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第二章

2-1

硅突变结二极管的掺杂浓度为：$N_d = 10^{15}cm^{-3}$，$N_a = 4\times10^{20}cm^{-3}$，在室温下

计算：

(1)自建电势；(2)耗尽层宽度；(3)零偏压下的最大内建电场。

解：

（1）饱和电离区有：

N型区：$n = N_d = n_i\exp(\dfrac{E_{fN}-E_i}{kT}) \implies E_{fN}-E_i = kT\ln\dfrac{N_d}{n_i}$

P型区：$p = N_a = n_i\exp(\frac{E_i-E_{fP}}{kT}) \implies E_i - E_{fP} = kT\ln\frac{N_a}{n_i}$

两式相减可得：$q\psi_0 = E_{fN}-E_{fP} = kT\ln\dfrac{N_dN_a}{n_i^2}$

代入数据：

$$\psi_0 = \frac{kT}{q}\ln\frac{N_dN_a}{n_i^2} = V_T\ln\frac{N_aN_d}{n_i^2} = 0.026V\times\ln\frac{4\times10^{20}\times10^{15}}{(1.5\times10^{10})^2} \approx 0.91V$$

（2）由

$$N_ax_p = N_dx_n \implies x_n = \frac{N_a}{N_d}x_p = \frac{4\times10^{20}}{10^{15}}x_p = 4\times10^5x_p \implies x_n \gg x_p$$

则有 $W\approx x_n$，且内建电势几乎全部降落在N侧的空间电荷区上。

由N侧泊松方程：$\dfrac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = -\dfrac{\rho}{\epsilon} = -\dfrac{qN_d}{\epsilon}$，得通解为

$$\psi(x) = -\frac{qN_d}{2\epsilon}x^2 + C_1x + C_2,\quad \text{其中}C_1, C_2\text{为常数}$$

代入边界条件:

$$\begin{cases} -\left.\dfrac{\mathrm{d}\psi}{\mathrm{d}x}\right|_{x=x_n} = 0, & N\text{侧中性区无电场} \\ \psi(0) = 0, & \text{设两侧交界即}x=0\text{处电势为}0 \end{cases} \implies \begin{cases} C_1 = \dfrac{qN_dx_n}{\epsilon} \\ C_2 = 0 \end{cases}$$

则有

$$\psi(x) = -\frac{qN_d}{2\epsilon}x^2 + \frac{qN_dx_n}{\epsilon}x$$

内建电势差$\psi_0=\psi(x_n)-\psi(0)=\frac{qN_dx_n^2}{2\epsilon} \implies x_n=\sqrt{\frac{2\psi_0\epsilon}{qN_d}}$

耗尽区宽度

$$W\approx x_n=\sqrt{\frac{2\psi_0\epsilon}{qN_d}}=\sqrt{\frac{2\times 0.91\times 11.9\times 8.85\times 10^{-14}}{1.6\times 10^{-19}10^{15}}}cm\approx 1.09\times 10^{-4}cm$$

（3）有$E(x)=-\frac{\mathrm{d}\psi}{\mathrm{d}x}=-\frac{qN_d}{\epsilon}x+\frac{qN_dx_n}{\epsilon}$，显然当$x=0$时$E$取最大值

$$\begin{aligned}E_{max}=E(0)&=\frac{qN_dx_n}{\epsilon}\\&=\frac{1.6\times 10^{-19}\times 10^{15}\times 1.09\times 10^{-4}}{11.9\times 8.85\times 10^{-14}}V/cm\\&\approx 1.66\times 10^4V/cm\end{aligned}$$

2-2

若突变结两边的掺杂浓度为同一数量级，试证明自建电势和耗尽层宽度可用下式表示：

$$\psi_0=\frac{qN_aN_d(x_n+x_p)^2}{2k\epsilon_0(N_a+N_d)}\quad x_n=\sqrt{\frac{2k\epsilon_0\psi_0N_a}{qN_d(N_a+N_d)}}\quad x_p=\sqrt{\frac{2k\epsilon_0\psi_0N_d}{qN_a(N_a+N_d)}}$$

解：

令PN结交界处为$x=0$，P侧中性区边界为$x=-x_p$，N侧中性区边界为$x=x_n$，则有泊松方程：

$$\begin{cases}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}=\frac{qN_a}{\epsilon}, & -x_p\le x\le 0\\ \frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}=-\frac{qN_d}{\epsilon}, & 0\le x\le x_n\end{cases}$$

通解为：

$$
\begin{cases}
\psi(x) = \dfrac{qN_a}{2\epsilon}x^2 + C_1 x + C_2, & -x_p \le x \le 0 \\
\psi(x) = -\dfrac{qN_d}{2\epsilon}x^2 + C_3 x + C_4, & 0 \le x \le x_n
\end{cases}
$$

其中C_1, C_2, C_3, C_4为常数

边界条件:

- 令PN结交界处即$x=0$处为内建电势的零点，即$\psi(0)=0$;
- 两侧中性区电场为零且连续:

$$
E(-x_p) = -\left.\frac{\mathrm{d}\psi(x)}{\mathrm{d}x}\right|_{x=-x_p} = 0 \ , \ E(x_n) = -\left.\frac{\mathrm{d}\psi(x)}{\mathrm{d}x}\right|_{x=x_n} = 0
$$

代入以上边界条件可得$C_1 = \dfrac{qN_d x_p}{\epsilon}$, $C_3 = \dfrac{qN_a x_n}{\epsilon}$, $C_2 = C_4 = 0$

代入ψ表达式中

$$
\begin{cases}
\psi(x) = \dfrac{qN_a}{2\epsilon}x^2 + \dfrac{qN_d x_p}{\epsilon}x, & -x_p \le x \le 0 \\
\psi(x) = -\dfrac{qN_d}{2\epsilon}x^2 + \dfrac{qN_a x_n}{\epsilon}x, & 0 \le x \le x_n
\end{cases}
$$

有$N_d x_n = N_a x_p$, $W = x_n + x_p$，可得

$$
x_n = \frac{N_a}{N_a + N_d}W \ , \ x_p = \frac{N_d}{N_a + N_d}W
$$

内建电势差:

$$
\begin{aligned}
\psi_0 &= \psi(x_n) - \psi(-x_p) \\
&= -\frac{qN_d}{2\epsilon}x_n^2 + \frac{qN_a x_n}{\epsilon}x_n - \frac{qN_a}{2\epsilon}x_p^2 + \frac{qN_d x_p}{\epsilon}x_p \\
&= \frac{qN_d}{2\epsilon}x_n^2 + \frac{qN_a}{2\epsilon}x_p^2 \\
&= \frac{qN_a N_d}{2\epsilon(N_a + N_d)}(x_n W + x_p W) \\
&= \frac{qN_a N_d (x_n + x_p)^2}{2\epsilon(N_a + N_d)}
\end{aligned}
$$

