MAS 433: Cryptography

Lecture 13

Message Authentication Code

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Lecture Outline

- Classical ciphers
- Symmetric key encryption
- Hash function and Message Authentication Code
  - Birthday attack
  - Hash function
  - **Message Authentication Code**
    - **CBC-MAC, CMAC**
    - **HMAC**
  - **Unconditionally Secure MAC**
- Public key encryption
- Digital signature
- Key establishment and management
- Introduction to other cryptographic topics

Recommended Reading

- CTP Section 4.4
- HAC Section 9.5
- NIST documents
  - CMAC: http://csrc.nist.gov/publications/nistpubs/800-38B/SP_800-38B.pdf
  - HMAC: http://csrc.nist.gov/publications/fips/fips198/fips-198a.pdf
- Wikipedia
  - Message Authentication Code
    http://en.wikipedia.org/wiki/Message_authentication_code
  - CBC-MAC
    http://en.wikipedia.org/wiki/CBC-MAC
  - CMAC
    http://en.wikipedia.org/wiki/CMAC
  - HMAC
    http://en.wikipedia.org/wiki/HMAC

Message Authentication

- Whether the received message is really sent from the sender
  - Message authentication implies data integrity (whether there is unauthorized modification to the message)

Message Authentication

- Inconvenient to use hash function itself for message authentication
  - A message digest represents only one message (computationally, for strong hash function)
  - But how to authenticate the message digest?

Message Authentication

- Two approaches to authenticate message
  - Symmetric key approach:

    Message Authentication Code

    - Compress a message together with a secret key
    - The person who knows the secret key can verify the authenticity of the message
  - Public key approach: **digital signature**

Message Authentication Code

- Message Authentication Code
  - An algorithm that compresses a **secret key** and a **message** with arbitrary length into a fixed length output (we call this output as message authentication tag, or MAC)
  - Verification: After receiving a message together with the attached authentication tag, repeat the above process to generate an authentication tag, compare it with the tag attached to the message, and check whether the message is sent from the person who posses the same secret key

Note: another frequently used MAC refers to Media Access Control address (MAC address). For example, each network card in a computer has a 48-bit unique identifier (MAC address).

Message Authentication Code

- Two security parameters
  - The size of the secret key
    - Should be large enough to resist brute force attack
  - The size of the message authentication tag (MAC)
    - For *n*-bit MAC, the probability of modifying (or forging) a message without being detected is at least 2^{-n}.
      - The size of MAC should not be too small
      - Normally, the MAC size should be at least 64 bits

Message Authentication Code

- Security requirements on MAC algorithm\
  - Attacker knows messages and their MACs
  - The key should remain secret even if it has been used to generate many MACs
    - Large key size
    - The MAC algorithm should resist other key recovery attacks
  - Without knowing the secret key, any forged or modified message can pass verification with negligible probability
    - An attacker may obtain the MACs of many messages, then try to forge only one message (generate the MAC for a forged message without knowing the secret key)

Message Authentication Code

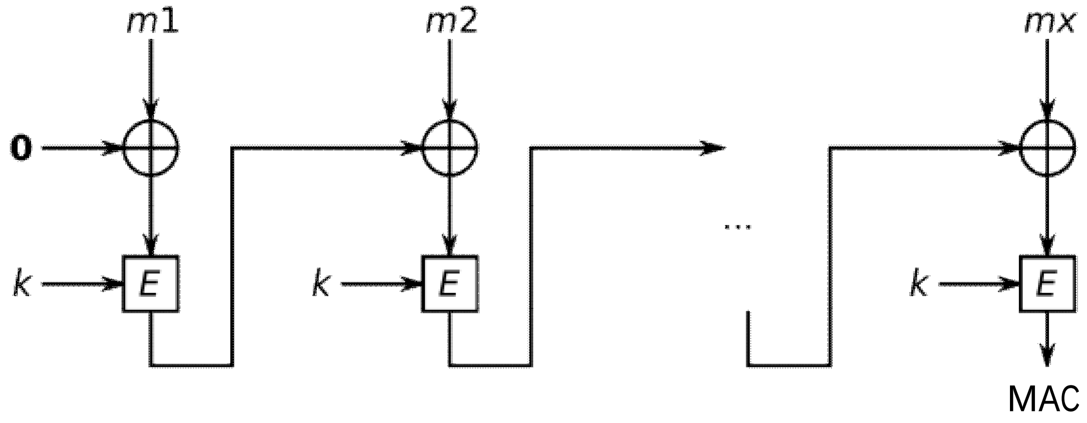
- In general, it is required to use different keys for encryption & authentication
  - Reason: the security of encryption & authentication can be separated
  - It is not advised to use the same key for different purposes
- If both symmetric key encryption & authentication are required
  - It is recommended to encrypt the message, then authenticate the ciphertext
    - So that authentication would not affect the security of encryption

