3.36pt

Safe Labeling of Graphs with Minimum Span

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Bangladesh University of Engineering and Technology

September 7, 2019

Outline

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k-safe Labeling of Graph

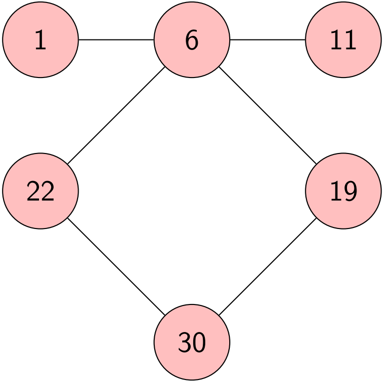
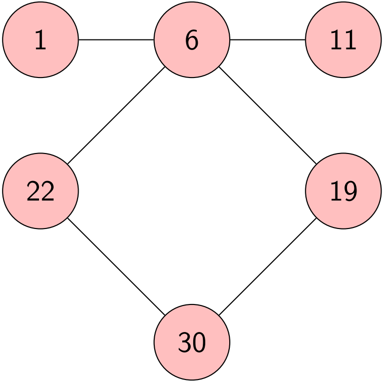


Figure : Safe labeling of graph for k=5

k-safe Labeling of Graph



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- Span of k-safe labeling $= l_l - l_s + 1$

k-safe Labeling of Graph

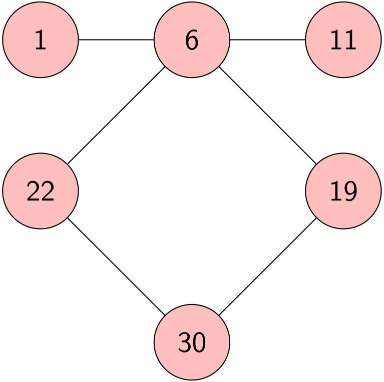
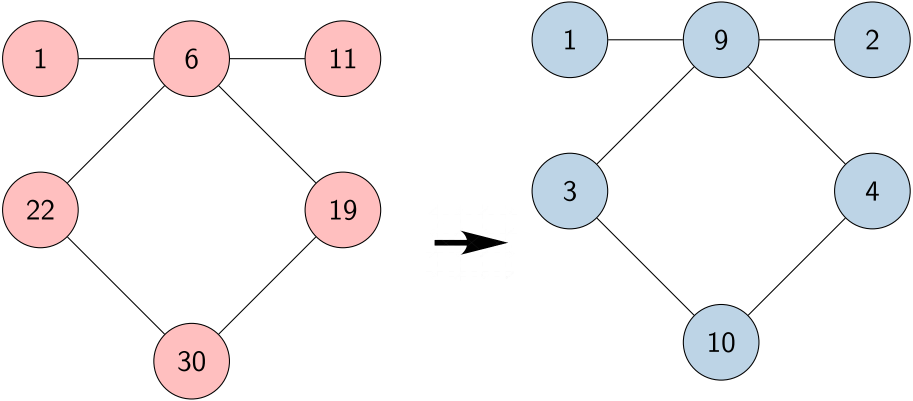


Figure : Safe labeling of graph for k=5

- Span of k-safe labeling $= l_l - l_s + 1$
- For this graph , Span $= 30 - 1 + 1 = 30$

Minimum Span



Span = 30

Span = 10
(Minimum Span)

k-safe Labeling Problem

Problem Statement

The *k-safe labeling problem* asks to find a *k*-safe labeling of a graph with minimum span

What is Antibandwidth problem ?

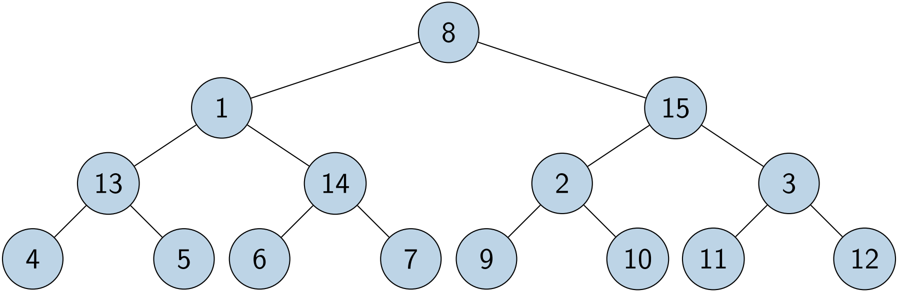


Figure : An Antibandwidth labeling of a tree

What is Antibandwidth problem ?

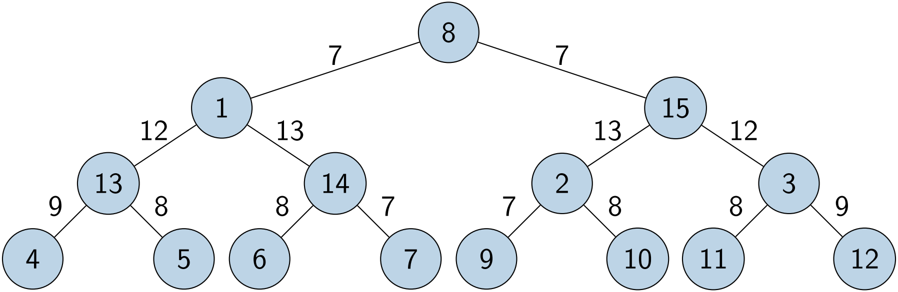


Figure : Calculating difference of adjacent nodes

What is Antibandwidth problem ?

