MAS 433: Cryptography

Lecture 8

Block Cipher

(Part 5: Attacks on Block Cipher)

Wu Hongjun

Lecture Outline

- Classical ciphers
- Symmetric key encryption
  - One-time pad & information theory
  - Block cipher
    - DES, Double DES, Triple DES
    - AES
    - Modes of Operation
    - **Attacks: double DES, differential cryptanalysis, linear cryptanalysis**
  - Stream cipher
- Hash function and Message Authentication Code
- Public key encryption
- Digital signature
- Key establishment and management
- Introduction to other cryptographic topics

Recommended Reading

- CTP Section 3.3, 3.4
- Wikipedia
  - Meet-in-the-middle attack
    http://en.wikipedia.org/wiki/Meet-in-the-middle_attack
  - Differential cryptanalysis
    http://en.wikipedia.org/wiki/Differential_cryptanalysis
  - Linear cryptanalysis
    http://en.wikipedia.org/wiki/Linear_cryptanalysis

Meet-in-the-middle attack on double DES

- Double DES: $C = E_{K_2}(E_{K_1}(P))$
- Attack (Given a plaintext P and ciphertext C)
  - Re-write the above equation as $E_{K_2}^{-1}(C) = E_{K_1}(P)$
  - Guess all the possible values of K_2, decrypt C and obtain a table T_2 (2^{56} elements, each element is (A_y, K_y))
  - Guess all the possible values of K_1, encrypt P and obtain a table T_1 (2^{56} elements, each element is (B_x, K_x))
  - Now compare those two tables: if $A_i = B_j$, then maybe $K_i = K_2$; $K_j = K_1$

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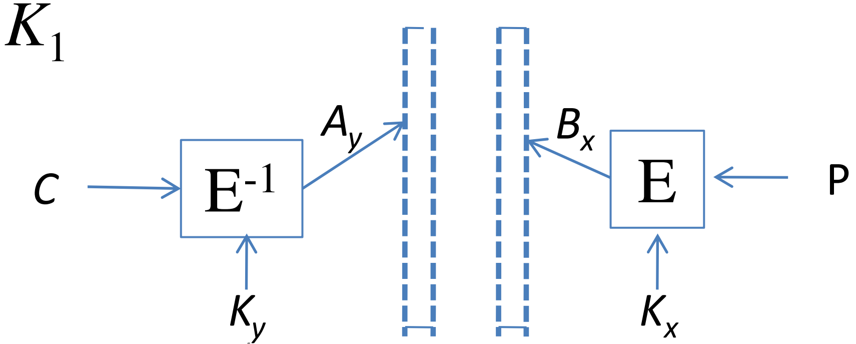
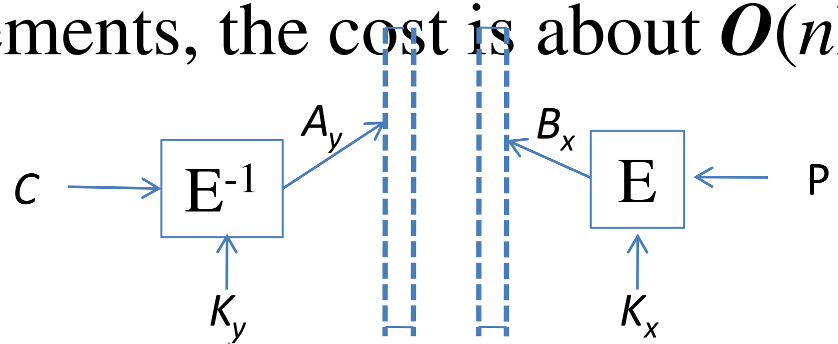


Meet-in-the-middle attack on double DES

- Two tables
  - T_2 (2^{56} elements, each element is (A_y, K_y))
  - T_1 (2^{56} elements, each element is (B_x, K_x))

  How to compare these two tables and find out identical elements, i.e., $A_y = B_x$?
- Sort those two tables first, then compare.
  - Sorting n elements, the cost is about $\boldsymbol{O}(n\log n)$



