HW1

2023年2月20日 22:13

P1. Base case: $i=2$

$$P(x_1, x_2) = P(X_1, X_2) \cdot P(X_1)$$

Induction

suppose $p(x_1, \dots, x_k) = p(a) = \prod_{i=1}^{k} P(X_i | X_1, \dots, X_{i-1})$

$$P(x_1, \dots, x_{k+1}) = p(a, x_{k+1}) = P(x_{k+1} | a) \cdot p(a)$$
$$= p(x_{k+1} | x_1, \dots, x_k) \prod_{i=1}^{k} P(x_i | x_1, \dots, x_{i-1})$$
$$= \prod_{i}^{k+1} p(x_i | x_1, \dots, x_{i-1})$$

Conclusion:

$$p(X_1, \dots, X_k) = \prod_{i=1}^{k} p(x_i | x_1, \dots, x_{i-1})$$
$$\Rightarrow p(x_1, \dots, x_{k+1}) = \prod_{i=1}^{k+1} p(X_i | X_1, \dots, X_{i-1}).$$

Problem 2

$$P(x, y | z) = \frac{P(x, y, z)}{P(z)} = \frac{P(x|y, z) * P(y, z)}{P(z)} = P(x|y, z) \times \frac{P(y, z)}{P(z)}$$
$$= P(x|y, z) \times P(y|z)$$

$$P(x|y, z) = P(x|y, z) \times P(y|z) = P(x|z) \times P(y|z)$$

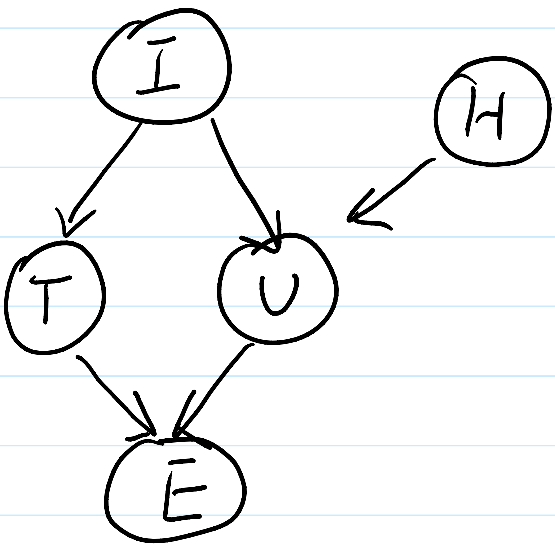
Problem 3

1): False, $X \to Z \leftarrow Y$, they are dependent after Z is given

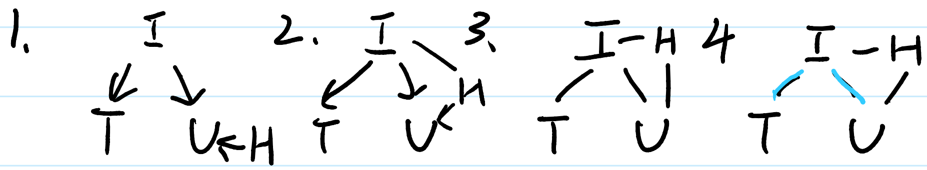
1): False, $X \to Z \leftarrow Y$, they are dependent after Z is given

2): False, $X \leftarrow Z \to Y$, they are unconditionally dependent.