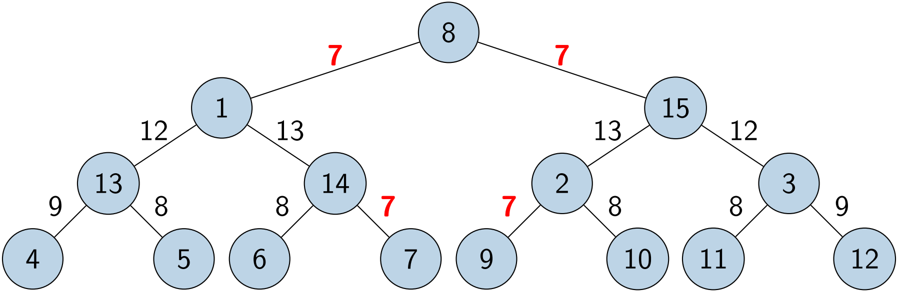


Figure : Labeling of a Tree with Antibandwidth=7

Motivation

Outline

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Hardness of Safe Labeling Problem

k-safe labeling problem is NP-Hard

We can reduce the Antibandwidth problem to k-safe labeling problem and thus successfully prove that k-safe labeling problem is NP-Hard.

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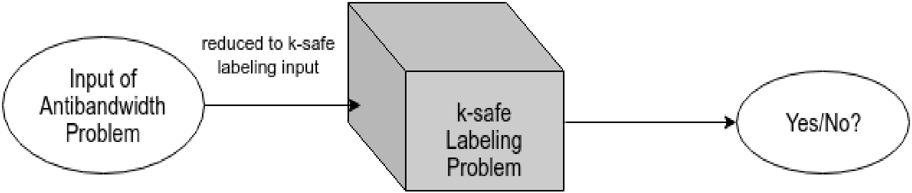
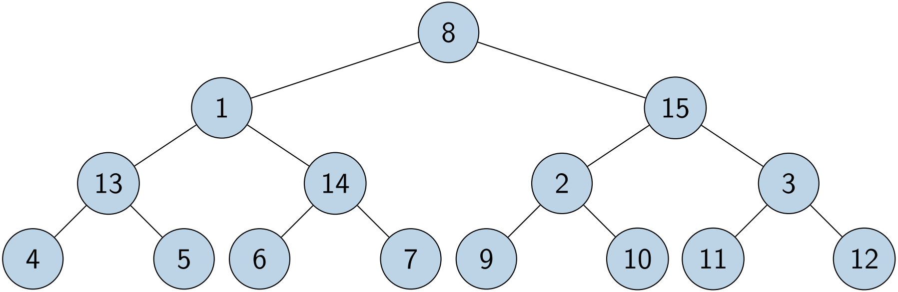
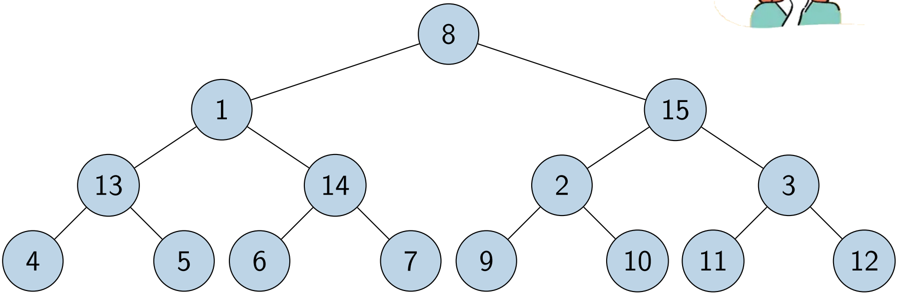


Decision version of Antibandwidth problem



★ An Antibandwidth labeling with $n = 15$ and $k = 7$;

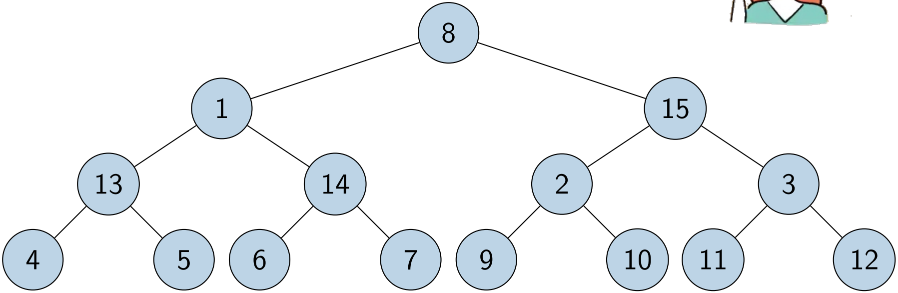
Decision version of Antibandwidth problem



★ An Antibandwidth labeling with $n = 15$ and $k = 7$;

But how can we reduce this to k-safe labeling problem?

Reduction to k-safe Labeling Problem



★ An Antibandwidth labeling with $n = 15$ and $k = 7$;

✓ If we set $s = n = 15$ and $k = 7$

Outline

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What is Bipartiate Graph?

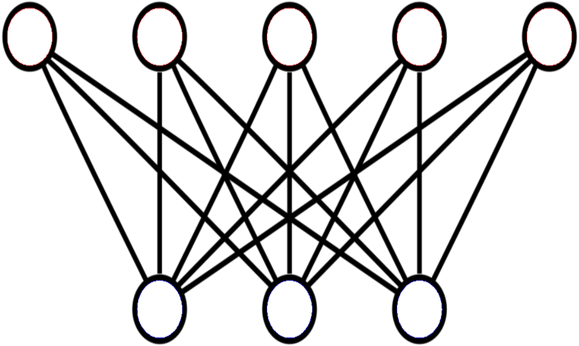


Figure:A Bipartite Graph

Two coloring of a Bipartiate Graph

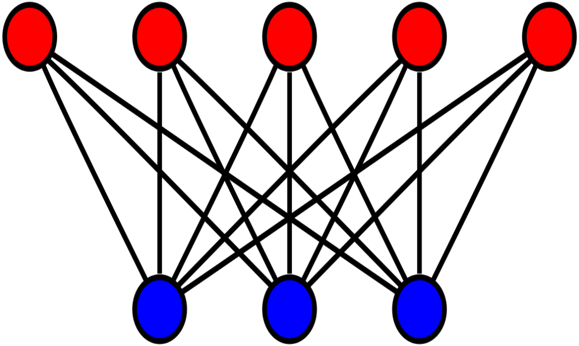
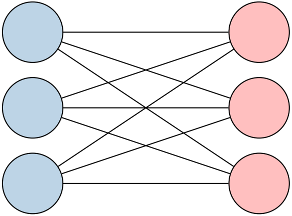


Figure:A Bipartite Graph is a Two-Coloring Graph

Upper Bound on the Span of Bipartite Graphs

k-safe labeling of Bipartite Graphs

A Bipartite Graph of n vertices admits a k-safe labeling of $n+k$-1 , and such a labeling can be computed in linear time.



