practical 1 md

qvns53

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My main section

Here's some maths:

$$\log(x^3) \neq \frac{\exp(3x)}{x}$$

Align subsection

Adding maths in an "align" environment makes it easier to line things up.

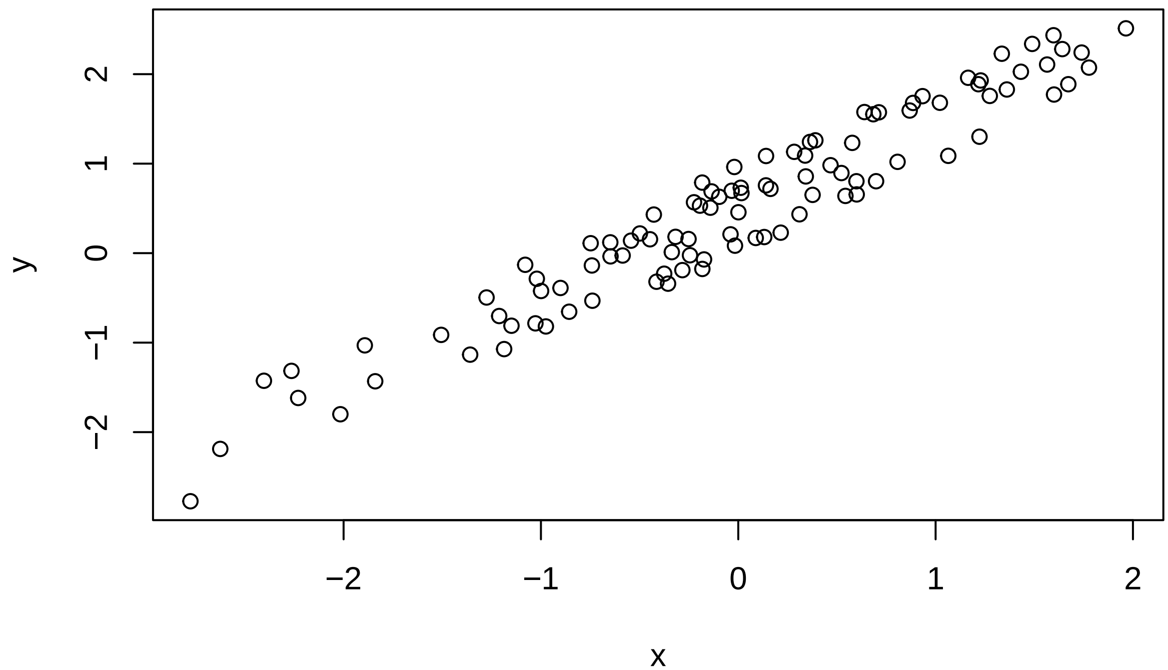
$$\begin{aligned}
\mu &\sim \mathrm{N}(0, 1), \\
X_i|\mu &\sim \mathrm{N}(\mu, 1), \quad i = 1, \ldots, n.
\end{aligned}$$

R section

Here's some very simple R code.

```
x <- rnorm(100)
y <- runif(100) + x
```

```
plot(x, y)
```

