第三章

3-2

一个NPN硅晶体管，具有下列参数：$x_B = 2\mu m$，在均匀掺杂基区 $N_a = 5\times10^{16}cm^{-3}, \tau_n = 1\mu s, A = 0.01cm^2$。若集电结被反向偏置，$I_{nE} = 1mA$，计算在发射结基区一边的过量电子浓度，发射结电压以及基区输运因子。

解：

$$D_n = V_T\mu_n = 26mV \times 1350cm^2/V\cdot s = 35.1cm^2/s$$
$$L_n = \sqrt{D_n\tau_n} = \sqrt{35.1cm^2/s\times 1\mu s} = 5.9\times10^{-3}cm$$
$$n_{p0} = \frac{n_i^2}{N_a} = \frac{(1.5\times10^{10})^2}{5\times10^{16}}cm^{-3} = 4.5\times10^3cm^{-3}$$

当$x_B \ll L_n$即$x_B/L_n \ll 1$时，有

$$\coth(x_B/L_n) \approx \frac{1}{\sinh(x_B/L_n)} \approx \frac{L_n}{x_B}$$

且有集电结反偏即$V_C < 0$，则

$$
\begin{aligned}
I_{nE} &= -\frac{qAD_n n_{p0}}{L_n}[(e^{V_E/V_T}-1)\coth(x_B/L_n)-(e^{V_C/V_T}-1)\frac{1}{\sinh(x_B/L_n)}] \\
&\approx -\frac{qAD_n n_{p0}}{L_n}[(e^{V_E/V_T}-1)\frac{L_n}{x_B}+\frac{L_n}{x_B}] \\
&= -\frac{qAD_n n_{p0}e^{V_E/V_T}}{x_B}
\end{aligned}
$$

则有：

$$
\begin{aligned}
e^{V_E/V_T} &= -\frac{x_B I_{nE}}{qAD_n n_{p0}} \\
&= -\frac{2\mu m \times 1mA}{-1.6\times 10^{-19}C \times 0.01cm^2 \times 35.1cm^2/s \times 4.5\times 10^3 cm^{-3}} \\
&\approx 7.91\times 10^8
\end{aligned}
$$

解得发射结电压

$$
V_E = V_T \ln e^{V_E/V_T} = 0.026V \times \ln(7.91\times 10^8) \approx 0.5327V
$$

电子浓度

$$
n_p(0) = n_{p0}e^{V_E/V_T} = 4.5\times 10^3 cm^{-3} \times 7.91\times 10^8 = 3.56\times 10^{12} cm^{-3}
$$

基区输运因子

$$
\begin{aligned}
\frac{x_B}{L_n} &= \frac{2\mu m}{5.9\times 10^{-3}cm} \approx 0.0339 \\
\beta_T = \frac{I_{nC}}{I_{nE}} &= \frac{1}{\cosh(x_B/L_n)} = \frac{1}{\cosh 0.0339} \approx 0.9989
\end{aligned}
$$

3-3

在习题3-2的晶体管中，假设发射极的掺杂浓度为$10^{18}cm^{-3}$，$x_E = 2\mu m$，$\tau_{pE} = 10ns$，发射结空间电荷区中$\tau = 0.1\mu s$。计算在$I_{nE} = 1mA$时的发射效率和h_{FE}。

解：

发射结内建电势差

$$\psi_0 = V_T \ln \frac{N_a N_d}{n_i^2} = 0.026V \times \ln \frac{5 \times 10^{16} \times 10^{18}}{(1.5 \times 10^{10})^2} \approx 0.8598V$$

发射结势垒区宽度

$$\begin{aligned} W_E &= \sqrt{\frac{2\epsilon(\psi_0 - V_E)}{qN_a}} \\ &= \sqrt{\frac{2 \times 1.053 \times 10^{-12} \times (0.8598 - 0.5327)}{1.6 \times 10^{-19} \times 5 \times 10^{16}}} cm \\ &\approx 9.24 \times 10^{-6} cm \end{aligned}$$

发射结反向电流

$$\begin{aligned} I_{RE} &= \frac{qAn_i W}{2\tau_0} e^{V_E/2V_T} \\ &= \frac{1.6 \times 10^{-19} \times 0.01 \times 1.5 \times 10^{10} \times 9.24 \times 10^{-6}}{2 \times 10^{-7}} \times \sqrt{7.91 \times 10^8} \\ &\approx 3.12 \times 10^{-6} A = 3.12 \times 10^{-2} mA \end{aligned}$$