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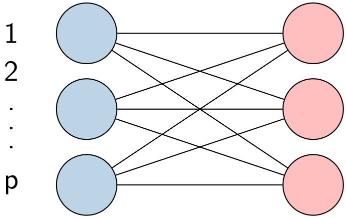

Total vertices $n=p+q$

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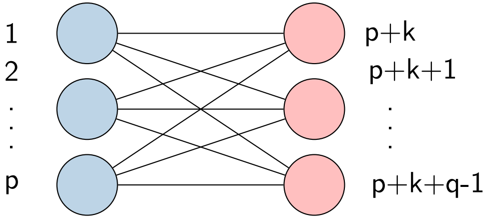

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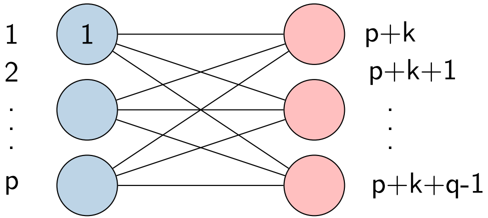

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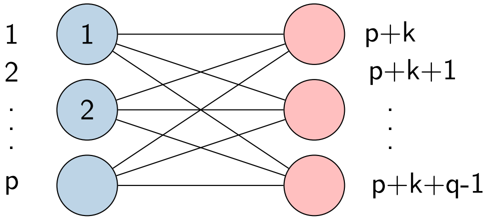

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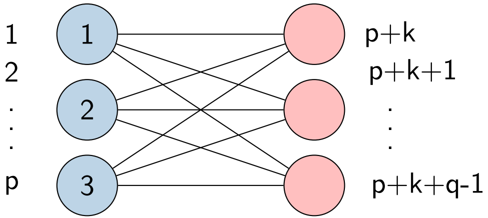

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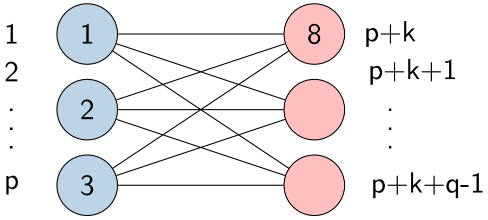

k=5

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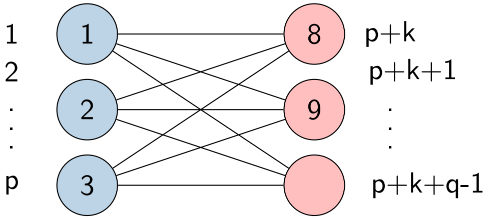

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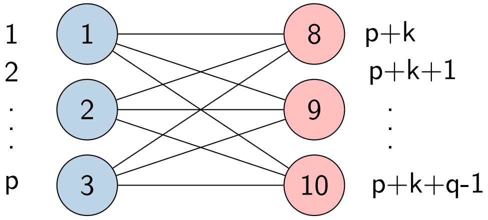

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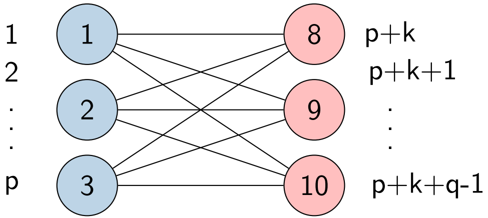

- Minimum distance of labels of two adjacent vertices is at least $p+k-p=k$

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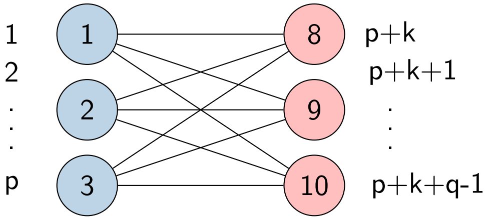

- Minimum distance of labels of two adjacent vertices is at least $p+k-p=k$
- Span $= p+k+q-1=n+k-1$

Outline

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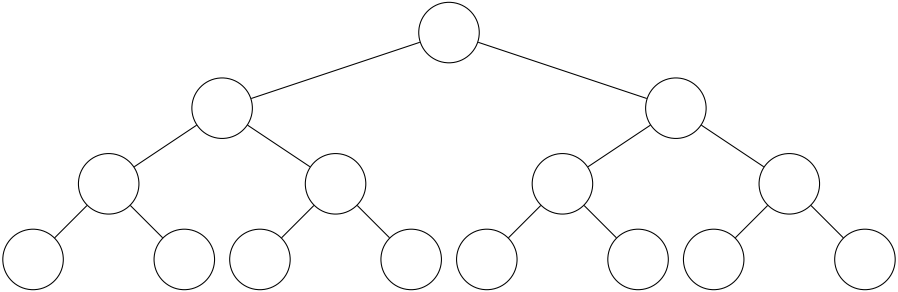
Upper Bound on the Span of Trees

k-safe labeling of Trees

Every Tree with n vertices admits a k-safe labeling of $n+k-1$, and such a labeling can be computed in linear time.



