

第二章

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第二章

2-1

硅突变结二极管的掺杂浓度为： $N_d = 10^{15} \text{ cm}^{-3}$ ， $N_a = 4 \times 10^{20} \text{ cm}^{-3}$ ，在室温下计算：

(1)自建电势；(2)耗尽层宽度；(3)零偏压下的最大内建电场。

解：

(1) 饱和电离区有：

$$\text{N型区: } n = N_d = n_i \exp\left(\frac{E_{fN} - E_i}{kT}\right) \Rightarrow E_{fN} - E_i = kT \ln \frac{N_d}{n_i}$$

$$\text{P型区: } p = N_a = n_i \exp\left(\frac{E_i - E_{fP}}{kT}\right) \Rightarrow E_i - E_{fP} = kT \ln \frac{N_a}{n_i}$$

$$\text{两式相减可得: } q\psi_0 = E_{fN} - E_{fP} = kT \ln \frac{N_d N_a}{n_i^2}$$

代入数据：

$$\psi_0 = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2} = V_T \ln \frac{N_a N_d}{n_i^2} = 0.026V \times \ln \frac{4 \times 10^{20} \times 10^{15}}{(1.5 \times 10^{10})^2} \approx 0.91V$$

$$(2) \text{ 由 } N_a x_p = N_d x_n \implies x_n = \frac{N_a}{N_d} x_p = \frac{4 \times 10^{20}}{10^{15}} x_p = 4 \times 10^5 x_p \implies x_n \gg x_p$$

则有 $W \approx x_n$, 且内建电势几乎全部降落在N侧的空间电荷区上。

由N侧泊松方程: $\frac{d^2 \psi}{dx^2} = -\frac{\rho}{\epsilon} = -\frac{qN_d}{\epsilon}$, 得通解为

$$\psi(x) = -\frac{qN_d}{2\epsilon} x^2 + C_1 x + C_2, \quad \text{其中 } C_1, C_2 \text{ 为常数}$$

$$\text{代入边界条件: } \begin{cases} -\frac{d\psi}{dx} \Big|_{x=x_n} = 0, & \text{N侧中性区无电场} \\ \psi(0) = 0, & \text{设两侧交界即 } x=0 \text{ 处电势为 } 0 \end{cases} \implies \begin{cases} C_1 = \frac{qN_d x_n}{\epsilon} \\ C_2 = 0 \end{cases}$$

则有

$$\psi(x) = -\frac{qN_d}{2\epsilon} x^2 + \frac{qN_d x_n}{\epsilon} x$$

$$\text{内建电势差 } \psi_0 = \psi(x_n) - \psi(0) = \frac{qN_d x_n^2}{2\epsilon} \implies x_n = \sqrt{\frac{2\psi_0 \epsilon}{qN_d}}$$

$$\text{耗尽区宽度 } W \approx x_n = \sqrt{\frac{2\psi_0 \epsilon}{qN_d}} = \sqrt{\frac{2 \times 0.91 \times 11.9 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times 10^{15}}} \text{ cm} \approx 1.09 \times 10^{-4} \text{ cm}$$

$$(3) \text{ 有 } E(x) = -\frac{d\psi}{dx} = -\frac{qN_d}{\epsilon} x + \frac{qN_d x_n}{\epsilon}, \text{ 显然当 } x=0 \text{ 时 } E \text{ 取最大值}$$

$$E_{max} = E(0) = \frac{qN_d x_n}{\epsilon} = \frac{1.6 \times 10^{-19} \times 10^{15} \times 1.09 \times 10^{-4}}{11.9 \times 8.85 \times 10^{-14}} \text{ V/cm} \approx 1.66 \times 10^4 \text{ V/cm}$$

2-2

若突变结两边的掺杂浓度为同一数量级, 试证明自建电势和耗尽层宽度可用下式表示:

$$\psi_0 = \frac{qN_a N_d (x_n + x_p)^2}{2k\epsilon_0 (N_a + N_d)} \quad x_n = \sqrt{\frac{2k\epsilon_0 \psi_0 N_a}{qN_d (N_a + N_d)}} \quad x_p = \sqrt{\frac{2k\epsilon_0 \psi_0 N_d}{qN_a (N_a + N_d)}}$$

解:

令PN结交界处为 $x=0$, P侧中性区边界为 $x=-x_p$, N侧中性区边界为 $x=x_n$, 则有泊松方程:

$$\begin{cases} \frac{d^2 \psi}{dx^2} = \frac{qN_a}{\epsilon}, & -x_p \leq x \leq 0 \\ \frac{d^2 \psi}{dx^2} = -\frac{qN_d}{\epsilon}, & 0 \leq x \leq x_n \end{cases}$$