MAC Constructions

- MAC algorithm based on block cipher
  - CBC-MAC
  - CMAC (NIST recommendation)
- MAC algorithm based on hash function
  - HMAC (NIST standard)
- Dedicated MAC algorithm

CBC-MAC

- Construct MAC algorithm from block cipher in CBC mode



- Differences between CBC-MAC and CBC encryption
  - CBC-MAC
    - Authentication, fixed public IV (0), output is the MAC
  - CBC encryption:
    - Encryption, random public IV, output is the ciphertext

CBC-MAC

- CBC-MAC is insecure
  - Attack 1. With the MACs of two messages, the MAC of a new message can be generated without knowing the secret key
    - Example:

$M = m_0 \| m_1$ CBC-MAC(M) = t

$M' = m'_0 \| m'_1 \| m'_2$ CBC-MAC(M') = t'

The new message: $m_0 \| m_1 \| (m'_0 \oplus t) \| m'_1 \| m'_2$

The forged tag: t'

CBC-MAC

- CBC-MAC is insecure
  - Attack 2. After with message length being appended to message, message can still be forged
    - Example:

Let M_1' represents M_1 with length padding

M_2' represents M_2 with length padding

Let $M_3 = M_1' || a_0 || a_1 || a_2$, where each a_i represents one message block.

Suppose now an attacker knows that the MACs of M_1, M_2, M_3 are t_1, t_2, t_3, then an attacker can generate the MAC for the following message without knowing the secret key:

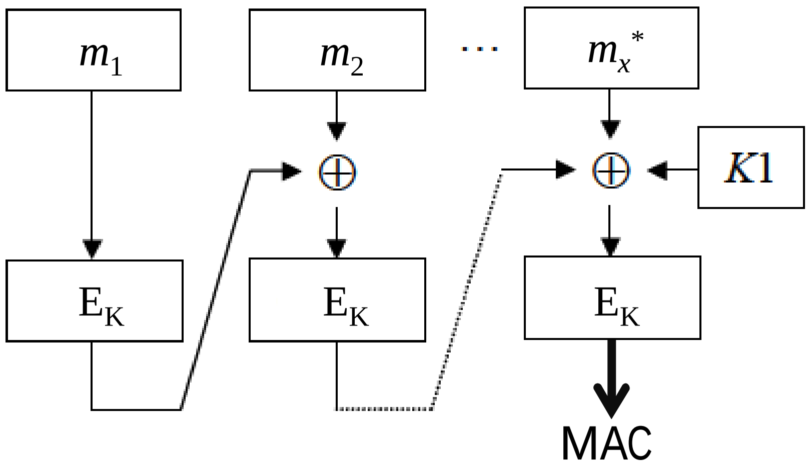
$$M_4 = M_2' || (a_0 \oplus t_1 \oplus t_2) || a_1 || a_2, \quad \text{and the MAC is } t_3$$

CMAC (Cipher-based Message Authentication Code)

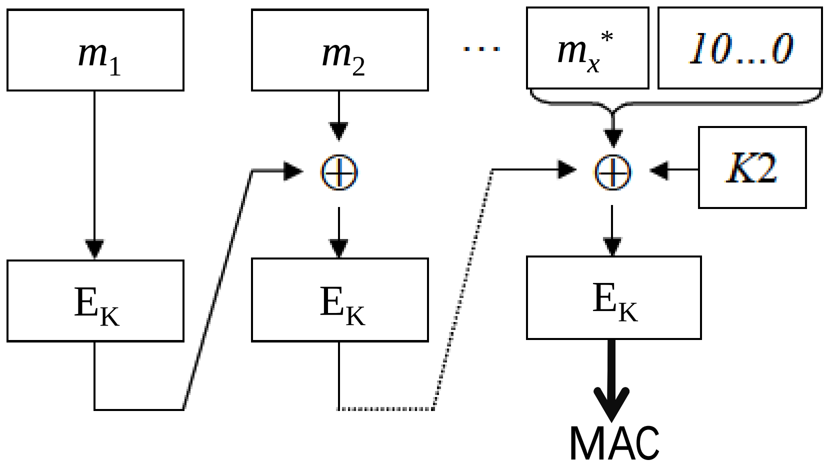
- NIST recommendation
- Strengthen the CBC-MAC
  - Use **an additional key (K_1 or K_2) for the last message block** to thwart the attacks on CBC-MAC
  - The last message block (to eliminate the ambiguity in the last block: full or partial, how ‘partial’)
    - **K_1 is used if it is full block**
    - **K_2 is used if it is partial block**
      - **Pad the partial block by bit ‘1’ followed by some zero bits**
  - Derive K_1 and K_2 from the secret key K
  - If the MAC size is less than the block size of block cipher, truncate some less significant bits