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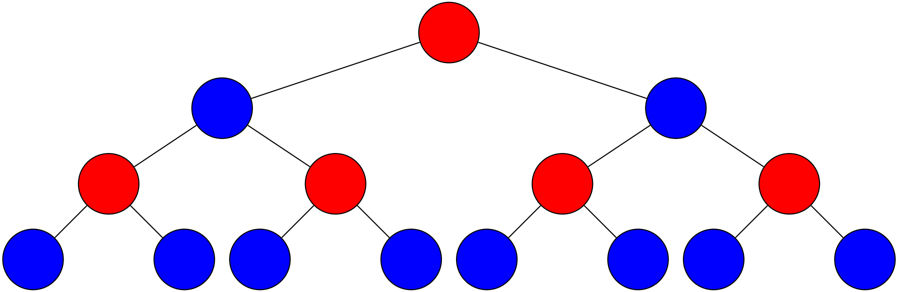

Outline

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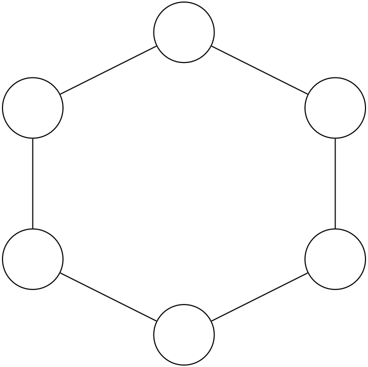
Upper Bound on the Span of Even Cycles

k-safe labeling of Even Cycles

Every Even Cycle with n vertices admits a k-safe labeling of $n+k\text{-}1$, and such a labeling can be computed in linear time.



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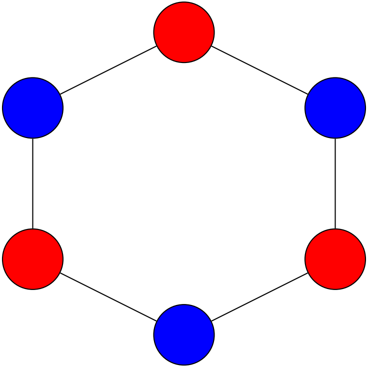

Upper Bound on the Span of Odd Cycles

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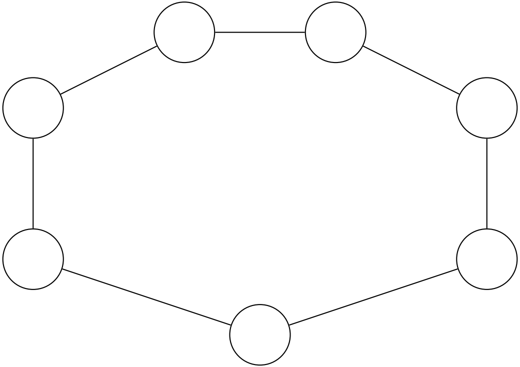
Every Odd Cycle with n vertices admits a k-safe labeling of $n+2k\text{-}2$, and such a labeling can be computed in linear time.



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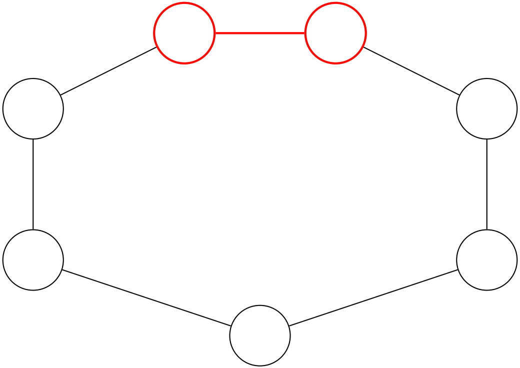

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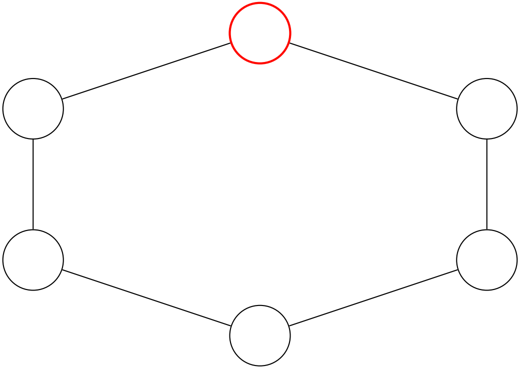

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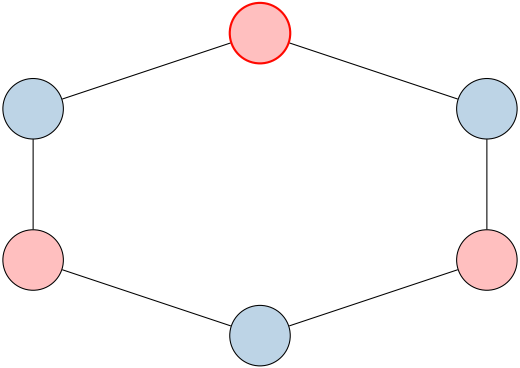

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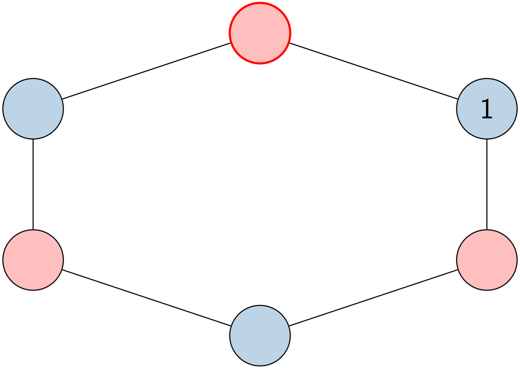

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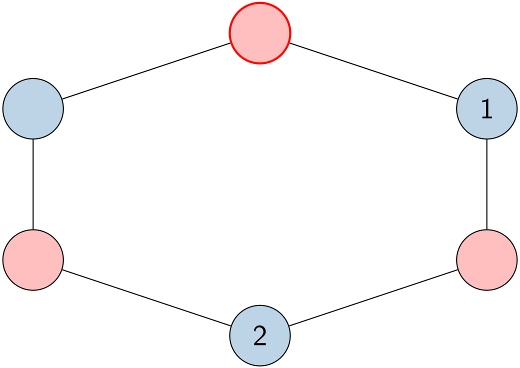

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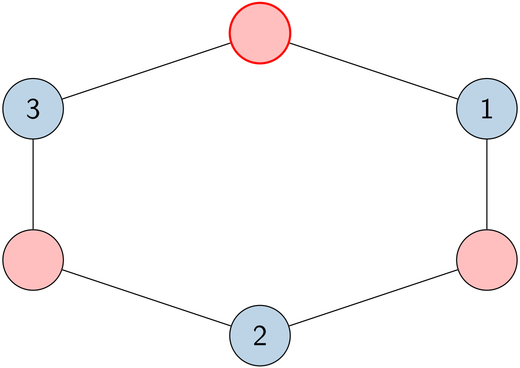