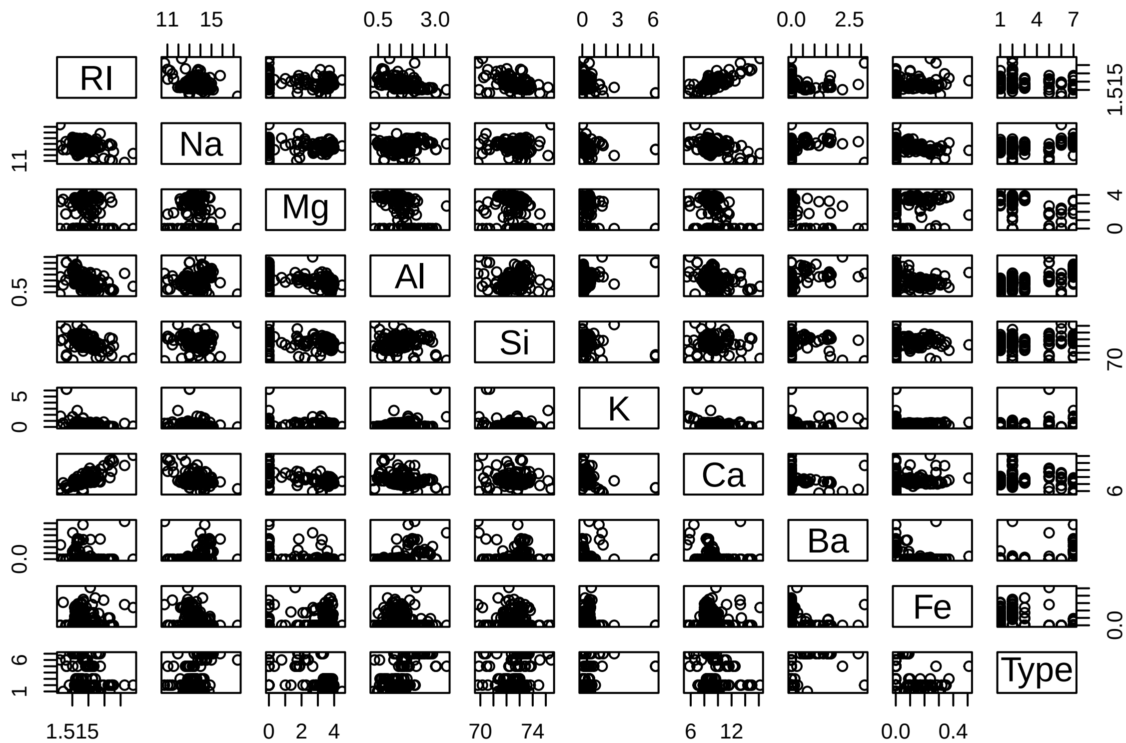


Task 1

```r
# Load in the data
Glass <- read.csv("https://www.maths.dur.ac.uk/users/john.p.gosling/MATH3431_practicals/Glass.csv")

# Look at the first few rows
head(Glass)
```

```
##        RI    Na   Mg   Al    Si    K   Ca Ba   Fe Type
## 1 1.52101 13.64 4.49 1.10 71.78 0.06 8.75  0 0.00    1
## 2 1.51761 13.89 3.60 1.36 72.73 0.48 7.83  0 0.00    1
## 3 1.51618 13.53 3.55 1.54 72.99 0.39 7.78  0 0.00    1
## 4 1.51766 13.21 3.69 1.29 72.61 0.57 8.22  0 0.00    1
## 5 1.51742 13.27 3.62 1.24 73.08 0.55 8.07  0 0.00    1
## 6 1.51596 12.79 3.61 1.62 72.97 0.64 8.07  0 0.26    1
```

```r
# Look at a pairs plot
pairs(Glass)
```

