

Problem 1.

1. Plurality

a: 10, b: 3, c: 6, d: 7 Winner (a)

2. Borda

$$S_{\text{Borda}}(P, a) = 10 \times (4-1) + 7 \times (4-2) + 6 \times (4-3) + 3 \times (4-4) = 50$$

$$S_{\text{Borda}}(P, d) = 10 \times (4-4) + 7 \times (4-1) + 6 \times (4-2) + 3 \times (4-3) = 36$$

$$S_{\text{Borda}}(P, c) = 10 \times (4-3) + 7 \times (4-4) + 6 \times (4-1) + 3 \times (4-2) = 34$$

$$S_{\text{Borda}}(P, b) = 10 \times (4-2) + 7 \times (4-3) + 6 \times (4-4) + 3 \times (4-1) = 36$$

Winner = a

3. Veto

$$S_{\text{veto}}(P, a) = 10 + 7 + 6 = 23$$

$$S_{\text{veto}}(P, d) = 7 + 6 + 3 = 16$$

$$S_{\text{veto}}(P, c) = 10 + 6 + 3 = 19$$

$$S_{\text{veto}}(P, b) = 10 + 7 + 3 = 20$$

Winner = a

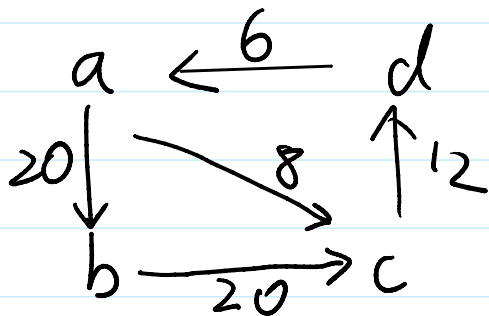
4. plurality

round 1: a: 10, d: 7, c: 6, b: 3

round 2: a: 10, d: 7 + 6 + 3 = 16

winner = d

5.



6. copeland.

a:2 b:1.5 c:1 d:1.5

a vs b 23:3 a+1

a vs c 17:9 a+1

a vs d 10:16 d+1

b vs c 20:6 b+1

b vs d 13:13 b+0.5, d+0.5

c vs d 19:7 c+1

winner (a)

Problem 2

1. Initial

	a	c	b
R1	27	42	24
R2	27	66	
Winner		c	

After.

23 @ [a > b > c] 46 @ [c > a > b] 24 @ [b > c > a]

	a	c	b
R1	22	146	24

	a	c	b
R1	23	46	24
R2		46	47
Winner			b

The winner will be b instead of a.

2.

23 @ [a > b > c] 42 @ [c > a > b] 24 @ [b > c > a]

	a	c	b
R1	23	42	24
R2		42	47
Winner			b

In the first scenario, the winner is C, after four voters changed, the winner is b. The voting system can produce different outcomes based on small changes.

Problem 3

Let $N = w(a,b) + w(b,a)$ be the total # of voters.

If N is even, both $w(a,b), w(b,a)$ have same parity.

So $|w(a,b) - w(b,a)|$ with same parity must be even.

If N is odd, both $w(a,b), w(b,a)$ have different parity.

So, $|w(a,b) - w(b,a)|$ with different parity must be odd.

Problem 4.

$U(5, 1, 4)$

$C(2, 1, 3)$

$D(0, 1, 5)$

↓

$C < U$

eliminate C

$$\Downarrow$$

	L	M	R
U	5, 0	1, 3	4, 0
D	0, 1	1, 0	5, 0

$$\Downarrow$$

U (5, 1, 4)
D (0, 1, 5)

$$\Downarrow$$

	L	M	R
U	0	3	0
D	1	1	1
	1	0	0

$L \&\& M > R$
eliminate R

	L	M
U	5, 0	1, 3
D	0, 1	1, 0

\Downarrow check row

U (5, 1)
D (0, 1)

$U > D$
eliminate D

$$\Downarrow$$

	L	M
U	5, 0	1, 3

\Downarrow check column

	L	M
U	0	3

$M > L$ eliminate L

M
U 1, 3 \Rightarrow NE.