The Analysis of PDEs

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Contents

CHAPTER 1

Introduction

24 lectures, with 3-4 example classes.

1. What is a PDE?

- PDE several variables and partial derivatives
- ODE in infinite dimensions - functional analysis
- Unfortunately, **no** general results exist for all PDEs
- **Guiding principles** - don't look at random - use fundamental equations of physics

(i) The CauchyKowalevski theorem
(ii) Laplace/Poisson equation
(iii) Heat equation
(iv) The transport equation
(v) The wave and Schrodiner equation
(vi) nonlinear pdes

2. The Mathematical Problem

"Inverting differentiation" - solve $\frac{du}{dx} = F(x), F : \mathbb{R} \to \mathbb{R}$. Solved by

$$u(x) = \int_{x_0}^{x} F(y)dy + u(x_0) \tag{1.1}$$

More generally,

$$F(x, y_1, \dots, y_n) : R^{n+1} \to R \tag{1.2}$$

$$\forall x \in \Omega, F(x, u(x), u'(x), \dots, u^{n-1}(x)) = 0 \tag{1.3}$$

The fundamental problem in PDE theory

(i)

$$u(x_0, x_1, \dots, x_l), l \geq 1 \tag{1.4}$$

(ii) A differential relationship involving

$$\frac{\partial u}{\partial x_i}, \frac{\partial^2 u}{\partial x_i \partial x_j}, \dots \tag{1.5}$$

given as a function $F : R \to R$, with

$$\forall (x_0, \dots, x_l) \in \Omega, F(x_0, x_1, \dots, x_l, \frac{\partial u}{\partial x_0}, \dots) = 0 \tag{1.6}$$

(iii) Boundary data to prescribe

(iv) Useful and conventional to write $x_0 = t$ "time".

$$\begin{cases} \frac{\partial u}{\partial t} & = g(x_1, \dots, x_l, \frac{\partial u}{\partial x_1}) \\ & = G[u_t] \end{cases} \tag{1.7}$$

Assume that $u_t(\cdot) \in C^0(R^l)$. Say $G : C^0(R^n) \to C^0(R^n)$

3. The Cauchy Problem for ODEs

Bibliography