通解为:

$$\begin{cases} \psi(x) = \frac{qN_a}{2\epsilon} x^2 + C_1 x + C_2, & -x_p \leq x \leq 0 \\ \psi(x) = -\frac{qN_d}{2\epsilon} x^2 + C_3 x + C_4, & 0 \leq x \leq x_n \end{cases} \quad \text{其中 } C_1, C_2, C_3, C_4 \text{ 为常数}$$

边界条件:

- 令PN结交界处即 $x=0$ 处为内建电势的零点, 即 $\psi(0) = 0$;

- 两侧中性区电场为零且连续:

$$E(-x_p) = -\frac{d\psi(x)}{dx}\bigg|_{x=-x_p} = 0, \quad E(x_n) = -\frac{d\psi(x)}{dx}\bigg|_{x=x_n} = 0$$

代入以上边界条件可得 $C_1 = \frac{qN_d x_p}{\epsilon}$, $C_3 = \frac{qN_a x_n}{\epsilon}$, $C_2 = C_4 = 0$

代入 ψ 表达式中

$$\begin{cases} \psi(x) = \frac{qN_a}{2\epsilon}x^2 + \frac{qN_d x_p}{\epsilon}x, & -x_p \leq x \leq 0 \\ \psi(x) = -\frac{qN_d}{2\epsilon}x^2 + \frac{qN_a x_n}{\epsilon}x, & 0 \leq x \leq x_n \end{cases}$$

有 $N_d x_n = N_a x_p$, $W = x_n + x_p$, 可得 $x_n = \frac{N_a}{N_a + N_d}W$, $x_p = \frac{N_d}{N_a + N_d}W$

内建电势差:

$$\begin{aligned} \psi_0 &= \psi(x_n) - \psi(-x_p) \\ &= -\frac{qN_d}{2\epsilon}x_n^2 + \frac{qN_a x_n}{\epsilon}x_n - \frac{qN_a}{2\epsilon}x_p^2 + \frac{qN_d x_p}{\epsilon}x_p \\ &= \frac{qN_d}{2\epsilon}x_n^2 + \frac{qN_a}{2\epsilon}x_p^2 \\ &= \frac{qN_a N_d}{2\epsilon(N_a + N_d)}(x_n W + x_p W) \\ &= \frac{qN_a N_d (x_n + x_p)^2}{2\epsilon(N_a + N_d)} \end{aligned}$$

则可求得:

$$\begin{aligned} W &= \sqrt{\frac{2\epsilon\psi_0(N_a + N_d)}{qN_a N_d}} \\ x_n &= \frac{N_a}{N_a + N_d}W = \sqrt{\frac{2\epsilon\psi_0 N_a}{qN_d(N_a + N_d)}} \\ x_p &= \frac{N_d}{N_a + N_d}W = \sqrt{\frac{2\epsilon\psi_0 N_d}{qN_a(N_a + N_d)}} \end{aligned}$$

2-3

推导出线性缓变PN结的下列表示式: (1)电场(2)电势分布(3)耗尽层宽度(4)内建电势差。

解:

令PN结交界处为 $x = 0$, P侧中性区边界为 $x = -x_p$, N侧中性区边界为 $x = x_n$

对于线性缓变结, 空间电荷区关于 $x = 0$ 左右对称, 即 $x_n = x_p = \frac{W}{2}$

耗尽层电荷浓度分布 $N(x) = N_d - N_a = \alpha x$, 代入泊松方程

$$\frac{d^2\psi}{dx^2} = -\frac{\rho}{\epsilon} = -\frac{q\alpha x}{\epsilon} \implies \psi(x) = -\frac{q\alpha}{6\epsilon}x^3 + C_1x + C_2, \quad \text{其中 } C_1, C_2 \text{ 为常数}$$

边界条件:

令PN结交界处即 $x = 0$ 处为内建电势的零点, 即 $\psi(0) = 0$;

两侧中性区电场为零且连续, 即 $E(-\frac{W}{2}) = E(\frac{W}{2}) = -\frac{d\psi(x)}{dx}\bigg|_{x=W/2} = 0$