则可求得:

$$W=\sqrt{\frac{2\epsilon\psi_0(N_a+N_d)}{qN_aN_d}}$$

$$x_n=\frac{N_a}{N_a+N_d}W=\sqrt{\frac{2\epsilon\psi_0N_a}{qN_d(N_a+N_d)}}$$

$$x_p=\frac{N_d}{N_a+N_d}W=\sqrt{\frac{2\epsilon\psi_0N_d}{qN_a(N_a+N_d)}}$$

2-3

推导出线性缓变PN结的下列表示式: (1)电场(2)电势分布(3)耗尽层宽度(4)内建电势差。

解:

令PN结交界处为$x=0$，P侧中性区边界为$x=-x_p$，N侧中性区边界为$x=x_n$

对于线性缓变结，空间电荷区关于$x=0$左右对称，即$x_n=x_p=\frac{W}{2}$

耗尽层电荷浓度分布$N(x)=N_d-N_a=\alpha x$，代入泊松方程

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}=-\frac{\rho}{\epsilon}=-\frac{q\alpha x}{\epsilon}\implies\psi(x)=-\frac{q\alpha}{6\epsilon}x^3+C_1x+C_2\ ,\quad \text{其中}C_1,C_2\text{为常数}$$

边界条件:

令PN结交界处即$x=0$处为内建电势的零点，即$\psi(0)=0$;

两侧中性区电场为零且连续，即

$$E(-\frac{W}{2})=E(\frac{W}{2})=-\frac{\mathrm{d}\psi(x)}{\mathrm{d}x}\bigg|_{x=W/2}=0$$

代入以上边界条件可得$C_1=\frac{q\alpha W^2}{8\epsilon},\quad C_2=0$，即

$$\psi(x)=-\frac{q\alpha}{6\epsilon}x^3+\frac{q\alpha W^2}{8\epsilon}x$$

(1)

$$E(x) = -\frac{\mathrm{d}\psi}{\mathrm{d}x} = \frac{q\alpha}{2\epsilon}x^2 - \frac{q\alpha W^2}{8\epsilon}$$

(2)

$$\psi(x) = -\frac{q\alpha}{6\epsilon}x^3 + \frac{q\alpha W^2}{8\epsilon}x$$

(3)

$$\psi_0 = \psi(\frac{W}{2}) - \psi(-\frac{W}{2}) = 2\psi(\frac{W}{2}) = 2[-\frac{q\alpha}{6\epsilon}(\frac{W}{2})^3 + \frac{q\alpha W^2}{8\epsilon}\frac{W}{2}] = \frac{q\alpha W^3}{12\epsilon}$$
$$\Longrightarrow W = \sqrt[3]{\frac{12\epsilon\psi_0}{q\alpha}}$$

(4)

$$\psi_0 = \frac{q\alpha W^3}{12\epsilon}$$

或用浓度表示

P侧N侧中性区杂质浓度：$p_{p0} = n_{n0} = \dfrac{\alpha W}{2}$，代入$\psi_0$浓度表达式

$$\psi_0 = V_T \ln\frac{p_{p0}n_{n0}}{n_i^2} = 2V_T \ln\frac{\alpha W}{2n_i}$$

2-4

推导出N^+N结(常称为高低结)内建电势差表达式。

解：

法一：热平衡时费米能级一致：

设N^+区杂质浓度N_d^+，N区杂质浓度N_d，饱和电离时有：

$$E_{fN^+} - E_i = kT\ln\frac{N_d^+}{n_i}\ ,\ E_{fN} - E_i = kT\ln\frac{N_d}{n_i}$$
$$\Longrightarrow\ q\psi_0 = E_{fN^+} - E_{fN} = kT\ln\frac{N_d^+}{N_d}$$

则有

$$\psi_0 = E_{fN^+} - E_{fN} = V_T\ln\frac{N_d^+}{N_d}$$

法二：静电势概念：

$$\psi^+ = V_T\ln\frac{N_d^+}{n_i}\ ,\ \psi = V_T\ln\frac{N_d}{n_i}$$
$$\Longrightarrow\ \psi_0 = \psi^+ - \psi = V_T\ln\frac{N_d^+}{N_d}$$

2-5

P^+N结空间电荷区边界分别为$-x_p$和x_n，利用$np = n_i^2 e^{V/V_T}$导出一般情况下的$p_n(x_n)$表达式。给出N区空穴为小注入和大注入两种情况下的$p_n(x_n)$表达式。

解：

在N侧空间电荷区和中性区的边界，即$x = x_n$处

$$\begin{cases} p_n = p_{n0} + \Delta p \approx \Delta p, & \Delta p \gg p_{n0} \\ n_n = n_{n0} + \Delta n \approx n_{n0} + p_n\ , & \Delta n = \Delta p \end{cases}$$

代入$np = n_i^2 e^{V/V_T}$可得$p_n n_n = p_n(n_{n0} + p_n) = p_n n_{n0} + p_n^2 = n_i^2 e^{V/V_T}$，解得

$$p_n = \frac{-n_{n0} + \sqrt{n_{n0}^2 + 4n_i^2 e^{V/V_T}}}{2} \quad 或 \quad p_n = \frac{-n_{n0} - \sqrt{n_{n0}^2 + 4n_i^2 e^{V/V_T}}}{2}(舍去)$$

小注入时：$p_n \approx \Delta p \ll n_{n0}$ ，则 $p_n^2 \ll p_n n_{n0}$，则有

$$p_n n_{n0} = n_i^2 e^{V/V_T} \implies p_n = \frac{n_i^2}{n_{n0}} e^{V/V_T} = p_{n0} e^{V/V_T}$$

大注入时：$p_n \approx \Delta p \gg n_{n0}$ ，则$p_n^2 \gg p_n n_{n0}$，则有

$$p_n^2 = n_i^2 e^{V/V_T} \implies p_n = \sqrt{n_i^2 e^{V/V_T}} = n_i e^{V/2V_T}$$

2-6

根据电子电流公式$I_n = qA(n\mu_n E + D_n \frac{\partial n}{\partial x})$推导方程

$\psi_0 = \psi_n - \psi_p = V_T \ln \frac{N_d N_a}{n_i^2}$。

解：

热平衡时，电子电流为零，即$I_n = qA(n\mu_n E + D_n \frac{\partial n}{\partial x}) = 0$

$$E = -\frac{1}{n\mu_n} D_n \frac{\mathrm{d}n}{\mathrm{d}x} = -\frac{V_T \mathrm{d}\ln n}{\mathrm{d}x} = -\frac{\mathrm{d}\psi}{\mathrm{d}x} \implies \mathrm{d}\psi = V_T \mathrm{d}\ln n$$