空穴扩散系数$D_{pE} = V_T \mu_p = 0.026V \times 480cm^2/V \cdot s = 12.48cm^2/s$

空穴浓度$p_{E0} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{18}} cm^{-3} = 225cm^{-3}$

空穴电流

$$\begin{aligned} I_{pE} &= -\frac{qAD_{pE}p_{E0}}{x_E - W_E}(e^{V_E/V_T} - 1) \\ &= \frac{1.6 \times 10^{-19} \times 0.01 \times 12.48 \times 225}{2 \times 10^{-4} - 9.24 \times 10^{-6}} \times (7.91 \times 10^8 - 1)A \\ &\approx 1.68 \times 10^{-5} A = 1.68 \times 10^{-2} mA \end{aligned}$$

发射效率

$$\gamma = \frac{I_{nE}}{I_E} = \frac{I_{nE}}{I_{nE} + I_{pE} + I_{RE}} = \frac{1}{1 + 1.68 \times 10^{-2} + 3.12 \times 10^{-2}} \approx 0.954$$

共基极直流电流增益

$$\alpha_0 = \gamma \beta_T = 0.954 \times 0.9989 \approx 0.953$$

共发射极直流电流增益

$$h_{FE} = \frac{\alpha_0}{1 - \alpha_0} = \frac{0.953}{1 - 0.953} \approx 20.28$$

3-4

（1）根据公式(3-3-5)或(3-3-6),证明对于任意的x_B/L_n值， 公式(3-4-9)和(3-4-11)变成

$$a_{11} = -qAn_i^2\left[\frac{D_n}{N_aL_n}\coth\frac{x_B}{L_n} + \frac{D_{PE}}{N_{dE}x_E}\right]$$
$$a_{12} = a_{21} = \frac{qAD_nn_i^2}{N_aL_n}\csc h\frac{x_B}{L_n}$$
$$a_{22} = -qAn_i^2\left[\frac{D_n}{N_aL_n}\coth\frac{x_B}{L_n} + \frac{D_{PC}}{N_{dC}L_{PC}}\right]$$

（2）证明，若$x_B/L_n << 1$， （1）中的表达式约化为(3-4-9) 和(3-4-11)。

$$I_E = -I_{F0}(e^{V_E/V_T} - 1) + \alpha_R I_{R0}(e^{V_C/V_T} - 1) \qquad (3\text{-}4\text{-}5)$$

$$I_C = \alpha_F I_{F0}(e^{V_E/V_T} - 1) - I_{R0}(e^{V_C/V_T} - 1) \qquad (3\text{-}4\text{-}6)$$

$$a_{11} = -qAn_i^2\left(\frac{D_n}{N_ax_B} + \frac{D_{PE}}{N_{dE}x_E}\right), \quad a_{12} = \frac{qAD_nn_i^2}{N_ax_B} \qquad (3\text{-}4\text{-}9)$$

$$a_{21} = \frac{qAD_nn_i^2}{N_ax_B}, \quad a_{22} = -qAn_i^2\left(\frac{D_n}{N_ax_B} + \frac{D_{PC}}{N_{dC}L_{PC}}\right) \quad (3\text{-}4\text{-}11)$$

解:

（1）对于E-M方程，有$I_E = I_{pE} + I_{nE}$，其中

$$I_{nE} = -\frac{qAD_nn_{p0}}{L_n}[(e^{V_E/V_T} - 1)\coth(x_B/L_n) - (e^{V_C/V_T} - 1)\frac{1}{\sinh(x_B/L_n)}]$$
$$I_{pE} = -qAn_i^2\frac{D_{PE}}{N_{dE}x_E}(e^{V_E/V_T} - 1)$$

两项相加并对比E-M方程系数可得

$$a_{11} = -qAn_i^2\left[\frac{D_n}{N_aL_n}\coth\frac{x_B}{L_n} + \frac{D_{PE}}{N_{dE}x_E}\right]$$
$$a_{12} = \frac{qAD_nn_i^2}{N_aL_n}\csc h\frac{x_B}{L_n}$$

同理可得

$$a_{21} = \frac{qAD_nn_i^2}{N_aL_n}\csc h\frac{x_B}{L_n}$$
$$a_{22} = -qAn_i^2\left[\frac{D_n}{N_aL_n}\coth\frac{x_B}{L_n} + \frac{D_{PC}}{N_{dC}L_{PC}}\right]$$