代入以上边界条件可得 $C_1 = \frac{q\alpha W^2}{8\epsilon}$, $C_2 = 0$, 即

$$\psi(x) = -\frac{q\alpha}{6\epsilon}x^3 + \frac{q\alpha W^2}{8\epsilon}x$$

$$(1) E(x) = -\frac{d\psi}{dx} = \frac{q\alpha}{2\epsilon}x^2 - \frac{q\alpha W^2}{8\epsilon}$$

$$(2) \psi(x) = -\frac{q\alpha}{6\epsilon}x^3 + \frac{q\alpha W^2}{8\epsilon}x$$

$$(3) \psi_0 = \psi(\frac{W}{2}) - \psi(-\frac{W}{2}) = 2\psi(\frac{W}{2}) = 2[-\frac{q\alpha}{6\epsilon}(\frac{W}{2})^3 + \frac{q\alpha W^2}{8\epsilon}\frac{W}{2}] = \frac{q\alpha W^3}{12\epsilon}$$

$$W = \sqrt[3]{\frac{12\epsilon\psi_0}{q\alpha}}$$

$$(4) \psi_0 = \frac{q\alpha W^3}{12\epsilon}$$

或用浓度表示

P侧N侧中性区杂质浓度: $p_{p0} = n_{n0} = \frac{\alpha W}{2}$, 代入 ψ_0 浓度表达式

$$\psi_0 = V_T \ln \frac{p_{p0}n_{n0}}{n_i^2} = 2V_T \ln \frac{\alpha W}{2n_i}$$

2-4

推导出 N^+N 结(常称为高低结)内建电势差表达式。

解:

法一: 热平衡时费米能级一致:

设 N^+ 区杂质浓度 N_d^+ , N 区杂质浓度 N_d , 饱和电离时有:

$$E_{fN^+} - E_i = kT \ln \frac{N_d^+}{n_i}, E_{fN} - E_i = kT \ln \frac{N_d}{n_i}$$

$$\text{两式相减可得: } q\psi_0 = E_{fN^+} - E_{fN} = kT \ln \frac{N_d^+}{N_d}$$

$$\text{则有 } \psi_0 = E_{fN^+} - E_{fN} = V_T \ln \frac{N_d^+}{N_d}$$

法二: 静电势概念:

$$\psi^+ = V_T \ln \frac{N_d^+}{n_i}, \psi = V_T \ln \frac{N_d}{n_i}$$

$$\psi_0 = \psi^+ - \psi = V_T \ln \frac{N_d^+}{N_d}$$

2-5

P^+N 结空间电荷区边界分别为 $-x_p$ 和 x_n ，利用 $np = n_i^2 e^{V/V_T}$ 导出一般情况下的 $p_n(x_n)$ 表达式。给出N区空穴为小注入和大注入两种情况下的 $p_n(x_n)$ 表达式。

解：

在N侧空间电荷区和中性区的边界，即 $x = x_n$ 处

$$\begin{cases} p_n = p_{n0} + \Delta p \approx \Delta p, & \Delta p \gg p_{n0} \\ n_n = n_{n0} + \Delta n \approx n_{n0} + p_n, & \Delta n = \Delta p \end{cases}$$

代入 $np = n_i^2 e^{V/V_T}$ 可得 $p_n n_n = p_n (n_{n0} + p_n) = p_n n_{n0} + p_n^2 = n_i^2 e^{V/V_T}$ ，解得

$$p_n = \frac{-n_{n0} + \sqrt{n_{n0}^2 + 4n_i^2 e^{V/V_T}}}{2} \quad \text{或} \quad p_n = \frac{-n_{n0} - \sqrt{n_{n0}^2 + 4n_i^2 e^{V/V_T}}}{2} \quad (\text{舍去})$$

小注入时： $p_n \approx \Delta p \ll n_{n0}$ ，则 $p_n^2 \ll p_n n_{n0}$ ，则有

$$p_n n_{n0} = n_i^2 e^{V/V_T} \implies p_n = \frac{n_i^2}{n_{n0}} e^{V/V_T} = p_{n0} e^{V/V_T}$$

大注入时： $p_n \approx \Delta p \gg n_{n0}$ ，则 $p_n^2 \gg p_n n_{n0}$ ，则有

$$p_n^2 = n_i^2 e^{V/V_T} \implies p_n = \sqrt{n_i^2 e^{V/V_T}} = n_i e^{V/2V_T}$$

2-6

根据电子电流公式 $I_n = qA(n\mu_n E + D_n \frac{\partial n}{\partial x})$ 推导方程 $\psi_0 = \psi_n - \psi_p = V_T \ln \frac{N_d N_a}{n_i^2}$ 。

解：

热平衡时，电子电流为零，即 $I_n = qA(n\mu_n E + D_n \frac{\partial n}{\partial x}) = 0$

$$E = -\frac{1}{n\mu_n} D_n \frac{dn}{dx} = -\frac{V_T d \ln n}{dx} = -\frac{d\psi}{dx} \implies d\psi = V_T d \ln n$$

