Day 2 Practice Problem Solutions

① $f(x) = -3x^2 + 1$

$$\frac{df}{dx} = \lim_{\Delta x \to 0}\left(\frac{-3(x+\Delta x)^2 + 1 - (-3x^2+1)}{\Delta x}\right)$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0}\left(\frac{-3(x^2 + 2x\Delta x + \Delta x^2) + 1 + 3x^2 - 1}{\Delta x}\right)$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0}\left(\frac{\cancel{-3x^2} - 6x\Delta x - 3\Delta x^2 \cancel{+1} \cancel{+3x^2} \cancel{-1}}{\Delta x}\right)$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0}\left(\frac{-6x\Delta x - 3\Delta x^2}{\Delta x}\right)$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(-6x - 3\Delta x^2)}{\cancel{\Delta x}}$$

$$\boxed{\frac{df}{dx} = -6x}$$

② a) $f(z) = 2z^4 + 6z - 3$

$$\boxed{\frac{df}{dz} = 8z^3 + 6}$$

b) $f(x) = \dfrac{(-3x^2+1)}{(4x-2x^2)}$

$$\boxed{\frac{df}{dx} = \frac{(4x-2x^2)(-6x) - (-3x^2+1)(4-4x)}{(4x-2x^2)^2}}$$

... can simplify ...

c) $f(x) = (2x+4)(x^3 - 5x)$

$$\boxed{\frac{df}{dx} = (2)(x^3-5x) + (3x^2-5)(2x+4)}$$

d) $f(b) = b + (3b^2-2)^4$

$$\frac{df}{db} = 4(3b^2-2)^3(6b) = \boxed{24b(3b^2-2)^3}$$

e) $f(x) = -4(x^2+8)^2$

$$\frac{df}{dx} = -8(x^2+8)^1(2x)$$

$$\boxed{\frac{df}{dx} = -16x(x^2+8)}$$

③ $f(x) = 3x^2 - (x+1)^3$

$$\frac{df}{dx} = 6x - 3(x+1)^2$$

@ $x = 2$, $\frac{df}{dx} = 6(2) - 3(2+1)^2$

$$= 12 - 3(3)^2$$

$$= 12 - 3(9) = 12 - 27 = \boxed{-15}$$

④ $f(x) = -3x^2 + 18x + 4$

$$\frac{df}{dx} = -6x + 18$$

@ maximum, $\frac{df}{dx} = 0$

$$-6x + 18 = 0$$

$$18/6 = 6x/6$$

$$\boxed{x = 3}$$

⑤ $f(y) = 4y^2 - 16y + 12.5$

$$\frac{df}{dy} = 8y - 16$$

$$8y - 16 = 0$$

$$8y = 16$$

minimum at: $\boxed{y = 2}$

(6) Find the second derivative:

a) $f(x) = -5x^4 + 2x$

$f'(x) = -20x^3 + 2$

$\boxed{f''(x) = -60x^2}$

b) $f(a) = 2(a^5+7)^3 - 9a$

$f'(a) = 6(a^5+7)^2(5a^4) - 9$

$f''(a) = \frac{d}{da}\left(\left[30a^4(a^5+7)^2\right] - 9\right)$

$f''(a) = 30a^4(2(a^5+7)(5a^4)) + (a^5+7)^2(120a^3)$

$$\boxed{f''(a) = 300a^8(a^5+7) + 120a^3(a^5+7)^2}$$