（2）若$x_B/L_n << 1$，则有$\coth\frac{x_B}{L_n}\approx\frac{L_n}{x_B}$代入可得

$$
\begin{aligned}
a_{11} &= -qAn_i^2\left(\frac{D_n}{N_a x_B}+\frac{D_{PE}}{N_{dE}x_E}\right)\\
a_{12} = a_{21} &= \frac{qAD_n n_i^2}{N_a x_B}\\
a_{22} &= -qAn_i^2\left(\frac{D_n}{N_a x_B}+\frac{D_{PC}}{N_{dC}L_{PC}}\right)
\end{aligned}
$$

3-5

证明在正向有源模式，晶体管发射极电流-电压特性可用下式表示

$I_E\approx\frac{I_{E0}}{1-\alpha_F\alpha_R}e^{V_E/V_T}+\frac{qAn_iW_E}{2\tau_0}e^{V_E/2V_T}$，其中$I_{E0}$为集电极开路时发射结反向饱和电流。

提示:首先由EM方程导出$I_{F0}=\frac{I_{E0}}{1-\alpha_F\alpha_R}$。

证明：有EM方程

$$
\begin{cases}
I_E = -I_{F0}(e^{V_E/V_T}-1)+\alpha_R I_{R0}(e^{V_C/V_T}-1)\\
I_C = \alpha_F I_{F0}(e^{V_E/V_T}-1)-I_{R0}(e^{V_C/V_T}-1)
\end{cases}
$$

当集电极开路，发射极反偏时，$I_C=0$，$e^{V_E/V_T}\ll 1$

$$
\begin{cases}
I_{E0} = I_{F0}+\alpha_R I_{R0}(e^{V_C/V_T}-1)\\
0 = -\alpha_F I_{F0}-I_{R0}(e^{V_C/V_T}-1)
\end{cases}
$$

二式联立可得$I_{E0}=I_{F0}-\alpha_R\alpha_F I_{F0}\implies I_{F0}=\frac{I_{E0}}{1-\alpha_F\alpha_R}$

正向有源模式下，$e^{V_C/V_T}\ll 1$，$e^{V_E/V_T}\gg 1$，则有

$$
\begin{aligned}
I_E &= -I_{F0}(e^{V_E/V_T}-1)+\alpha_R I_{R0}(e^{V_C/V_T}-1)\\
&\approx -I_{F0}e^{V_E/V_T}-\alpha_R I_{R0}\\
&\approx -I_{F0}e^{V_E/V_T}
\end{aligned}
$$

EM方程忽略了复合电流$I_{RE}=\frac{qAn_iW_E}{2\tau_0}e^{V_E/2V_T}$

且EM方程中I_E方向与实际方向相反。代入可得

$$I_E=\frac{I_{E0}}{1-\alpha_F\alpha_R}e^{V_E/V_T}+\frac{qAn_iW_E}{2\tau_0}e^{V_E/2V_T}$$

3-6

（1）忽略空间电荷区的复合电流，证明晶体管共发射极输出特性的精确表达式为

$$V_{CE}=-V_T\ln\frac{I_{R0}(1-\alpha_F\alpha_R)+\alpha_FI_B-I_C(1-\alpha_F)}{I_{F0}(1-\alpha_F\alpha_R)+I_B+I_C(1-\alpha_R)}-V_T\ln\frac{\alpha_R}{\alpha_F}$$

提示：首先求出用电流表示结电压的显式解。

（2）若$I_B\gg I_{E0}$且$\alpha_FI_B\gg I_{R0}(1-\alpha_F\alpha_R)$，证明上式化为：

$$V_{CE}=V_T\ln\frac{1/\alpha_R+I_C/I_Bh_{FER}}{1-I_C/I_Bh_{FEF}}\qquad \text{其中}h_{FEF}=\frac{\alpha_F}{1-\alpha_F},h_{FER}=\frac{\alpha_R}{1-\alpha_R}$$

证明：

（1）有EM方程

$$\begin{cases}I_E=-I_{F0}(e^{V_E/V_T}-1)+\alpha_RI_{R0}(e^{V_C/V_T}-1)\\ I_C=\alpha_FI_{F0}(e^{V_E/V_T}-1)-I_{R0}(e^{V_C/V_T}-1)\end{cases}$$

两式联立可解得：

$$\begin{cases}V_E=V_T\ln\dfrac{I_E+\alpha_RI_C+I_{F0}(\alpha_R\alpha_F-1)}{I_{F0}(\alpha_R\alpha_F-1)}\\ V_C=V_T\ln\dfrac{I_C+\alpha_FI_E+I_{R0}(\alpha_R\alpha_F-1)}{I_{R0}(\alpha_R\alpha_F-1)}\end{cases}$$