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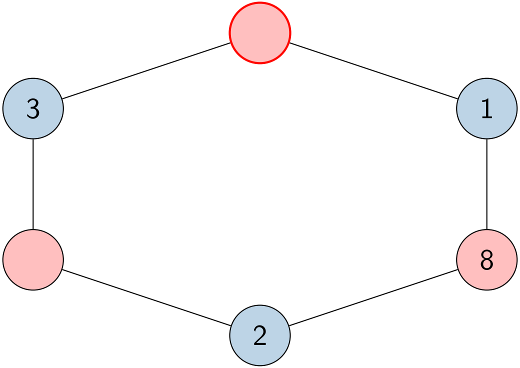

$k=5$

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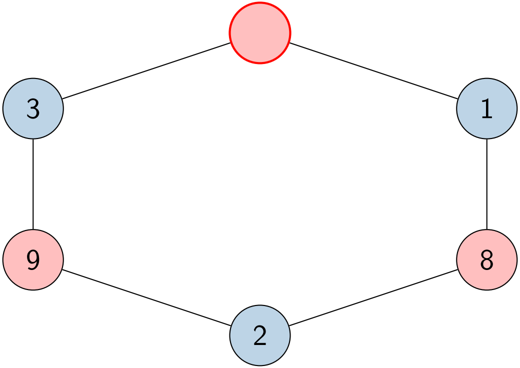

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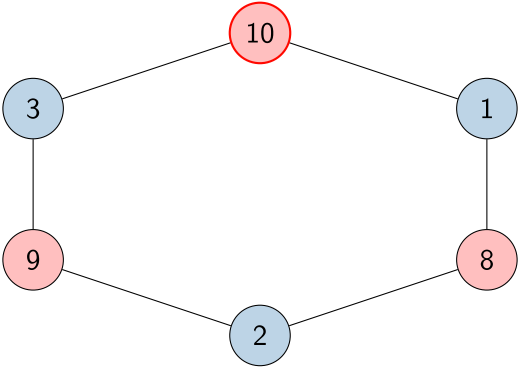

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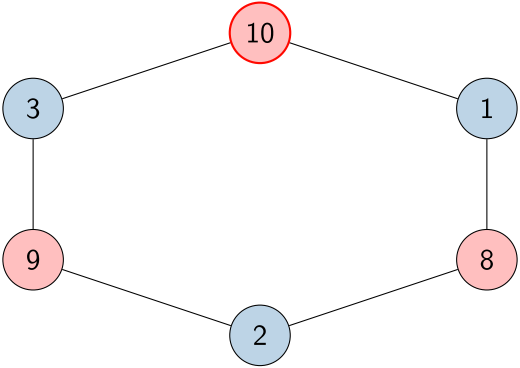

Span = $(n\text{-}1)+k\text{-}1$

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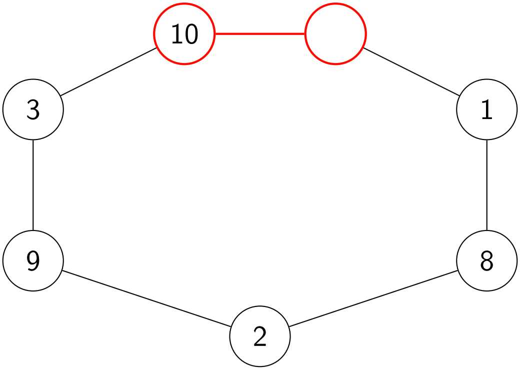

One end is labeled with the label of Unified Vertex $(n\text{-}1)+k\text{-}1 = 10$

Upper Bound on the Span of Odd Cycles

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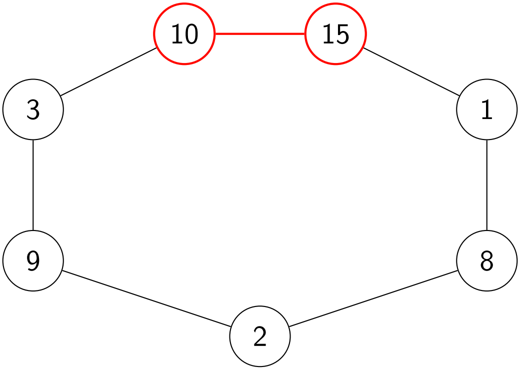

Other end is labeled with $(n\text{-}1)+k\text{-}1+k = 7\text{-}1+5\text{-}1+5 = 15$

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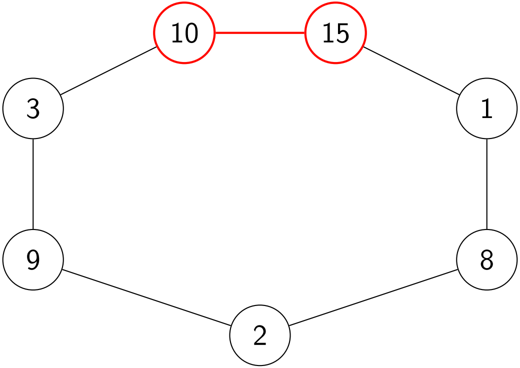

Span is $(n-1)+k-1+k = n+2k-2$

Outline

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What is Biconnected Graph?

