THE MOST IMPORTANT THING IN MATH: YOU CAN REWRITE EQUATIONS HOWEVER YOU WANT, BUT YOU HAVE TO DO THE **SAME THING** TO BOTH SIDES OF THE EQUATION!

The Natural Exponential and Natural Log:

$$e \approx 2.718$$

Exponential Growth:

$$f(t) = Ce^{kt}$$

$$e^{b} = x$$

$$\log_e(e^{b}) = \log_e(x)$$

$$b = \log_e(x) = \ln(x)$$

Natural Log Properties:

1. $\ln(e^{x}) = x$
2. $e^{\ln(x)} = x$
3. $\ln(xy) = \ln x + \ln y$
4. $\ln\left(\dfrac{x}{y}\right) = \ln(x) - \ln(y)$
5. $\ln(x^{n}) = n\ln x$

Derivatives: Find Instantaneous Rates of Change (Slope of the Tangent at any point on a function!

"The long way:"

$$m = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The derivative gives you the slope at any point on a continuous function. When the x-axis is time, this is a rate.

The Power Rule: $\dfrac{d}{dx}[x^{n}] = nx^{n-1}$ $\xrightarrow{\text{example}}$ $\dfrac{d}{dx}[x^{3}] = 3x^{2}$

Addition/Subtraction Rules:

$$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$$

$$\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$$

A critical point, i.e. maximum, minimum or 'saddle', occurs when the first derivative = 0. To find critical points, take the first derivative, set equal to zero, and solve for x(can be multiple solutions). To determine whether maximum, minimum, or neither, find whether slope to left and right of the point is increasing or decreasing!

Product Rule:

$\dfrac{d}{dx}[f(x)\cdot g(x)] = f(x)g'(x) + g(x)f'(x)$ $\xrightarrow{\text{example}}$ $\dfrac{d}{dx}[x^{2}(3x-2)] = x^{2}(3) + (3x-2)(2x)$

Quotient Rule:

$\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^{2}}$ $\xrightarrow{\text{example}}$ $\dfrac{d}{dx}\left[\dfrac{3x}{x^{2}-1}\right] = \dfrac{(x^{2}-1)(3) - 3x(2x)}{[x^{2}-1]^{2}}$

Chain Rule:

$\dfrac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$ $\xrightarrow{\text{example}}$ $\dfrac{d}{dx}[(3x^{2}-1)^{3}] = 3(3x^{2}-1)^{2}(6x)$

Derivatives of e^{x} and ln(x):

if u is a function of x,

$$\frac{d}{dx}e^{u} = e^{u}\frac{du}{dx} \quad \text{and} \quad \frac{d}{dx}\ln(u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}(e^{3x}) = 3e^{3x} \qquad \frac{d}{dx}(\ln(3x^{3}-2x)) = \frac{1}{3x^{3}-2x}(9x-2) = \frac{9x-2}{3x^{3}-2x}$$

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Partial derivatives: If f(x,y) is a function of multiple variables (e.g. x and y), we find the derivative with respect to each variable (treating any others as constants) to find the *partial with respect to that variable*

Function f(x,y)

$$f(x,y) = 2xy + y^{2} - 8$$

Partial with respect to *x*

$$\frac{\partial f}{\partial x} = 2y$$

Partial with respect to *y*

$$\frac{\partial f}{\partial y} = 2x + 2y$$

Integration (Solve Differential Equations, Find AREAS under curves!)

$$\frac{d}{dx}\left[\int f(x)dx\right] = f(x) \longrightarrow$$ Differentiation is the inverse of integration.

$$\int f'(x)dx = f(x) + C \longrightarrow$$ Integration is the inverse of differentiation.

Rules of Integration

1. if k is a constant, $\int kdx = kx + C$
2. if k is a constant, $\int kf(x)dx = k\int f(x)dx$
3. $\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$
4. $\int [f(x)-g(x)]dx = \int f(x)dx - \int g(x)dx$
5. $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq 1 \xrightarrow{\text{example}} \int x^3 dx = \frac{x^4}{4} + C$

Methods of Integration:

The General Power Rule: $\int u^n \frac{du}{dx} dx = \int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq 1$

example $\int 2x(x^2+4)^3 dx = \frac{(x^2+4)^4}{4} + C$

Exponential Functions:

$$\int e^{ax} dx = \frac{e^{ax}}{a} + C \xrightarrow{\text{example}} \int e^{2x} dx = \frac{e^{2x}}{2} + C$$

$$\int e^u \frac{du}{dx} dx = \int e^u du = e^u + C \xrightarrow{\text{example}} \int 3e^{3x} dx = e^{3x} + C$$

Natural Log as an Integral Solution:

$$\int \frac{1}{x} dx = \ln|x| + C \xrightarrow{\text{example}} \int \frac{1}{2x} dx = \frac{1}{2}\ln|x| + C$$

$$\int \frac{du/dx}{u} dx = \int \frac{1}{u} du = \ln|u| + C \xrightarrow{\text{example}} \int \frac{6x}{3x^2-2} dx = \ln|3x^2-2| + C$$

Definite Integral: Area under a curve!

$$\int f(x) = F(x) + C$$

$$\int_a^b f(x)dx = F(a) - F(b) \xrightarrow{\text{example}} \int_2^4 2xdx = x^2\Big|_2^4 = 4^2 - 2^2 = 12$$

In words: the total area under the curve y=2x between x=2 and x=4 is 12.

