NATIONAL RESEARCH UNIVERSITY
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Shumeiko Lyubov

Measuring the effectiveness of various aggregation procedures

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Academic Supervisor
Veselova Julia Alexandrovna,
Associate Professor

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Table of contents

INTRODUCTION

The quest for innovative methods in preference, a pursuit dating back to the 17th century, however researchers are finding new and creative ways to combine and adapt existing methods for preference analysis. New technologies and challenges have led to a renewed interest in this field, with researchers breathing new life into old techniques.

Goal of the work:optimize the mechanism for aggregating individual preferences into collective ones by Kemeny distance, while maintaining the computational ease of simple aggregation methods

The task is to:

- Obtaining and comparing the results of various aggregation methods
- Evaluation of the results of applying preference aggregation methods based on audience satisfaction using Kemeny distance
- Finding the best optimization of existing simple aggregation methods by combining aggregation methods

Try to optimize aggregation methods in accordance with the selected parameters:

1) depending on the average level of satisfaction of the individual

2) depending on the complexity and practical implementation of the algorithm (computational complexity (time-consuming))

Relevance of the research topic and practical value of the study: the mechanism of a technically simple, but individually effective method of preference aggregation can find wide application in recommendation systems focused on processing large volumes of data in real time. This approach is especially suitable for decisions that require immediate decision-making based on a large level of incoming data and do not require long-term storage. A high focus on practical issues determines a fairly high level of theoretical attention to the issues of optimizing algorithms, and in particular methods for ranking preferences over many years.

Research methods. Methods of voting theory (social choice), mathematical statistics, combinatorics, measurement theory and preference aggregation were used.[1]Optimization methods, data analysis and machine learning methods are partially covered.

Novelty and practical value: the mechanism of a technically simple, but individually effective aggregation method, preference can be reflected in the use of recommendation mechanisms focused on large data flows that require immediate solutions and non-long-term storage.

Structure and volume.

[1] AGGREGATION OF PREFERENCES BASED ON AN EXACT SOLUTION TO THE PROBLEM OF RANKING KEMENI Dissertation for the degree of Candidate of Technical Sciences. Tomsk – 2022

The introduction substantiates the relevance of the dissertation topic, formulates the purpose of the research, defines the tasks to be solved, and indicates the scientific novelty and practical value of the results of the work.

To achieve the set objectives in this coursework, within the framework of Literature Overview, a brief study of already existing decision-making methods will be conducted, and the issues of the effectiveness of the decision made and satisfaction with it, other issues the significance of which affects the quality of decision-making, will be raised and considered.

Next, we will analyze and systematize works that directly aim to study problems associated with the use of aggregation methods to solve selection problems.

As part of the theoretical review, the necessary terms and notations will be introduced, the rules used in the practical part for aggregating preferences will be reviewed and described, special attention will be paid to the Kemeny distance.

The third chapter “Problem Statement” is devoted to a description of the progress in solving the research problem: the first block is devoted to a description of the selected datasets and the requirements for them, explanations are given for the operating procedures of the code algorithms corresponding to the selected aggregation methods.

The methods used in the work to assess the technical complexity of using a particular algorithm are also described. Additionally, explanations are given for the formulated understanding of efficiency by Kemeny distance.

Next, an analysis will be made of the results obtained as a result of applying the selected aggregation methods. As well as analysis and interpretation of the results obtained by combining methods in various ways.

The practical research part of the work consists of the following stages:

- Obtaining and comparing the results of various aggregation methods (compensatory and non-compensatory methods)

- Calculating audience satisfaction using the Kemeny distance (Obtaining the average result and calculating the deviation distance of each individual solution from the obtained average value?)

- Finding the most effective (according to Kemeny) combination by combining aggregation methods

LITERATURE OVERVIEW

Definition of scientific field when making decisions. Methods for making individual and collective decisions. Decision Making Problems.

In view of the fact that decision-making methods and solutions in general exist and in almost any field there are quite a lot of sciences and framework approaches that have formulated the problem in different ways and focused their attention on solving a specific part of it. At the moment, there is such an approximate list of sciences that differently answer the question of decision making or are involved in the calculation and formation of such a decision: Modern methods of decision theory, optimization methods, game theory and economic mechanisms, graph theory, multicriteria and collective theories choice, intelligent methods of data analysis, theories of probability and imprecise probabilities, theories of trust functions.

The leader in the theory of individual decision making is “THE UTILITY MAXIMIZATION”, formed by scientists around the question of how to solve the problem of utility maximization. In his work “Choice and preferences” by Faud Aleskerov[2] The presents of the classical model used to describe individual preferences and choices over alternatives and links between the model.. The same topic finds a new breathe in everyday scientific states, but in a little bit different situations Thershold utility Modell with applications to retailing and discrete Choice models by[3] Guillermo Gallego(and others) by revealing a novel approach

[2] Aleskerov, F. "Choice and Preferences."

[3] Gallego, G., et al. "Threshold Utility Models with Applications to Retailing and Discrete Choice Models."

to modeling consumer behavior under constraints through instruments of utility model. It is incredible that in fact this direction began withDaniel Bernoulli and Expected Utility Theory: In the 18th century, Bernoulli challenged the idea that people valued outcomes solely based on their objective rewards. He proposed that individuals consider the probability of achieving different outcomes when making decisions, prioritizing options with higher expected utility. Until the more modern may was invented by John von Neumann and Oskar Morgenstern described and formalized as Axiomatic Utility Theory Building upon Bernoulli's work, von Neumann and Morgenstern formalized through a set of axioms. These axioms define rational behavior based on preferences, ensuring consistency and measurability of utility values.

When considering the greatest axiom, one cannot fail to mention The Impossibility Theorem. “KJ Arrow has described five apparently reasonable properties which any voting system or other "social welfare function" should have. He has demonstrated mathematically that none could possibly have all these properties”[4]- thats what the Harry Markowitz writes about his job. The most important of these are:

1. Axiom of Universal Domain: This axiom ensures inclusivity of all possible individual preferences, making sure that the social welfare function can process and aggregate any set of preferences.

[4]Social Welfare Functions Based on Individual Rankings
Leo A. Goodman and Harry Markowitz

2. **Axiom of Unanimity**: This axiom guarantees that if everyone agrees on a preference order between two alternatives, the group's decision should reflect that agreement.
3. **Axiom of Independence of Irrelevant Alternatives**: This axiom ensures that the group's preference between two alternatives should not be affected by the introduction or removal of other alternatives.[5]

Another way to look at the problem is to turn your attention to algorithm optimization methods centered around a mathematical and technical solution. For example, often when the theory of decision making arises and is discussed, topics such as “Linear optimization models and Linear Programming (LP)” arise. Examples. Geometry and Algebra of LP problems. Simplex method. Duality in LP. Integer LP. Nonlinear optimization models and Nonlinear Programming (NLP). “Which, on the one hand, would seem to have nothing to do with Preferences but provide a method to achieve the best outcome in a mathematical model whose requirements are represented by linear relationships. As a rule, in the aspect of searching for optimal individual and collective solutions, this topic is rather applied, for example, in the final report of the National Research University Higher School of Economics in 2019 “On scientific research work on the study of decision-making models and analysis of complex

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[5] von Neumann, J., & Morgenstern, O. "Axiomatic Utility Theory."

structured data[6]. Conducted applied research tasks side by side; “based on previously developed models of influence in networks, an effective algorithm for calculating indices of short- and long-range interactions is proposed; and the computational complexity of this algorithm is estimated”; and then, within the framework of “the analysis of methods for selecting optimal options and procedures for ranking alternatives, an empirical study of the stability of aggregated ratings constructed using ordinal methods based on paired comparisons according to the majority rule was carried out”

Procedural details of decision-making

Let's a little bit dave at two key disciplines that delve into the procedural details of decision-making:

Permutations and their properties: Permutations play an important role in decision theory. Studying the properties of permutations allows you to understand how changes in the order of elements affect the final decisions. A classic example is Kendall's tau distance and Kemeny distance, which measure the number of inversions or permutations required to move from one ranking to another. Controlling such permutations helps optimize decision-making processes.

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[6] Research leaders: F.T. Aleskerov, E.S. MaskinHSE 2019 ABOUT RESEARCH WORK RESEARCH OF DECISION MAKING MODELS AND ANALYSIS OF COMPLEX STRUCTURED DATA
Higher School of Economics, National Research University. "On Scientific Research Work on Decision-Making Models and Analysis of Complex Structured Data.

As for example, it is in this topic that Kornienko's work is deepened In P, called the optimized method for selecting the resulting ranking of objects represented in the ranking school. In the course of the research result, he set himself the task of “ranking objects represented by matrices of binary relations proposed by J. Kemeny and J. We refer to the class of combinatorial NP-complete problems for which there is currently no optimal method for finding the resulting ranking according to the matrix criterion.

Improving decision patterns: Modern methods of data analysis and machine learning allow you to identify and improve decision patterns. Intelligent data analysis methods use algorithms to recognize patterns and predict results based on large amounts of data. These methods help to identify hidden dependencies and improve the accuracy of solutions.

Disciplines at the intersection

There are several disciplines that investigate the issues of the greatest intersection between the properties of permutations and the improvement of decision patterns:

Optimization theory: Includes linear and nonlinear programming, methods for finding optimal solutions under constraints. Examples include the simplex method and duality in linear programming.

Graph Theory: Studies data structures that help visualize and analyze decision patterns.

Machine learning and Data Analysis: Techniques such as clustering and classification use data to improve decision patterns and identify optimal strategies.

The influence of the human factor

The third most far-reaching approach is the approach centered around paying attention to the human characteristics of decision-making, so Daniel Kahneman[7], who popularized this direction, received the Nobel Prize for comparing the characteristics of thinking and subsequent errors and subsequent affect on decision-making. More accessible Afanasieff Larichev, Lin his book Larichev, L. "The Theory and Methods of Decision Making and the Chronicles in Wizard Countries[8]about how much our subjective perception influences the weight rating in the multi-Choice criteria and how a person or decision-making subject can make mistakes in their own assignment of their own parameters when making a decision.

Current trends in decision-making include the use of pattern analysis and data mining to get answers without explicit questions. These approaches allow us to identify new patterns and

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[7] Kahneman, D. "Thinking, Fast and Slow."

[8] Larichev, L. "The Theory and Methods of Decision Making and the Chronicles in Wizard Countries

patterns that can improve decision-making processes. The large flow of data opens up new possibilities for analysis and interpretation, which highlights the interdisciplinary nature of research in this area.

Thus, we can say that the problem of decision making, namely methods, can be looked at from different concepts.

Ranking methods in political life.

Ranking and decision-making methods play an important role in political life, contributing to the formation of public opinion, prioritization and strategic decision-making. Various approaches are used in this field, such as voting theory, multi-criteria decision-making methods and optimization algorithms.

Voting theory

Voting theory studies the mechanisms of aggregation of individual preferences into a collective decision. The most well-known methods include the majority rule, the Borda method, and the Condorcet method. These methods make it possible to take into account multiple preferences and ensure fair representation of the interests of all participants in the process.