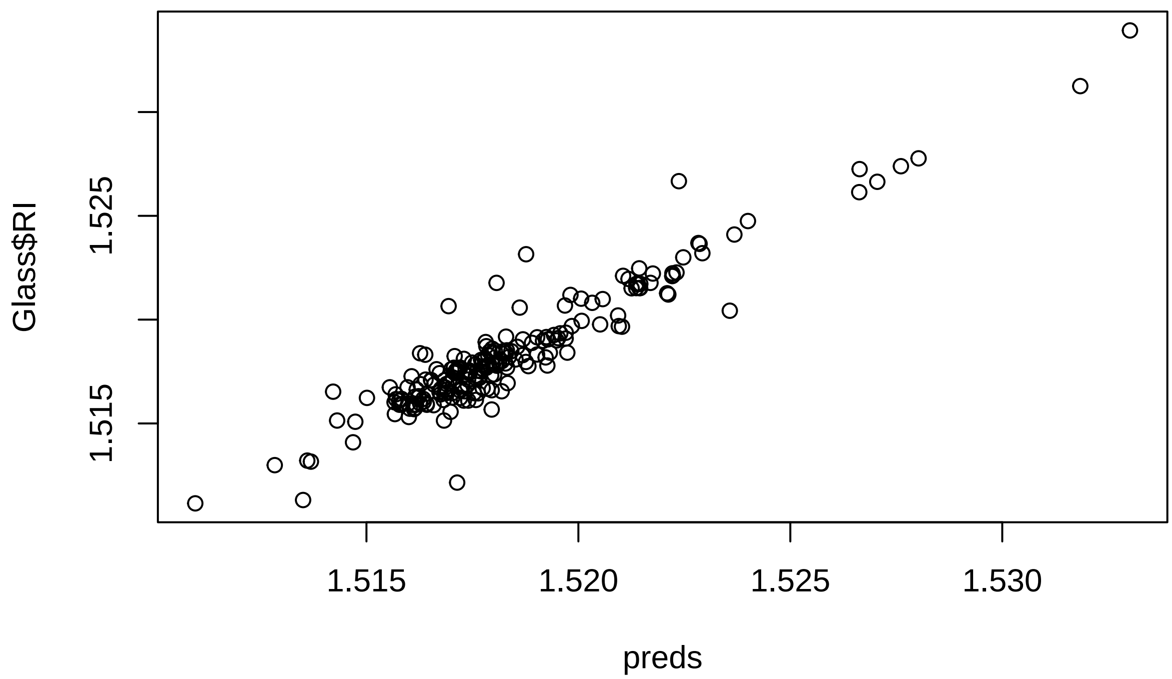



```r
# Fit a linear model
lm1 <- lm(RI ~ . - Type,
          data = Glass)

# Summarise the model
summary(lm1)
```

```
##
## Call:
## lm(formula = RI ~ . - Type, data = Glass)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0049898 -0.0004273 -0.0000264  0.0004187  0.0043833
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.453e+00  6.704e-02  21.678  < 2e-16 ***
## Na          1.395e-03  6.551e-04   2.130  0.03436 *
## Mg          1.844e-03  6.755e-04   2.730  0.00688 **
## Al          3.262e-05  6.983e-04   0.047  0.96278
## Si          1.685e-04  6.774e-04   0.249  0.80380
## K           1.383e-03  6.900e-04   2.004  0.04636 *
## Ca          3.117e-03  6.684e-04   4.663 5.61e-06 ***
## Ba          2.983e-03  6.760e-04   4.412 1.65e-05 ***
## Fe          4.263e-04  7.787e-04   0.547  0.58468
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.001004 on 205 degrees of freedom
## Multiple R-squared:  0.8948, Adjusted R-squared:  0.8907
## F-statistic: 217.9 on 8 and 205 DF,  p-value: < 2.2e-16
```

```r
# Make predictions
preds <- predict(lm1)

# Plot the predictions against the actual values
plot(preds, Glass$RI)
```




```r
# Calculate the MSE
mse <- mean((preds - Glass$RI)^2)
mse
```

```
## [1] 9.657919e-07
```

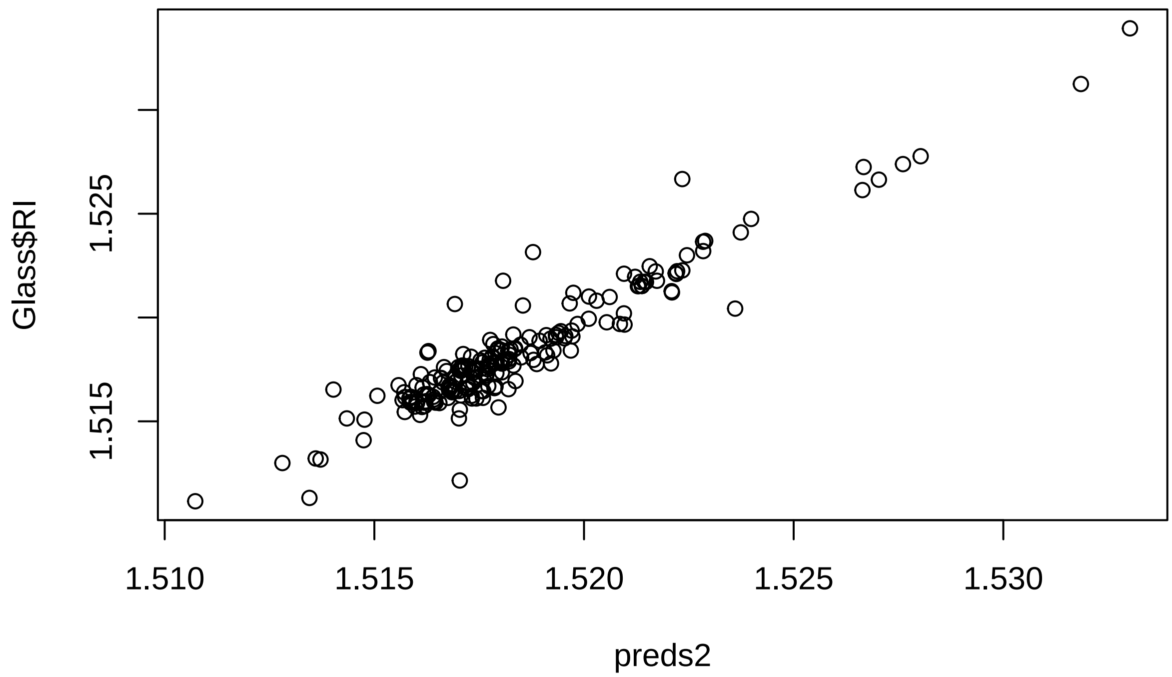
```r
# Calculate the MAE
mae <- mean(abs(preds - Glass$RI))
mae
```

```
## [1] 0.0006399008
```

```r
# Fit a linear model
lm2 <- lm(RI ~ Na + Mg + K + Ca + Ba,
          data = Glass)

# Summarise the model
summary(lm2)
```

```
##
## Call:
## lm(formula = RI ~ Na + Mg + K + Ca + Ba, data = Glass)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0048871 -0.0004520 -0.0000279  0.0004198  0.0043659
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.469e+00  2.270e-03 647.364  < 2e-16 ***
## Na          1.247e-03  1.159e-04  10.758  < 2e-16 ***
## Mg          1.717e-03  8.093e-05  21.221  < 2e-16 ***
## K           1.208e-03  1.360e-04   8.882 3.12e-16 ***
## Ca          2.986e-03  8.102e-05  36.851  < 2e-16 ***
## Ba          2.813e-03  1.803e-04  15.604  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0009985 on 208 degrees of freedom
## Multiple R-squared:  0.8944, Adjusted R-squared:  0.8919
## F-statistic: 352.5 on 5 and 208 DF,  p-value: < 2.2e-16
```

```r
# Make predictions
preds2 <- predict(lm2)

plot(preds2, Glass$RI)
```




```r
# Calculate the MSE
mse2 <- mean((preds2 - Glass$RI)^2)
mse2
```

```
## [1] 9.689946e-07
```

```r
# Calculate the MAE
mae2 <- mean(abs(preds2 - Glass$RI))
mae2
```