在EM方程中有$I_E+I_C+I_B=0,\alpha_RI_{R0}=\alpha_FI_{F0}$，代入则有：

$$\begin{aligned}V_{CE}&=V_E-V_C\\&=V_T\ln\frac{[I_E+\alpha_RI_C+I_{F0}(\alpha_R\alpha_F-1)]I_{R0}}{[I_C+\alpha_FI_E+I_{R0}(\alpha_R\alpha_F-1)]I_{F0}}\\&=V_T\ln\frac{-(I_C+I_B)+\alpha_RI_C+I_{F0}(\alpha_R\alpha_F-1)}{I_C-\alpha_F(I_C+I_B)+I_{R0}(\alpha_R\alpha_F-1)}+V_T\ln\frac{I_{R0}}{I_{F0}}\\&=-V_T\ln\frac{I_{R0}(1-\alpha_R\alpha_F)+\alpha_FI_B-I_C\ (1-\alpha_F)}{I_{F0}(1-\alpha_R\alpha_F)+I_B+I_C(1-\alpha_R)}-V_T\ln\frac{\alpha_R}{\alpha_F}\end{aligned}$$

（2）由3-5可得$I_{E0}=I_{F0}-\alpha_R\alpha_FI_{F0}=I_{F0}(1-\alpha_R\alpha_F)\ll I_B$，代入$V_{CE}$

$$
\begin{aligned}
V_{CE} &= -V_T \ln \frac{I_{R0}(1-\alpha_R\alpha_F)+\alpha_F I_B - I_C(1-\alpha_F\ )}{I_{F0}(1-\alpha_R\alpha_F)+I_B+I_C(1-\alpha_R)} - V_T \ln \frac{\alpha_R}{\alpha_F} \\
&\approx V_T \ln \frac{[I_B+I_C(1-\alpha_R)]\alpha_F}{[\alpha_F I_B - I_C(1-\alpha_F)]\alpha_R} \\
&= V_T \ln \frac{1/\alpha_R + I_C(1-\alpha_R)/(I_B\alpha_R)}{1-I_C(1-\alpha_F)/(I_B\alpha_F)} \\
&= V_T \ln \frac{1/\alpha_R + I_C/(I_B h_{FRR})}{1-I_C/(I_B h_{FEF})}
\end{aligned}
$$

其中$h_{FEF}=\dfrac{\alpha_F}{1-\alpha_F}, h_{FER}=\dfrac{\alpha_R}{1-\alpha_R}$

3-7

一个用离子注入制造的NPN晶体管，其中性区内浅杂质浓度为$N_a(x)=N_0e^{-x/L}$，其中$N_0=2\times10^{18}cm^{-3}$ ，$L=0.3\mu m$。

（1）求宽度为$0.8\mu m$的中性区内单位面积的杂质总量；

（2）求出中性区内的平均杂质浓度；

（3） 若$L_{pE}=1\mu m$，$N_{dE}=10^{19}cm^{-3}$，$D_{pE}=1cm^2/s$，基区内少子平均寿命为$10^{-6}s$，基区的平均扩散系数和(2)中的杂质浓度相应，求共发射极电流增益。

解：

（1）单位面积杂质总量：

$$
\begin{aligned}
\int_0^{x_B} N_a(x)\,dx = \int_0^{x_B} N_0 e^{-x/L}\,dx &= LN_0(1-e^{-x_B/L}) \\
&= 0.3\mu m \times 2\times10^{18}cm^{-3} \times (1-e^{-0.8/0.3}) \\
&= 5.58\times10^{13}cm^{-2}
\end{aligned}
$$

（2）中性区平均杂质浓度：

$$
\overline{N_a} = \frac{\int_0^{x_B} N_a(x)\,dx}{x_B} = \frac{5.58\times10^{13}cm^{-2}}{0.8\mu m} = 6.975\times10^{17}cm^{-3}
$$

（3）共发射极电流增益取倒数：

$$
h_{FE}^{-1} = \frac{N_a x_B D_{pE}}{N_{dE} x_E D_n} + \frac{x_B^2}{2L_n^2} + \frac{N_a x_B W_E}{2D_n n_i \tau_0} e^{-V_E/2V_T} \overset{V_E \gg V_T}{\approx} \frac{N_a x_B D_{pE}}{N_{dE} x_E D_n} + \frac{x_B^2}{2L_n^2}
$$