The relevance reflects the maintenance and creation of sites aimed at studying the methods of aggregation using the example of an open date containing socio-political knowledge.

For example, the Voting matters[9] website has been studying the specifics of using open access ranking methods since 1994. Here is an example of an analysis of the problems raised as an article that raised the problem of feedback for a method of counting votes that is more time-consuming than modern methods, but with the help of computers it is possible to effectively cope with this complexity. Critics in this article argued that the results of such methods may differ from existing methods, but preliminary tests show differences between the selected candidates. Supporters of the Single Transferable Vote (STV) system, which values equal treatment, may find it advisable to automatically count reviews. Although the feedback method works within the limitations of STV, it needs to be adapted for voters who rate only some candidates. This adaptation, discussed later in another article, increases the accuracy of the will of voters.

Application of the Kemeny median and Kemeny distance methods so far.

One of the most preferable for me not so young works was about a “new distance between rankings”.

One of the most interesting work from my point of view is Preference segregation analysis for sorting problems in the context of group decision-making with uncertain and inconsistent preferences, where the goal is to study a value-based preference disaggregation method for solving problems of sorting by several criteria with the participation of several decision makers.

[9] https://www.votingmatters.org.uk/ISSUE1/P1.HTM

Preference disaggregation analysis effectively infers decision makers' preference models from decision examples. While multiple criteria sorting often involves group decision-making, estimating a collective sorting model for groups with uncertain and inconsistent preferences remains underexplored. This paper introduces a threshold-based, value-driven preference disaggregation method to address this. It reduces cognitive effort by allowing decision-makers to express preferences through reference alternatives or pairwise comparisons with uncertainty. A two-stage procedure identifies and eliminates inconsistencies, both within individual and group preferences. Using linear programming, an additive value function and thresholds are estimated. A practical example of government-funded financial investments demonstrates the method's effectiveness in improving preference disaggregation accuracy by handling uncertainty and inconsistency.

In general, the whole hit is the procedure of group multicriteria analysis of solutions, presented by numerous works. In the context of the “Application of a hybrid Delphi and aggregation–segregation procedure for group decision-making”[10], it combines a managed Delphi process with an autonomous aggregation-disaggregation mechanism.
Theese paper presents algorithms ME and ME-RCW for calculating Kemeny rankings for profiles with n=12 where no Condorcet winner exists. The study finds that ME-RCW significantly reduces execution time compared to ME, particularly at lower ω values. For profiles with n=13 and n=14, ME-RCW maintains manageable execution times without memory issues. The results demonstrate that these algorithms outperform previous exact algorithms, emphasizing the impact of ranking profile characteristics on execution time. Future work will focus on refining these characteristics and enhancing solution exploration techniques.

If you sit and watch longer, then these works form whole clusters of questions (with examples provided below)

1. **Kemeny Median in Rank Aggregation**:

10

Kemeny median is known for its robustness in aggregating rankings by minimizing the distance to individual rankings. It ensures neutrality, consistency, and adheres to the Condorcet criterion.

Recent research has focused on improving algorithms to find the Kemeny ranking efficiently despite the NP-hard nature of the problem. Enhancements involve pruning the search space using Condorcet properties (Pérez-Fernández & Díaz, 2021)[11]

2. **Kemeny Distance in Clustering and Prediction**:

Kemeny distance has been used for clustering and predicting rankings. This method is valuable for handling incomplete and tied rankings, making it suitable for various applications including bioinformatics and computational social choice (SpringerLink)[12]

3. **Robust Fuzzy Clustering**:

A Kemeny distance-based robust fuzzy clustering method has been proposed to handle ranking data more effectively. This approach provides a novel way to manage uncertainty and variability in data, which is critical for applications in recommendation systems and decision support (SpringerLink).

4. **Group Decision Support**:

The application of the Kemeny median in group decision support systems helps in synthesizing individual preferences into a collective decision, maintaining fairness and minimizing individual dissatisfaction. This has been applied in various organizational and strategic decision-making scenarios (ResearchGate)[13]

5. **Optimization Algorithms**:

New algorithms, such as differential evolution algorithms, have been developed to find the median ranking efficiently under the Kemeny rule. These

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[11] https://www.mdpi.com/2227-7390/9/12/1380

[12] https://bilder.buecher.de/myracloud-blocked/?r=ZGRkMmQwYjQ5MzA4MDdhN2ExZGQ3NzJjMDZhY2Y0MTgzMDU4YmY0YTU5YzMxY2I3&t=YzhhYTA5MTE4YjQ5NjllNTE1L0p1bi8yMDI0OjIyOjMzOjAzICswMjAw&w=ODk4OTZkMGI0ZTAwNGZjYWI5YmZlN2Q0NWNiYTRmNDhiZGE1YzlkY2YwZjdkYzFh

[13]

advancements are crucial for practical applications in fields where ranking large sets of alternatives is necessary (ScienceDirect)[14]

Conclusion

The Kemeny median and Kemeny distance continue to be significant in research and practical applications for preference aggregation and ranking problems. The recent advancements in algorithms and their applications in diverse fields highlight their importance and potential for future research.

[14] https://www.mdpi.com/2227-7390/9/12/1380

THEORETICAL PART
"General terminology. "

This chapter introduces the necessary terms and notations, in particular the concept of ranking as a form of representing preferences; An analysis of the concept of alternatives and ranking was carried out.

One of the stages of forming a collective decision is the process of translating individual preferences (taking into account individual opinions) into a single decision. Those based on the decisions of a set of voters, given the set of possible alternatives A(|), construct preferences$N = \{1,... n\}|A| = m$[15]in such a way that, based on the resulting data, it is possible to make a decision.

There are many options for expressing individual preferences: value function, choice function, etc., one of the simplest is the construction of binary relations - identifying preferences regarding alternatives by pairwise comparison of them according to m criteria, and their further ranking - construction of binary relations of preferences () of each voter on set of alternatives A in the form of a chain of ordered alternatives (preference profile) using a ranking system. $A\ (a_1,\ a_2... an\ )P,\ P_1,.... PnN = \{1,... n\}$

In different terminology[16]the problem of ordering A by preference is understood as the problem of ranking X taking into account the following ^

"By ranking X is a complete transitive binary relation R on X, so that for any alternatives x, y, z ∈ X two properties are satisfied:

- "xRy or yRx" (R completeness)
- "from xRy and yRz it follows that xRz" (transitivity of R)"

[15]Veselova YA "On the representation of the threshold aggregation rule in the form of a representation of the scoring rule"
[16] V.A. Kalyagin, V.V. ChistyakovAXIOMATIC MODEL OF NON-COMPENATORY AGGREGATION

[17]

18

Определение 1.5. *Общим рангом* r_i *альтернативы* a_i в профиле предпочтения $\Lambda(m,\ n)$ называется сумма ее рангов во всех ранжированиях этого профиля, т.е.

$$r_i = \sum_{k=1}^{m} r_i^k . \qquad (1.10)$$

Ясно, что любой профиль $\Lambda(m,\ n)$ может быть однозначно представлен *таблицей рангов*.

[18]

The overall rank ri of an alternative in a preference profile is the sum of its ranks in all profile rankings

Theoretical part (main)
"Rules for aggregation of preferences and their axiomatic model"

In this work, the following basic classical aggregation methods will be applied based on the ranking of individual preferences expressed in the form of binary linear relationships, as well as partial orders.

Majority (Plurality rule)

This rule counts the number of votes for the leading position and only if the alternative has more than half the votes does it recognize it as the winner. This leads to multiple situations where it is impossible to determine a winner.

STV(single transferable vote)

[17] Kalyagin V.A., Chistyakov V.V. An axiomatic model of non-compensatory aggregation: Preprint WP7/2009/01. - M.: Publishing house. House of State University Higher School of Economics, 2009 - 76 p.

[18] Soc Choice Welf (2010) 35:627–646DOI 10.1007/s00355-010-0454-9
Social threshold aggregations
Fuad T. Aleskerov · Vyacheslav V. Chistyakov · Valery A. Kalyagin

The STV model is also a system that determines only the winner, without arranging the remaining elements in preferences, but unlike Majority (Plurality rule), it allows it to be determined in much more variety of situations.

At step zero, we take the reflected individual preferences expressed as binary linear relations, as well as partial orders, and write them into a preference matrix.

Since STV allows us to find only the absolute winner, we determine whether the absolute victory of one of the alternatives is obvious.

In this step, the procedure for determining the winner is similar to determining the winner using the absolute majority rule: for this, the total number of votes that determined this alternative as the most preferable is calculated, and then the winner is determined if one of the alternatives received an overwhelming number of votes.

If a winner is not found, the following algorithm of actions is performed: the alternative that received the least number of votes is determined, and then it is removed from the matrix, followed by reformation of the matrix to its original state: all preferences are rewritten into a new matrix, without changes, minus the least preferred one alternatives. As a result of repeated repetition of the procedure, a winner is determined - the only remaining alternative.

Let us imagine the collective preferences of voters as P(S), then the individual preferences of voters will look like this: (P1, P2 .... P i), where P1 are the preferences of voter 1, P2 are the preferences of voter 2.

Since preferences represent a ranking of alternatives (a, b, c) in the form of binary linear relationships as well as partial orders, they can be displayed in a preference matrix as follows:

P1	P2	P3
a	V	A
V	A	With

Wi th	Wi th	V

In this case, alternative a, which received n votes (in this case 2, 1, 0) on the preference vector, or the aggregate set of preferences of individuals in relation to any alternative will look like this:

$$v(a) = (n)$$

$$v(a) = (2, 1, 0)$$

Condorce (+ Copeland rule)

The following two types of aggregation are quite often considered together due to the fact that in order to determine the winner, they are primarily guided by the determination of the winner using pairwise construction of dominance relations.

A dominance relationship is a hierarchically constructed relationship when an alternative (a) turns out to be preferable to another alternative (b), regardless of its general location in the matrix of alternatives or ranking relative to other alternatives (c).

Alternative (a) is recognized as dominant relative to (b) if, when comparing the ratio of dominated (a) to (b), and (b) to (a), the ratio of (a) to (b) received more votes, in such In this case, it can also be considered that alternative (a) is preferable to (b).

Based on certain pairwise relationships, a dominance graph is built. (Fig. 1). According to the Condorcet rule, the winning alternative is the one that is dominant in relation to all other alternatives (or has the largest number of dominant relations along with neutral relations) Sometimes Conorce rule known as a condorce hair[19].

However, it is pairwise comparison that often leads to a situation where it is impossible to determine the winner: first of all, due to the situation when, when determining the dominant relations, there is no more preferable alternative

[19] https://civs.cs.cornell.edu/proportional.html

(equality of votes), and secondly, due to a situation called the Condorcet paradox.