对该式从$-x_p$到x_n积分：$\psi(x_n) - \psi(-x_p) = V_T[\ln n(x_n) - \ln n(-x_p)]$

则内建电势差$\psi_0 = \psi_n - \psi_p = V_T \ln \frac{n_{n0}}{n_{p0}} = V_T \ln \frac{N_a N_d}{n_i^2}$

2-7

根据修正欧姆定律和空穴扩散电流公式证明，在外加正向偏压V作用下，PN结N侧空穴扩散区准费米能级的改变量为$\Delta E_{FP} = qV$。

解：

空穴电流修正欧姆定律$I_p=-qAp\mu_p\dfrac{\mathrm{d}\phi_p}{\mathrm{d}x}$

空穴扩散电流 $I_p=-qAD_p\dfrac{\mathrm{d}p_n}{\mathrm{d}x}$

两式电流相等，即

$$-qAp\mu_p\frac{\mathrm{d}\phi_p}{\mathrm{d}x}=-qAD_p\frac{\mathrm{d}p_n}{\mathrm{d}x}\implies \mathrm{d}\phi_p=\frac{D_p}{\mu_p}\frac{\mathrm{d}p_n}{p_n}=V_T\mathrm{d}(\ln p_n)$$

有准费米势：$\phi_p=-\dfrac{E_{FP}}{q}\implies \mathrm{d}\phi_p=-\dfrac{1}{q}\mathrm{d}E_{FP}$ ，代入上式得

$\mathrm{d}E_{FP}=-kT\mathrm{d}(\ln p_n)$

对其在扩散区进行积分，由于扩散区外无扩散电流，可从x_n到W_x积分

$$\begin{aligned}\Delta E_{FP}=\int_{x_n}^{W_n}\mathrm{d}E_{FP}&=\int_{x_n}^{W_n}-kT\mathrm{d}(\ln p_n)\\&=-kT[\ln p_n(W_n)-\ln p_n(x_n)]\\&=-kT\ln\frac{p_{n0}}{p_{n0}e^{V/V_T}}\\&=\frac{kTV}{V_T}\\&=qV\end{aligned}$$

2-8

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(1) PN结的空穴注射效率定义为在$x=0$处的I_p/I，证明此效率可写成

$\gamma=\dfrac{I_p}{I}=\dfrac{1}{1+\sigma_nL_p/\sigma_pL_n}$;

(2)在实际的二极管中怎样才能使γ接近1。

解：

(1) $I_p(x_n)=\dfrac{qAD_p}{L_p}p_{n0}(e^{V/V_T}-1)$, $I_n(-x_p)=\dfrac{qAD_n}{L_n}n_{p0}(e^{V/V_T}-1)$

总电流$I=qA(\dfrac{p_{n0}D_p}{L_p}+\dfrac{n_{p0}D_n}{L_n})(e^{V/V_T}-1)$

$$\gamma=\frac{I_p}{I}=\frac{I_p(x_n)}{I}=\frac{\frac{qAD_p}{L_p}p_{n0}(e^{V/V_T}-1)}{qA(\frac{p_{n0}D_p}{L_p}+\frac{n_{p0}D_n}{L_n})(e^{V/V_T}-1)}=\frac{1}{1+\frac{n_{p0}D_n}{L_n}\frac{L_p}{p_{n0}D_p}}$$

有$D_n=\mu_nV_T$，$D_p=\mu_pV_T$，$\sigma_n=nq\mu_n$，$\sigma_p=pq\mu_p$，代入上式

$$\gamma=\frac{1}{1+\frac{n_{p0}D_n}{L_n}\frac{L_p}{p_{n0}D_p}}=\frac{1}{1+\frac{n_{p0}\mu_nV_TL_p}{p_{n0}\mu_pV_TL_n}}=\frac{1}{1+\frac{qn_{p0}\mu_nL_p}{qp_{n0}\mu_pL_n}}=\frac{1}{1+\frac{\sigma_nL_p}{\sigma_pL_n}}$$

（2）为使γ接近1，应使$\frac{\sigma_nL_p}{\sigma_pL_n}\ll 1$

$$\frac{\sigma_nL_p}{\sigma_pL_n}=\frac{qn_{p0}\mu_n\sqrt{\mu_pV_T\tau_p}}{qp_{n0}\mu_p\sqrt{\mu_nV_T\tau_n}}=\frac{N_dn_i^2\sqrt{\mu_n}\sqrt{\tau_p}}{N_an_i^2\sqrt{\mu_p}\sqrt{\tau_n}}=\frac{N_d}{N_a}\cdot\frac{\sqrt{\mu_n}}{\sqrt{\mu_p}}\cdot\frac{\sqrt{\tau_p}}{\sqrt{\tau_n}}$$

μ_n，μ_p相差不大，τ_n，τ_p相差不大，所以只需使$N_a\gg N_d$即PN结为P^+N结即可

2-9

长PN结二极管处于反偏压状态：
(1)解扩散方程求少子分布$n_p(x)$和$p_n(x)$并画出它们的分布示意图。
(2)计算扩散区内少子贮存电荷。
(3)证明反向电流$I=-I_0$为PN结扩散区内的载流子产生电流。

解：

（1）P侧电子连续方程$\frac{\partial n_p}{\partial t}=D_n\frac{\partial n_p}{\partial x}-\frac{n_p-n_{p0}}{\tau_n}$

稳态时，有$\frac{\partial n_p}{\partial t}=0$，即$D_n\frac{\partial n_p}{\partial x}-\frac{n_p-n_{p0}}{\tau_n}=0$

令$L_n^2=D_n\tau_n$，代入上式可得：$\frac{\mathrm{d}(n_p-n_{p0})}{\mathrm{d}x}-\frac{n_p-n_{p0}}{L_n^2}=0$，其通解为：

$$n_p = n_{p0} + Ae^{x/L_n} + Be^{-x/L_n}\ ,\ -W_p \le x \le -x_p$$ 其中A, B为常数

设PN结空间电荷区P侧边界为$-x_p$，外部接触$-W_p$，可得
边界条件：

反偏状态时有反向抽取，即当$x = -x_p$时有$n_p = 0$；

在外部接触位置即当$x = -W_p$时有$n_p = n_{p0}$

代入以上边界条件：

$$\begin{cases} n_p(-x_p) = n_{p0} + Ae^{-x_p/L_n} + Be^{x_p/L_n} = 0 \\ n_p(-W_p) = n_{p0} + Ae^{-W_p/L_n} + Be^{W_p/L_n} = n_{p0} \end{cases}$$

长PN结中$W_n \gg L_n$，则可得$A = -n_{p0}e^{x_p/L_n}$，$B = 0$，代入通解

$$n_p = n_{p0} - n_{p0}e^{(x_p+x)/L_n} = n_{p0}[1 - e^{(x_p+x)/L_n}]\ ,\ -W_p \le x \le -x_p$$

