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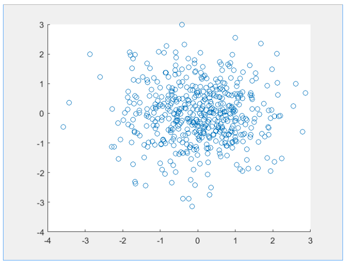
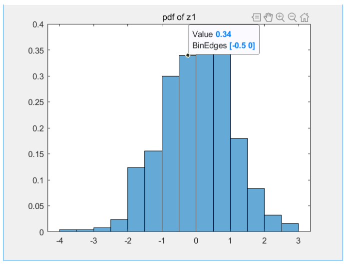
Lab2

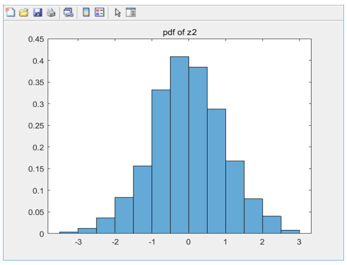
Part I: Use the streams in Lab 1 exercise to convert the generated uniform streams to follow a Gaussian distribution of zero mean and unit variance using the Box-Muller approach:

a. Convert the generated streams to a standard normal distribution
b. Visualize the data samples. Are they normally distributed with mean 0 and 1 variance?
c. Compute the normalized histogram (pdfs) of the data samples from a and visualize the distribution, comment.

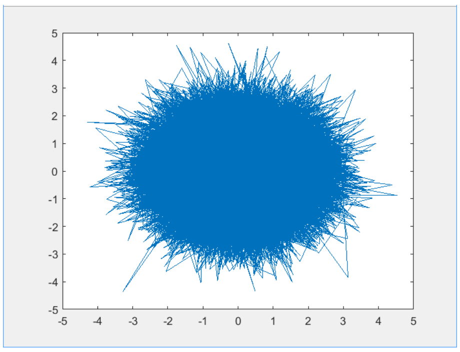
1. For 1D stream, we get z1 and z2 by Box-Muller approach. Then we plot combination of z1, z2 for better visualization, we could see from the pdfs below that they are approximately standard normal distribution. For z1, MuEstimated = 0.0212, CovEstimated =1.0541.

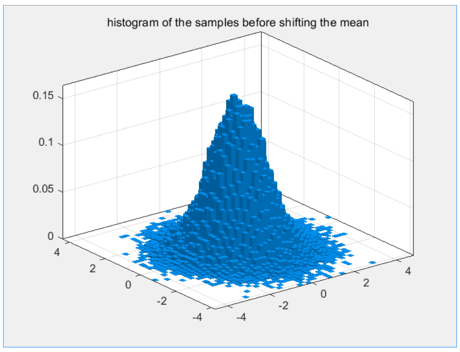






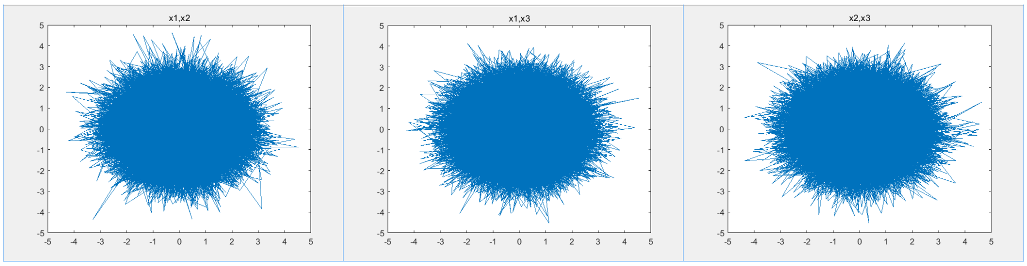
2. For 2D stream, we calculate the standard normal distribution by converting 1D stream separately.





3. For 3D stream, we calculate the standard normal distribution similarly to 2D case. We

visualize by plot 2 dimensions components.