Condorcet's paradox is a relationship between alternatives when (a) is preferable to (b), (b) is preferable to (c), (c) is preferable to (a)

In this case, it makes sense to apply Copland's rule. In accordance with it, after determining the dominance relations, it is necessary to calculate the difference between the sum of dominant votes and the sum of dominance votes.

The sum of dominance is the result of adding the votes for cases of dominance over other alternatives in the aggregate, and the sum of dominance votes is the result of counting votes when this particular alternative turned out to be less preferable relative to other alternatives.
The winner according to Copland's rule is the alternative (a) that receives the most votes in pairwise comparisons.

Often, comparisons with a tournament are used to describe these procedures, where scoring functions are used based on the results of paired comparisons of participants. For a victory in a doubles competition, one point is given, for a loss, 0, for a draw, alpha points. And the required ranking is the ordering of alternatives by the number of points ta(x)[20]

Threshold (leximin and leximax)

The threshold rule, unlike previous methods, allows you to rank individual preferences into public ones with the definition of a specific position for each alternative. These two types of aggregation are included in a single group of threshold aggregation, since they are essentially a single algorithm, with the only difference being the scope of their application (the beginning of the algorithm): if leximin determines the position of a certain alternative depending on how many people classified it as the worst (from the end)[21], then leximax -, on the contrary, as the best (from the beginning).

[20]page 64RESEARCH REPORT ON DECISION MAKING MODELS AND ANALYSIS OF COMPLEX STRUCTURED DATA

[21] https://hturner.github.io/PlackettLuce/articles/Overview.html

Preferences in alternatives are compared by correlating the number of votes that determined a given alternative as the best (most preferred) with other alternatives and then ranking the alternatives relative to each other by the number of votes for the most preferred place.

Thus, alternative a is considered preferable to alternative b, if the number of best estimates in the set of preferences .... is greater than the number of best estimates in the set of preferences ....$aPbr = (r_1 (a_1, P_1))r = (r_1 (a_1, P_1))$ $r = (rn (a_1, Pn))r = (r_1 (b_1, P_1))r = (r_1 (b_1, P_1))r = (rn (b_1, Pn))$

If several alternatives have the same values for the best position (or have no votes at all) in the individual ranking, then the number of votes for the alternative in the following positions is compared.

This type of aggregation is completely non-compensatory - it does not allow alternatives that initially received fewer votes with the highest possible rating to be compensated with further high performance. That is, it does not take into account further multiple distributions of votes with average (subsequent) ratings, which leads to a relatively random assignment of seats on the scale of the majority of participants.

Borda rule

Absolutely opposite or absolutely compensatory is the aggregation method in accordance with Borda's rule. It, like the previous one, allows you to completely rank individual preferences into collective ones, but it takes into account not only the number of votes, but is also more sensitive to any fluctuation in preferences throughout the entire individual ranking.

The Borda method, like many previously listed methods, uses the scoring method as an auxiliary tool: assigning a certain numerical value to an alternative depending on the specified parameters. The position in the resulting collective social ranking is determined depending on the number of points scored: the most preferred is the one with the most points.

The parameters for scoring in this case are determined by two variables - the number of votes received for a certain place and the position (place) of this alternative in the ranking. For each alternative, the total number of points obtained as a result of multiplying the received votes by the number of the place in the ranking and (the more remote the position in the ranking, the fewer points it brings) and their subsequent addition is calculated.

Thus, in addition to the rule of adding ranks), where the total rank R(A) of an alternative in the preference profile Pn is the sum of its ranks in all rankings of the profile. $(rn\ (an,\ Pn))$

$$R(\mathrm{a}) = \sum_{k=1}^{n} r_i \quad (a_r P_n)$$

For the Borda rule, the sum becomes the sum of its ranks, taking into account the number of votes and the distance from the highest rank.

$$R(\mathrm{a}) = \sum_{i=1}^{n} r_{r_{a_1} - r_i} \quad (a_{r_i} P_n)$$

where is the number of votes for alternative a with rating ri,Pn

$$r_{r_{a_1} - r_i}$$

is the distance in the preference ranking, the distance of the counted vote of alternative ai at position ri from the most preferred place in the ranking ra1.

The Borda method allows all available opinions to be taken into account due to the compensation effect, but does not always guarantee that the winner will be preferred by the majority of voters.

"Kemeny Distance and Kemeny Median"

$$D(A,B) = \sum |a_{ij} - b_{ij}|,$$

Kemeny's rule has the positive properties of the Condorcet rule and does not lead to a paradox, which makes it possible to recommend it for use in various situations of preference aggregation, for example, in multi-criteria (or group) decision making, in the field of artificial intelligence and, in particular, machine learning.

Appropriate methods are used to solve problems such as crowdsensing/labeling, sentiment analysis, creation of meta-search engines, etc.

However, there are two serious obstacles to the practical use of Kemeny's rule: (1) the Kemeny ranking problem (KRP) is NP-hard and (2) Kemeny's rule can lead to multiple consensus rankings

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Kemeny distance is a metric that counts "total number of discrepancies among all the voters in their pairwise preferences between all candidates"[22]

The search for the Kemeny distance is carried out between two preference profiles. () by forming all possible pairs of candidates in each ranking.$P,\ P_1$,

$P,$	$A \prec B \prec C$
P_1	$B \prec A \prec C$

Based on this, a preference matrix is formed that displays how many times one candidate is preferred to another among all voters. It is the pairwise difference in its contents (the number of mismatched pairs and their order relative to each other) that forms the number that is the distance in unique preferences for two given sets.

	A	B	C
A	0	1	1
B	0	0	1
C	0	0	0

[22] William H. Press University of Texas at Austin August 6, 2012 C++ Program for Kemeny-Young Preference Aggregation https://numerical.recipes/whp/ky/kemenyyoung.html#usage

	A	B	C
A	0	0	1
B	1	0	1
C	0	0	0

A and B	: $A \prec B$ (1) : $B \prec A$ (0)	Difference:1
A and C	$A \prec C$ (1) : $A \prec C$ (1)	Difference:0
B and C	$B \prec C$ (1) $B \prec C$ (1)	Difference:0

PROBLEM STATEMENT

Since the practical task is to find the most optimal (in terms of Kemeny distance) and least computationally expensive method among the existing simple compensatory and non-compensatory aggregation methods.

The most important first task is to objectively formulate optimization and efficiency criteria.

The optimization criterion will be determined by the time costs calculated during the execution of the aggregation process.

The time cost directly depends on the computational complexity of the function written to rank group preferences.

To make it easier to understand, the details of the practical progress of the study, as well as the description of the problems encountered along the way, will be chronological rather than logical.

The implementation of practical research consists of the following stages:

IMPLEMENTATION AND APPLICATION

1) Evaluating, writing and testing the preference aggregation function
2) Determining requirements for the dataset and generating multiple preference profiles.
3) Methods for calculating Kemeny distance. Calculation of maximum satisfaction and dissatisfaction.
4) Methods for determining efficiency and the principle of Pareto efficiency
5) Schematic representation of the experiment.
6) Proposing a hypothesis for combining aggregation methods and their effectiveness according to Kemeny

7) The course of the experiment. The results and conclusions obtained.

Evaluating and writing and testing the preference aggregation function.

This description is necessary not only due to the understanding of the correct technology of operation of the aggregation method in practice, but also for the purpose of assessing the computational complexity, which affects the amount of work of the compiler, as well as the necessary memory to perform a particular action. In this work, these variables are understood as optimization.

Outside of theoretical analysis, in practice these factors will be expressed in the form of data processing speed, and measured by the metric - time.

Computational complexity: O(n), where n is the number of candidates.

Traditionally, computational complexity is measured in orders of magnitude, and in principle, the greatest complexity that it assumes in any part is considered to be computationally complex. Since these functions are mainly represented by linear orders with computational complexity O(n) or tending (coupland and partly board) to that is, second order. Each stage will be evaluated and the algorithm will be evaluated in its entirety.

Borda method

To write the board function, variables are initialized, the number of candidates is determined based on the length of the first list of preferences \(O(n)\), a dictionary `scores` is created, in which each candidate is assigned an initial score value of zero.

```python
num_candidates = len(preferences[0])
```

```
scores = {candidate: 0 for candidate in range(1, num_candidates + 1)}
```
Then the compiler goes to an outer loop that iterates through all the preference lists(a). And it enters an internal loop that goes through each candidate in the current preference list, where, taking into account the rank in the preference list, the corresponding number of points (b) is added. (O(m * n)\)
-Where \(m\) is the number of voters, and \(n\) is the number of candidates. Nested loops add points for each candidate.

Based on the received data, sorts the candidates in descending order\(O(n log n)\)
A)
```python
for preference in preferences:
```
b)
```python
for rank, candidate in enumerate(preference):
scores[candidate] += (num_candidates - rank)
```
Computational complexity:

- **Initialization**: \(O(n)\)
- **Scoring**: \(O(m \cdot n)\)
- **Sorting candidates**: \(O(n \log n)\)

Leximin method

To write the Leximin function, variables are initialized, the number of candidates is determined based on the length of the first O(n) preference list, and a scores dictionary is created, in which each candidate is assigned an empty list to store the ranks.

The compiler then moves to an outer loop that iterates through all the preference lists. And enters an inner loop that goes through each candidate in the current preference list, and adds its rank to the list of the corresponding candidate. (O(m*n))

Based on the received data, the compiler sorts the lists of ranks for each candidate O(n·mlogm)O(n*m log m)O(n·mlogm). Then sorts the candidates in the lexicographic order of their sorted rank lists O(n·m)O(n*m)O(n·m).

Computational complexity:

- **Initialization**: $O(n)$
- **Filling ranks**: $O(m \cdot n)$
- **Sorting ranks**: $O(n \cdot m \log m)$
- **Sorting candidates**: $O(n \cdot m)$

Leximak method

To write the Leximak function, variables are initialized, the number of candidates is determined based on the length of the first O(n) preference list, and a scores dictionary is created, in which each candidate is assigned an empty list to store the ranks.

The compiler then moves to an outer loop that iterates through all the preference lists. And enters an inner loop that goes through each candidate in the current preference list, and adds its rank to the list of the corresponding candidate. (O(m* n)). Nested loops populate the scores dictionary, adding a rank for each candidate.

Based on the received data, the compiler sorts the lists of ranks for each candidate in reverse order O(n·mlogm)O(n*m \log m)O(n·mlogm). Then sorts the candidates in the lexicographic order of their sorted rank lists O(n·m)O(n \cdot m)O(n·m).

Computational complexity:

- **Initialization**: $O(n)$
- **Filling ranks**: $O(m \cdot n)$
- **Sorting ranks**: $O(n \cdot m \log m)$
- **Sorting candidates**: $O(n \cdot m)$

Copeland method

To write the Copeland function, variables are initialized, the number of candidates is determined based on the length of the first preference list O(n)O(n)O(n), and a scores dictionary is created in which each candidate is assigned an initial score value of zero.

The compiler then moves on to an outer loop that iterates through all possible candidate pairs. And it enters an inner loop that goes through each list of preferences, and determines the voter's preference between the two candidates. (O(n^2 * m))

Based on the received data, the compiler sorts the candidates in descending order of their scores O(nlogn)O(n \log n)O(nlogn).