其中：

$$D_n = \mu_n V_T = 700cm^2/V \cdot s \times 0.026V = 18.2cm^2/s$$
$$L_n^2 = D_n \tau_n = 18.2cm^2/s \times 10^{-6}s = 1.82 \times 10^{-7}cm^2$$

代入电流增益倒数方程:

$$\begin{aligned} h_{FE}^{-1} &= \frac{N_a x_B D_{pE}}{N_{dE} x_E D_n} + \frac{x_B^2}{2L_n^2} \\ &= \frac{6.975 \times 10^{17} \times 0.8 \times 10^{-4} \times 1}{10^{19} \times 10^{-4} \times 18.2} + \frac{(0.8 \times 10^{-4})^2}{2 \times 1.82 \times 10^{-7}} \\ &\approx 0.00318 \end{aligned}$$

则电流增益:$h_{FE} = \dfrac{1}{0.00318} \approx 314$

3-8

若在公式$I_n = \dfrac{qAD_n n_i^2}{\int_0^{x_B} N_a dx} e^{V_E/V_T}$中假设$I_C = I_n$，则可在集电极电流$I_C \sim V_E$曲线计算出根梅尔数。求出图 3-12 中晶体管中的根梅尔数。采用$D_n = 35cm^2/s$、$A = 0.1cm^2$ 、以及$n_i = 1.5 \times 10^{10}cm^{-3}$。

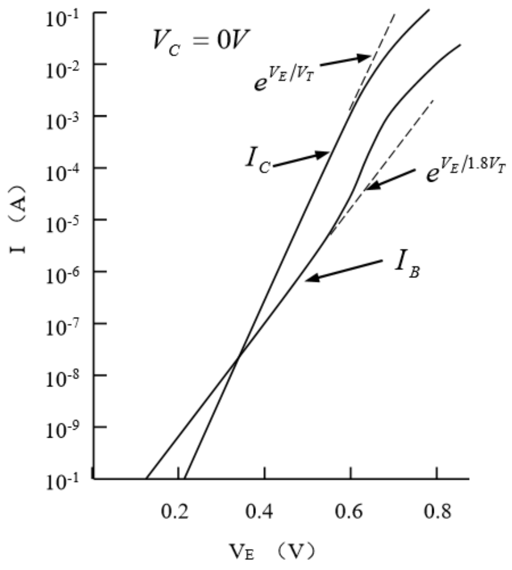

图 3-12 NPN 晶体管的静态电流—电压特性

解:令$G_M = \int_0^{x_B} N_a dx$，$I_0 = \dfrac{qAD_n n_i^2}{G_M}$，则有

$$I_n = I_0 e^{V_E/V_T} \implies \lg I_n = \frac{V_E}{V_T} \lg e + \lg I_0$$

图像取两点$(V_E, \lg I_n) = (0.22, -10)$和$(V_E, \lg I_n) = (0.44, -6)$
代入直线方程 $\lg I_n = KV_E + B$

$$\begin{cases} -10 & = 0.22K + B \\ -6 & = 0.44K + B \end{cases} \implies \begin{cases} K = 18.18 \\ B = -14 \end{cases}$$

即 $\lg I_0 = B = -14 \implies I_0 = 10^{-14}A$，代入$G_M$中：

$$\begin{aligned} G_M &= \frac{qAD_n n_i^2}{I_0} \\ &= \frac{1.6 \times 10^{-19}C \times 0.1cm^2 \times 35cm^2/s \times (1.5 \times 10^{10}cm^{-3})^2}{10^{-14}A} \\ &= 1.26 \times 10^{16}cm^{-2} \end{aligned}$$

3-9

（1）证明对于均匀掺杂的基区，式$\beta_T = 1 - \frac{1}{L_n^2}\int_0^{x_B}(\frac{1}{N_a}\int_x^{x_B}N_a dx)dx$简化为$\beta_T = 1 - \frac{x_B^2}{2L_n^2}$；
（2） 若基区杂质为指数分布，即$N_a = N_0 e^{-\alpha x/x_B}$，推导出基区输运因子的表示式。

解：

（1）均匀掺杂即N_a与x无关，此时有

$$\begin{aligned} \beta_T &= 1 - \frac{1}{L_n^2}\int_0^{x_B}(\frac{1}{N_a}\int_x^{x_B}N_a dx)dx \\ &= 1 - \frac{1}{L_n^2}\int_0^{x_B}\int_x^{x_B}dxdx \\ &= 1 - \frac{1}{L_n^2}\int_0^{x_B}x_B - xdx \\ &= 1 - \frac{1}{L_n^2}(x_B x - \frac{1}{2}x^2)\Big|_0^{x_B} \\ &= 1 - \frac{x_B^2}{2L_n^2} \end{aligned}$$