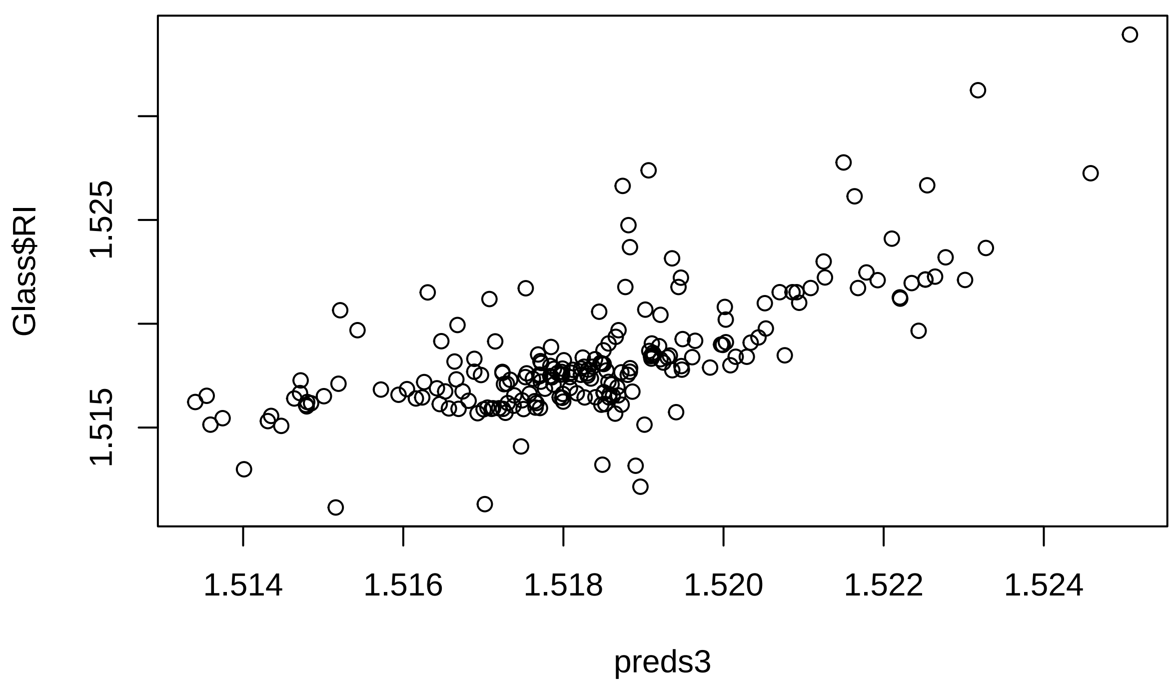
```
## [1] 0.0006393474
```

```r
# Fit a linear model
lm3 <- lm(RI ~ Si + Al + Fe,
          data = Glass)

# Summarise the model
summary(lm3)
```

```
##
## Call:
## lm(formula = RI ~ Si + Al + Fe, data = Glass)
##
## Residuals:
```

```
##        Min         1Q     Median         3Q        Max
## -0.0068121 -0.0011799 -0.0003764  0.0007744  0.0088538
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.6751971  0.0144519 115.915  < 2e-16 ***
## Si          -0.0021111  0.0001986 -10.629  < 2e-16 ***
## Al          -0.0024676  0.0003076  -8.022 7.24e-14 ***
## Fe           0.0019356  0.0015832   1.223    0.223
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.002235 on 210 degrees of freedom
## Multiple R-squared:  0.466,  Adjusted R-squared:  0.4584
## F-statistic: 61.08 on 3 and 210 DF,  p-value: < 2.2e-16
```

```r
# Make predictions
preds3 <- predict(lm3)

plot(preds3, Glass$RI)
```




```r
# Calculate the MSE
mse3 <- mean((preds3 - Glass$RI)^2)
mse3
```

```
## [1] 4.901921e-06
```

```r
# Calculate the MAE
mae3 <- mean(abs(preds3 - Glass$RI))
mae3
```

```
## [1] 0.001525121
```