1. Computational complexity:
2. Initialization: $O(n)$
3. Pairwise comparison of candidates: $O(n^2 \cdot m)$
4. Sorting of candidates: $O(n \log n)$

Determining requirements for the dataset and generating multiple preference profiles.

Types of data files and their descriptions

Data files for preference aggregation traditionally have the following types: SOC, SOI, TOC, TOI

SOC - Strict Orders - Complete List
The SOC extension includes preferences represented by a strict and complete linear order (transitive and asymmetric relation) among alternatives. Cannot be equal and incomplete.[23]

SOI - Strict Orders - Incomplete List
A preference that does not include the entire set of alternatives. However, the alternatives cannot be equal.

TOC - Orders with Ties - Complete List
Full set of candidates, but alternatives may be equal

TOI - Orders with Ties - Incomplete List
The TOI extension includes preferences represented by a transitive and possibly incomplete relation among alternatives.

For this research work, SOC data was chosen because it helps avoid errors due to missing data or ambiguity due to equating different alternatives.

Generate multiple preference profiles.

[23] https://preflib.simonrey.fr/

Alternative profiles of the set of preferences in this work are represented by a set of randomly generated m! ordered combinations of possible individual preferences using the permutations function and the formation of n rankings randomly using the random - function (in an alternative profile of a set of preferences) (Original Preferences)

Calculating the distance of Kemeny. Calculation of maximum satisfaction and dissatisfaction.

Despite its computational complexity, the Kemeny distance described in the theoretical part, in the practical part in Python is calculated absolu[24]tely exactly as the lexicographic form suggests.

Afterwards, the discrepancy counter of the function dist = 0 is initialized, based on the input data represented by the two rankings, pairs of elements are formed, in theory represented by a discrepancy matrix, and then the distance is calculated by adding one for each discrepancy.

```
        if (rank1[i] < rank1[j]) != (rank2[i] < rank2[j]):
            dist += 1
```

In practice, there is also a ready-made function that automatically calculates satisfaction for a person based on the distance of the person, as well as information about the number of voters and the number of possible alternatives. There is also a separate library on github "kemeny_ranking"to calculate the distance of kemini.

However, for some reason this function did not work correctly in my dataset due to the dislike of my interpretation of Python for the "combs, perms" libraries as a result of their implementation,

24

negative values were obtained, so I had to consider the satisfaction and dissatisfaction of the system with the actual written function.

Determining the satisfaction of an individual regarding social ranking is the value of the maximum distance along the kemeny with the subtraction of the resulting kemeny distance. The absolute satisfaction of an individual is taken as zero according to the Kemeny distance, since in this case the opinion of the crowd absolutely coincides with his opinion. When the maximum discrepancy, or the maximum possible distance, is displeasure.

The maximum Kemeny distance for a permutation of length n can be calculated as follows:

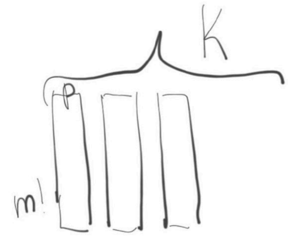
$$\boxed{R(k)= \frac{n(n-1)}{2}}$$

Further explanations will be given regarding the application of the Kemeny distance to specific quantities, since, from my point of view, the theoretical description in the practical part makes it easier to perceive.

Schematic representation of the progress of research work.

Generation m! ordered alternatives

Random placement of m ordered alternatives into a set of preference profiles



Applying the aggregation rule and forming a ranking of alternatives

Ranked alternatives according to a specific rule now have the appearance of social preferences according to a specific rule.

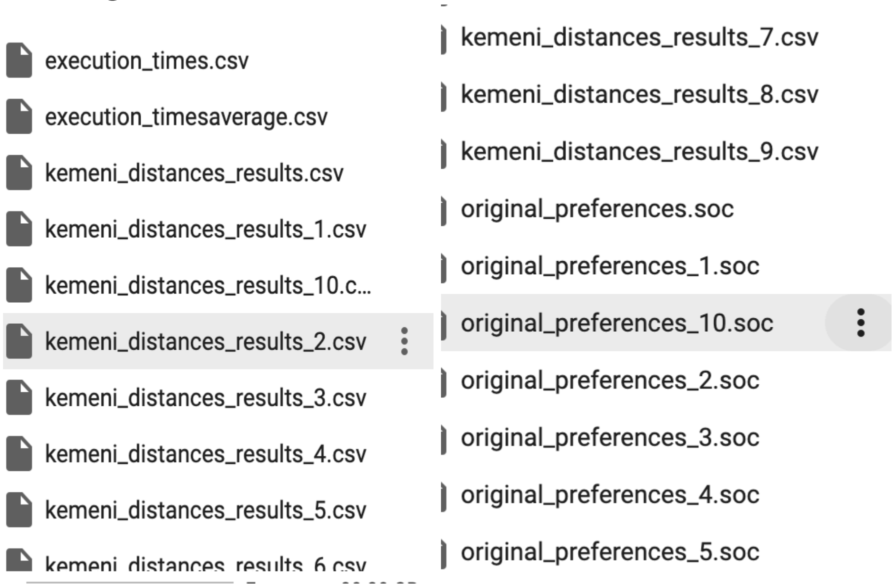
A summary calculation of the distance between each specific individual preference and the social choice ranking

d(F1())+d(F1())....P P_1,P $P2$,
where d(F1()P P_1),is the distance between the collective preference resulting from applying the aggregation method and the first individual preference in the dataset

and whered(F1()) -P $P2$,this is the distance between the collective preference resulting from applying the aggregation method and the second individual preference in the dataset

Determining the average kemen distance for each aggregation method

Saving received data to files.



Construction of graphs reflecting the average statistical time spent by the aggregation method and the level of satisfaction with the result obtained

Determining the effectiveness and degree of optimization of the aggregation method using the Pareto principle

Creation of combined aggregation methods and their evaluation

Comparison of the results obtained

Methods and technologies

The main technologies used were imported libraries and individual functions removed from the open source Github code.

For visualization	Matplotlib	From Github
For data repurchase	NumPy Pandas	[25]https://github.com/PrefLib [26]https://github.com/erelsgl/PrefLib-Tools

[25] https://github.com/PrefLib
[26] https://github.com/erelsgl/PrefLib-Tools

	SciPy itertools collections and many others	[27]https://github.com/erelsgl/PrefLib-Tools/blob/master/preflibtools/notebooks/MSS_16_Tutorial.ipynb [28]https://github.com/mikedillion/PrefLib-Tools [29]https://github.getafreenode.com/django/django [30]https://hturner.github.io/PlackettLuce/reference/preflib.html [31] https://github.com/logc/borda [32]https://github.com/johnh865/election_sim

Introduction of the Pareto principle to address the issue of efficiency and optimization of the algorithm.

The Pareto principle or Pareto optimality is a concept in economics and decision theory that describes the allocation of resources or outcomes in which it is impossible to improve the position of one participant without making another participant worse off.

This principle is applied as a ready-made imported library function.

Proposing a hypothesis for combining aggregation methods and their effectiveness according to Kemeny.

Hypothesis 1. The Borda ranking method will be most effective since it is compensatory and takes into account the ranking of each candidate by each voting participant.

[27] https://github.com/erelsgl/PrefLib-Tools/blob/master/preflibtools/notebooks/MSS_16_Tutorial.ipynb
[28] #https://github.com/mikedillion/PrefLib-Tools
[29] https://github.getafreenode.com/django/django
[30] https://hturner.github.io/PlackettLuce/reference/preflib.html
[31] https://github.com/logc/borda
[32] https://github.com/johnh865/election_sim

Hypothesis 2. The Leximax method will be most effective, since it spends the least amount of time on execution.

Several ideas have been proposed as a way to improve operational efficiency in terms of distance:

1) using an ensemble approach to increase the likelihood of effective
2) application of several aggregation methods and search for an effective empirical combination
3) increasing the accuracy of the method using other various metrics

The use of ensemble approaches was regarded by me as the most likely. Increasing the accuracy of the method using other various metrics would lead to a change in the essence of the research work and a deviation from the study of the effectiveness of aggregation methods precisely in accordance with the kemeny distance; the application and search for an effective empirical solution is based on the implementation of self-learning systems and would vary from specific data.

Therefore, I chose to use ensemble methods based on the following idea:

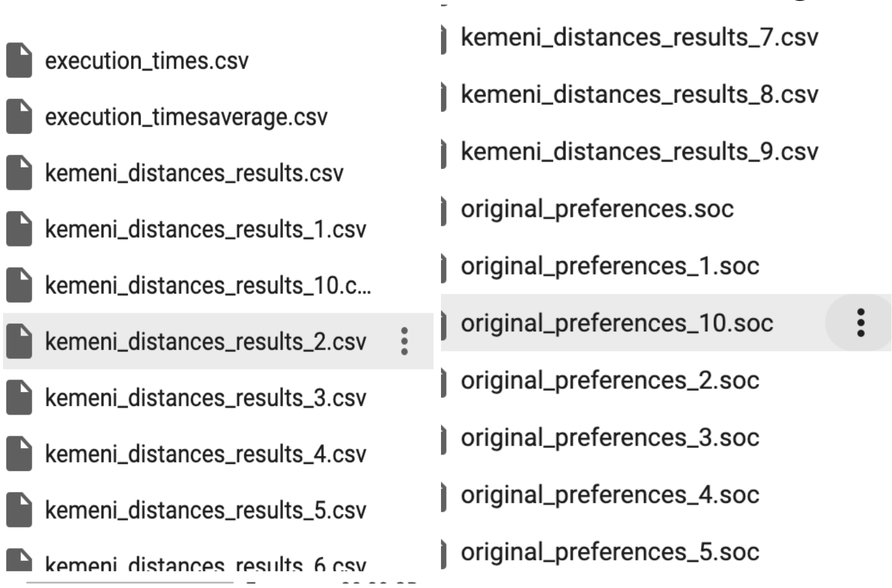
Find the most optimized and effective aggregation methods from a quantitative point of view and as a result of applying the ranking of alternatives using three different methods, fixing the place in the ranking of the alternative in case of receiving matching ratings.

For alternatives that do not have the same scores, you choose one of the remaining methods to determine their final ranking.

