MAS 433: Cryptography

Lecture 17

Digital Signature

Wu Hongjun

Lecture Outline

- Classical ciphers
- Symmetric key encryption
- Hash function and Message Authentication Code
- Public key encryption
- Digital signature
  - RSA signature scheme
  - ElGamal signature scheme
  - Digital Signature Standard (DSS)
    - Digital Signature Algorithm (DSA)
    - RSA Digital Signature Algorithm
- Key establishment and management
- Introduction to other cryptographic topics

Recommended Reading

- CTP: Chapter 7
- HAC: Chapter 11
- FIPS 186-3 (2009)
  - Digital Signature Standard (DSS)

    http://csrc.nist.gov/publications/fips/fips186-3/fips_186-3.pdf
- Wikipedia
  - Digital signature

    http://en.wikipedia.org/wiki/Digital_signature
  - ElGamal signature scheme

    http://en.wikipedia.org/wiki/ElGamal_signature_scheme
  - Digital signature algorithm (DSA)

    http://en.wikipedia.org/wiki/Digital_Signature_Algorithm

Digital Signature

- Handwritten signature
  - for personal/object authentication (identification)
    - signature verification at bank for account access
    - signatures on attendance sheet
    - ….
  - for message authentication
    - signing a document, contract
  - for personal authentication & message authentication
    - signing a credit card bill
- Security requirements on signature
  - Difficult to forge a signature
    - As long as the same handwritten signature is fixed (used for many times), theoretically it is **not difficult to forge a handwritten signature**
  - Combining the signature with the document being signed

Digital Signature

- How to authenticate a digital document/ contract?
  - handwritten signature
    - Inconvenient
    - Not strong
      - Easy to forge: copy & paste
      - Difficult to combine the signature together with the document
  - digital signature is needed

Digital Signature

- Digital signature
  - Message authentication with public key cryptosystem
  - Public key being used:
    - Alice signs documents using her private key
    - Everyone can verify Alice's signatures using Alice's public key
  - Strong security, easy to verify
    - **Message authentication**
    - **Non-repudiation**
  - Different from message authentication code
    - Digital signature: public/private key; non-repudiation
    - MAC: symmetric key

Digital Signature

- Digital signature
  - Key generation
    - Public key/private key
  - Signature generation
  - Signature verification

Digital Signature

- Digital signature schemes
  - RSA signature scheme
  - ElGamal signature scheme
  - **Digital Signature Standards (DSS)**
    - Digital Signature Algorithm (DSA)
    - RSA Digital Signature Algorithm
    - Elliptic Curve Digital Signature Algorithm (ECDSA)

RSA Signature Scheme

- Public key: (n, e)
- Private key: d
- Signature generation:

  $s = (H(m))^d \bmod n$ H: hash function
- Signature verification:

  $s^e \bmod n \stackrel{?}{=} H(m)$

Avoid using the same public key and private key for both signature & encryption

RSA Signature Scheme

$$s = (H(m))^d \bmod n$$

- Insecure against multiplicative forgery for typical hash sizes (say, 160-bit)
  - Vulnerable to chosen-message attack
    - a 160-bit random integer is 2^{10}-smooth with prob. about 2^{-40}
    - An attacker generates some messages with "smooth" message digests, then request for the signatures of these messages
    - Then the attacker can forge the signature of other messages
- Padding the message digest before signing
  - To learn later

ElGamal Signature Scheme

- Public key: (p, g, y)
- Private key: x
- Signature generation
  - Choose a random secret k with $\gcd(k, p\text{-}1) = 1$
  - Computer $r = g^k \bmod p$
  - Computer $s = (H(M)\text{-}xr)\, k^{-1} \pmod{p\text{-}1}$
  - If $s = 0$, start over again

$y = g^x \bmod p$

The signature is: (r, s)

ElGamal Signature Scheme

$r = g^k \bmod p$

$s = (H(M) - xr)\, k^{-1} \pmod{p-1}$

- Signature verification
  - $0 < r < p$, $0 < s < p-1$
  - $g^{H(m)} \equiv y^r r^s \pmod{p}$

ElGamal Signature Scheme

$$r = g^k \bmod p$$
$$s = (H(M) - xr)\, k^{-1} \pmod{p-1}$$

- Security
  - use each k only once
  - Otherwise, x can be recovered

Digital Signature Standard (DSS)

- Digital Signature Standard (FIPS 186-3, 2009)
  - **Digital Signature Algorithm (DSA)**
    - Based on discrete logarithm
    - A variant of ElGamal signature algorithm
      - The size of signature of DSA smaller that of Elgamal Signature Scheme
  - **RSA Digital Signature Algorithm**
  - Elliptic Curve Digital Signature Algorithm (ECDSA)