Task 2

```r
# Fit a linear model without interactions
lm4 <- lm(RI ~ Na + Ba,
          data = Glass)
# Summarise the model
summary(lm4)
```

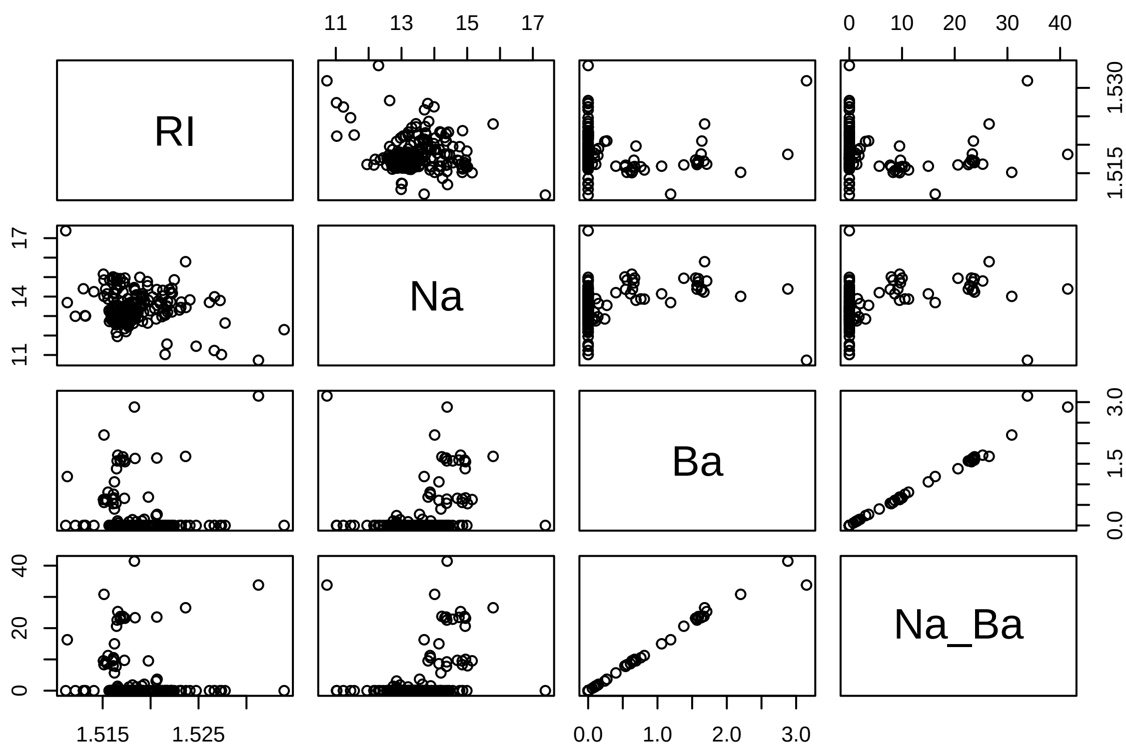
```
##
## Call:
## lm(formula = RI ~ Na + Ba, data = Glass)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0072624 -0.0018338 -0.0008541  0.0012016  0.0147547
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.5289942  0.0035380 432.160  < 2e-16 ***
## Na          -0.0007983  0.0002652  -3.010  0.00293 **
## Ba           0.0004258  0.0004356   0.978  0.32941
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.002988 on 211 degrees of freedom
## Multiple R-squared:  0.04116,    Adjusted R-squared:  0.03207
## F-statistic: 4.529 on 2 and 211 DF,  p-value: 0.01186
```

```r
# Fit a model with the interaction
lm5 <- lm(RI ~ Na*Ba,
          data = Glass)

# Summarise the model
summary(lm5)
```

```
##
## Call:
## lm(formula = RI ~ Na * Ba, data = Glass)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0073689 -0.0018427 -0.0006707  0.0010093  0.0151228
##
## Coefficients:
```

```
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.5235728  0.0039060 390.061  < 2e-16 ***
## Na          -0.0003874  0.0002935  -1.320  0.18823
## Ba           0.0124291  0.0039871   3.117  0.00208 **
## Na:Ba       -0.0008827  0.0002915  -3.028  0.00277 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.002932 on 210 degrees of freedom
## Multiple R-squared:  0.08127,    Adjusted R-squared:  0.06815
## F-statistic: 6.192 on 3 and 210 DF,  p-value: 0.0004739
```

```r
# Create a proxy for the interaction term
Glass$Na_Ba <- Glass$Na * Glass$Ba

# Look at the relationships between the variables
pairs(Glass[, c("RI", "Na", "Ba", "Na_Ba")])
```