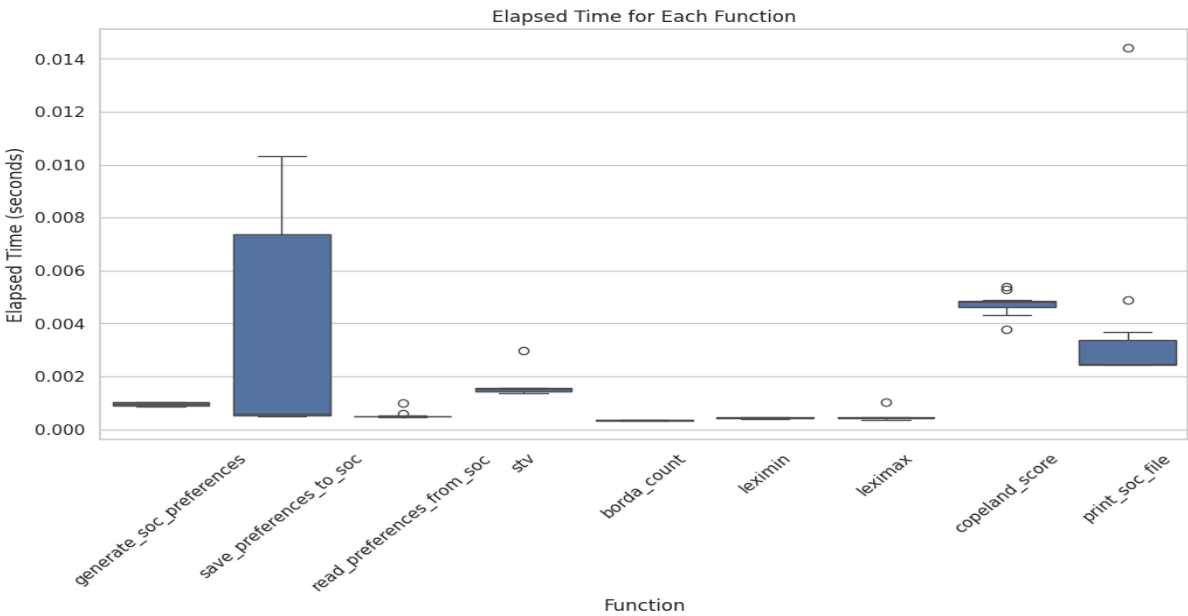
Hypothesis 3. Combining effective aggregations in a way that avoids the random placement of an alternative in a certain place will increase satisfaction with a certain value.

Description of the progress of the project and the results obtained.

At the first auxiliary stage, I wrote the code and generated a list of necessary materials for further study of the features of aggregation, taking into account the peculiarities of the code and the generated database of various sizes. As the main and standard dataset, a size consisting of 10 possible alternatives and 100 different choices was used, which were integrated 10 times.

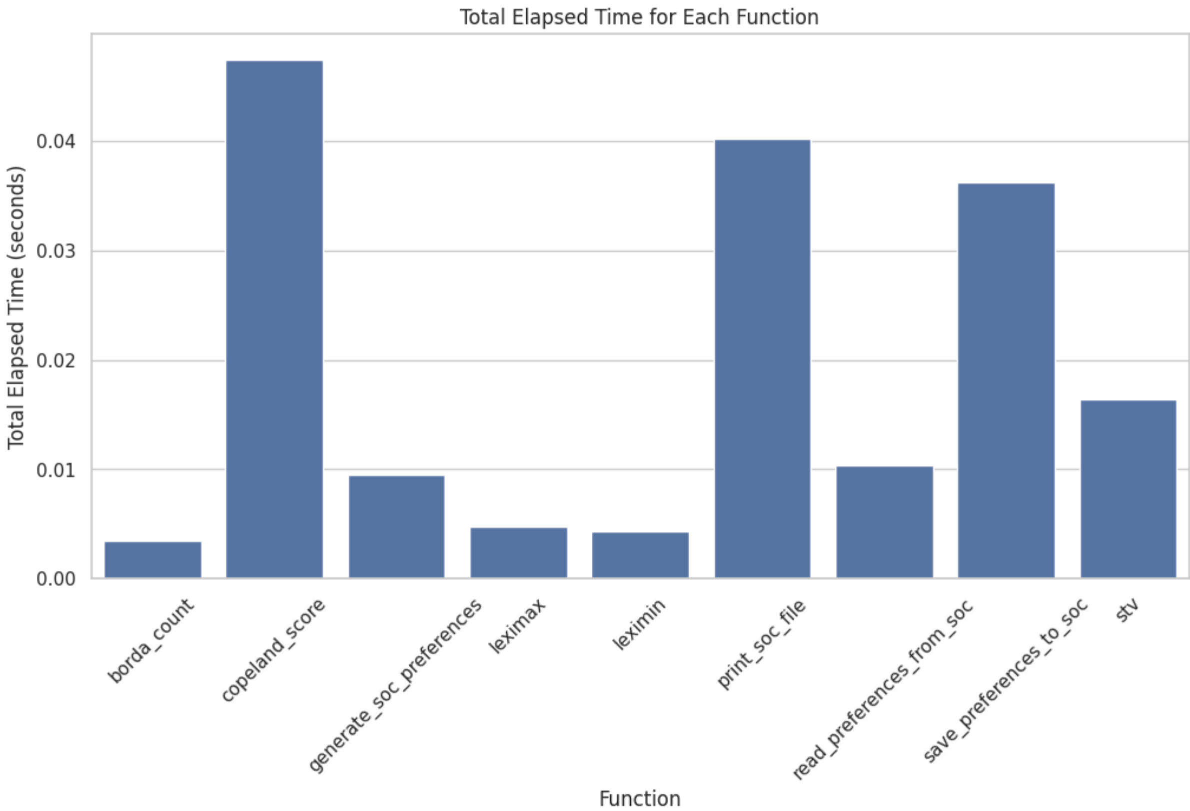


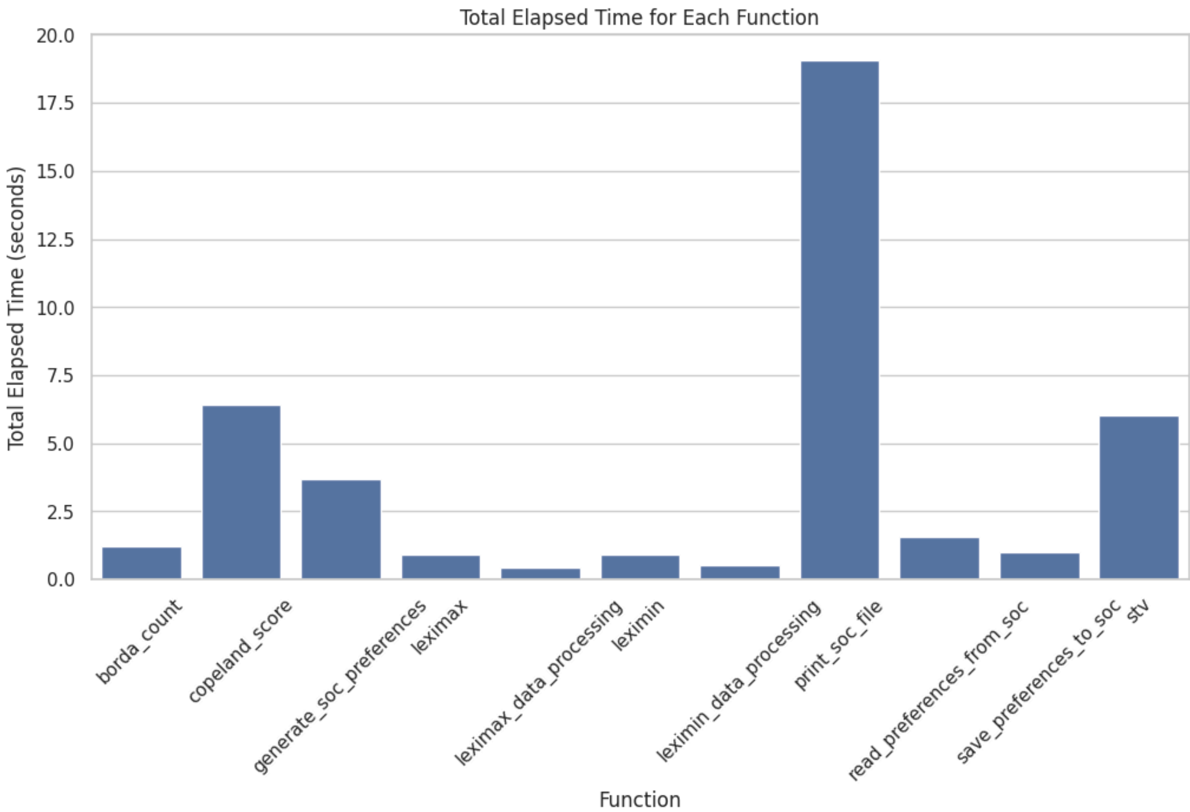
For a more accurate representation of the work of aggregation methods, a practical comparison of the methods was carried out not only in the relationship between time and efficiency, but also these parameters relative to each other independently. The study of the operation of individual aggregation methods was carried out from the beginning with an assessment of the time consumption relative to all parts of the function.



-
-

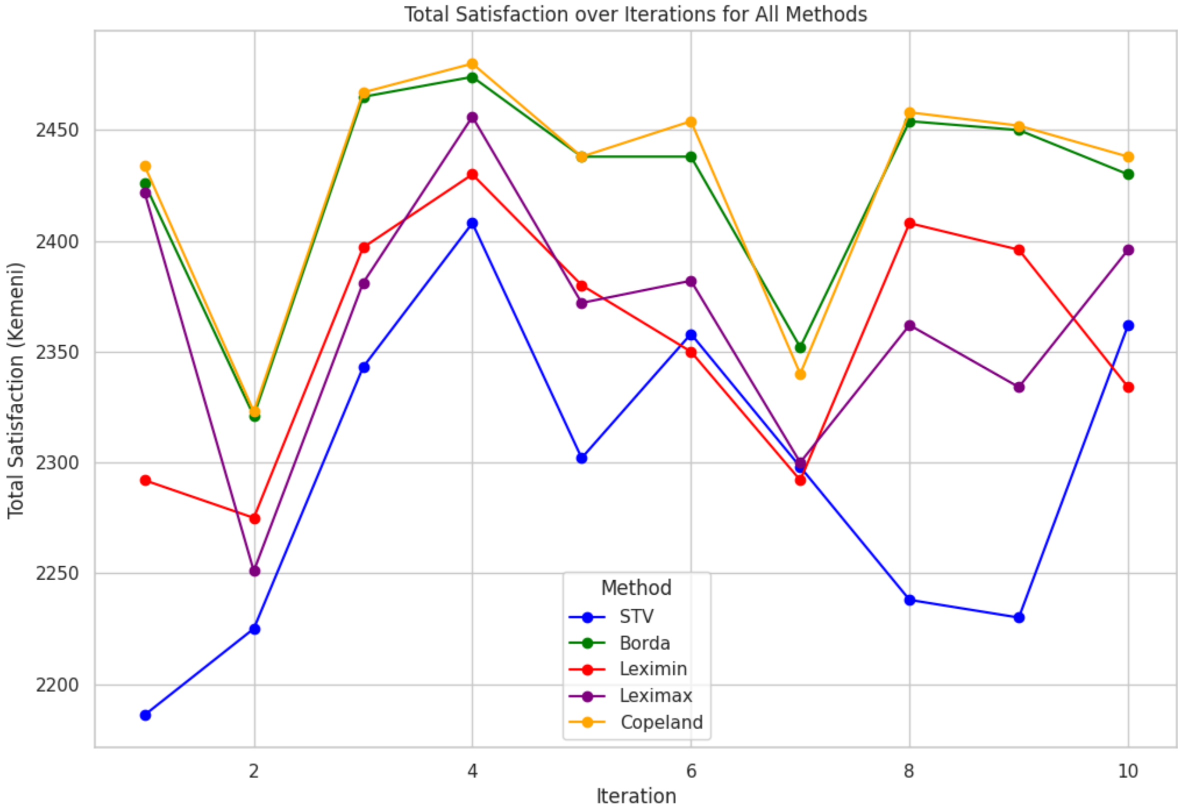
Although this stage is not mandatory, a defect was discovered, for example, at the stage of applying the Leximin Lexmax rule, it turned out that despite the execution of the code function to determine candidates, the rest of the date still continued to be read, which technically unjustifiably complicated the work of the method, relative to its theoretical representation.