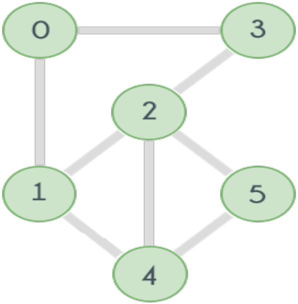

Figure : A Biconnected Graph

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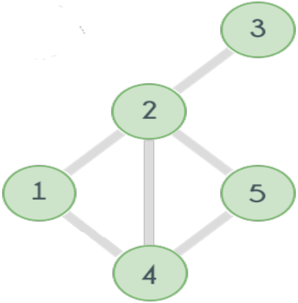

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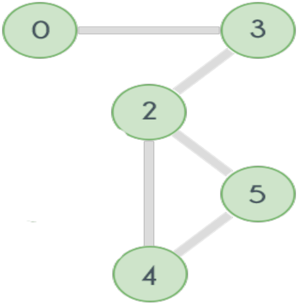

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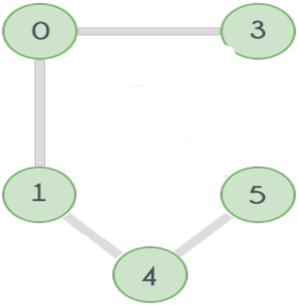

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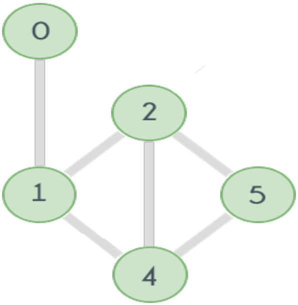

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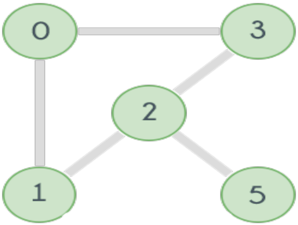

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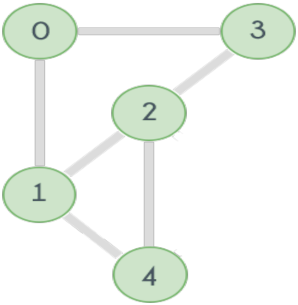
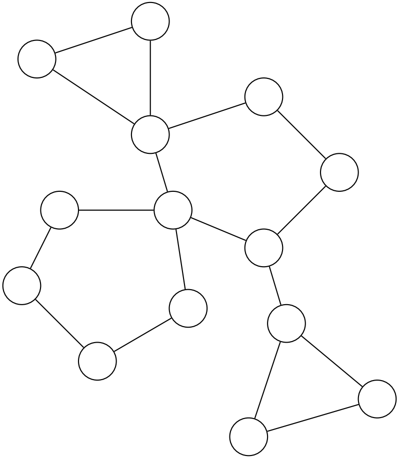


What is Biocnnected Component?



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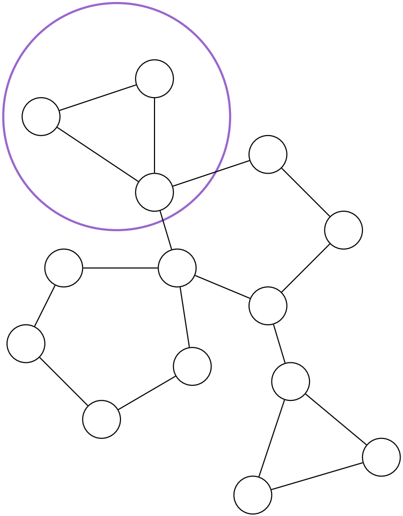

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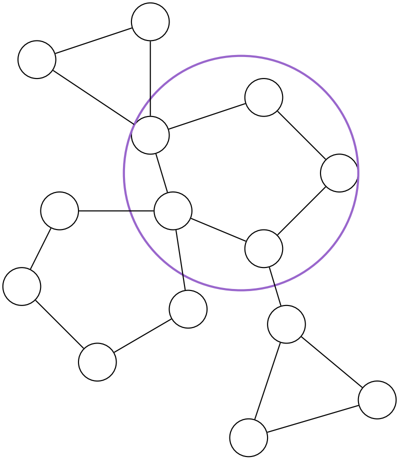

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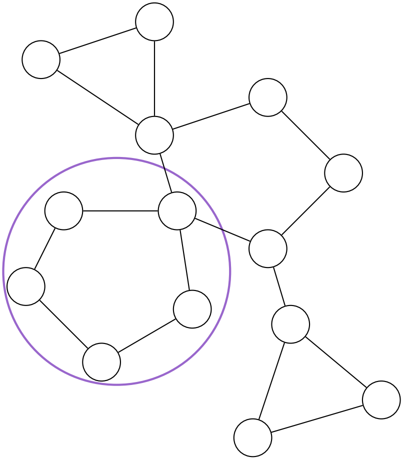

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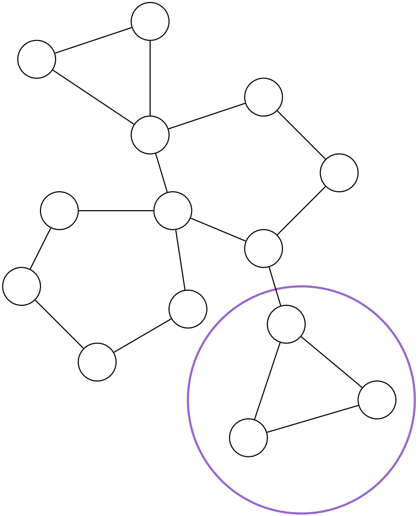

What is Cactus Graph?

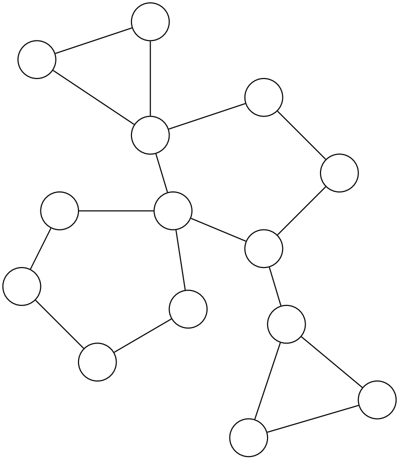

Figure : A Cactus Graph

Upper Bound on the Span of Cactus Graphs

k-safe labeling of Cactus Graphs

Every Cactus Graph with n vertices admits a k-safe labeling of $n+2k-2$, and such a labeling can be computed in linear time.