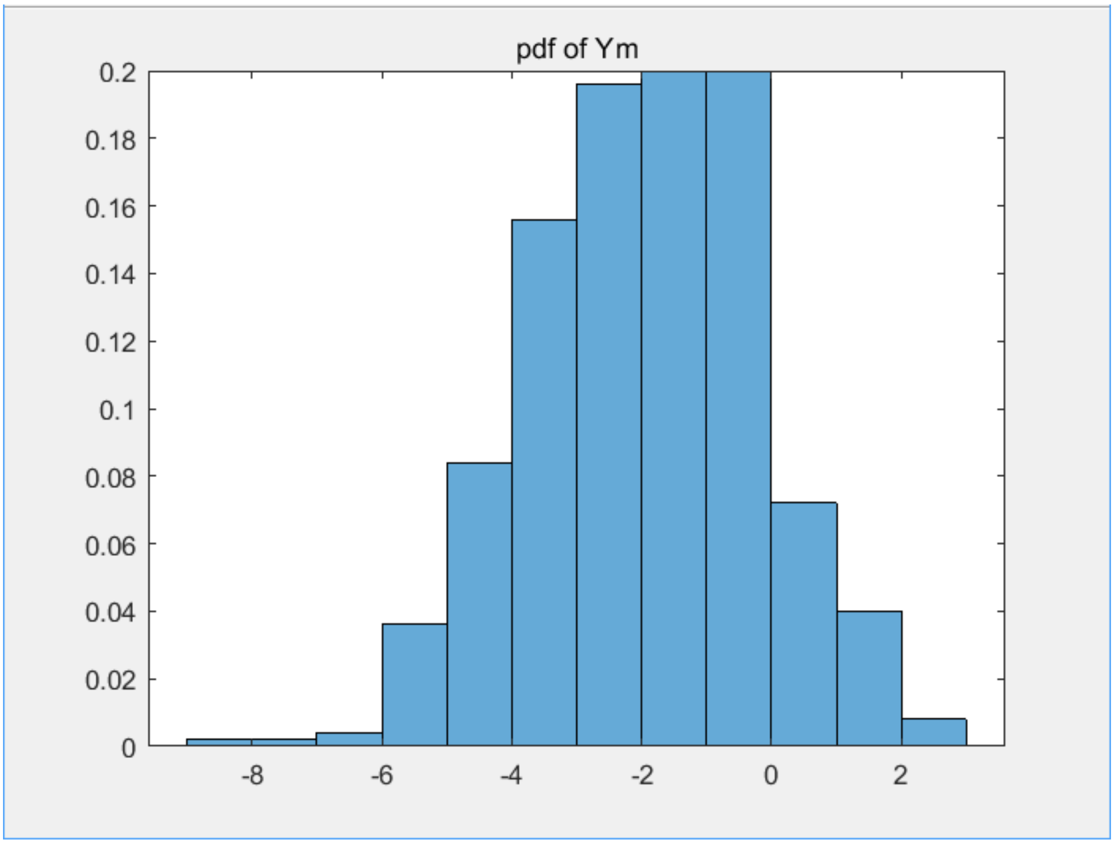
Part II: Use the streams in 1 to do the following:

(a) Convert the 1D normal variate to a normal distribution with mean -2 and standard deviation 3.

We convert the 1D normal variate by

$$Ym = \sqrt{3} * z_1 - 2$$

The pdf of Ym are shown below:



MuEstimatedYm = -1.9632, CovEstimatedYm =3.1624, as desired.

(b) Convert the 2D normal variate to a normal distribution with mean $M = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and covariance matrix $\sum = \begin{bmatrix} 4 & 4 \\ 4 & 9 \end{bmatrix}$.