同理可解得：

$$p_n = p_{n0}[1 - e^{(x_n-x)/L_p}]\ ,\ x_n \le x \le W_n$$

（2）P侧扩散区有$\Delta n_p = n_p - n_{p0} = -n_{p0}e^{(x_p+x)/L_n}$，对其从$-W_p$到$-x_p$积分

$$\begin{aligned} Q_n &= -qA\int_{-W_p}^{-x_p} -n_{p0}e^{(x_p+x)/L_n}\,\mathrm{d}x \\ &= qAL_n n_{p0}e^{(x_p+x)/L_n}\Big|_{-W_p}^{-x_p} \\ &= qAL_n n_{p0}[1 - e^{(x_p-W_p)/L_n}] \end{aligned}$$

长PN结有$W_p \gg L_n$，则可近似得到$Q_n = qAL_n n_{p0}$，符号为正是少子电子被抽取的结果。

N侧扩散区有$\Delta p_n = p_n - p_{n0} = -p_{n0}e^{(x_n-x)/L_p}$，对其从$x_n$到$W_n$积分

$$\begin{aligned} Q_p &= qA\int_{x_n}^{W_n} -p_{n0}e^{(x_n-x)/L_p}\,\mathrm{d}x \\ &= qAL_p p_{n0}e^{(x_n-x)/L_n}\Big|_{x_n}^{W_n} \\ &= qAL_p p_{n0}[e^{(x_n-W_n)/L_p} - 1] \end{aligned}$$

长PN结有$W_n \gg L_p$，则可近似得到$Q_p = -qAL_p p_{n0}$，符号为负是少子空穴被抽取的结果。

（3）设P、N侧扩散区贮存电荷均匀分布在长为L_n、L_p的扩散区内。则有

$$\Delta n_p = \frac{Q_n}{-qAL_n} = -n_{p0} \text{ , } \Delta p_n = \frac{Q_p}{qAL_p} = -p_{n0}$$

P侧扩散区电子产生率$G_n = -U_n = -\frac{\Delta n_p}{\tau_n} = \frac{n_{p0}}{\tau_n} > 0$，有产生电流。

N侧扩散区空穴产生率$G_p = -U_p = -\frac{\Delta p_n}{\tau_p} = \frac{p_{n0}}{\tau_p} > 0$，有产生电流。

显然和$I = -I_0 = -qA(\frac{n_{p0}}{\tau_n}L_n + \frac{p_{n0}}{\tau_p}L_p)$一致，即反向电流为扩散区载流子产生电流。

2-10

若 PN结边界条件为$x = W_n$处$p = P_{n0}$，$x = -W_p$处$n = n_{po}$。其中W_p和W_n分别与L_p与L_n具有相同的数量级，求$n_p(x)$、$p_n(x)$以及$I_n(x)$、$I_p(x)$的表达式。

解：

P侧电子连续方程$\frac{\partial n_p}{\partial t} = D_n \frac{\partial n_p}{\partial x} - \frac{n_p - n_{p0}}{\tau_n}$

稳态时，有$\frac{\partial n_p}{\partial t} = 0$，即$D_n \frac{\partial n_p}{\partial x} - \frac{n_p - n_{p0}}{\tau_n} = 0$

令$L_n^2 = D_n \tau_n$，代入上式可得：$\frac{\mathrm{d}(n_p - n_{p0})}{\mathrm{d}x} - \frac{n_p - n_{p0}}{L_n^2} = 0$，其通解为：

$$n_p = n_{p0} + Ae^{x/L_n} + Be^{-x/L_n} \text{ , } -W_p \le x \le -x_p \quad \text{其中} A, B \text{为常数}$$

边界条件：

正偏时有正向注入，即当$x = -x_p$时有$n_p = n_{p0}e^{V/V_T}$；

在外部接触位置即当$x=-W_p$时有$n_p=n_{p0}$

代入以上边界条件：

$$\begin{cases} n_p(-x_p)=n_{p0}+Ae^{-x_p/L_n}+Be^{x_p/L_n}=n_{p0}e^{V/V_T} \\ n_p(-W_p)=n_{p0}+Ae^{-W_p/L_n}+Be^{W_p/L_n}=n_{p0} \end{cases}$$

联立可解得$A=\dfrac{e^{W_p/L_n}n_{p0}(e^{V/V_T}-1)}{2\sinh\dfrac{W_p-x_p}{L_n}}$，$B=\dfrac{e^{-W_p/L_n}n_{p0}(e^{V/V_T}-1)}{2\sinh\dfrac{x_p-W_p}{L_n}}$，代入通解：

$$\begin{aligned} n_p &= n_{p0}+\frac{e^{W_p/L_n}n_{p0}(e^{V/V_T}-1)}{2\sinh\dfrac{W_p-x_p}{L_n}}e^{x/L_n}+\frac{e^{-W_p/L_n}n_{p0}(e^{V/V_T}-1)}{2\sinh\dfrac{x_p-W_p}{L_n}}e^{-x/L_n} \\ &= n_{p0}+n_{p0}(e^{V/V_T}-1)\frac{\sinh\dfrac{W_p+x}{L_n}}{\sinh\dfrac{W_p-x_p}{L_n}}, -W_p\le x\le -x_p \end{aligned}$$

同理可解得

$$p_n=p_{n0}+p_{n0}(e^{V/V_T}-1)\frac{\sinh\dfrac{W_n-x}{L_p}}{\sinh\dfrac{W_n-x_n}{L_p}}, x_n\le x\le W_n$$

电流强度分别是

$$I_n=qAD_n\frac{\mathrm{d}n_p}{\mathrm{d}x}=\frac{qAn_{p0}}{L_n}(e^{V/V_T}-1)\frac{\cosh\dfrac{W_p+x}{L_n}}{\sinh\dfrac{W_p-x_p}{L_n}}, -W_p\le x\le -x_p$$

$$I_p=-qAD_n\frac{\mathrm{d}p_n}{\mathrm{d}x}=\frac{qAp_{n0}}{L_p}(e^{V/V_T}-1)\frac{\cosh\dfrac{W_n-x}{L_p}}{\sinh\dfrac{W_n-x_n}{L_p}}, x_n\le x\le W_n$$

2-11

在P^+N结二极管中，N区的宽度W_n远小于L_p，用$I_p|_{x=W_n} = qS\Delta p_n A$ (S为表面复合速度)作为N侧末端的少数载流子电流，并以此为边界条件之一，推导出载流子和电流分布。绘出在$S=0$和$S=\infty$时N侧少数载流子的分布形状(计算机解)。

解：

设空间电荷区与N侧中性区交界为$x=0$的点，此时N侧外部接触点$x=W_n'=W_n-x_n$。

N侧少子连续方程为
$\frac{\partial p_n}{\partial t} = D_p\frac{\partial p_n}{\partial x} - \frac{p_n - p_{n0}}{\tau_p} \implies \frac{\partial \Delta p_n}{\partial t} = D_p\frac{\partial \Delta p_n}{\partial x} - \frac{\Delta p_n}{\tau_p}$。