（2）代入$N_a = N_0 e^{-\alpha x/x_B}$

$$
\begin{aligned}
\beta_T &= 1 - \frac{1}{L_n^2}\int_0^{x_B}\left(\frac{1}{N_0 e^{-\alpha x/x_B}}\int_x^{x_B} N_0 e^{-\alpha x/x_B}dx\right)dx \\
&= 1 - \frac{1}{L_n^2}\int_0^{x_B}\left[e^{\alpha x/x_B}\left(-\frac{x_B}{\alpha}e^{-\alpha x/x_B}\right)\Big|_x^{x_B}\right]dx \\
&= 1 - \frac{1}{L_n^2}\int_0^{x_B}\left[e^{\alpha x/x_B}\left(-\frac{x_B}{\alpha}e^{-\alpha} + \frac{x_B}{\alpha}e^{-\alpha x/x_B}\right)\right]dx \\
&= 1 - \frac{1}{L_n^2}\int_0^{x_B}\left(\frac{x_B}{\alpha} - \frac{x_B}{\alpha}e^{\alpha x/x_B-\alpha}\right)dx \\
&= 1 - \frac{1}{L_n^2}\left(\frac{x_B}{\alpha}x - \frac{x_B^2}{\alpha^2}e^{\alpha x/x_B-\alpha}\right)\Big|_0^{x_B} \\
&= 1 - \frac{1}{L_n^2}\left(\frac{x_B^2}{\alpha} - \frac{x_B^2}{\alpha^2} + \frac{x_B^2}{\alpha^2}e^{-\alpha}\right) \\
&= 1 - \frac{x_B^2}{\alpha^2 L_n^2}(\alpha - 1 + e^{-\alpha})
\end{aligned}
$$

3-10

基区直流扩展电阻对集电极电流的影响可表示为$I_C = I_0 exp[(V_E - I_B r_{bb'})/V_T]$，用公式以及示于图3- 12的数据估算出$r_{bb'}$。

解：由$I_C = I_0 e^{(V_E - I_B r_{bb'})/V_T}$可得

$$
\begin{cases}
I_{C1} = I_0 exp[(V_{E1} - I_{B1} r_{bb'})/V_T] \\
I_{C2} = I_0 exp[(V_{E2} - I_{B2} r_{bb'})/V_T]
\end{cases}
$$

两式相除：

$$
\frac{I_{C2}}{I_{C1}} = exp\frac{(V_{E2} - V_{E1}) - (I_{B2} - I_{B1})r_{bb'}}{V_T}
$$

$$
\Longrightarrow r_{bb'} = \frac{1}{I_{B2} - I_{B1}}[(V_{E2} - V_{E1}) - V_T \ln\frac{I_{C2}}{I_{C1}}]
$$

取两点

$$
\begin{cases}
V_{E2} = 0.7V, I_{C2} = 10^{-2}A, I_{B2} = 5\times10^{-4}A \\
V_{E1} = 0.8V, I_{C1} = 10^{-1}A, I_{B1} = 6\times10^{-3}A
\end{cases}
$$

代入$r_{bb'}$表达式

$$
r_{bb'} = \frac{1}{5\times10^{-4}A - 6\times10^{-3}A}[(0.7V - 0.8V) - 0.026V\times\ln\frac{10^{-2}A}{10^{-1}A}] \approx 7.30\Omega
$$

注：据老师说图不准确，取不同点结果相差很大

3-11

(1) 推导出均匀掺杂基区晶体管的基区渡越时间表达式。假设$x_B/L_n \ll 1$。
(2) 若基区杂质分布为$N_a = N_0 e^{-ax/x_B}$， 重复 (1) 。

解:

基区渡越时间

$$\tau_B = \int_0^{\tau_B} \mathrm{d}t = \int_0^{x_B} \frac{\mathrm{d}x}{v(x)} = \frac{Aq}{I_n}\int_0^{x_B} n_p(x)\,\mathrm{d}x = \int_0^{x_B} \frac{\mathrm{d}x}{D_n N_a(x)} \int_x^{x_B} N_a(x)\,\mathrm{d}x$$