Task 3

```r
# Load in the data
LetterRecognition <- read.csv("https://www.maths.dur.ac.uk/users/john.p.gosling/MATH3431_practicals/Let
```

```r
# Look at the first few rows
head(LetterRecognition)
```

```
##   lettr x.box y.box width high onpix x.bar y.bar x2bar y2bar xybar x2ybr xy2br
## 1     T     2     8     3    5     1     8    13     0     6     6    10     8
## 2     I     5    12     3    7     2    10     5     5     4    13     3     9
## 3     D     4    11     6    8     6    10     6     2     6    10     3     7
## 4     N     7    11     6    6     3     5     9     4     6     4     4    10
## 5     G     2     1     3    1     1     8     6     6     6     6     5     9
## 6     S     4    11     5    8     3     8     8     6     9     5     6     6
##   x.ege xegvy y.ege yegvx
## 1     0     8     0     8
## 2     2     8     4    10
## 3     3     7     3     9
## 4     6    10     2     8
## 5     1     7     5    10
## 6     0     8     9     7
```

```r
# Look at the structure of the data
str(LetterRecognition)
```

```
## 'data.frame':    20000 obs. of  17 variables:
##  $ lettr: chr  "T" "I" "D" "N" ...
##  $ x.box: int  2 5 4 7 2 4 4 1 2 11 ...
##  $ y.box: int  8 12 11 11 1 11 2 1 2 15 ...
##  $ width: int  3 3 6 6 3 5 5 3 4 13 ...
##  $ high : int  5 7 8 6 1 8 4 2 4 9 ...
##  $ onpix: int  1 2 6 3 1 3 4 1 2 7 ...
##  $ x.bar: int  8 10 10 5 8 8 8 8 10 13 ...
##  $ y.bar: int  13 5 6 9 6 8 7 2 6 2 ...
##  $ x2bar: int  0 5 2 4 6 6 6 2 2 6 ...
##  $ y2bar: int  6 4 6 6 6 9 6 2 6 2 ...
##  $ xybar: int  6 13 10 4 6 5 7 8 12 12 ...
##  $ x2ybr: int  10 3 3 4 5 6 6 2 4 1 ...
##  $ xy2br: int  8 9 7 10 9 6 6 8 8 9 ...
##  $ x.ege: int  0 2 3 6 1 0 2 1 1 8 ...
##  $ xegvy: int  8 8 7 10 7 8 8 6 6 1 ...
##  $ y.ege: int  0 4 3 2 5 9 7 2 1 1 ...
##  $ yegvx: int  8 10 9 8 10 7 10 7 7 8 ...
```