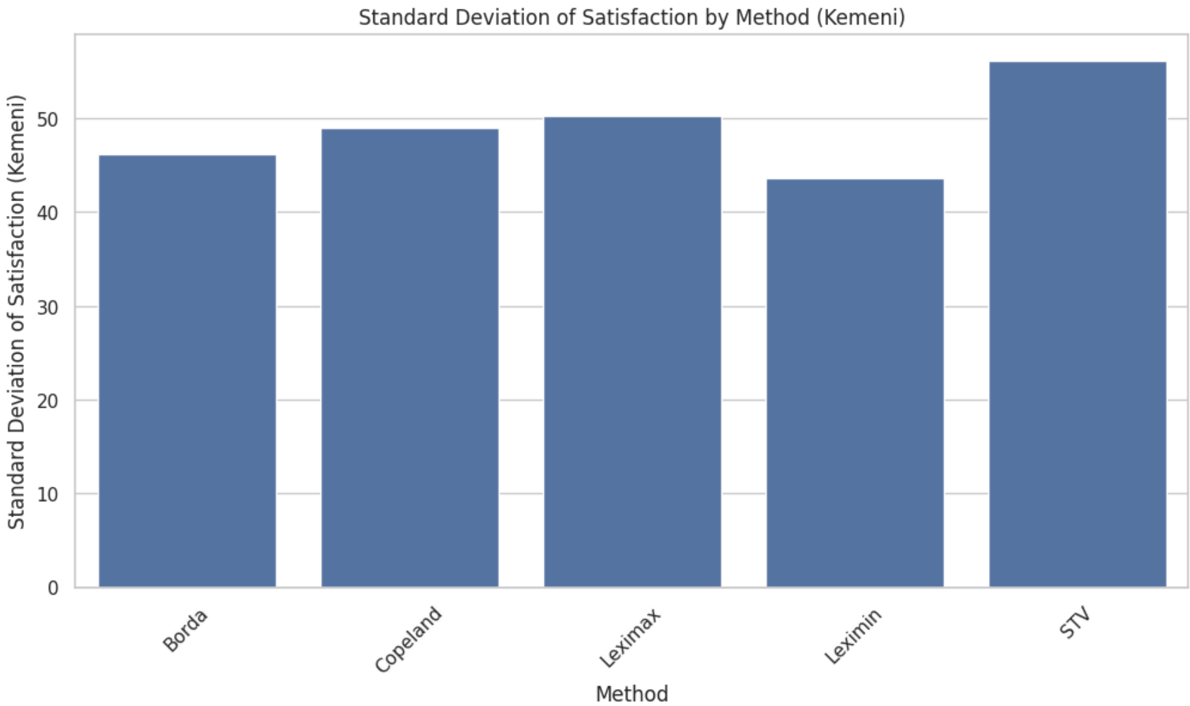


Next, a comparative study was carried out on the effectiveness of ranking methods relative to Kemeny distance in relation to each other, both on small and large values, and the average efficiency of aggregation methods by Kemeny varied from each other by less than 10% of the maximum distance by Kemeny.

And the best readings of the indicators, not surprisingly, were compensatory aggregation methods.



With the standard deviation of the work of aggregation methods relative to their own indicators, it was approximately 1% of the maximum distance by kemen for all aggregation methods.



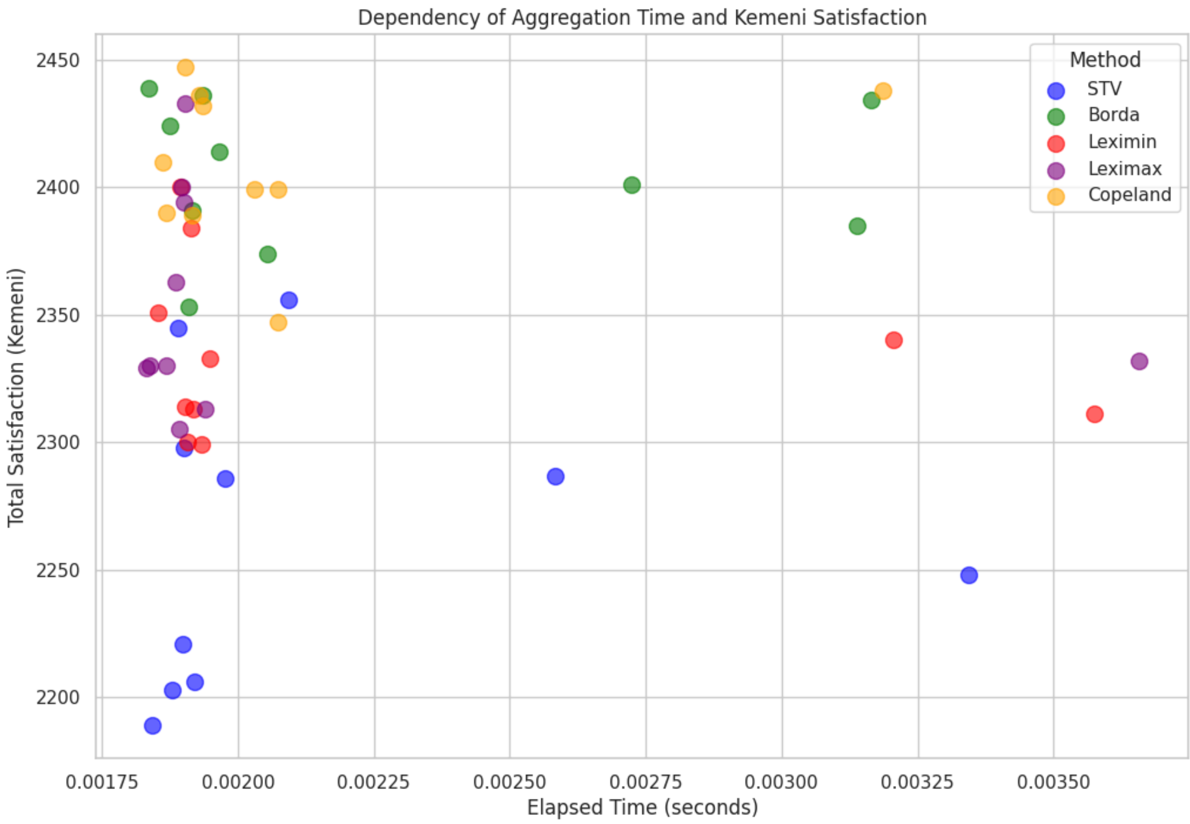
	Method	Total Satisfaction
0	STV	2209
1	Borda	2431
2	Leximin	2351
3	Leximax	2387
4	Copeland	2449

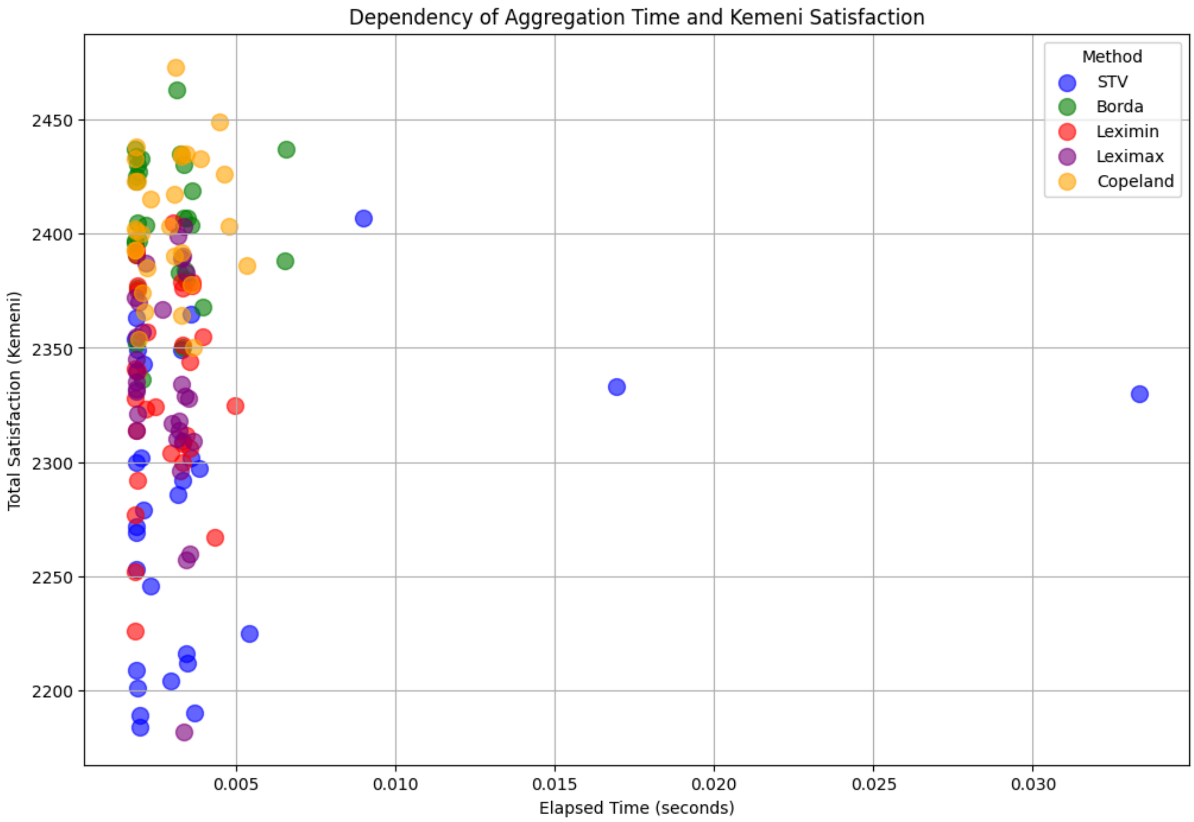
	Method	Standard Deviation of Satisfaction
0	Borda	46.197763
1	Copeland	49.042499
2	Leximax	50.312910
3	Leximin	43.651270
4	STV	56.214767

Standard Deviation of Satisfaction by Method (Kemeni)

An attempt was also made (3 attempts, 10 iterations of 100 voting 10 alternatives) to separately study the reasons for the sharp change in the kemen distance depending on individual date sets within specific aggregation methods, however, relative to each other, the aggregation methods reacted almost identically (as can be seen in the graph above) no obvious interesting relationships were identified.