DSS: Digital Signature Algorithm (DSA)

Key generation (Phase 1):

- Choose an N-bit prime q (example: $N = 256$, $L = 3072$)
- Choose an L-bit prime modulus p such that $p–1$ is a multiple of q
- Choose g whose multiplicative order modulo p is q
  - Set $g = h^{(p–1)/q} \bmod p$ for some integer h, then test whether g is 1 or not.
- The algorithm parameters (p, q, g) may be shared between different users of the system

- key length L and N
  - (1024,160)
  - (2048,224)
  - (2048,256)
  - (3072,256)

DSS: Digital Signature Algorithm (DSA)

Key generation (Phase 2):

- Computes private and public keys for a single user:
  - Choose a random secret integer x, where $0 < x < q$
  - Calculate $y = g^x \bmod p$

  Public key: (p, q, g, y)
  Private key: x

.

DSS: Digital Signature Algorithm (DSA)

Signature Generation:

- Generate a random per-message secret integer k where $0 < k < q$
- Calculate $r = (g^k \bmod p) \bmod q$
- Calculate $s = (k^{-1}(H(m) + xr)) \bmod q$
- Recalculate the signature if $r = 0$ or $s = 0$

The signature is: (r, s)

DSS: Digital Signature Algorithm (DSA)

$r = (g^k \bmod p) \bmod q$

$s = (k^{-1}(H(m) + xr)) \bmod q$

Signature Verification:

- $0 < r <$ q and $0 < s < q$
- Calculate $u1 = H(m)s^{-1} \bmod q$
- Calculate $u2 = rs^{-1} \bmod q$
- Calculate $v = ((g^{u1}\ y^{u2}) \bmod p) \bmod q$
- $v \stackrel{?}{=} r$

DSS: Digital Signature Algorithm (DSA)

- Hash function being used in DSS
  - SHA-1
  - SHA-2

DSS: RSA Digital Signature Algorithm

- Message padding is used to resist multiplicative forgery
- Two versions
  - One version specified in ANSI X9.31
    - ANSI: American National Standard Institute (private organization)
  - Another version specified in PKCS#1 v2.1
    - PKCS: Public-Key Cryptography Standard
      - Published by RSA lab

DSS: RSA Digital Signature Algorithm

ANSI X9.31

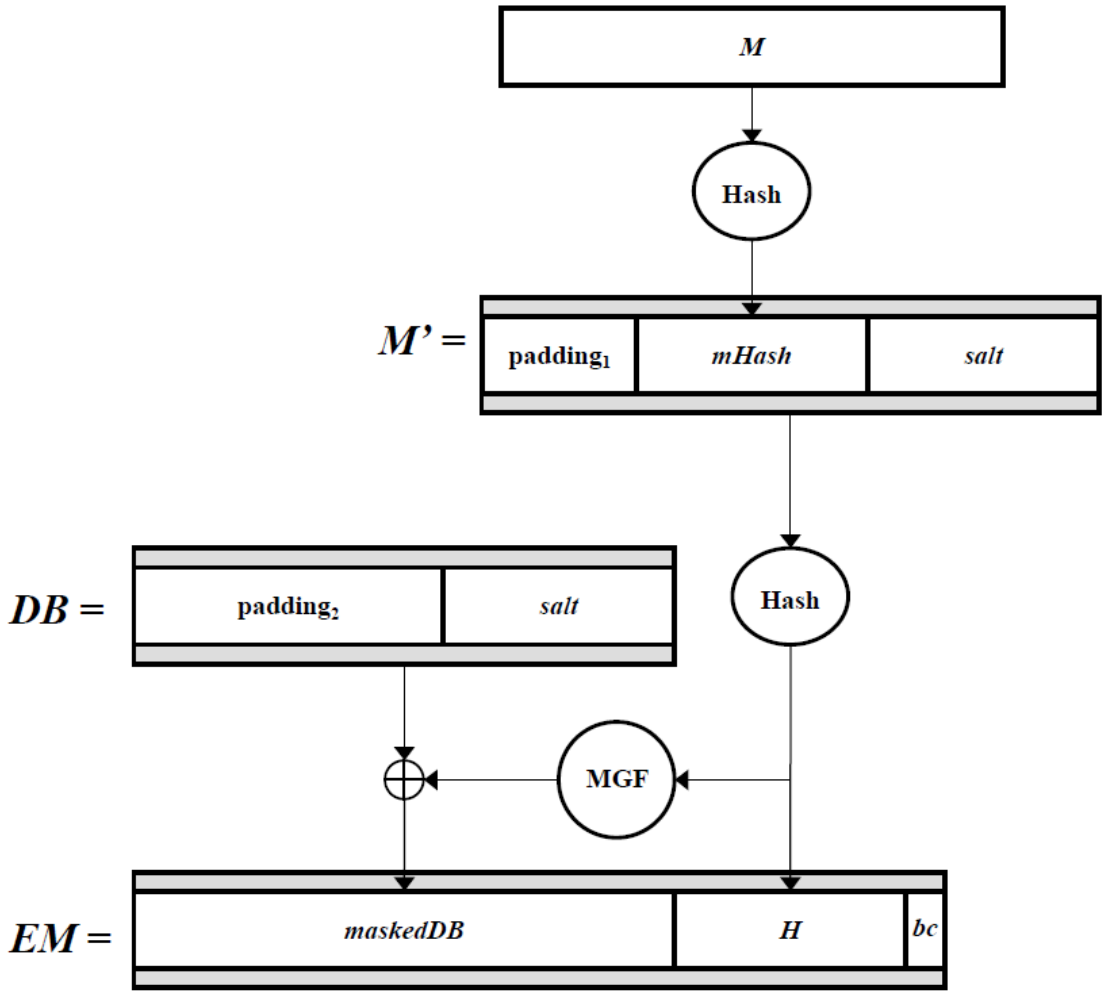
- Message padding:

  6b bb … bb ba || Hash(*M*) || 3x cc

  where $x = 3$ for SHA-1, 1 for RIPEMD-160
- Resistant to multiplicative forgery
- Some moduli are more at risk, but still out of range
- Widely standardized
  - IEEE P1363, ISO/IEC 14888-3
  - FIPS 186-3

DSS: RSA Digital Signature Algorithm

- Padding in RSASSA-PSS in PKCS#1 v2.1 (not required for exam)



Digital Signature & Hash Function

- Hash function is necessary for digital signature
  - Main Reason:
    - Fixed message digest size for signing
  - Another reason:
    - Randomizing the message before signing,
      - Resist forgery attack

Digital Signature & Hash Function

- The security of hash function &
  the security of digital signature
  - Second-preimage resistance
    - Hash function being second-preimage resistance
      => important for using digital signature for authentication
  - Collision resistance
    - Hash function being collision resistance
      => important for the non-repudiation property of digital signature

Applications of Digital Signature

- Authenticate public keys
  - The most widely used application of digital signature
  - Important for resisting impersonation attack

- How to authenticate a public key over internet?
  - Normally a Certificate Authority (CA) is needed
    - The public key of CA is known to every computer
      - Such as in the web browser (IE, Firefox …)
    - CA sign a user's public key + the user's information
    - After receiving a public key of a user (together with the signature generated by CA), use CA's public key to verify the signature to check whether the public key belongs to that user

Applications of Digital Signature

- Authenticate email
  - PGP

```
-----BEGIN PGP SIGNED MESSAGE-----
Hash: SHA1

(content)

-----BEGIN PGP SIGNATURE-----
Version: GnuPG/MacGPG2 v2.0.14 (Darwin)

iEYEARECAAYFAkzbeO0ACgkQPntiHUJ4tFn66wCfZq+6Kj4
oXjPQK3fSp+NmV1m9YLIAnR60gstcXldFNSmVDhAiAOOQnG
o8=o8bb
-----END PGP SIGNATURE-----
```

Applications of Digital Signature

- Digital time stamping
  - A trusted party sign a user's document + current time
  - The signature provides the evidence that the document is available at that time

Applications of Digital Signature

- Authenticate electronic passport
  - The data (passport information + personal particulars: photo, fingerprint) stored in a e-passport is signed by the government.

    The signature is also stored in the e-passport.
  - The public key of the government is given to (local/foreign) customs for passport verification
  - Forgery of electronic passport is virtually impossible
- Sign contract
  - Expected to be widely used in the future

Summary

- Digital Signature
  - Authentication
    - Everyone can verify
  - Schemes
    - RSA signature scheme
      - padding is needed for message digest
    - ElGamal signature scheme
    - Digital Signature Standards
      - Digital Signature Algorithm
      - RSA digital signature algorithm
      - Elliptic curve digital signature algorithm
- Use different keys for digital signature and public key encryption
  - RSA
  - ElGamal
- Application
  - Authenticate digital documents (public key, e-passport …)
  - Signing contract …