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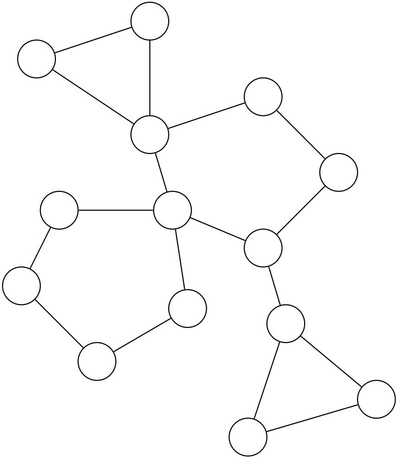

n=14

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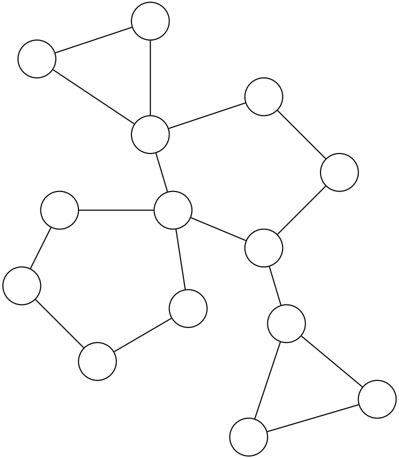

$c=4$

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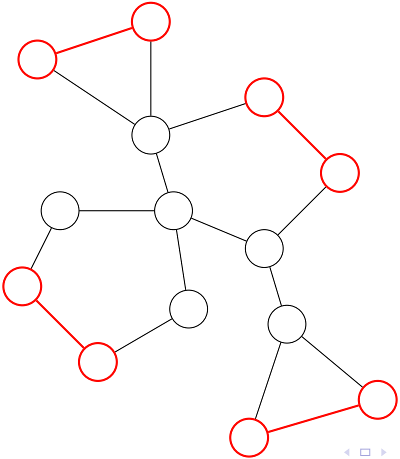

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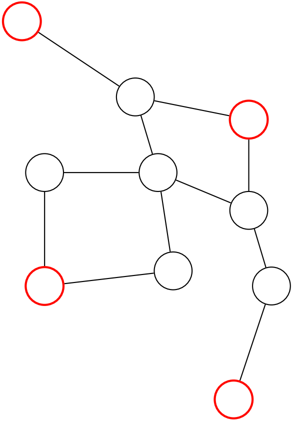

n-c vertices

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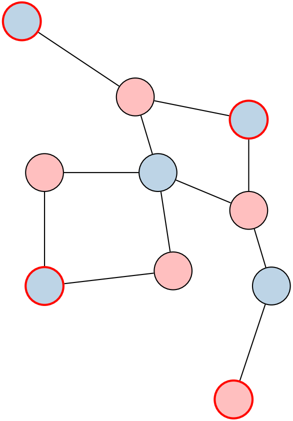

can be labeled from *1* to $(n\text{-}c)+k\text{-}1$

1 to 14 ($k=5$)

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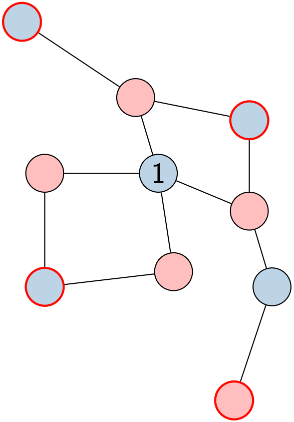

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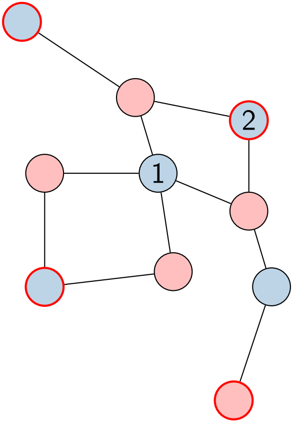

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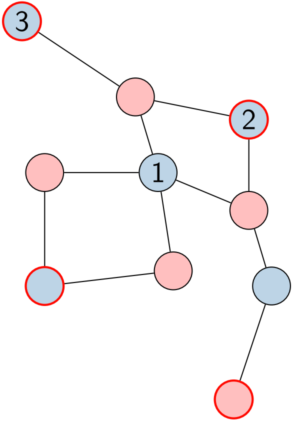

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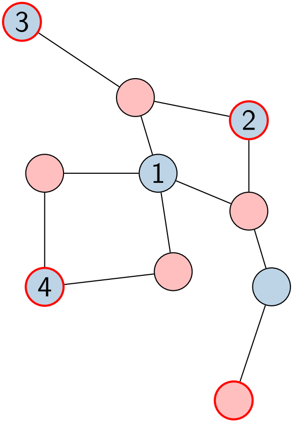

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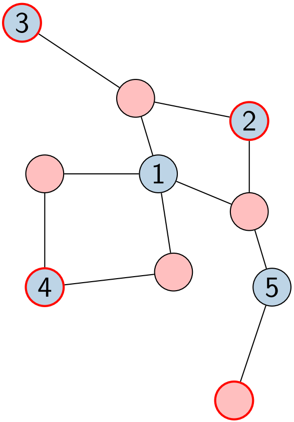

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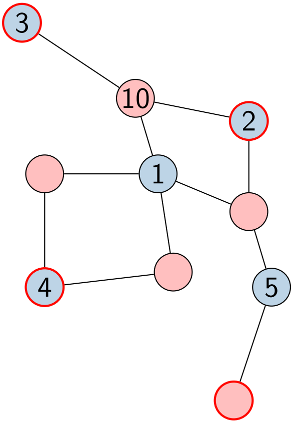

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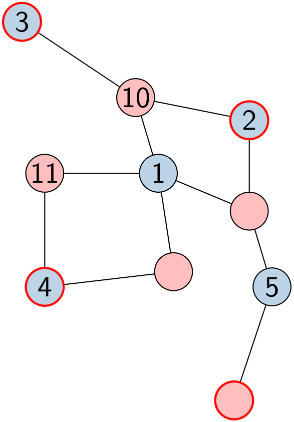

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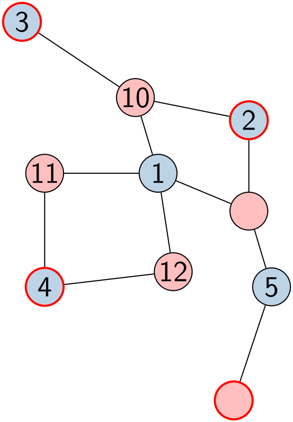