CMAC

- CMAC without considering truncation



No partial message block



with partial message block

CMAC

- Derive K_1 and K_2 from K
  - Let $W = E_K(0)$
  - Represent W as polynomial over GF(2)
    - If the block size of block cipher is 128 bits:
      - $K_1 = W \cdot x \mod (x^{128} + x^7 + x^2 + x + 1)$
      - $K_2 = K_1 \cdot x \mod (x^{128} + x^7 + x^2 + x + 1)$
    - If the block size of block cipher is 64 bits:
      - $K_1 = W \cdot x \mod (x^{64} + x^4 + x^3 + x + 1)$
      - $K_2 = K_1 \cdot x \mod (x^{64} + x^4 + x^3 + x + 1)$

K_1 should not be set as W!

Not a good idea to let K_1 and K_2 linearly dependent

CMAC

- Security of CMAC
  - With $2^{n/2}$ MACs being generated from the same key, if the messages are with certain pattern, an attacker can generate the MAC of a new message without knowing the secret key
    - Example:

Find two messages $M = m_0 \| m_1 \| m_2 \| m_3 \| m_4$

$M' = m'_0 \| m'_1 \| m_2 \| m_3 \| m_4$

with the same MAC t (birthday attack). Then

$$E_K(m_0) \oplus m_1 = E_K(m'_0) \oplus m'_1$$

Now an attacker requests for the MAC of

$M'' = m_0 \| (m_1 \oplus x) \| y$ (for any x and y)

the attacker can forge the new message

$M''' = m'_0 \| (m'_1 \oplus x) \| y$

(the MAC of M''' is the same as that of M'')

HMAC

- The Keyed-Hash Message Authentication Code (HMAC)
  - NIST standard (2002)
  - Construct a secure MAC from a strong hash function

HMAC

- How to construct a secure MAC algorithm from a strong hash function?
  - Method 1: key-prefix
    - The key is prepended to the message, then hash

      $\mathrm{MAC}_K(M) = \mathrm{Hash}(K \parallel M)$

      ('$\parallel$' indicates concatenation)
    - Insecure
      - An attacker can extend the message and generate new MAC for the extended message (if there is no finalization stage in the Merkle-Damgard construction, such as SHA-1, SHA-2).
      - Details: Suppose that $(K \parallel M)$ after padding becomes $(K \parallel M \parallel p)$, and the attacker knows the MAC value $\mathrm{Hash}(K \parallel M \parallel p)$. Then an attacker can compute $\mathrm{Hash}(K \parallel M \parallel p \parallel x)$ for any x due to the iterated nature of has function

HMAC

- How to construct a secure MAC algorithm from a strong function?
  - Method 2: Key-suffix
    - A secret key is appended to the end of message, then hash

      $\mathrm{MAC}_K(M) = \mathrm{Hash}(M \,\|\, K)$
    - Not strong
      - A MAC can be forged with $2^{n/2}$ hash function computations
      - Details:
        - » An attacker can apply birthday attack to find two messages *M* and *M'* (with the same length) satisfying Hash(*M*)= Hash(*M'*)
        - » Then the attacker requests for the MAC of *M*
        - » The attacker immediately knows the MAC of *M'*

          (The forgery is successful, since the people who know the secret key *K* has never generated MAC for *M'*)

HMAC

- How to construct a secure MAC algorithm from a strong function?
  - Method 3: Key-prefix + key-suffix (envelope)
    - A secret key is used as both prefix and suffix, then hash

$$\mathrm{MAC}_K(M) = \mathrm{Hash}(K \parallel M \parallel K)$$

    - Not strong(suppose *n*-bit key, and hash function with *n*-bit message digest)
      - The attacker requests the MACs of more than $2^{n/2}$ messages, then recovers the *n*-bit secret key
        - » Reason: depending on the message length, a secret key may be separated into two parts, and those two parts appear in two compression functions