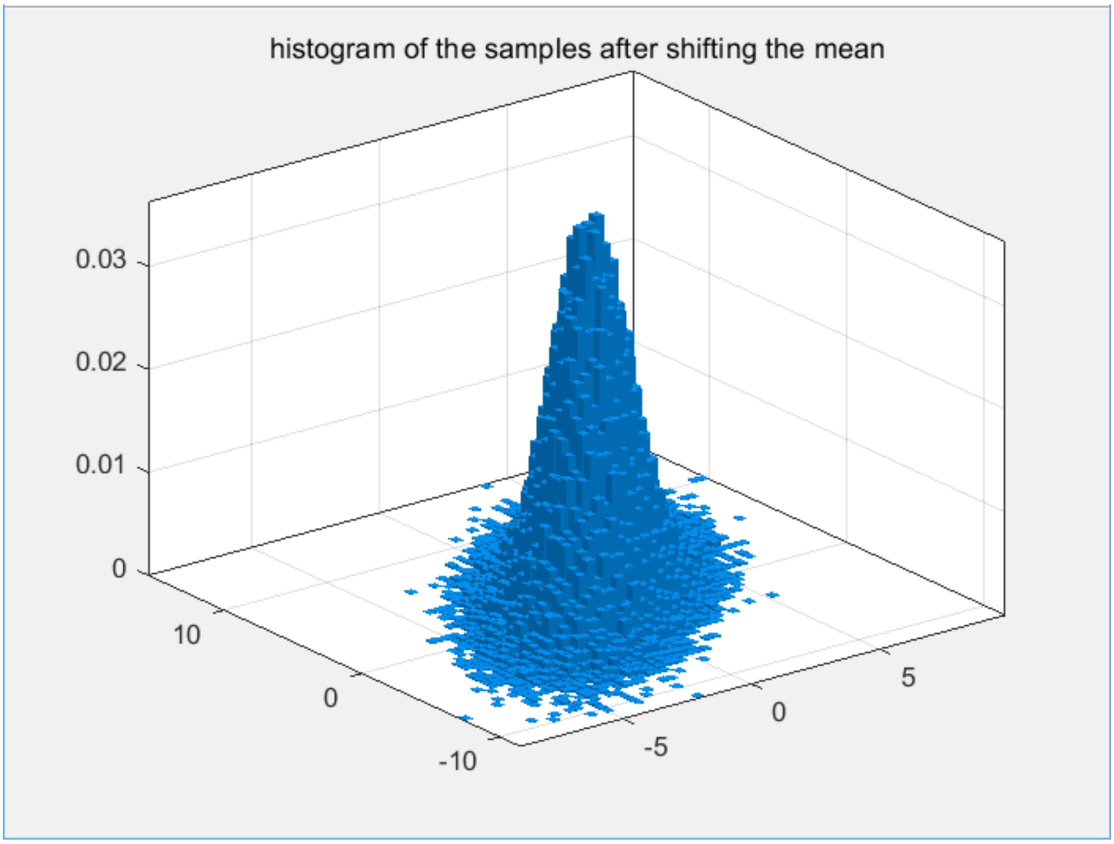
We covert by

$$\mathbf{x} = \mathbf{R}^T\mathbf{z} + \boldsymbol{\mu}$$

$d \times d$ matrix $\mathbf{R}^T\mathbf{R} = \boldsymbol{\Sigma}$

$d \times 1$

$d \times 1 \sim N(0,1)$



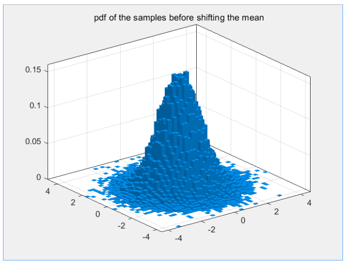
$MuEstimated = \begin{bmatrix} 0.9987 \\ 1.9979 \end{bmatrix}$, covariance matrix $CovEstimated = \begin{bmatrix} 3.9773 & 3.9962 \\ 3.9962 & 9.0066 \end{bmatrix}$.

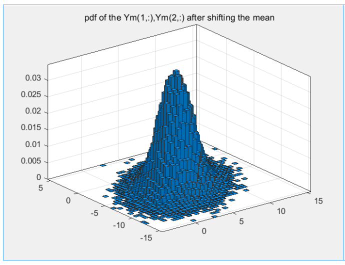
Very similar to the ground truth.

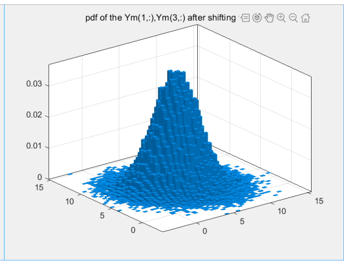
(c) Convert the 3D normal variate to a normal distribution with mean vector

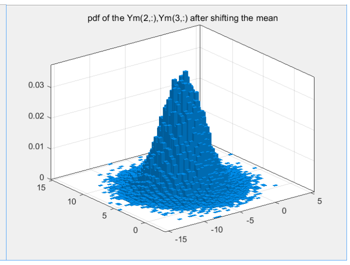
. $M = \begin{bmatrix} 5 \\ -5 \\ 6 \end{bmatrix}$ and covariance matrix $\Sigma = \begin{bmatrix} 5 & 2 & -1 \\ 2 & 5 & 0 \\ -1 & 0 & 4 \end{bmatrix}$.

We convert the 3D variate similar to (b). The pdfs are shown below.









(d) For (a), (b) and (c) estimate the mean and covariance matrix and compare the estimated values with the ground truth ones. Comment.

For 1D, MuEstimatedYm = -1.9632, CovEstimatedYm =3.1624, groud truth is M = -2 $\sigma = 3$.

For 2D, $M = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and covariance matrix $\sum = \begin{bmatrix} 4 & 4 \\ 4 & 9 \end{bmatrix}$.

$MuEstimated = \begin{bmatrix} 0.9987 \\ 1.9979 \end{bmatrix}$, covariance matrix $CovEstimated = \begin{bmatrix} 3.9773 & 3.9962 \\ 3.9962 & 9.0066 \end{bmatrix}$. Very similar to the ground truth.

For 3D . $M = \begin{bmatrix} 5 \\ -5 \\ 6 \end{bmatrix}$ and covariance matrix $\sum = \begin{bmatrix} 5 & 2 & -1 \\ 2 & 5 & 0 \\ -1 & 0 & 4 \end{bmatrix}$ and

$MuEstimated = \begin{bmatrix} 5.0078 \\ -4.9939 \\ 6.0019 \end{bmatrix}$ and covariance matrix $CovEstimated =$ $\begin{bmatrix} 5.0273 & 2.0283 & -0.9966 \\ 2.0283 & 5.0236 & 0.0000 \\ -0.9966 & 0.0000 & 4.0325 \end{bmatrix}$, they are very similar.

We could see the validity of the converting method.