稳态时有$\frac{\partial \Delta p_n}{\partial t} = D_p\frac{\partial \Delta p_n}{\partial x} - \frac{\Delta p_n}{\tau_p} = 0$，通解为：

$$\Delta p_n = C_1 e^{x/L_p} + C_2 e^{-x/L_p}, \quad \text{其中} C_1\text{，}C_2\text{为常数}$$

边界条件：

当$x=0$时有$\Delta p_n|_{x=0} = C_1 + C_2 = p_{n0}(e^{V/V_T}-1)$;

当$x=W_n'$时$I_p|_{x=W_n'} = -qAD_p\frac{\mathrm{d}p_n}{\mathrm{d}x} = -qAD_p\frac{\mathrm{d}\Delta p_n}{\mathrm{d}x}q = S\Delta p_n A|_{x=W'}$

即：

$$-\frac{qAD_p}{L_p}(C_1 e^{W_n'/L_p} - C_2 e^{-W_n'/L_p}) = qSA(C_1 e^{W_n'/L_p} + C_2 e^{-W_n'/L_p})$$

联立边界条件可得

$$C_1 = \frac{(D_p - L_pS)e^{-W_n'/L_p}}{2[D_p\cosh(\frac{W_n'}{L_p}) + L_pS\sinh(\frac{W_n'}{L_p})]} p_{n0}(e^{V/V_T}-1)$$
$$C_2 = \frac{(D_p + L_pS)e^{W_n'/L_p}}{2[D_p\cosh(\frac{W_n'}{L_p}) + L_pS\sinh(\frac{W_n'}{L_p})]} p_{n0}(e^{V/V_T}-1)$$

代入通解：

$$\Delta p_n=\frac{D_p\cosh(\frac{W_n'-x}{L_p})+L_pS\sinh(\frac{W_n'-x}{L_p})}{D_p\cosh(\frac{W_n'}{L_p})+L_pS\sinh(\frac{W_n'}{L_p})}p_{n0}(e^{V/V_T}-1)$$

$$I_p=\frac{qAD_p}{L_p}\frac{D_p\sinh(\frac{W_n'-x}{L_p})+L_pS\cosh(\frac{W_n'-x}{L_p})}{D_p\cosh(\frac{W_n'}{L_p})+L_pS\sinh(\frac{W_n'}{L_p})}p_{n0}(e^{V/V_T}-1)$$

当有$W_n\ll L_p$时，$W_n'\ll L_p$，

$$\cosh(\frac{W_n'-x}{L_p})\approx 1\quad \sinh(\frac{W_n'-x}{L_p})\approx\frac{W_n'-x}{L_p}$$
$$\cosh(\frac{W_n'}{L_p})\approx 1\qquad \sinh(\frac{W_n'}{L_p})\approx\frac{W_n'}{L_p}$$

则通解化为

$$\Delta p_n=\frac{D_p+S(W_n'-x)}{D_p+SW_n'}p_{n0}(e^{V/V_T}-1)$$
$$I_p=\frac{qAD_p}{L_p^2}\frac{D_p(W_n'-x)+L_p^2S}{D_p+SW_n'}p_{n0}(e^{V/V_T}-1)$$

当$S=0$时

$$\Delta p_n=p_{n0}(e^{V/V_T}-1)$$
$$I_p=\frac{qAD_p}{L_p^2}p_{n0}(e^{V/V_T}-1)(W_n'-x)$$

当$S=\infty$时

$$\Delta p_n=\frac{(W_n'-x)}{W_n'}p_{n0}(e^{V/V_T}-1)$$
$$I_p=\frac{qAD_pp_{n0}}{W_n'}(e^{V/V_T}-1)$$

2-12

推导公式(2-6-7) 和(2-6-8)。

正偏压下，给定电流，电压随温度线性地减小：

$$\frac{dV}{dT}=\frac{V-E_{g0}/q}{T} \tag{2-6-7}$$

给定电压，电流随温度升高而迅速增加：

$$\frac{1}{I}\frac{dI}{dT}=\frac{E_{g0}-qV}{KT^2} \tag{2-6-8}$$

解：

(2-6-7)有$I_0=qA(\frac{D_p}{L_pN_d}+\frac{D_n}{L_nN_a})n_i^2$，其中括号内参量对温度变化不敏感，则可得到

$$I_0\propto n_i^2\propto T^3e^{-E_{g0}/KT}$$

对温度T求导并除以I_0可得

$$\frac{1}{I_0}\frac{\mathrm{d}I_0}{\mathrm{d}T}=\frac{3}{T}+\frac{E_{g0}}{KT^2}\approx\frac{E_{g0}}{KT^2}$$

正偏情况下$I=I_0e^{V/V_T}$，当I_0为常数时，对T求导

$$\left.\frac{\mathrm{d}V}{\mathrm{d}T}\right|_{I_0=\text{常数}}=\frac{\mathrm{d}V_T}{\mathrm{d}T}\ln\frac{I}{I_0}-V_T(\frac{1}{I_0}\frac{\mathrm{d}I_0}{\mathrm{d}T})=\frac{V}{T}-\frac{V_TE_{g0}}{KT^2}=\frac{V-E_{g0}/q}{T}$$

(2-6-8)当V为常数时，对T求导并除以I

$$\frac{1}{I}\frac{\mathrm{d}I}{\mathrm{d}T}=\frac{1}{I}(\frac{\mathrm{d}I_0}{\mathrm{d}T}e^{V/V_T}-\frac{I_0V}{V_T^2}e^{V/V_T}\frac{\mathrm{d}V_T}{\mathrm{d}T})=\frac{E_{g0}}{KT^2}-\frac{V}{V_TT}=\frac{E_{g0}-qV}{KT^2}$$

2-13

把一个硅二极管用做变容二极管。在结的两边掺杂浓度分别为
$N_a=10^{19}cm^{-3}$以及$N_d=10^{15}cm^{-3}$。二极管的面积为100平方密尔。
(1)求在$V_R=1$和$5V$时的二极管的电容。
(2)计算用此变容二极管及$L=2mH$的储能电路的共振频率。

(注:密耳(mil) 为长度单位，$1mil = 10^{-3} in$(英寸) $= 2.54 \times 10^{-5} m$)

解:

(1)
$A = 100mil^2 = 100 \times (2.54 \times 10^{-5})^2 m^2 = 6.4516 \times 10^{-8} m^2 = 6.4516 \times 10^{-4} cm^2$

内建电势差:

$$\psi_0 = V_T \ln \frac{N_a N_d}{n_i^2} = 0.026V \times \ln \frac{10^{19} \times 10^{15}}{(1.5 \times 10^{10})^2} \approx 0.817V$$