Multiple Integrals: To find multiple integrals, complete the innermost integral (with respect to the appropriate variable), then take the integral of *that* solution with respect to the appropriate variable

$$\int_1^2 \int_0^3 (2x)\, dx\, dy$$

First (innermost) integral:

$$\int_0^3 2x \ dx = x^2 \,|_0^3 = 3^2 - 0^2 = 9$$

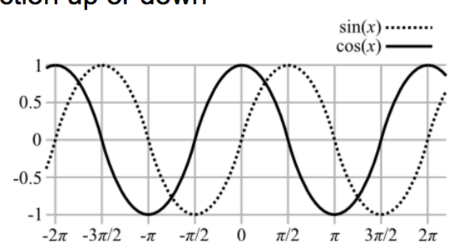
Second (outermost) integral:

$$\int_1^2 9 \ dy = 9y \,|_1^2 = 9 * 2 - 9 * 1 = 9$$

Trigonometry: SOHCAHTOA

2π radians = 360 degrees

f(x) = a sin (bx) + c
f(x) = a cos (bx) + c
a = amplitude, period = 2π/b, c shifts entire function up or down



Derivatives of sine and cosine functions:

if u is a function of x, then:

d/dx (sin u) = cos u * du/dx
d/dx (cos u) = -sin u * du/dx

Examples:
d/dx (2 cos ($4x^2$)) = 2 (-sin ($4x^2$))*(8x)
= -16x sin ($4x^2$)
d/dx(-5 sin (3x)) = -5 cos (3x) * 3
= -15 cos (3x)

General Mass Balance

Accumulation = Input + Generation – Output - Consumption

Steady State

Input + Generation = Output + Consumption

Steady State No Consumption/Generation

Input = Output

Metric Prefixes

Symbol	Prefix	Multiplication Factor	
E	exa	10^{18}	1,000,000,000,000,000,000
P	peta	10^{15}	1,000,000,000,000,000
T	tera	10^{12}	1,000,000,000,000
G	giga	10^{9}	1,000,000,000
M	mega	10^{6}	1,000,000
k	kilo	10^{3}	1,000
h	hecto	10^{2}	100
da	deka	10^{1}	10
d	deci	10^{-1}	0.1
c	centi	10^{-2}	0.01
m	milli	10^{-3}	0.001
μ	micro	10^{-6}	0.000,001
n	nano	10^{-9}	0.000,000,001
p	pico	10^{-12}	0.000,000,000,001
f	femto	10^{-15}	0.000,000,000,000,001
a	atto	10^{-18}	0.000,000,000,000,000,001

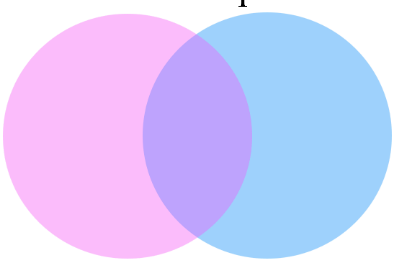
Significant Figures

- All non-zero numbers are significant
- Zeroes before non-zero numbers are NOT significant
- All zeroes between non-zero numbers are significant
- Zeroes after non-zero numbers are only significant if a decimal point is included

- *Addition/Subtraction: Round result to least number of decimal places*
- *Multiplication/Division: Result significant figures Should match lowest number of significant figures in values multiplied or divided*
- *MAKE REASONABLE DECISIONS ABOUT SIG. FIGS. + BE CONSISTENT*

Basic Probability Theory

1. Draw a picture!



Think of these as physical layers.

How many times is each layer accounted for?

2. Define events + probabilities

Event B: I eat a burrito on a Monday

{B} = Burrito on Monday

$P\{B\} = 0.4$

$P\{B^C\} = 0.6$

3. Calculate

Intersection: A and B occur

INTERSECTION: $P\{A \cap B\} = P\{A\} * P\{B\}$

Union: A or B occur (at least one)

UNION: $P\{A \cup B\} = P\{A\} + P\{B\} - P\{A \cap B\}$

Conditional Probability (non-independent events)

$$P\{B|A\} = \frac{P\{A \cap B\}}{P\{A\}}$$

Resources

UCSB Counseling & Psychological Services
http://caps.sa.ucsb.edu/
(805) 893-4411 (available 24 hrs)

Graduate Student Resource Center
http://www.graddiv.ucsb.edu/profdev/home
(805) 893-2277

Non-Traditional Student Resource Center
http://wgse.sa.ucsb.edu/nontrad/
(805) 893-5869

Resource Center for Sexual & Gender Diversity
http://wgse.sa.ucsb.edu/RCSGD/home
(805) 893-5847

CARE Office for Sexual and Gender Based Violence
http://wgse.sa.ucsb.edu/care/
(805) 893-3778

UCSB Health & Wellness
http://wellness.sa.ucsb.edu/
(805) 893-2630

Graduate Student Resource Center
http://www.graddiv.ucsb.edu/profdev/home
(805) 893-2277

Non-Traditional Student Resource Center
http://wgse.sa.ucsb.edu/nontrad/
(805) 893-5869

Graduate Student Association
http://www.gsa.ucsb.edu/
gsavpcommunication@gmail.com

UCSB Multicultural Center
http://mcc.sa.ucsb.edu/
(805) 893-8411