(y_{d,i}, y_{d,i-1}, \mathbf{x}_d) - \sum_{\mathbf{y}} \big(P(\mathbf{y} | \mathbf{x_d}) \sum_{i=1}^{n} \mathbf{f}(y_i, y_{i-1}, \mathbf{x}_d) \big) \Big) \\ &= \sum_{d=1}^{N} \Big(\sum_{i=1}^{n} \big(\mathbf{f}(y_{d,i}, y_{d,i-1}, \mathbf{x}_d) - \sum_{y_i, y_{i-1}} \alpha'(y_i, y_{i-1}) \mathbf{f}(y_i, y_{i-1}, \mathbf{x}_d) \big) \Big) \end{aligned}$$

- Learning can now be done using gradient ascent:

$$\begin{aligned} \lambda^{(t+1)} &= \lambda^{(t)} + \eta \nabla_\lambda L(\lambda^{(t)}, \mu^{(t)}) \\ \mu^{(t+1)} &= \mu^{(t)} + \eta \nabla_\mu L(\lambda^{(t)}, \mu^{(t)}) \end{aligned}$$

CRF learning

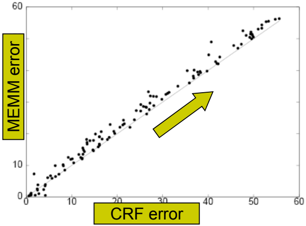
- In practice, we use a Gaussian Regularizer for the parameter vector to improve generalizability

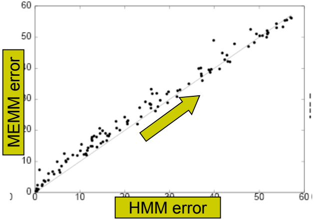
$$\lambda*, \mu* \quad = \quad \arg\max_{\lambda,\mu} \sum_{d=1}^{N} \log P(\mathbf{y}_d | \mathbf{x}_d, \lambda, \mu) - \frac{1}{2\sigma^2}(\lambda^T \lambda + \mu^T \mu)$$

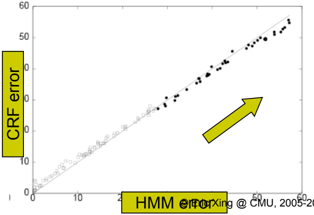
- In practice, gradient ascent has very slow convergence
  - Alternatives:
    - Conjugate Gradient method
    - Limited Memory Quasi-Newton Methods

CRFs: some empirical results

- Comparison of error rates on synthetic data







Data is increasingly higher order in the direction of arrow

CRFs achieve the lowest error rate for higher order data

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CRFs: some empirical results

- Parts of Speech tagging

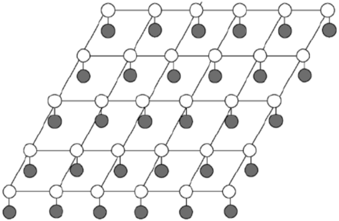
model	*error*	*oov error*
HMM	5.69%	45.99%
MEMM	6.37%	54.61%
CRF	5.55%	48.05%
MEMM$^+$	4.81%	26.99%
CRF$^+$	4.27%	23.76%

$^+$Using spelling features

- Using same set of features: HMM >=< CRF > MEMM
- Using additional overlapping features: CRF$^+$ > MEMM$^+$ >> HMM

Other CRFs

- So far we have discussed only 1-dimensional chain CRFs
  - Inference and learning: exact
- We could also have CRFs for arbitrary graph structure
  - E.g: Grid CRFs
  - Inference and learning no longer tractable
  - Approximate techniques used
    - MCMC Sampling
    - Variational Inference
    - Loopy Belief Propagation
  - We will discuss these techniques soon



Content

1. Sequence labeling problems
2. Word window classification
3. Hidden Markov models (HMMs)
4. Graphical models for sequence labeling