(1) 当掺杂均匀，即$N_a =$ 常数时有

$$\tau_B = \int_0^{x_B} \frac{\mathrm{d}x}{D_n N_a(x)} \int_x^{x_B} N_a(x)\,\mathrm{d}x = \frac{1}{D_n}\int_0^{x_B}\int_x^{x_B} \mathrm{d}x = \frac{x_B^2}{2D_n}$$

(2) 当杂质分布为$N_a = N_0 e^{-ax/x_B}$时，代入τ_B公式

$$\begin{aligned}
\tau_B &= \int_0^{x_B} \frac{\mathrm{d}x}{D_n N_a(x)} \int_x^{x_B} N_a(x)\,\mathrm{d}x \\
&= \int_0^{x_B} \frac{\mathrm{d}x}{D_n N_0 e^{-ax/x_B}} \int_x^{x_B} N_0 e^{-ax/x_B}\,\mathrm{d}x \\
&= \int_0^{x_B} \frac{e^{ax/x_B}}{D_n}\,\mathrm{d}x\,\left(-\frac{x_B}{a}e^{-ax/x_B}\right)\Big|_x^{x_B} \\
&= \int_0^{x_B} \frac{e^{ax/x_B}}{D_n}\left(-\frac{x_B}{a}e^{-a} + \frac{x_B}{a}e^{-ax/x_B}\right)\mathrm{d}x \\
&= \int_0^{x_B} -\frac{x_B e^{-a} e^{ax/x_B}}{aD_n} + \frac{x_B}{aD_n}\,\mathrm{d}x \\
&= -\frac{x_B^2 e^{-a} e^{ax/x_B}}{a^2 D_n}\Big|_0^{x_B} + \frac{x_B}{aD_n}(x_B - 0) \\
&= -\frac{x_B^2}{a^2 D_n} + \frac{x_B^2 e^{-a}}{a^2 D_n} + \frac{x_B^2}{aD_n} \\
&= \frac{x_B^2}{a^2 D_n}(e^{-a} + a - 1)
\end{aligned}$$

3-12

硅NPN晶体管在300K具有如下参数：

$I_E = 1mA, C_{TE} = 1pF, x_B = 0.5\mu m, D_n = 25cm^2/s, x_m = 2.4\mu m, r_{SC} = 20\Omega, C_{TC} = 0.1pF$

。 求发射区—集电区渡越时间和截止频率。

解：

$$\begin{aligned}
\tau_E &= r_E C_{TE} = \frac{V_T}{I_E} C_{TE} = \frac{0.026V}{1mA} \times 1pF = 26ps \\
\tau_B &= \frac{x_B^2}{2D_n} = \frac{(0.5\mu m)^2}{2 \times 25cm^2/s} = 50ps \\
\tau_d &= \frac{x_m}{v_s} = \frac{2.4\mu m}{10^7 cm/s} = 24ps \\
\tau_C &= r_{SC} C_{TC} = 20\Omega \times 0.1pF = 2ps
\end{aligned}$$

则总的渡越时间

$$\tau_{EC} = \tau_E + \tau_B + \tau_d + \tau_C = 26ps + 50ps + 24ps + 2ps = 102ps$$

截止频率：

$$f_\alpha = \frac{1}{2\pi\tau_{EC}} = \frac{1}{2 \times 3.14 \times 102ps} = 1.56 \times 10^9 Hz = 1.56GHz$$