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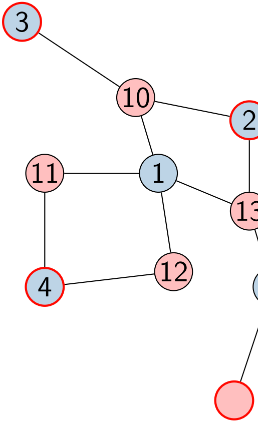

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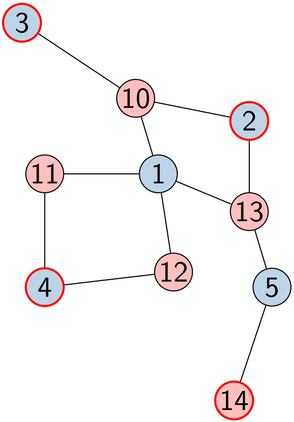

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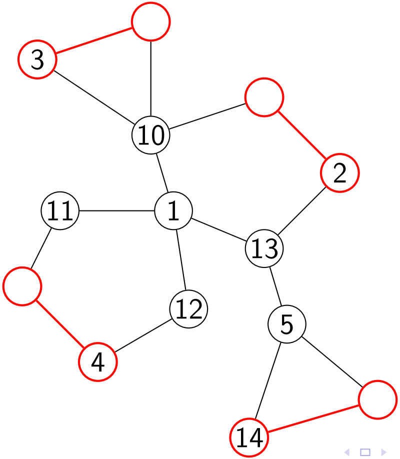

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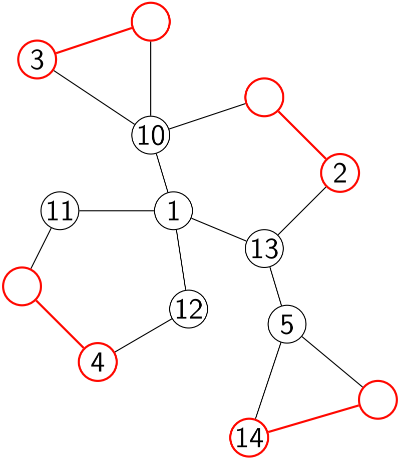

$(n\text{-}c)+k\text{-}1+k$

$(n\text{-}c)+k\text{-}1+k+1$

$(n\text{-}c)+k\text{-}1+k+2$

$\vdots$

$(n\text{-}c)+k\text{-}1+k+(c\text{-}1)$

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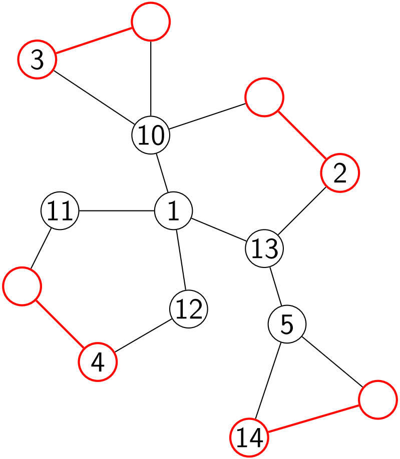

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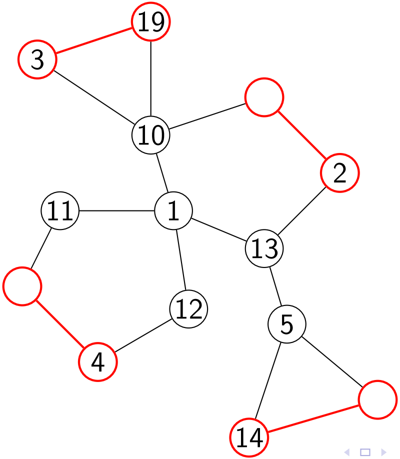

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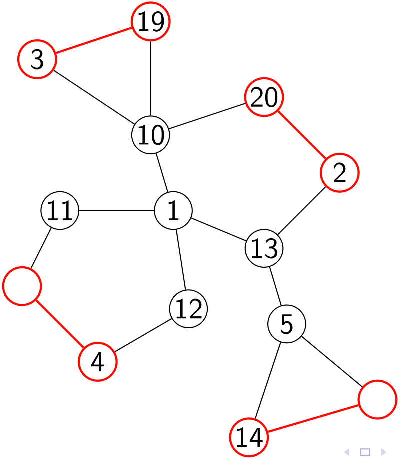

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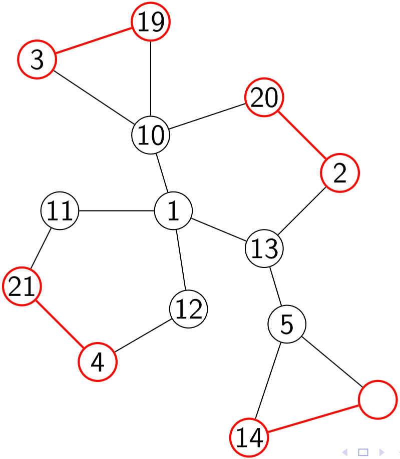

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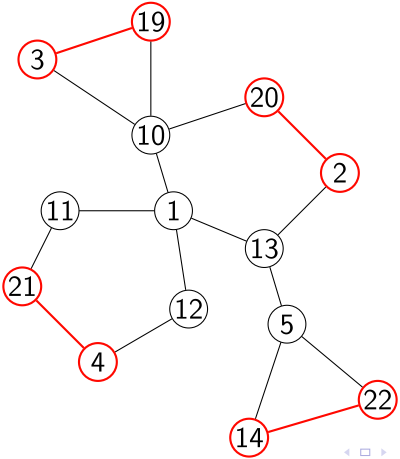

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★ Span is $n-c+k-1+k+c-1 = n+2k-2$

Outline

3.36pt

Future Interests

★ Finding non-trivial bounds for general graphs and other subclasses of graphs.

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★ Proving NP-hardness for subclasses of graphs

Thank You !!!