将势垒区宽度$W = \sqrt{\frac{2\epsilon(\psi_0 + V_R)}{qN_d}}$代入变容二极管电容公式

$$C_T = \frac{A\epsilon}{W} = A\sqrt{\frac{q\epsilon_r\epsilon_0 N_d}{2(\psi_0 + V_R)}}$$

当$V_R = 1V$ 时

$$\begin{aligned} C_T = & 6.4516 \times 10^{-4} cm^2 \\ & \times \sqrt{\frac{1.6 \times 10^{-19} C \times 11.9 \times 8.85 \times 10^{-14} F/cm \times 10^{15} cm^{-3}}{2 \times (0.817V + 1V)}} \\ \approx & 4.39 \times 10^{-12} F = 4.39pF \end{aligned}$$

当$V_R = 5V$时

$$\begin{aligned} C_T = & 6.4516 \times 10^{-4} cm^2 \\ & \times \sqrt{\frac{1.6 \times 10^{-19} C \times 11.9 \times 8.85 \times 10^{-14} F/cm \times 10^{15} cm^{-3}}{2 \times (0.817V + 5V)}} \\ \approx & 2.46 \times 10^{-12} F = 2.46pF \end{aligned}$$

(2) 有$\omega_r = \frac{1}{\sqrt{LC}}$，分别代入$V_R = 1V$和$V_R = 5V$时的电容可得:

$$\omega_r|_{V_R=1V} = \frac{1}{\sqrt{2mH \times 4.39pF}} \approx 1.07 \times 10^7 rad/s$$

$$\omega_r|_{V_R=5V} = \frac{1}{\sqrt{2mH \times 2.46pF}} \approx 1.43 \times 10^7 rad/s$$

2-14

P^+N结杂质分布N_a=常数，$N_d = N_{d0}e^{-x/L}$，导出$C—V$特性表达式。

解：

在P^+N结中，有$x_n \approx W$，势垒大部分降落在N侧。

设PN结交界$x = 0$为内建电场零点。

在N侧有泊松方程

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = -\frac{\rho}{\epsilon} = -\frac{qN_{d0}}{\epsilon}e^{-x/L} \quad (0 \le x \le x_n)$$

通解：

$$\psi = -\frac{qN_{d0}L^2}{\epsilon}e^{-x/L} + C_1x + C_2 \quad (0 \le x \le x_n) \quad \text{其中}C_1\text{，}C_2\text{为常数}$$

边界条件：

在$x = 0$处定义为内建电场零点：$\psi(0) = 0$

在$x = x_n$处电场连续，即$E|_{x=x_n} = -\left.\frac{\mathrm{d}\psi}{\mathrm{d}x}\right|_{x=x_n} = 0$

代入边界条件可解得

$$C_1 = -\frac{qN_{d0}L}{\epsilon}e^{-x_n/L} \quad C_2 = \frac{qN_{d0}L^2}{\epsilon}$$

可得电势分布方程

$$\psi = -\frac{qN_{d0}L^2}{\epsilon}e^{-x/L} - \frac{qN_{d0}L}{\epsilon}xe^{-x_n/L} + \frac{qN_{d0}L^2}{\epsilon} \quad (0 \le x \le x_n)$$

内建电势差：

$$\psi_0 = \psi(x_n) - \psi(0) = -\frac{qN_{d0}L^2}{\epsilon}e^{-x_n/L} - \frac{qN_{d0}L}{\epsilon}x_ne^{-x_n/L} + \frac{qN_{d0}L^2}{\epsilon}$$

当$x_n \ll L$时有$e^{-x_n/L} \approx 1 - \frac{x_n}{L}$代入上式

$$\begin{aligned}\psi_0 &= -\frac{qN_{d0}L^2}{\epsilon}(1-\frac{x_n}{L}) - \frac{qN_{d0}L}{\epsilon}x_n(1-\frac{x_n}{L}) + \frac{qN_{d0}L^2}{\epsilon} \\ &= -\frac{qN_{d0}L^2}{\epsilon} + \frac{qN_{d0}Lx_n}{\epsilon} - \frac{qN_{d0}Lx_n}{\epsilon} + \frac{qN_{d0}x_n^2}{\epsilon} + \frac{qN_{d0}L^2}{\epsilon} \\ &= \frac{qN_{d0}x_n^2}{\epsilon}\end{aligned}$$

在P^+N结中，有$x_n \approx W$，代入可得$W = \sqrt{\frac{\psi_0\epsilon}{qN_{d0}}}$

在偏压V_R下，$W = \sqrt{\frac{\epsilon(\psi_0+V_R)}{qN_{d0}}}$

空间电荷区N侧电荷

$$\begin{aligned}Q &= qA\int_0^{x_n} N_{d0}e^{-x/L}\,\mathrm{d}x = -qALN_{d0}e^{-x/L}\Big|_0^{x_n} = -qALN_{d0}e^{-x_n/L} + qALN_{d0} \\ &= qALN_{d0}(1-e^{-x_n/L}) = qALN_{d0}\frac{x_n}{L} = qAN_{d0}W \\ &= A\sqrt{\epsilon(\psi_0+V_R)qN_{d0}}\end{aligned}$$

有$C = \frac{\mathrm{d}Q}{\mathrm{d}V_R}$可得

$$C = \frac{\mathrm{d}Q}{\mathrm{d}V_R} = \frac{A\sqrt{\epsilon qN_{d0}}}{2\sqrt{\psi_0+V_R}} = \frac{A\epsilon}{2}\sqrt{\frac{qN_{d0}}{\epsilon(\psi_0+V_R)}} = \frac{A\epsilon}{2W}$$

2-15

若P^+N二极管N区宽度W_n是和扩散长度同一数量级，推导小信号交流空穴分布和二极管导纳，假设在$x = W_n$处表面复合速度无限大。

解：

设外加偏压$v = V + v_a e^{\imath\omega t}$，电流为$i = I + i_a e^{\imath\omega t}$，N侧少子分布
$p_n(x,t) = P_n(n,t) + p_a e^{\imath\omega t}$

则连续方程有:

$$\left.\begin{aligned}\frac{\partial p_n}{\partial t}=D_p\frac{\partial^2 p_n}{\partial x^2}-\frac{p_n-p_{n0}}{\tau_p}\\ 0=D_p\frac{\partial^2 P_n}{\partial x^2}-\frac{P_n-p_{n0}}{\tau_p}\end{aligned}\right\}$$
$$\Longrightarrow\ \imath\omega p_a e^{\imath\omega t}=D_p\frac{\partial^2 p_a}{\partial x^2}e^{\imath\omega t}-\frac{p_a e^{\imath\omega t}}{\tau_p}$$
$$\Longrightarrow\ \frac{\partial^2 p_a}{\partial x^2}-p_a\frac{\imath\omega\tau_p+1}{D_p\tau_p}=0$$