Problem 4

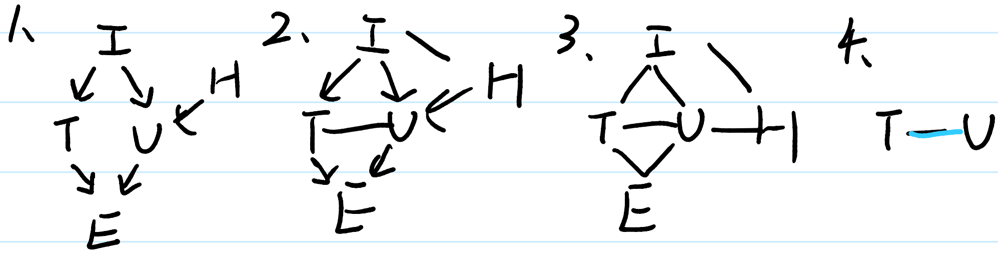


1、$T \perp U$



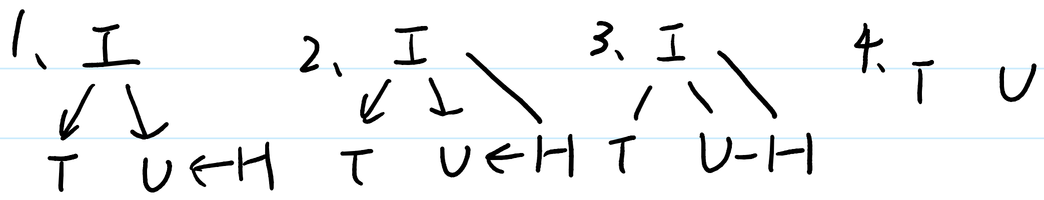
not garuanteed to be independent, ∴ F

2、$T \perp U \mid I, E, H$



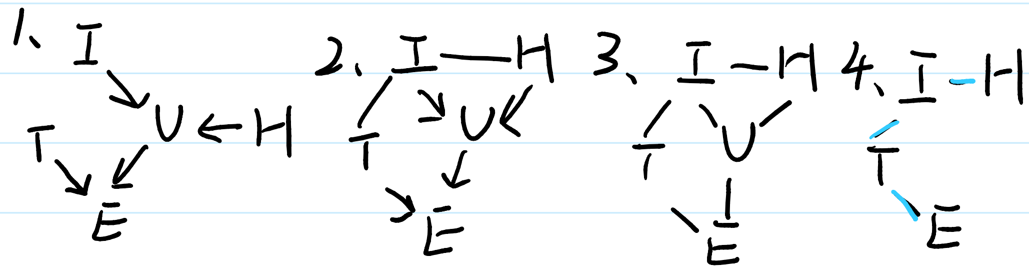
not garuanteed to be independent ∴ F

3、$T \perp U \mid I, H$



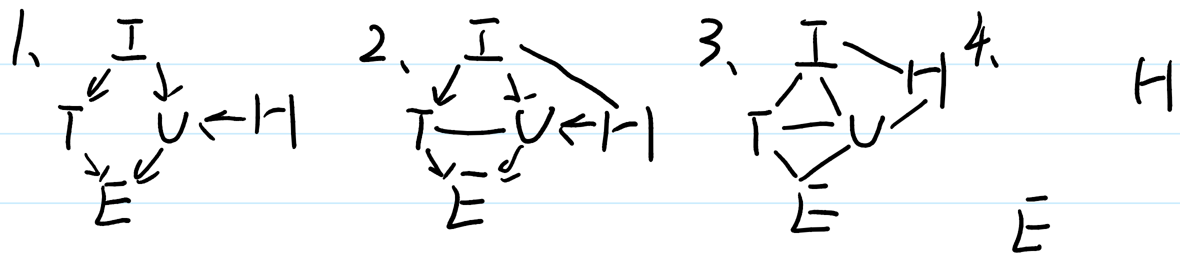
garuanteed to be independent , ∴ T

4、E⊥H | U



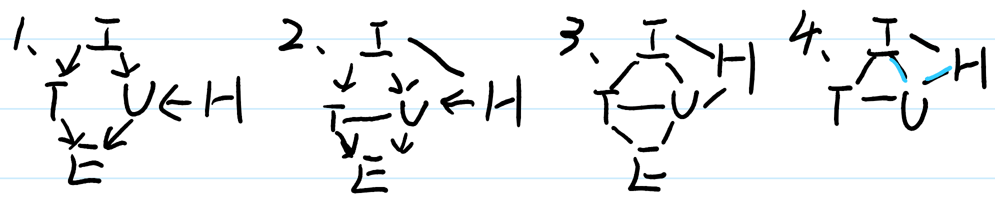
not garuanteed to be independent ∴ F

5、E⊥H | U, I, T



garuanteed to be independent. ∴ T

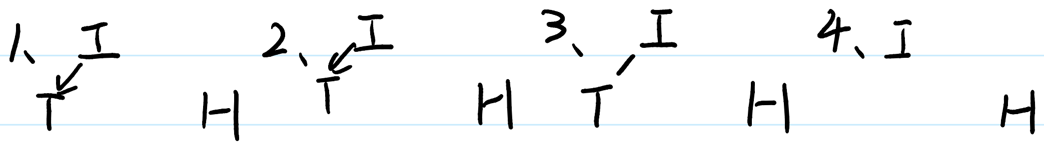
6、I⊥H | E



not garuanteed to be independent , ∴ F

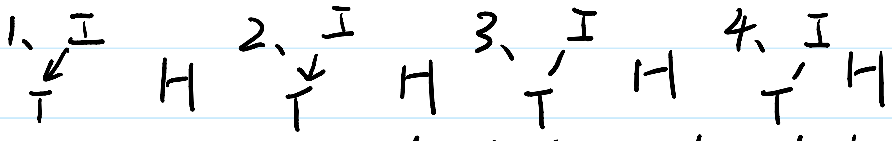
7、I⊥H | T

7、 $I \perp H \mid T$



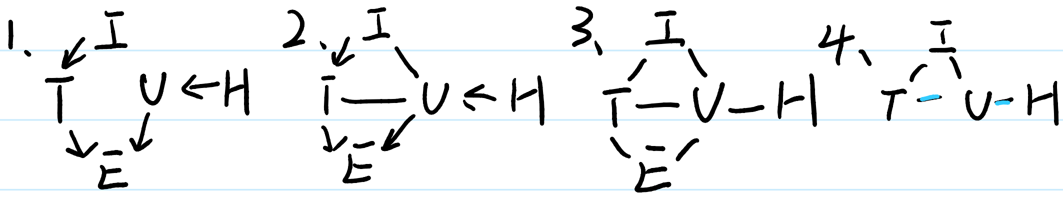
garuanteed to be independent, $\therefore$ T