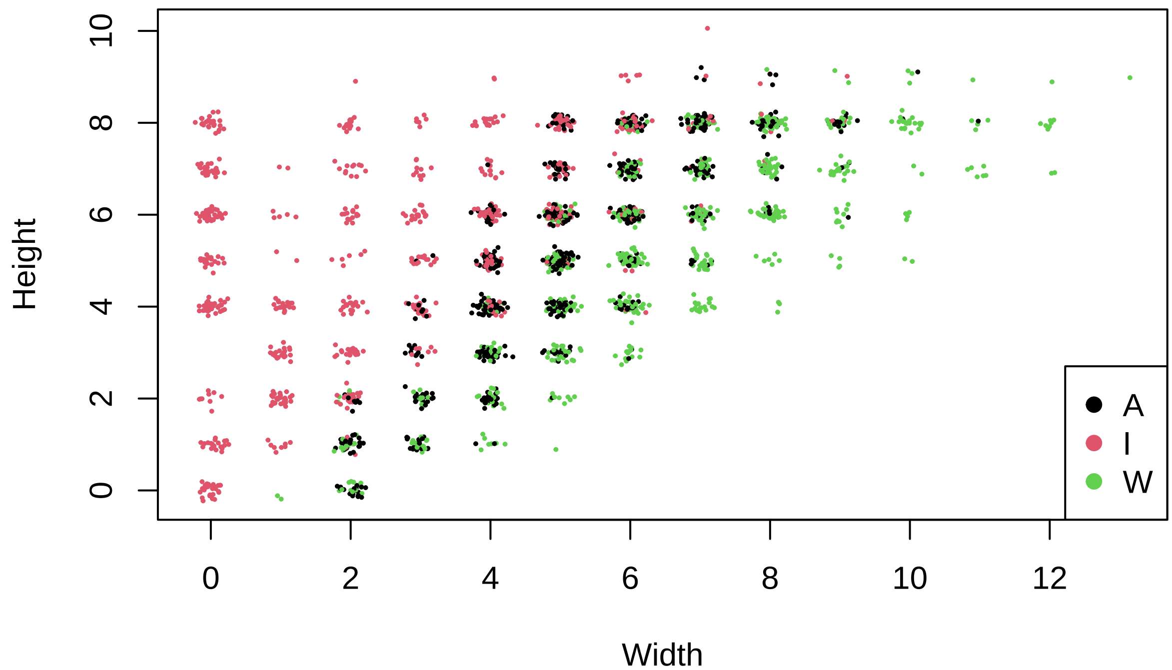
```r
# Look at the levels of the letter variable
levels(LetterRecognition$lettr)
```

```
## NULL
```

```r
# Create a subset of the data
ltrs <- subset(LetterRecognition,
               lettr %in% c("I", "A", "W"))

# Reset the levels of the letter variable
ltrs$lettr <- factor(ltrs$lettr)
str(ltrs)
```

```
## 'data.frame':    2296 obs. of  17 variables:
##  $ lettr: Factor w/ 3 levels "A","I","W": 2 1 3 3 1 3 3 3 2 2 ...
##  $ x.box: int  5 1 12 5 3 3 4 2 2 3 ...
##  $ y.box: int  12 1 14 9 7 4 8 1 9 9 ...
##  $ width: int  3 3 12 6 5 4 5 3 3 4 ...
##  $ high : int  7 2 8 7 5 3 6 1 7 7 ...
##  $ onpix: int  2 1 5 8 3 2 3 1 2 3 ...
##  $ x.bar: int  10 8 9 7 12 9 6 7 8 7 ...
##  $ y.bar: int  5 2 10 9 2 10 8 8 7 7 ...
##  $ x2bar: int  5 2 4 5 3 3 4 4 0 0 ...
##  $ y2bar: int  4 2 3 3 2 2 1 0 7 7 ...
##  $ xybar: int  13 8 5 7 10 5 7 7 13 13 ...
##  $ x2ybr: int  3 2 10 9 2 9 8 8 6 6 ...
##  $ xy2br: int  9 8 7 8 9 7 8 8 9 8 ...
##  $ x.ege: int  2 1 10 6 2 6 8 6 0 0 ...
##  $ xegvy: int  8 6 12 8 6 11 9 10 8 8 ...
##  $ y.ege: int  4 2 2 3 3 0 0 0 1 1 ...
##  $ yegvx: int  10 7 6 8 8 8 8 8 8 8 ...
```

```r
# A jittered scatter plot of the width vs the height with the letters coloured
plot(ltrs$width+rnorm(nrow(ltrs), 0, 0.1),
     ltrs$high+rnorm(nrow(ltrs), 0, 0.1),
     col = as.numeric(ltrs$lettr),
     pch = 19, cex = 0.2,
     xlab = "Width", ylab = "Height")

# Add a legend
legend("bottomright", legend = levels(ltrs$lettr), col = 1:3, pch = 19)
```