令$L_p'^2=\dfrac{L_p^2}{\imath\omega\tau_p+1}=\dfrac{D_p\tau_p}{\imath\omega\tau_p+1}$则连续方程可化为

$$\frac{\partial^2 p_a}{\partial x^2}-\frac{p_a}{L_p'^2}=0$$

可得通解为

$$p_a=C_1e^{x/L_p'}+C_2e^{-x/L_p'}\quad(0\le x\le x_n),\text{其中}C_1,C_2\text{为常数}$$

在$x=x_n$处

$$p_n(x_n)=p_{n0}e^{v/V_T}=p_{n0}e^{V/V_T}exp(\frac{v_a e^{\imath\omega t}}{V_T})$$
$$\approx p_{n0}e^{V/V_T}(1+\frac{v_a e^{\imath\omega t}}{V_T})=P(x_n)+p_{a1}e^{\imath\omega t}$$

其中$P(x_n)=p_{n0}e^{V/V_T}$,　$p_{a1}=\dfrac{v_a p_{n0}}{V_T}e^{V/V_T}$。

可得边界条件

$$p_a=\begin{cases}p_{a1} & x=x_n\\ 0 & x=W_n\end{cases}$$

代入边界条件可得

$$C_1=\frac{p_{a1}e^{-W_n/L_p'}}{2\sinh(\frac{x_n-W_n}{L_p'})},\quad C_2=\frac{p_{a1}e^{W_n/L_p'}}{2\sinh(\frac{W_n-x_n}{L_p'})}$$

代入通解

$$p_a = p_{a1}\frac{\sinh(\frac{W_n - x}{L_p'})}{\sinh(\frac{W_n - x_n}{L_p'})} \quad (0 \le x \le x_n)$$

可得N侧交流电流分布

$$i_a = -qAD_p\frac{\mathrm{d}p_a}{\mathrm{d}x} = \frac{qAD_p p_{a1}}{L_p'}\frac{\cosh(\frac{W_n - x}{L_p'})}{\sinh(\frac{W_n - x_n}{L_p'})}$$

交流少子电流

$$i \approx i_{pmax} = i_p(x_n) = \frac{qAD_p p_{a1}}{L_p'}\coth(\frac{W_n - x}{L_p'}) = \frac{qAD_p v_a p_{n0}}{V_T L_p'}e^{V/V_T}\coth(\frac{W_n - x}{L_p'})$$

可得交流导纳:

$$Y = \frac{i}{v} = \frac{qAD_p p_{n0}}{V_T L_p'}e^{V/V_T}\coth(\frac{W_n - x}{L_p'})$$

2-16

一个硅二极管工作在$0.5V$的正向电压下，当温度从$25^\circ C$上升到$150^\circ C$时，计算电流增加的倍数。假设$I \approx I_0 e^{V/2V_T}$，I_0每$10^\circ C$增加一倍。

解:

法一：有$V_T(27^\circ C) = 0.026V$可得

$$V_T(25^\circ C) = \frac{298}{300} \times 0.026V \quad V_T(150^\circ C) = \frac{423}{300} \times 0.026V$$

I_0每$10^\circ C$增加一倍:

$$\frac{I_0(150^\circ C)}{I_0(25^\circ C)} = 2^{\frac{150-25}{10}} = 2^{12.5}$$

前后电流比

$$
\begin{aligned}
\gamma &= \frac{I(150^\circ C)}{I(25^\circ C)} = \frac{I_0(150^\circ C)}{I_0(25^\circ C)} exp[\frac{V}{2}(\frac{1}{V_T(150^\circ C)} - \frac{1}{V_T(25^\circ C)})] \\
&= 2^{12.5} \times exp[\frac{0.5 \times 300}{2 \times 0.026}(\frac{1}{423} - \frac{1}{298})] \\
&\approx 331
\end{aligned}
$$

则增加了$(331-1)=330$倍

法二：对T微分并除以I

$$
\frac{1}{I}\frac{\mathrm{d}I}{\mathrm{d}T} = \frac{1}{I}(\frac{\mathrm{d}I_0}{\mathrm{d}T}e^{V/2V_T} - I_0 e^{V/2V_T}\frac{V}{2}\frac{1}{V_T^2}\frac{\mathrm{d}V_T}{\mathrm{d}T}) = \frac{1}{I_0}\frac{\mathrm{d}I_0}{\mathrm{d}T} - \frac{V}{2V_T T}
$$

两边同乘dV并对其从$25^\circ C$到$150^\circ C$进行积分

$$
\begin{aligned}
\int_{25^\circ C}^{150^\circ C}\frac{1}{I}\mathrm{d}I &= \int_{25^\circ C}^{150^\circ C}\frac{1}{I_0}\mathrm{d}I_0 - \frac{V}{2V_T T}\mathrm{d}T \\
\Longrightarrow \ln\frac{I(150^\circ C)}{I(25^\circ C)} &= \ln\frac{I_0(150^\circ C)}{I_0(25^\circ C)} + \frac{Vq}{2kT}\bigg|_{298K}^{423K} \approx 5.8 \\
\Longrightarrow \frac{I(150^\circ C)}{I(25^\circ C)} &= e^{5.8} \approx 331
\end{aligned}
$$

则增加了$(331-1)=330$倍

2-17

采用电容测试仪在$1MHz$测量GaAs P^+N结二极管的电容反偏压关系。下面是从$0—5V$每次间隔$0.5V$测得的电容数据，以微微法为单位：19.9，17.3，15.6，14.3，13.3，12.4，11.6，11.1，10.5，10.1，9.8。计算ψ_0和N_d。二极管的面积为$4\times10^{-4}cm^2$。

解：P^+N结中有

$$
\frac{1}{C_T^2} = \frac{2}{A^2\epsilon qN_d}(\psi_0 + V_R)
$$

令

$$\frac{1}{C_T^2} = KV_R + B,\quad 其中K = \frac{2}{A^2\epsilon q N_d}, B = \frac{2}{A^2\epsilon q N_d}\psi_0$$

代入题目数据并进行线性拟合可得

$$K = 0.0016\,V^{-1}pF^{-2} \quad B = 0.0025\,pF^{-2}$$

可得

$$\begin{aligned} N_d &= \frac{2}{A^2\epsilon q K} \\ &= \frac{2}{(4\times10^{-4})^2\times1.17\times10^{-12}\times1.6\times10^{-19}\times1.6\times10^{24}} \\ &\approx 4.18\times10^{16}\,cm^{-3} \end{aligned}$$

$$\begin{aligned} \psi_0 &= \frac{A^2\epsilon q N_d B}{2} \\ &= \frac{(4\times10^{-4})^2\times1.17\times10^{-12}\times1.6\times10^{-19}\times4.18\times10^{16}\times2.5\times10^{21}}{2} \\ &\approx 1.56V \end{aligned}$$