Meet-in-the-middle attack on double DES

- Two tables
  - T_2 (2^{56} elements, each element is (A_y, K_y))
  - T_1 (2^{56} elements, each element is (B_x, K_x))
- If $A_y = B_x$, what is the probability that

$$K_y = K_2;\ K_x = K_1\ ?$$

  - There are $2^{56} \times 2^{56} = 2^{112}$ (A, B) pairs
  - The chance for two random A, B to be equal is 2^{-64}
  - There are about $2^{112-64} = 2^{48}$ cases that $A_y = B_x$, so we now have 2^{48} possible keys, one of them is correct.
  - Try all these keys with another (P', C') to find the correct key

Cryptanalysis of Block Cipher

Two main approaches:

- Solve algebraic equations
- Statistical approach
  - *Differential cryptanalysis
  - *Linear cryptanalysis
  - ……..

Solve Algebraic Equations

- Algebraic equation
  - Equation over a given field

- Example: two variables, two equations over GF(p)

$$x^2 + xy + y = 16 \mod p$$
$$x^2 + y^2 + x + y = 1 \mod p$$

$$x^2 + xy + y = 16 \quad \bmod\ p$$
$$x^2 + y^2 + x + y = 1 \quad \bmod\ p$$

- How to solve the above equations?
  - Brute force
    - Try all the possible values of x and y
      - Example: $p = 17$, only need to try 17^2 possible values
    - Impractical for many variables/large field
  - Linearization
    - if algebraic equations are over-defined
      (i.e., number of equations > number of variables)

Linearization of Overdefined Algebraic Equations

$$x^2 + xy + y = 16 \mod p \quad (1)$$
$$x^2 + y^2 + x + y = 1 \mod p \quad (2)$$
$$2x^2 + 3xy + y = 0 \mod p \quad (3)$$
$$x^2 + y^2 + 4x + y = 16 \mod p \quad (4)$$
$$x^2 + 2xy + 5y = 11 \mod p \quad (5)$$

let $z_1 = x^2, z_2 = xy, z_3 = y^2, z_4 = x, z_5 = y$

$$z_1 + z_2 + z_5 = 16 \mod p \quad (1)$$
$$z_1 + z_3 + z_4 + z_5 = 1 \mod p \quad (2)$$
$$2 z_1 + 3 z_2 + z_5 = 0 \mod p \quad (3)$$
$$z_1 + z_3 + 4 z_4 + z_5 = 16 \mod p \quad (4)$$
$$z_1 + 2 z_2 + 5 z_5 = 11 \mod p \quad (5)$$

Solve Algebraic Equations

- Two basic operations over GF(2)
  - Addition (XOR)   (in C programming language "^")

    $0+0 = 0$

    $0+1 = 1$

    $1+0 = 1$

    $1+1 = 0$
  - Multiplication (AND) (in C language, "&")

    $0 \cdot 0 = 0$

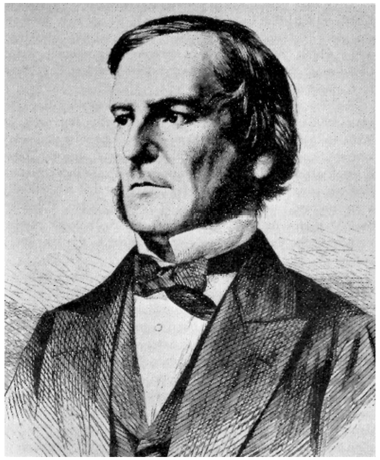
    $0 \cdot 1 = 0$

    $1 \cdot 0 = 0$

    $1 \cdot 1 = 1$

Solve Algebraic Equations

- Basis of digital computer: any digital computation can be carried out as computations over GF(2)
  - George Boole (1815-1864)
  - addition, multiplication, table lookup ….



- How to implement computations over GF(2)
  - Claude Shannon, 1937
    - Electrical relays can be used to construct logic gates to perform computations over GF(2)
    - "possibly the most important and famous master's thesis in the century"
  - Today transistors are used to build the logic gates to perform operations over GF(2)

Solve Algebraic Equations

- Any cipher can be expressed as algebraic equations involving plaintext, ciphertext and key
  - these algebraic equations are normally overdefined (since an attacker may obtain many plaintexts, ciphertexts)

- A natural approach to attack a cipher is to solve those overdefined equations
  - Linearization technique
    - How to defend: increase the number of monomials
      - High algebraic degree
      - Randomness: a lot of random monomials in the equations
  - Other methods
    - Some methods were proposed to attack AES by solving algebraic equations efficiently over GF(2) or GF(2^8), but not recognized (and not verified)
    - Any more ?