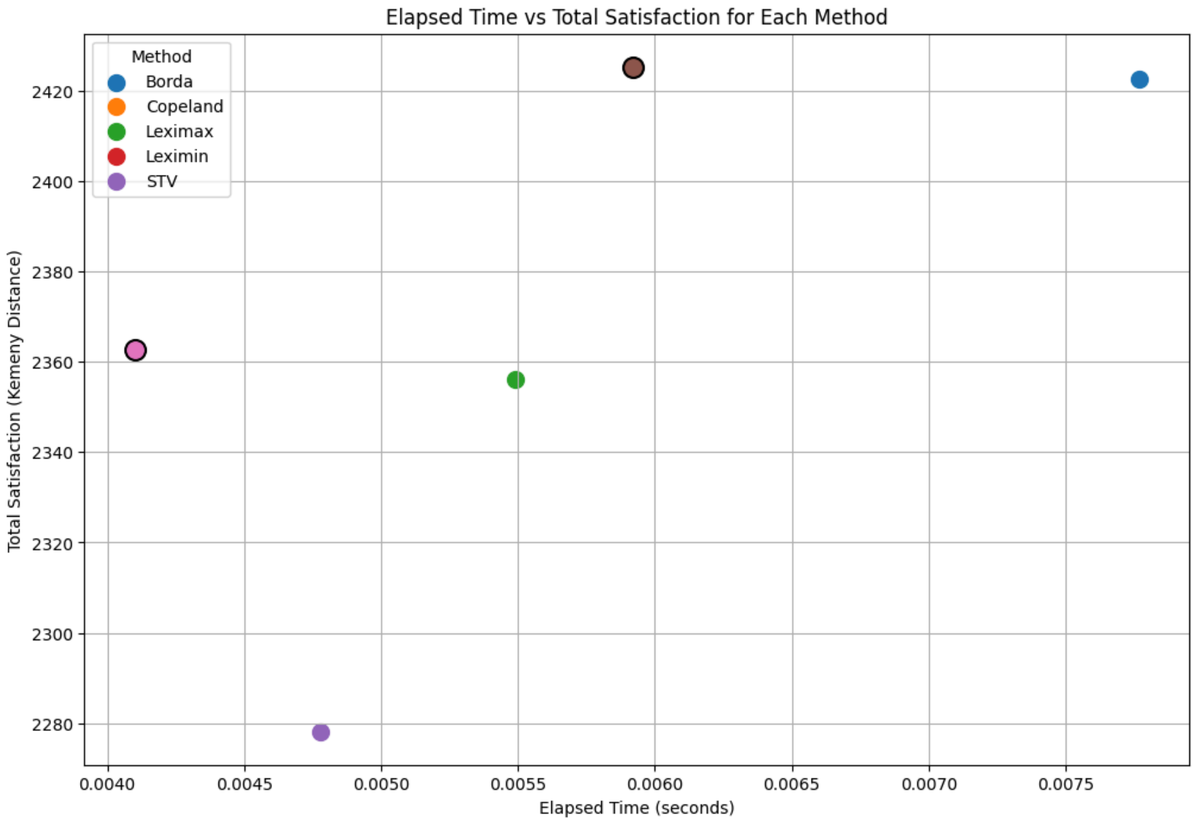
Therefore, it was decided to construct a graph that would reflect an understanding of the effectiveness of the aggregation method from the point of view of this work, namely the operating time and distance according to Kemeny.





The initial changes showed some uncharacteristic deviations in the operation of aggregation methods on certain iterations of datasets. Further study of it served as the basis for the assumption that the deviation is associated with instability in the operation of the Internet network, since the aggregation method that took the longest to compile was the most susceptible to this effect.

Applying a ready-made function to the received data determined the next winner in accordance with the Pareto principle.

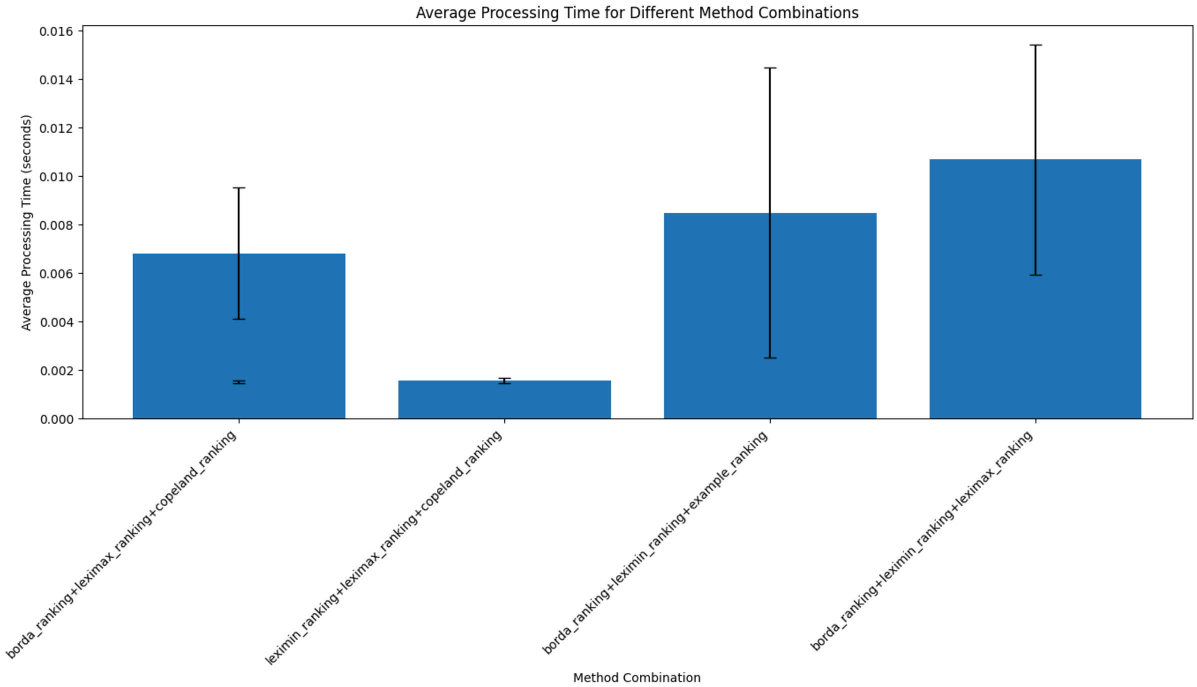


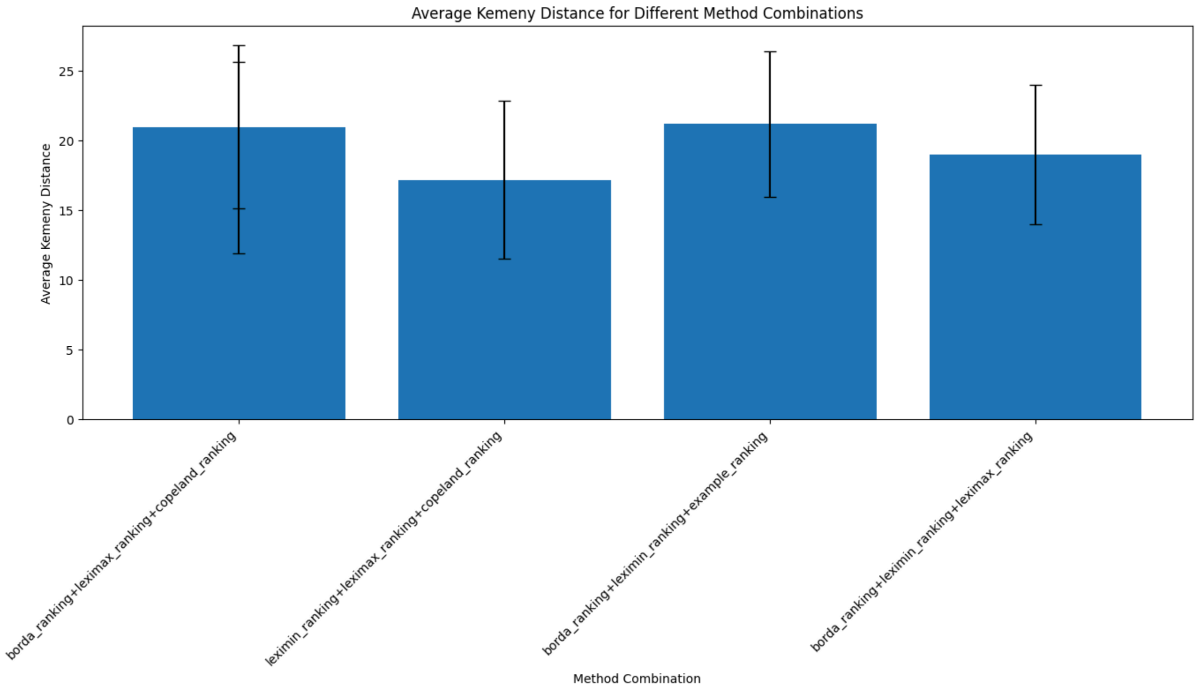
index	Method	Elapsed Time	Total Satisfaction
1	Copeland	0.0059208631515502924	2425.2
3	Leximin	0.004100203514099121	2362.6