8、 $T \perp H$



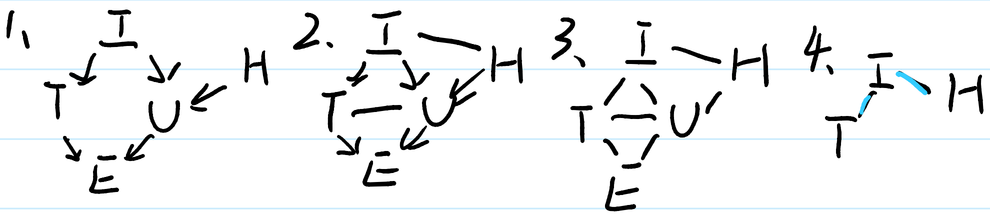
garuanteed to be independent. , $\therefore$ T

9、 $T \perp H \mid E$



not garuanteed to be independent. , $\therefore$ F

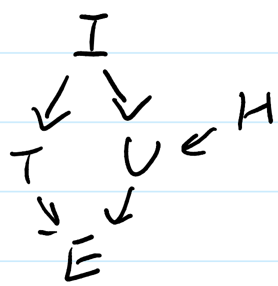
10、 $T \perp H \mid E, U$



garuanteed to be independent. , $\therefore$ F

Problem 5.

$$P(+u \mid +e) = \frac{P(+u, +e)}{P(+e)}$$



E

$$P(+u,+e)=\sum_{h,i,t} P(+u,+e,i,t,h)$$

$$=\sum_{h,i,t} P(i)\times P(t|i)\times P(+u|i,h)\times P(h)\times P(+e|t,+u$$

$$=\sum_{h} P(h)\sum_{i} P(i)\cdot P(+u|i,h)\underbrace{\sum_{t} P(t|i)\times P(+e|t,+u)}_{f_2}$$

$$f_2(h,i)=\sum_{t} P(t|i)\times P(+e|t,+u)$$

$$f_2(h,+i)=P(+t|+i)\times P(+e|+t,+u)+P(+e|-t,+u)\times P(-t|+i)$$
$$=0.8\times 0.9+0.2\times 0.7=0.86$$