2-18

在$I_f = 0.5mA$，$I_r = 1.0mA$条件下测量P^+N长二极管恢复特性。得到的结果是$t_s = 350ns$。用严格解和近似公式两种方法计算τ_p。

解：

近似解：由$t_s = \tau_p \ln(1+\dfrac{I_f}{I_r})$可得

$$\tau_p = \frac{t_s}{\ln(1+\dfrac{I_f}{I_r})} = \frac{350ns}{\ln(1+\dfrac{0.5}{1.0})} \approx 863.2ns$$

严格解：有$erf\sqrt{\dfrac{t_s}{\tau_p}} = \dfrac{I_f}{I_f+I_r} = \dfrac{0.5}{0.5+1} = \dfrac{1}{3}$，查表可得$\sqrt{\dfrac{t_s}{\tau_p}} \approx 0.3046$

，可得

$$\tau_p = \frac{350ns}{0.3046^2} \approx 3.77\mu s$$

2-19

用二极管恢复法测量P^+N二极管空穴寿命。
(1)对于$I_f = 1mA$，$I_r = 2mA$，在具有$0.1ns$上升时间的示波器上测得$t_s = 3ns$，求τ_p。
(2)若(1)中快速示波器无法得到，只得采用一只具有$10ns$上升时间较慢的示波器，问怎样才能使测量精确？叙述你的结果。

解：

（1）由$t_s = \tau_p \ln(1+\frac{I_f}{I_r})$可得

$$\tau_p = \frac{t_s}{\ln(1+\frac{I_f}{I_r})} = \frac{3ns}{\ln(1+\frac{1}{2})} \approx 7.40ns$$

（2）$10ns$上升时间的示波器只可测$t_s \gg 10ns$的t_s，有$t_s = \tau_p \ln(1+\frac{I_f}{I_r})$，只需增大$I_f$或减小$I_r$即可

2-20

在硅中当最大电场接近$10^6 V/cm$时发生击穿。假设在P侧$N_a = 10^{20} cm^{-3}$，为要得到$2V$的击穿电压，采用单边突变近似，求N侧的施主浓度。

解：当雪崩击穿发生时有

$$1 = \frac{AW|\epsilon_m|}{B} exp\frac{-B}{|\epsilon_m|}\left[1 - exp\frac{-B}{|\epsilon_m|}\right]$$

可解得

$$
\begin{aligned}
W &= \frac{B}{A|\epsilon_m|}\exp\frac{B}{|\epsilon_m|}\Big/\left[1-\exp\frac{-B}{|\epsilon_m|}\right] \\
&= \frac{1.8\times 10^6 V/cm}{9\times 10^5 cm^{-1}\times 10^6 V/cm}\times\exp\frac{1.8\times 10^6}{10^6}\Big/\left[1-\exp\frac{-1.8\times 10^6}{10^6}\right] \\
&\approx 1.45\times 10^{-5} cm
\end{aligned}
$$

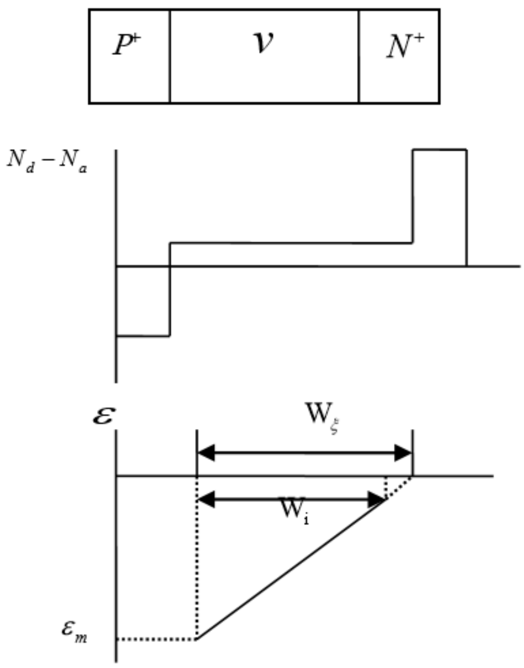
有$|\epsilon_m| = \dfrac{qN_dW}{\epsilon}$可得

$$
N_d = \frac{\epsilon|\epsilon_m|}{qW} = \frac{11.8\times 8.85\times 10^{-14}}{1.6\times 10^{-19}\times 1.45\times 10^{-5}} cm^{-3} \approx 4.50\times 10^{17} cm^{-3}
$$

2-21

对于下图中的$P^+ — v — N^+$二极管，假设P^+和N^+区不承受任何外加电压，证明雪崩击穿的条件可表示为：

$$
\frac{Ak\epsilon_0\epsilon_m^2}{qN_vB}exp(-\frac{B}{|\epsilon_m|})[1-exp(-\frac{qBN_vW_i}{k\epsilon_0\epsilon_m^2})] = 1
$$



解：P^+vN^+二极管的雪崩击穿临界电场$|\epsilon_m|$与P^+N结相当。

设标准P^+N结的SCR宽度为W_ξ，则有：

$$
\epsilon(x) = \epsilon_m(1-\frac{x}{W_\xi}) \text{，} \quad 其中\epsilon_m = \frac{qN_vW_\xi}{\epsilon}
$$

电离系数:

$$\alpha(x) = A\exp\left[-\frac{B}{|\epsilon_m|(1-\dfrac{x}{W_\xi})}\right]$$

当$x \to 0$时，取一阶泰勒展开

$$\alpha(x) \approx A\exp\left[-\frac{B}{|\epsilon_m|}(1+\frac{x}{W_\xi})\right]$$

对其从0到W_i积分

$$\begin{aligned}\int_0^{W_i}\alpha(x)\,\mathrm{d}x &= A\exp(-\frac{B}{|\epsilon_m|})\int_0^{W_i}\exp(-\frac{Bx}{|\epsilon_m|W_\xi})\,\mathrm{d}x \\ &= -\frac{A|\epsilon_m|W_\xi}{B}\exp(-\frac{B}{|\epsilon_m|})\exp(-\frac{Bx}{|\epsilon_m|W_\xi})\Big|_0^{W_i} \\ &= -\frac{A|\epsilon_m|W_\xi}{B}\exp(-\frac{B}{|\epsilon_m|})\left[\exp(-\frac{BW_i}{|\epsilon_m|W_\xi})-1\right]\end{aligned}$$

代入$W_\xi = \dfrac{|\epsilon_m|\epsilon}{qN_v}$，并代入雪崩击穿条件$\int_0^{W_i}\alpha(x)\,\mathrm{d}x = 1$，可得

$$\frac{A\epsilon_m^2\epsilon}{qN_vB}\exp(-\frac{B}{|\epsilon_m|})\left[1-\exp(-\frac{qBN_vW_i}{\epsilon_m^2\epsilon})\right] = 1$$