Day 5 Practice Problem Solutions

① a) $f(x) = 2x^3 + e^{5x^2} + \dfrac{\ln(3x-2)}{x^2}$

$$\frac{df}{dx} = 6x^2 + e^{5x^2}(10x) + \frac{x^2\left(\frac{1}{3x-2}\right)(3) - \ln(3x-2)(2x)}{x^4}$$

$$\boxed{\frac{df}{dx} = 6x^2 + 10x e^{5x^2} + \left(\frac{\frac{3x^2}{3x-2} - 2x\ln(3x-2)}{x^4}\right)}$$

b) $G(t) = (4e^{6t} + t^2 + \ln(t))^5$

$$\frac{dG}{dt} = 5(4e^{6t} + t^2 + \ln(t))^4 \left(4e^{6t}(6) + 2t + \frac{1}{t}\right)$$

$$\boxed{\frac{dG}{dt} = 5\left(24e^{6t} + 2t + \frac{1}{t}\right)(4e^{6t} + t^2 + \ln(t))^4}$$

② a) $\dfrac{dy}{dx} = 3x^2 + 2x - 4$

$y(x) = \int 3x^2 + 2x - 4 \, dx$

$$\boxed{y(x) = x^3 + x^2 - 4x + C}$$

b) $y(x) = \int 4x(\underbrace{x^2+1}_{u})\,dx$

$du = 2x\,dx$

$2\,du = 4x\,dx$

$\xrightarrow{\text{u-sub}} \int 2u\,du$

$= u^2 + C$

$$\boxed{= (x^2+1)^2 + C}$$

③ $\frac{df}{dx} = 4x-3$; $f(0)=6$

$f(x) = \int 4x-3 \, dx$

$f(x) = \frac{4x^2}{2} - 3x + C = 2x^2 - 3x + C$

@ $x=0, f(x)=6$:

$6 = 2(0)^2 - 3(0) + C$

$C = 6$

particular solution:

$\boxed{f(x) = 2x^2 - 3x + 6}$

④ a) $\int_1^3 (x^2+4x-1)\,dx$

$\frac{x^3}{3} + 2x^2 - x \Big|_1^3 = \left[\frac{(3)^3}{3} + 2(3)^2 - 3\right] - \left[\frac{(1)^3}{3} + 2(1)^2 - 1\right]$

$= [9 + 18 - 3] - [1/3 + 2 - 1]$

$= 24 - [1/3 + 1] = 23 - 1/3 = 22\,2/3$

$\boxed{\text{OR } 68/3}$

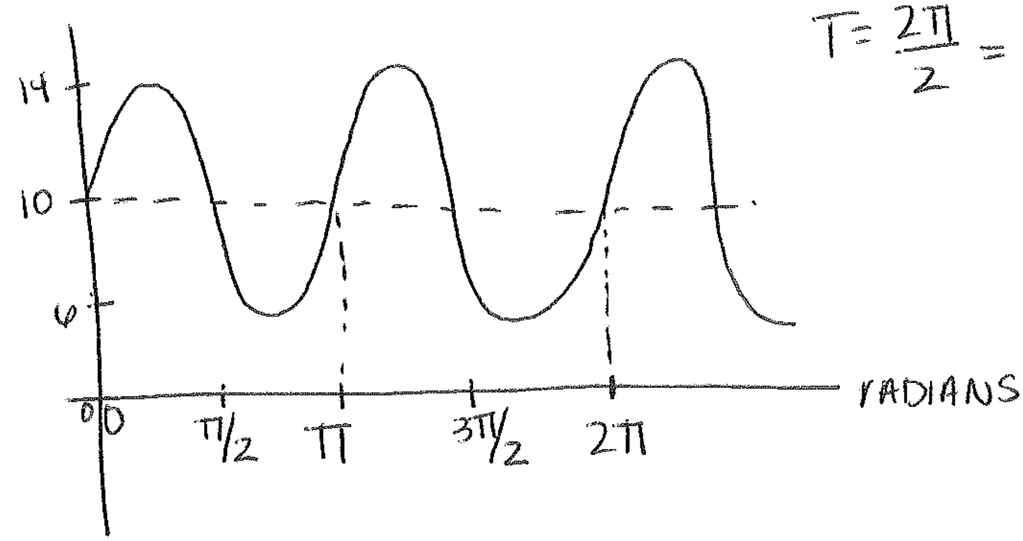
b) $\int_0^4 (5-x)\,dx$

$5x - \frac{x^2}{2}\Big|_0^4 = \left(5(4) - \frac{(4)^2}{2}\right) - \left(5(0) - \frac{(0^2)}{2}\right)$

$= 20 - 16/2 = 20 - 8 = \boxed{12}$

(5) $y = 4\sin(2x) + 10$

$T = \frac{2\pi}{2} = \pi$



(6) $f(x) = x(x-4)^{-5} + 2\sin(x^2+1)$

$$\frac{df}{dx} = x\left(-5(x-4)^{-6}\right) + (x-4)^{-5}(1) + 2\cos(x^2+1)(2x)$$

$$\boxed{\frac{df}{dx} = \frac{-5x}{(x-4)^6} + \frac{1}{(x-4)^5} + 4x\cos(x^2+1)}$$

(7) a) $f(x,y) = 2xy + 6y - 4$

$$\boxed{\frac{\partial f}{\partial x} = 2y \quad \frac{\partial f}{\partial y} = 2x + 6}$$

b) $f(i,j) = 3i^2j - 4i + \ln(j^3+6)$

$$\boxed{\frac{\partial f}{\partial i} = 6ij - 4 \quad \frac{\partial f}{\partial j} = 3i^2 + \frac{1}{j^3+6}(3j^2)}$$

⑧ $$\int_1^3 \underbrace{\int_0^2 (2x+y)\,dx}_{\text{innermost first!}}\,dy$$

$$x^2 + xy \Big|_0^2 = (2^2 + 2y) - 0$$

$$= 4 + 2y$$

next layer out:

$$\int_1^3 4+2y \; dy$$

$$4y + y^2 \Big|_1^3$$

$$= (4(3) + (3)^2) - (4(1) + (1)^2)$$

$$= (12+9) - (4+1)$$

$$= 21 - 5 = \boxed{16}$$