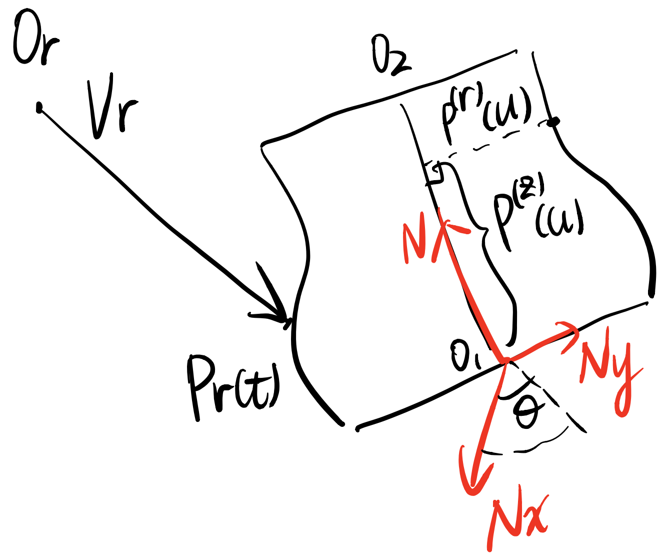

$u \in [0,1]$

$\theta \in [-\pi, \pi)$

1° 光线: $P_r(t) = O_r + tV_r$ (1)

2° Bezier曲线旋转面方程:

$$S(u,\theta) = O_1 + p^{(z)}(u)(O_2 - O_1) + p^{(r)}(u)\left(N_x \cos\theta + N_y \sin\theta\right) \qquad (2)$$

其中 Bezier曲线相关表达式如下:

$$P(u) = \sum_{i=0}^{n} b_{i,n}(u) P_i$$

$$\frac{dP(u)}{du} = n \sum_{i=0}^{n-1} b_{i,n-1}(u)\left(P_{i+1} - P_i\right) \qquad (3)$$

$$b_{i,n}(u) = \binom{n}{i} (1-u)^{n-i} u^{i}$$

3° 求解如下方程得交点：

$$F(t,u,\theta)$$
$$= S(u,\theta) - P_r(t)$$
$$= (O_1 - O_r) - tV_r + P^{(z)}(u)(O_2 - O_1) + P^{(r)}(u)(N_x\cos\theta + N_y\sin\theta) \qquad (4)$$

使用Newton迭代法求解，方法如下：

$$\begin{bmatrix} t \\ u \\ \theta \end{bmatrix}_{i+1} := \begin{bmatrix} t \\ u \\ \theta \end{bmatrix}_{i} - J^{-1}F \qquad (5)$$

其中 $J = \dfrac{\partial F}{\partial(t,u,\theta)} = \begin{bmatrix} \dfrac{\partial F}{\partial t} & \dfrac{\partial F}{\partial u} & \dfrac{\partial F}{\partial \theta} \end{bmatrix}$

$$= \begin{bmatrix} -V_r \\ (O_2 - O_1)\dfrac{dP^{(z)}(u)}{du} + \dfrac{dP^{(r)}(u)}{du}(N_x\cos\theta + N_y\sin\theta) \\ (-N_x\sin\theta + N_y\cos\theta)P^{(r)}(u) \end{bmatrix}^T \qquad (6)$$

4° 求解 (u_0, θ_0) 处的法向量：

$$\vec{N} = \left(\frac{\partial S}{\partial \theta} \times \frac{\partial S}{\partial u}\right)\bigg|_{(u_0,\theta_0)}$$

$$\frac{\partial S}{\partial \theta} = (O_2 - O_1)\frac{dP^{(z)}(u)}{du} + \frac{dP^{(r)}(u)}{du}(N_x\cos\theta + N_y\sin\theta) \qquad (7)$$

$$\frac{\partial S}{\partial \theta} = (-N_x\sin\theta + N_y\cos\theta)P^{(r)}(u)$$