1 Enumerating vs. Counting

1.1 Permutation of Letters

Example: How many words can we make out of the letters A B C using each letter once?

- ABC
- BAC
- CAB
- ACB
- BCA
- CBA

When we list all objects as above we call it **enumeration**, whereas **counting** is only concerned with the total number of objects. If we consider the example above, how many words would be possible for A B C D?

It's best to find a formula, as using it is a very efficient way to count Objects. For $n = 4$ letters we end up with 24 permutations.

The formula for the amount of different words with n letters is $n!$

1.2 Points in Convex Position

How many crossing-free spanning paths exist for n points on convex position?

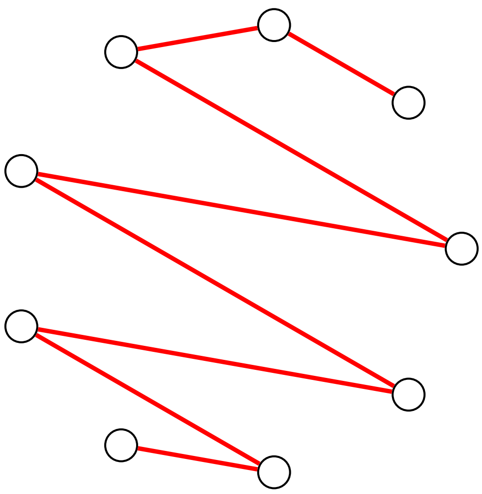

Figure 1: An example illustrating one possibility of a spanning path for $n = 9$ points

For $n = 1$ points the definition of the spanning path is unclear, in some cases it is considered as path with the size 1 and in others with size 0.

Let's look at some examples for $n > 1$ and try to determine a suitable formula.

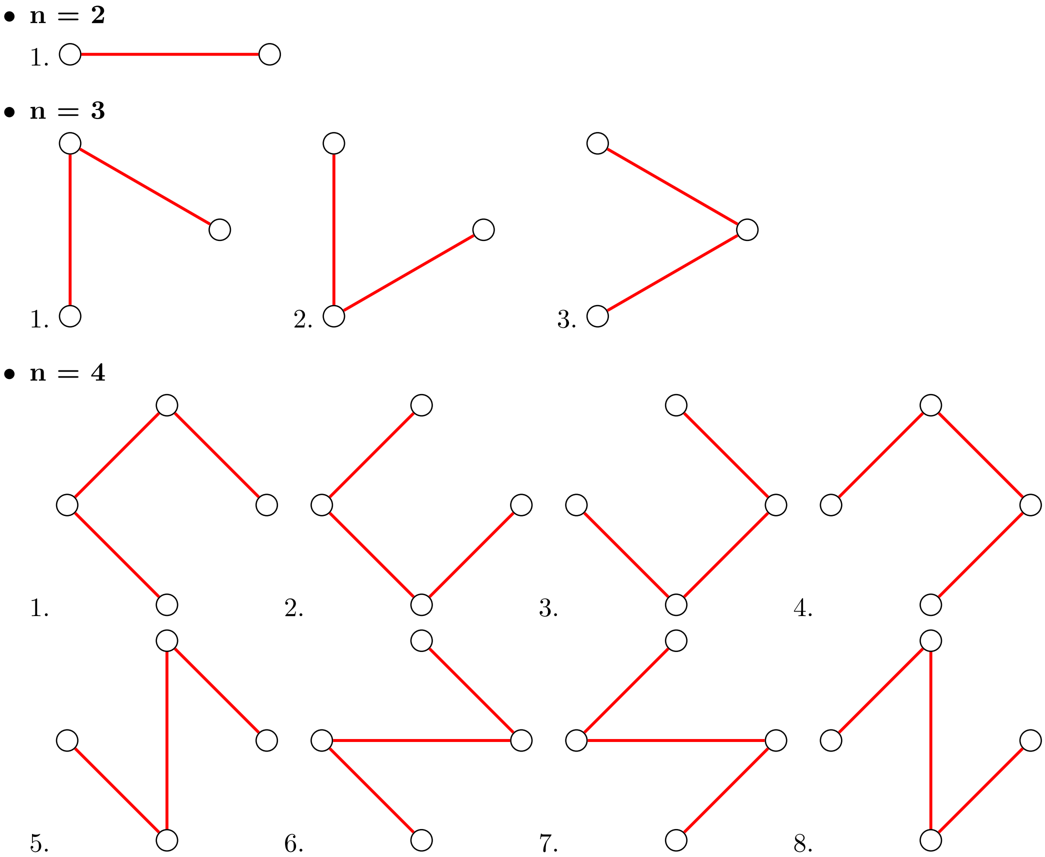

Figure 2: Enumeration of crossing free spanning paths up to $n = 4$

As we can see from this example, enumeration can become a tedious and error prone task very fast. Can you list all paths for $n = 5$?

It is better to abstract the problem and find an inductive solution. When constructing the path we start with a point, and from it we only see two immediate choices. After one of those points is added, we have two choices again. This goes on for a while until $n - 2$.

$$\underbrace{2 \cdot 2 \cdot 2 \cdots 2 \cdot 2}_{n-2 \text{ times}} = 2^{n-2}$$

Now in order to construct all paths we need to start at all possible points, when we do that however a double count occurs.

$$n \cdot 2^{n-2} \Rightarrow \frac{n \cdot 2^{n-2}}{2} \Rightarrow n \cdot 2^{n-3} \text{ for } n \geq 2$$

We can use this formula to find the number of crossing-free spanning paths for $n = 5$, which gives us $5 \cdot 2^2 = 20$.

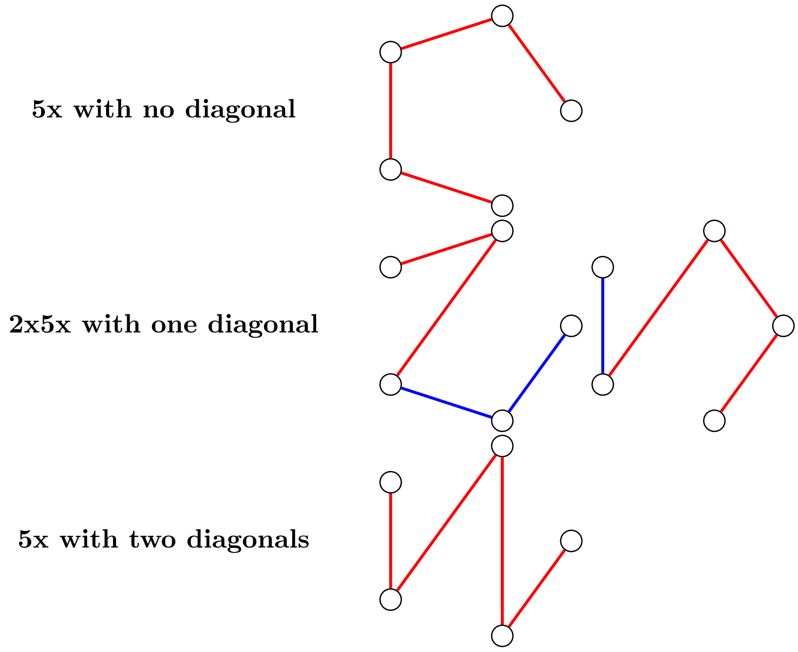

Figure 3: Another method of enumeration, do not list similar objects