Designing Online Contests

Andrew Tulloch

Contents

CHAPTER 1

Introduction

We'll look at a few problems in contest design:

- Standard all-pay contest
- Rank-order allocation of prizes
- Smooth allocation of prizes
- Simultaneous contest
- Sequential contests
- Credits for contributions and production

DEFINITION 1.1. (i) $N = \{1, \ldots, n\}$, $n \geq 2$ set of players.
(ii) b_i is the effort invested by player $i \in N$.
(iii) $b = (b_1, b_2, \ldots, b_n)$.
(iv) $X = (X_1(b), \ldots, X_n(b))$.
(v) $X_i(b) \geq 0$.
(vi) $\sum_{i=1}^{n} X_i(b) = 1$.
(vii) $N(b) = \{i \in N : b_i \geq b_j \forall j \in N\}$.
(viii)

$$X_i(b) = \begin{cases} \frac{1}{|N(b)|} & i \in N(b) \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

(ix) Payoff for player i is $s_i(b) = v_i X_i(b) - b_i$.

Any pure Nash equilibrium b satisfies

$$\forall i \in N, s_i(b) \geq s_i((b_{i'}, b_{-i})), \forall b_{i'} \in R \tag{1.2}$$

A mixed Nash equilibrium b satisfies

$$\forall i \in N, \mathbb{E}(s_i(b)) \geq \mathbb{E}(s_i((b_{i'}, b_{-i}))), \forall b_{i'} \in R \tag{1.3}$$

Consider the two player case, with $v_1 \geq v_2 > 0$. There exists a unique mixed Nash equilibrium such that $B_1(b) = \frac{b}{v_2}$, $B_2(b) = 1 - \frac{v_2}{v_1} + \frac{b}{v_1}$.

Fill in rest of notes from lecture material

Fill in missing lecture

1. Rank order allocation of prizes

Consider

(i) $N = \{1, 2, \ldots, n\}$, $n \geq 2$.
(ii) $w_1 \geq w_2 \geq \cdots \geq w_n \geq 0$.
(iii) Valuations $v_1, \ldots, v_n$ IID F
(iv) $s_i(b) = v_i \sum_{j=1}^{n} w_j X_{i,j}(b) - b_i$, $i \in N$
(v) Symmetric Bayes Nash equilibrium
(vi) $X_j(v) = \binom{n-1}{j-1} F(v)^{n-j}(1 - F(v))^{j-1}$

$F(v) = \mathbb{P}(v_i \leq v)$

2. Optimal Auction Design

Bibliography