Statistical Approach

- Basic idea: to find strong correlation between plaintext & ciphertext, then recover key

- Two basic and powerful techniques
  - Differential cryptanalysis
    - NSA, 19??, kept secret
    - IBM, around 1974--1976, kept secret
    - Eli Biham, 1990
  - Linear cryptanalysis
    - Mitsuru Matsui, 1993

Statistical Approach

- Differential cryptanalysis
  - Basic idea: if the input difference and output difference are statistically strongly correlated, differential attack can be applied!
  - Differential cryptanalysis is a type of chosen-plaintext attack
    - Chosen plaintext attack: the attacker is able to choose some plaintexts and obtain their ciphertexts

Statistical Approach

- Differential cryptanalysis
  - For a particular difference between plaintexts, there are many plaintext pairs with this difference. Now if the distribution of those ciphertext differences are not random, then differential cryptanalysis can be launched.

$$\begin{array}{c} \Delta P = P_i \oplus P_i' \\ \downarrow \\ \boxed{\mathrm{E}} \\ \downarrow \\ \Delta C_i = C_i \oplus C_i' \end{array}$$

ΔP is fixed, P_i is random chosen

For a strong cipher,

ΔC_i should appear with probability about 2^{-n}

Otherwise, insecure.

Statistical Approach

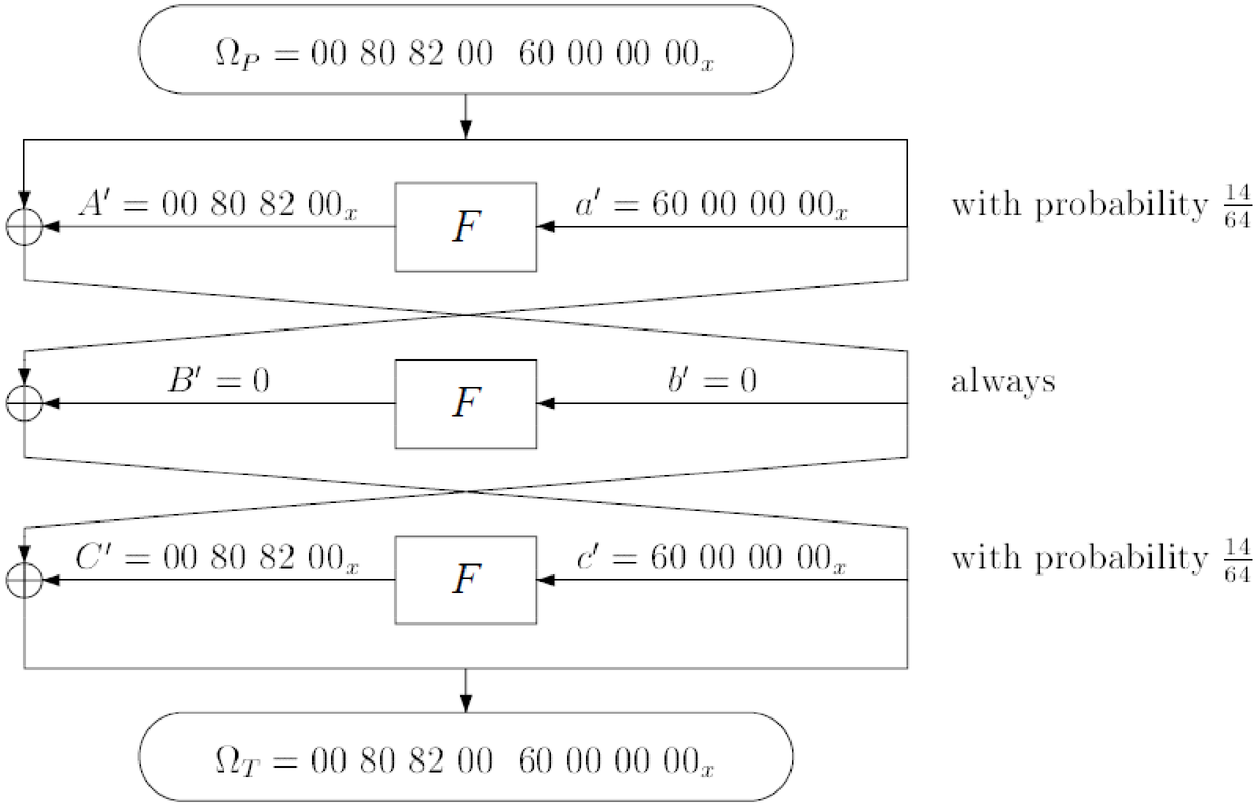
- Differential cryptanalysis
  - Basic steps of differential attack:
    - Suppose that $\Delta P \Rightarrow \Delta C$ with probability $p > 2^{-n}$
    - And suppose that within a cipher, the difference is propagated as follows to achieve the highest prob.:

$$\Delta P \Rightarrow \Delta_1 \Rightarrow \Delta_2 \Rightarrow \Delta_3 \cdots\cdots \Rightarrow \Delta_{r-1} \Rightarrow \Delta C$$

    - After observing 1/p ciphertext pairs,
      - we are expected to find one ciphertext pairs $C_i \oplus C_i' = \Delta C$;
      - then we know that likely for the two plaintexts $P_i \oplus P_i' = \Delta P$, the difference propagates as $\Delta P \Rightarrow \Delta_1 \cdots\cdots \Rightarrow \Delta C$
        - » We are able to attack the first round separately!

Statistical Approach

- Differential cryptanalysis
  - Example: differential propagation for 3-round DES



Statistical Approach

- Differential cryptanalysis
  - DES
    - Designed to resist differential attack
    - 2^{47} chosen plaintexts are required in the attack
  - AES
    - Strong against differential attack

Statistical Approach

- How to resist the differential attack
  - Strong Sbox
    - Reduce the maximal diff. prob. of each Sbox !
    - AES: for the Sbox, the maximal diff. prob. is 2^{-6}
  - Enforce the difference propagation to pass through many Sboxes
    - Diffusion should be properly designed
      - Example: AES
        - » ShiftRows +
          MixColumns(maximum distance separable code)
        - » At least 25 active Sboxes are involved in 4 rounds

Statistical Approach

- Linear cryptanalysis
  - Basic idea: for plaintext and ciphertext, if some input bits and output bits are statistically correlated, linear cryptanalysis may be applied!
  - We can have the following linear approximation equation that involves some plaintext bits, ciphertext bits and key bits:

$$k_a + k_b + \ldots = p_i + p_j + \ldots + c_{i'} + c_{j'} + \ldots \quad \text{with prob. } p = 0.5 + x$$

(*x* is a small value)

    - After collecting enough plaintext-ciphertext ($1/x^2$), we can obtain the equation with high confidence:

$$k_a + k_b + \ldots = 0$$

or $$k_a + k_b + \ldots = 1$$

Another way to solve overdefined nonlinear equations!

    - or you can use the method in the textbook to find other key bits!

Statistical Approach

- Linear cryptanalysis
  - Example:

    DES: 5-round linear approximation with p = 0.519

$$P_H[15] \oplus P_L[7,18,24,27,28,29,30,31] \oplus C_H[15] \oplus C_L[7,18,24,27,28,29,30,31]$$
$$= K_1[42,43,45,46] \oplus K_2[22] \oplus K_4[22] \oplus K_5[42,43,45,46].$$

Statistical Approach

- Linear cryptanalysis
  - DES
    - 2^{43} known plaintexts (not that strong)
  - AES
    - Strong against linear cryptanalysis

Statistical Approach

- Linear cryptanalysis
  - Strong Sbox
    - Let the prob. of linear approximation be close to 0.5!
    - For AES Sbox, the prob. of linear approximation is within 0.5 ± 2^{-3}
  - Enforce the linear approximation to pass through many Sboxes
    - Diffusion should be properly designed
      - AES: at least 25 Sboxes are involved in 4-round linear approximation

Summary

- Meet-in-the-middle attack on double DES

- Attacks on block cipher
  - Solving algebraic equations
  - Statistical approach
    - *Differential cryptanalysis
    - *Linear cryptanalysis
    - ……..

Important for block cipher design: Sbox (confusion), diffusion