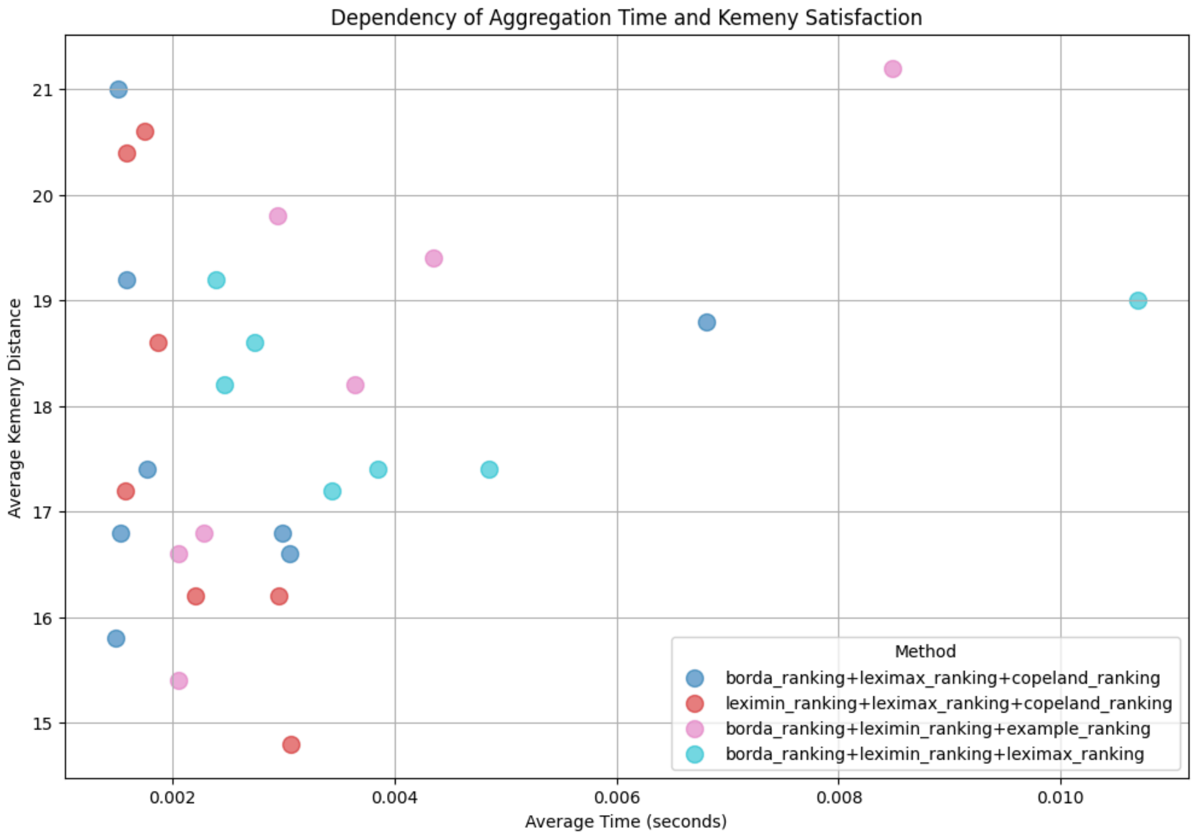
Creation of combined aggregation methods and their evaluation

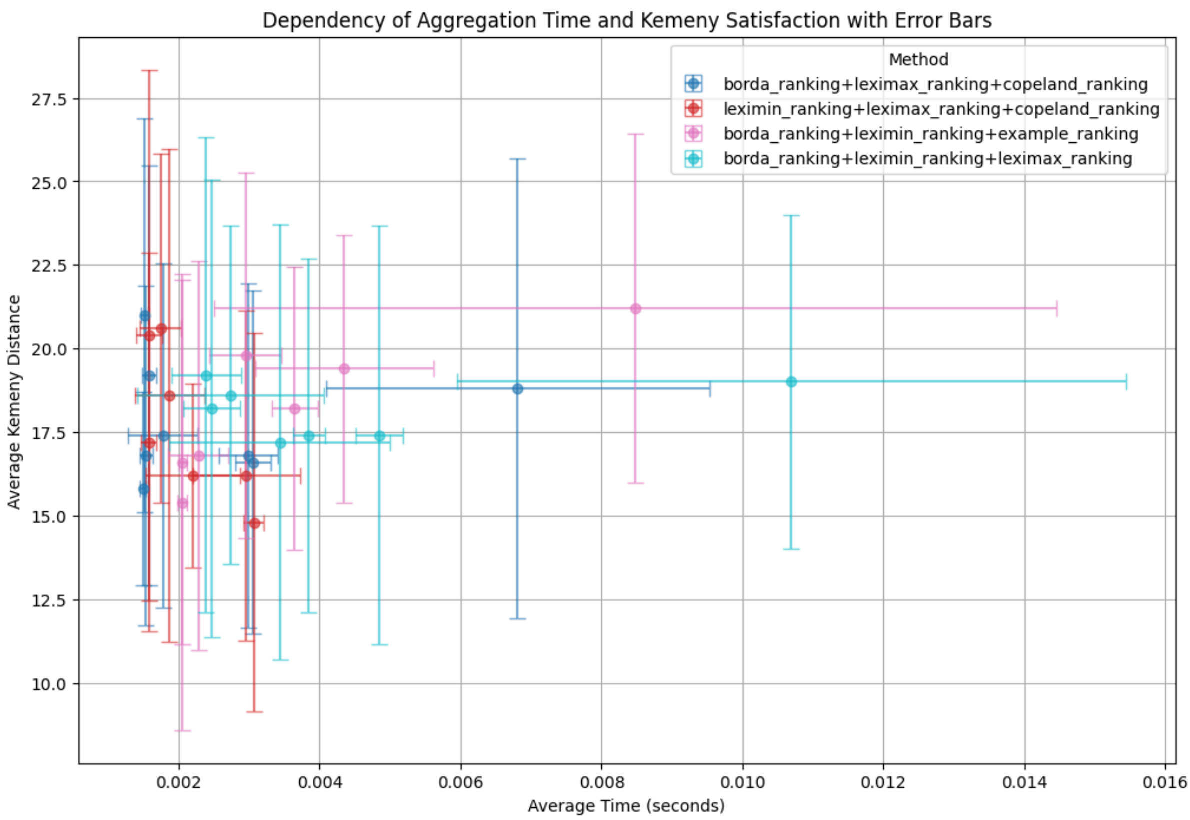
Since models with STV showed negative results both in speed of work and in the degree of satisfaction in terms of quality, it was not used in the future as a method that serves as the basis for using the combination.

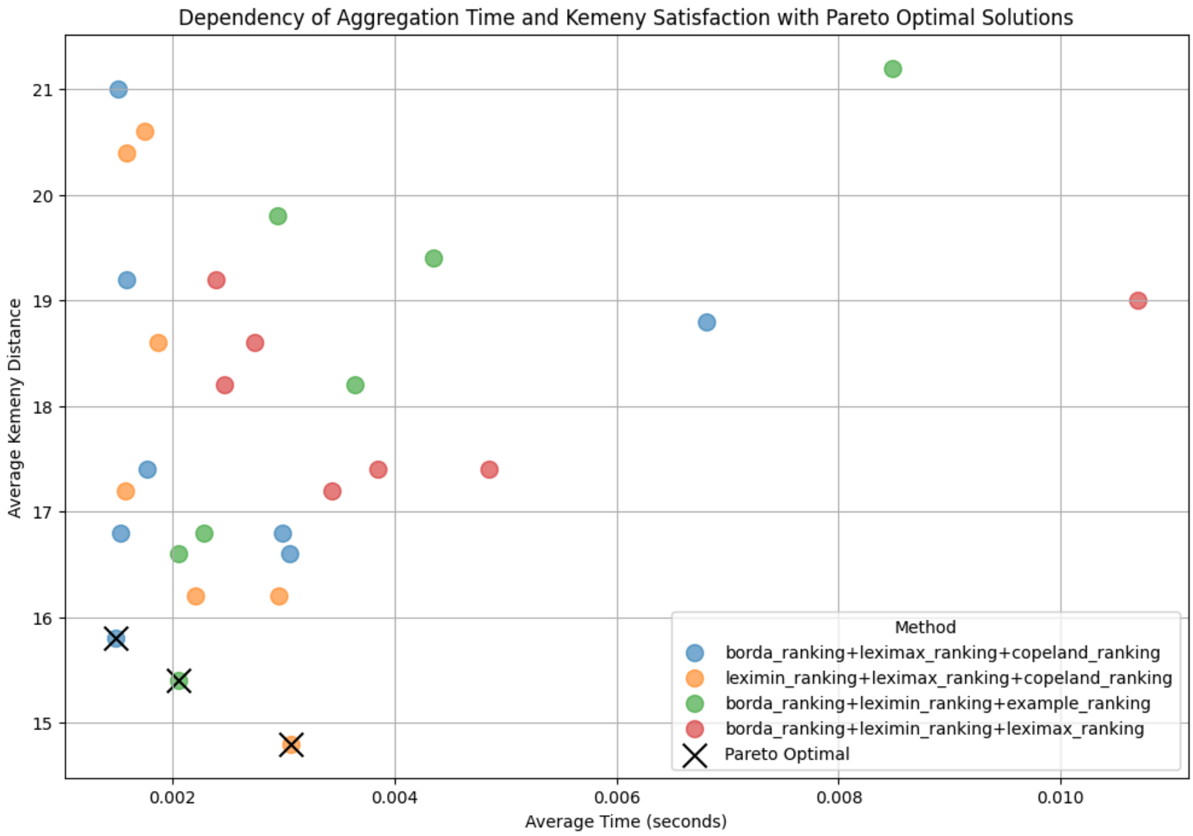
The rest of the assessment and the rest of the assessment model of analysis follow a similar structure to the previous part.






Dependency of Aggregation Time and Kemeny Satisfaction
Average Kemeny Distance
Average Time (seconds)
Method
borda_ranking+leximax_ranking+copeland_ranking
leximin_ranking+leximax_ranking+copeland_ranking
borda_ranking+leximin_ranking+example_ranking
borda_ranking+leximin_ranking+leximax_ranking


Dependency of Aggregation Time and Kemeny Satisfaction with Error Bars
Average Kemeny Distance
Average Time (seconds)
Method
borda_ranking+leximax_ranking+copeland_ranking
leximin_ranking+leximax_ranking+copeland_ranking
borda_ranking+leximin_ranking+example_ranking
borda_ranking+leximin_ranking+leximax_ranking



The graph shows that despite the selective combination of more effective methods, the distance along the road does not improve.

Thus, hypothesis one, hypothesis 2, hypothesis 3 were not confirmed. Despite all the assumptions about the effectiveness of the Bard or Xenin method due to the peculiarities of their compilation in time or due to their similarity in the evaluation method with whom and the most optimal is Copeland, which is quite unexpected.

However, the graphs show that the combination is better than conventional response methods, although not by much.

Conclusion and analysis of the results obtained

The main goal of the work of finding a method for optimizing the mechanisms of aggregation by Kemeny distance from individual preferences to collective preservation with computational ease was partially fulfilled: A solution was found that allows you to get rid of minor unnecessary deviations caused by random date deviations. , which is a positive effect, but it does not significantly improve the algorithm.

Of the three assigned tasks, all three have been completed: the results obtained from various aggregation methods have been compared. The operating time of each of the aggregation methods in various situations was calculated; the assessment of the data when finding the complexity of the algorithm was fully justified. a minus was found during the work and corrected for the better: when using the Leximax methods as an example, we managed to get rid of unnecessary reading of dates, which also wastes time (see graph above). As the date increases, the greatest increase in time spent is shown by the Bard method (see graph above)

The second task, which is to evaluate the results of applying aggregation methods based on the Kemeny distance, was also completed successfully.

It turned out that with the exception of the scv method in terms of kemen distance, such basic methods as Barda, Copeland, Leximin, Leximax in the long term do not have large significant differences in the satisfaction of individuals when aggregating public preferences. a stable indicator of the level of satisfaction of individuals on average ranges from 40 to 52-55% (using the previous date and visual assessment of graphs) and even in the case of significant deviation does not exceed these values. The lowest efficiency score was obtained by the method from STV and the highest by the Copeland method, which was a rather unexpected result.

The third task was partially completed; a search for optimization was carried out: a hypothesis was put forward, it was justified, and implemented in practice. However, its indicators remained neutral.

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