HMAC

- How to construct a secure MAC algorithm from a strong function?
  - Method 3: key-prefix + key-suffix (envelope)
    - Example: recovering 32 key bits (not required for exam)
      - Suppose that the messages lengths are chosen so that after padding, the last block contains only *n*-32 key bits, i.e., the previous block contains the first 32 bits of the key
      - An attacker requests for the MACs of about $2^{n/2+0.5}$ messages, with difference in the last message block (but the last padded block is the same).
        - » The attacker obtains two collisions MAC(*x*) = MAC(*x'*):
          - one collision happens before compressing the last padded block

            i.e., hash $(K \| x \| K_{32}) = \text{hash}(K \| x' \| K_{32})$

HMAC

- How to construct a secure MAC algorithm from a strong function?
  - Method 3: key-prefix + key-suffix (envelope)
    - Example: recovering 32 key bits (not required for exam) (contd.)
      - Then an attacker requests for MACs of MAC($x \parallel r \parallel y$) and MAC($x' \parallel r \parallel y$), where r is a 32-bit number, and y is a fixed arbitrary message.
        - » For all the 2^{32} values of r, if any particular r satisfying that $\mathrm{MAC}_k(x \parallel r \parallel y) = \mathrm{MAC}_k(x \parallel r \parallel y)$, then we know that this r is equal to the first 32 bits of the key with very high chance.
          - Reason: if hash ($K \parallel x \parallel K_{32}$) = hash ($K \parallel x' \parallel K_{32}$), and if some $r = K_{32}$, then hash ($K \parallel x \parallel r$) = hash ($K \parallel x' \parallel r$)

            => hash ($K \parallel x \parallel r \parallel y \parallel K$) = hash ($K \parallel x' \parallel r \parallel y \parallel K$),

            i.e., $\mathrm{MAC}_k(x \parallel r \parallel y) = \mathrm{MAC}_k(x \parallel r \parallel y)$

HMAC

- How to construct a secure MAC algorithm from a strong function?
  - Method 4: HMAC (NIST standard, FIPS 198a)
    - Suppose: key size less than or equal to message block size
    - Append ‘0’s to the end of key *K* so that the resulting ***K’*** is with one message block length
      - Example: 128-bit key, and 512-bit message block size, then 384 ‘0’ bits are appended to the key
    - Let opad and ipad be two constants with one block size
      - `opad = 0x5c5c5c5c5c5c` …………
      - `ipad = 0x363636363636` …………
    - Compute the MAC as

$$\mathrm{MAC}_K(M) = \underline{\mathrm{Hash}(\,(K' \oplus \mathrm{opad}) \,\|\, \underline{\mathrm{Hash}((K' \oplus \mathrm{ipad}) \,\|\, M)}\,)}$$

HMAC

- Security of HMAC
  - For a hash function with n-bit internal state (the size of H_i is n-bit), if a key is used to generate the MACs of $2^{n/2}$ messages with certain pattern, message can be forged successfully (birthday attack)
    - Similar to (but somehow different from) the security of CMAC

Message Authentication Code

- Repeated trials
  - For n-bit MAC, a forged message & its MAC can pass verification with probability at least 2^{-n}
  - However, if there is no mechanism that stops repeated trials, the chance to forge a message successfully can become higher
    - Example: 1024 trials, the probability that one of the forged messages & their MAC can pass the verification with probability at least 2^{-n+10}

Unconditionally Secure MAC

- Each key is used to generate the authentication tag of only one message
  - Example:

GF(p) with p being a large prime number (say, 64-bit)

```
message:   (m1, m2, m3, m4, …, mn), each mi ∈ GF(p)
key:   (k0, k1, k2, k3, k4, …, kn), each ki ∈ GF(p)
```

The message authentication tag is computed as:

$$t = (k_0 + \sum_{i=1}^{n} m_i \times k_i) \bmod p$$

Security: An attacker with **unlimited** computing power can generate a correct message tag (or modify a message successfully) with probability 1/p

Unconditionally Secure MAC

– Another example:

GF(2^s) with s being reasonably large (say, 64)

```
message:     (m1, m2, m3, m4, …, mn), each mi ∈ GF(2^s)
    key: (k0, k1, k2, k3, k4, …, kn), each ki ∈ GF(2^s)
```

The message authentication tag is computed as:

$$t = k_0 + \sum_{i=1}^{n} m_i \cdot k_i$$

Security: An attacker with **unlimited** computing power can generate a correct message tag (or modify a message successfully) with probability 2^{-s}

Unconditionally Secure MAC

- Unconditionally secure, but not practical
  - How to make it practical?
    - Generate the key for the MAC from a secret key and IV
    - The IV does not repeat

=>But it is no longer ‘unconditionally secure’

Summary

- Message Authentication Code
  - Compresses a secret key and a message into an authentication tag with fixed length
  - MAC based on block cipher
    - CBC-MAC
    - CMAC (NIST recommendation)
  - MAC based on hash function
    - HMAC (NIST standard)
- Unconditionally secure MAC
  - Each key is used only once