Linear Discriminant Analysis (LDA)

```r
# Load the MASS package
library(MASS)

# Fit the LDA model
lda1 <- lda(lettr ~ .,
            data = ltrs)

# Summarise the model
summary(lda1)
```

```
##         Length Class  Mode
## prior    3     -none- numeric
## counts   3     -none- numeric
## means   48     -none- numeric
## scaling 32     -none- numeric
## lev      3     -none- character
## svd      2     -none- numeric
## N        1     -none- numeric
## call     3     -none- call
## terms    3     terms  call
## xlevels  0     -none- list
```

```r
# Make predictions
preds <- predict(lda1)
```

```r
# Calculate the confusion matrix using the table function
table(ltrs$lettr, preds$class)
```

```
##
##        A   I   W
##    A 754   4  31
##    I  12 743   0
##    W   3   0 749
```

looks pretty good, the diagonals have most of the observations, so they are classified correctly.

```r
# Fit the LDA model
lda2 <- lda(lettr ~ x.box + y.box + width + high,
            data = ltrs)

# Summarise the model
summary(lda2)
```

```
##         Length Class  Mode
## prior    3     -none- numeric
## counts   3     -none- numeric
## means   12     -none- numeric
## scaling  8     -none- numeric
## lev      3     -none- character
## svd      2     -none- numeric
## N        1     -none- numeric
## call     3     -none- call
## terms    3     terms  call
## xlevels  0     -none- list
```

```r
# Make predictions
preds2 <- predict(lda2)

# Calculate the confusion matrix
conf <- table(ltrs$lettr, preds2$class)
conf
```

```
##
##        A   I   W
##    A 609  47 133
##    I 151 557  47
##    W 256   5 491
```

```r
# Calculate the overall accuracy
# (hint consider the elements of the confusion matrix)
acc <- sum(diag(conf))/sum(conf)*100
acc
```

```
## [1] 72.16899
```

```r
# Calculate the precision for `A`
prec_a <- conf[1,1]/(sum(conf[, 1]))*100
prec_a
```

```
## [1] 59.94094
```

```r
# Calculate the recall for `A`
recall_a <- conf[1,1]/(sum(conf[1,]))*100
recall_a
```

```
## [1] 77.18631
```

Task 4

```r
# Fit the LDA model
lda3 <- lda(lettr ~ I(width^0.5) + high,
            data = ltrs)

# Summarise the model
summary(lda3)
```

```
##         Length Class  Mode
## prior   3      -none- numeric
## counts  3      -none- numeric
## means   6      -none- numeric
## scaling 4      -none- numeric
## lev     3      -none- character
## svd     2      -none- numeric
## N       1      -none- numeric
## call    3      -none- call
## terms   3      terms  call
## xlevels 0      -none- list
```

```r
# Make predictions
preds3 <- predict(lda3)

# Calculate the confusion matrix
conf3 <- table(ltrs$lettr, preds3$class)
conf3
```

```
##
##        A   I   W
##   A 634   3 152
##   I 282 450  23
##   W 283   0 469
```

```r
# Calculate the overall accuracy
# (hint consider the elements of the confusion matrix)
acc3 <- sum(diag(conf3))/sum(conf3)*100
acc3
```

```
## [1] 67.63937
```

```r
# Calculate the precision for `A`
prec_a3 <- conf3[1,1]/(sum(conf3[, 1]))*100
prec_a3
```

```
## [1] 52.8774
```

```r
# Calculate the recall for `A`
recall_a3 <- conf3[1,1]/(sum(conf3[1,]))*100
recall_a3
```

```
## [1] 80.35488
```