3-13

（1）求出图3-24中输出短路时i_{out}/i_{in}的表达式；

（2）求出ω_β，它相应于i_{out}/i_{in}的数值下降了$3dB$的情况；

（3）求出ω_T。

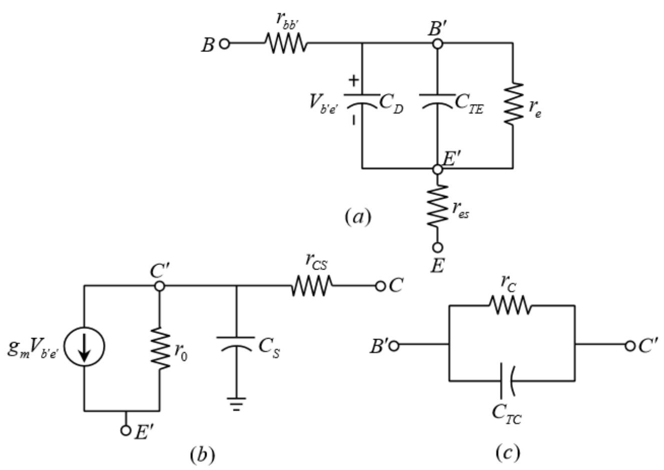

图 3-24 H-P 模型等效电路中的组成部分

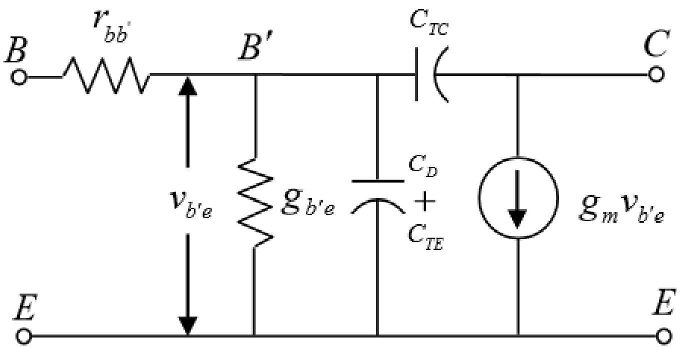

图 3-26 混接π型等效电路

解:

(1）当输出短路时，有

$$
\begin{aligned}
i_{in} &= v_{b'e}[g_{b'e} + \jmath\omega(C_D + C_{TE} + C_{TC})] \\
i_{out} &= g_m v_{b'e} + \jmath\omega C_{TC} v_{b'e} \approx g_m v_{b'e}(\text{低频})
\end{aligned}
$$

可得:

$$
\frac{i_{in}}{i_{out}} = \frac{g_m}{g_{b'e} + \jmath\omega(C_D + C_{TE} + C_{TC})}
$$

(2）下降$3dB$对应于原数值的$\dfrac{1}{\sqrt{2}}$倍，有

$$
h_{fe}^2|_{\omega=\omega_\beta} = \frac{g_m^2}{g_{b'e}^2 + \omega_\beta^2(C_D + C_{TE} + C_{TC})^2} = \frac{1}{2}h_{FE}^2
$$

代入$g_{b'e} = \dfrac{g_m}{h_{FE}}$可解得

$$
\omega_\beta = \frac{g_m}{h_{FE}(C_D + C_{TE} + C_{TC})}
$$

(3）当$C_D \gg C_{TE} + C_{TC}$时

$$
\omega_T = h_{FE}\omega_\beta = \frac{g_m}{C_D + C_{TE} + C_{TC}} \approx \frac{g_m}{C_D} = \frac{1}{\tau_B} = \frac{2D_n}{x_B^2}
$$

3-14

证明均匀基区BJT穿通击穿电压可表示为$BV_{BC} = \dfrac{qW_B^2}{2k\epsilon_0}\dfrac{N_a(N_a + N_{dc})}{N_{dc}}$

证明:

当基极穿通击穿时，集电极空间电荷区的基极部分宽度应约等于基极的冶金学宽度。即有

$$W_B \approx x_B = \sqrt{\frac{2k\epsilon_0(\psi_0 + V_R)}{q(N_a + N_{dc})}\frac{N_{dc}}{N_a}}$$

此时反偏电压即为穿通击穿电压，且有$V_R \gg \psi_0$，可得:

$$BV_{BC} \approx BV_{BC} + \psi_0 = \frac{qW_B^2}{2k\epsilon_0}\frac{N_a(N_a + N_{dc})}{N_{dc}}$$

3-15

一均匀基区硅BJT，基区宽度为0.5μm，基区杂质浓度$N_a = 10^{16} cm^{-3}$。若穿通电压期望值为$BV_{BC} = 25V$，集电区掺杂浓度为若干?如果不使集电区穿通，集电区宽度至少应大于多少?

解:

由3-14可得$BV_{BC} = \frac{qW_B^2}{2k\epsilon_0}\frac{N_a(N_a + N_{dc})}{N_{dc}}$，则有

$$\frac{N_a}{N_{dc}} = \frac{2k\epsilon_0 BV_{BC}}{qW_B^2 N_a} - 1 = \frac{2 \times 11.8 \times 8.85 \times 10^{-14} \times 25}{1.6 \times 10^{-19} \times (0.5 \times 10^{-4})^2 \times 10^{16}} - 1 = 12.05375$$

$$\implies N_{dc} = \frac{N_a}{12.05375} = \frac{10^{16} cm^{-3}}{12.05375} = 8.30 \times 10^{14} cm^{-3}$$

集电结空间电荷区的集电极区部分有:

$$x_C = \frac{N_a W_B}{N_{dc}} = 12.05375 \times 0.5\mu m \approx 6.03\mu m$$

即当$W_C > x_C = 6.03\mu m$即可保证集电区不被穿通。