$$f_2(h,-i)=P(+t|-i)\times P(+e|+t,+u)+P(-t|-i)\times P(+e|-t,+u)$$
$$=0.5\times 0.9+0.5\times 0.7=0.8$$

$$f_1(h)=\sum_{i} P(i)\cdot P(+u|i,h)\cdot f_2(h,i)$$

$$f(+h)=P(-i)\cdot P(+u|-i,+h)\cdot f_2(+h,i)+$$
$$P(+i)\cdot P(+u|+i,+h)\cdot f_2(+h,+i)$$
$$=0.3\times 0.5\times 0.8+0.7\times 0.9\times 0.86$$
$$=0.6618$$

$$f(-h)=P(-i)\cdot P(+u|-i,-h)\cdot f_2(-h,i)+$$
$$P(+i)\cdot P(+u|+i,-h)\cdot f_2(-h,i)$$
$$=0.3\times 0.1\times 0.8+0.7\times 0.3\times 0.86$$
$$=0.2046$$

$$P(+u,+e)=\sum_{h} P(h)\cdot f_1(h)$$

$$=P(h)\times f_1(h)+P(-h)\times f_1(-h)$$

$$=0.6\times 0.6618+0.4\times 0.2046$$

$= P(h) \times f_1(h) + P(-h) \times f_1(-h)$

$= 0.6 \times 0.6618 + 0.4 \times 0.2046$

$= 0.47892$

$$P(-u,+e) = \sum_{h,i,t} P(-u,+e,i,t,h)$$

$$= \sum_{h,i,t} P(i) \times P(t|i) \times P(-u|i,h) \times P(h) \times P(+e|t,-u)$$

$$= \sum_h P(h) \sum_i P(i) \cdot P(-u|i,h) \underbrace{\sum_t P(t|i) \times P(+e|t,-u)}_{f_2}$$

$$f_2(h,i) = \sum_t P(t|i) \times P(+e|t,-u)$$

$$f_2(h,+i) = P(+t|+i) \times P(+e|+t,-u) + P(+e|-t,-u) \times P(-t|+i)$$

$$= 0.8 \times 0.5 + 0.2 \times 0.3 = 0.46$$

$$f_2(h,-i) = P(+t|-i) \times P(+e|+t,-u) + P(-t|-i) \times P(+e|-t,-u)$$

$$= 0.5 \times 0.5 + 0.5 \times 0.3$$

$$f_1(h) = \sum_i P(i) \cdot P(-u|i,h) \cdot f_2(h,i)$$

$$f(+h) = P(-i) \cdot P(-u|-i,+h) \cdot f_2(+h,i) +$$
$$P(+i) \cdot P(-u|+i,+h) \cdot f_2(+h,+i)$$

$$= 0.3 \times 0.5 \times 0.4 + 0.7 \times 0.1 \times 0.46$$

$$= 0.0922$$

$$f(-h) = P(-i) \cdot P(-u|-i,-h) \cdot f_2(-h,i) +$$
$$P(+i) \cdot P(-u|+i,-h) \cdot f_2(-h,i)$$

$$= 0.3 \times 0.9 \times 0.4 + 0.7 \times 0.7 \times 0.46$$

$$= 0.2334$$

$= 0.3 \times 0.9 \times 0.4 + 0.7 \times 0.7 \times 0.46$

$= 0.3334$

$P(-u, +e) = \sum_{h} P(h) \cdot f_1(h)$

$= P(h) \times f_1(h) + P(-h) \times f_1(-h)$

$= 0.6 \times 0.0922 + 0.4 \times 0.3334$

$= 0.18868$

$P(+e) = P(+u) \cdot P(+e|+u) + P(-u) \cdot P(+e|-u)$

$= P(+u, +e) + P(-u, +e)$

$= 0.18868 + 0.47892$

$= 0.6676$

$P(+u|+e) = \dfrac{P(+u, +e)}{P(+e)} = 0.7112$