对该式从 $-x_p$ 到 x_n 积分： $\psi(x_n) - \psi(-x_p) = V_T [\ln n(x_n) - \ln n(-x_p)]$

则内建电势差 $\psi_0 = \psi_n - \psi_p = V_T \ln \frac{n_{n0}}{n_{p0}} = V_T \ln \frac{N_a N_d}{n_i^2}$

2-7

根据修正欧姆定律和空穴扩散电流公式证明，在外加正向偏压 V 作用下，PN结N侧空穴扩散区准费米能级的改变量为 $\Delta E_{FP} = qV$ 。

解：

$$\text{空穴电流修正欧姆定律 } I_p = -qAp\mu_p \frac{d\phi_p}{dx}$$

$$\text{空穴扩散电流 } I_p = -qAD_p \frac{dp_n}{dx}$$

两式电流相等，即

$$-qAp\mu_p \frac{d\phi_p}{dx} = -qAD_p \frac{dp_n}{dx} \implies d\phi_p = \frac{D_p}{\mu_p} \frac{dp_n}{p_n} = V_T d(\ln p_n)$$

$$\text{有准费米势：} \phi_p = -\frac{E_{FP}}{q} \implies d\phi_p = -\frac{1}{q} dE_{FP}, \text{ 代入上式得 } dE_{FP} = -kT d(\ln p_n)$$

对其在扩散区进行积分，由于扩散区外无扩散电流，可从 x_n 到 W_x 积分

$$\begin{aligned} \Delta E_{FP} &= \int_{x_n}^{W_n} dE_{FP} = \int_{x_n}^{W_n} -kT d(\ln p_n) \\ &= -kT [\ln p_n(W_n) - \ln p_n(x_n)] \\ &= -kT \ln \frac{p_{n0}}{p_{n0} e^{V/V_T}} \\ &= \frac{kTV}{V_T} \\ &= qV \end{aligned}$$

2-8

(1) PN结的空穴注射效率定义为在 $x=0$ 处的 I_p/I ，证明此效率可写成 $\gamma = \frac{I_p}{I} = \frac{1}{1 + \sigma_n L_p / \sigma_p L_n}$ ；

(2) 在实际的二极管中怎样才能使 γ 接近1。

解：

$$(1) I_p(x_n) = \frac{qAD_p}{L_p} p_{n0} (e^{V/V_T} - 1), I_n(-x_p) = \frac{qAD_n}{L_n} n_{p0} (e^{V/V_T} - 1)$$

$$\text{总电流 } I = qA \left(\frac{p_{n0} D_p}{L_p} + \frac{n_{p0} D_n}{L_n} \right) (e^{V/V_T} - 1)$$

$$\gamma = \frac{I_p}{I} = \frac{I_p(x_n)}{I} = \frac{\frac{qAD_p}{L_p} p_{n0} (e^{V/V_T} - 1)}{qA \left(\frac{p_{n0} D_p}{L_p} + \frac{n_{p0} D_n}{L_n} \right) (e^{V/V_T} - 1)} = \frac{1}{1 + \frac{n_{p0} D_n}{L_n} \frac{L_p}{p_{n0} D_p}}$$

有 $D_n = \mu_n V_T$, $D_p = \mu_p V_T$, $\sigma_n = nq\mu_n$, $\sigma_p = pq\mu_p$, 代入上式

$$\gamma = \frac{1}{1 + \frac{n_{p0} D_n}{L_n} \frac{L_p}{p_{n0} D_p}} = \frac{1}{1 + \frac{n_{p0} \mu_n V_T L_p}{p_{n0} \mu_p V_T L_n}} = \frac{1}{1 + \frac{qn_{p0} \mu_n L_p}{qp_{n0} \mu_p L_n}} = \frac{1}{1 + \frac{\sigma_n L_p}{\sigma_p L_n}}$$

(2) 为使 γ 接近1，应使 $\frac{\sigma_n L_p}{\sigma_p L_n} \ll 1$

$$\frac{\sigma_n L_p}{\sigma_p L_n} = \frac{qn_{p0} \mu_n \sqrt{\mu_p V_T \tau_p}}{qp_{n0} \mu_p \sqrt{\mu_n V_T \tau_n}} = \frac{N_d n_i^2 \sqrt{\mu_n} \sqrt{\tau_p}}{N_a n_i^2 \sqrt{\mu_p} \sqrt{\tau_n}} = \frac{N_d}{N_a} \cdot \frac{\sqrt{\mu_n}}{\sqrt{\mu_p}} \cdot \frac{\sqrt{\tau_p}}{\sqrt{\tau_n}}$$

μ_n , μ_p 相差不大, τ_n , τ_p 相差不大, 所以只需使 $N_a \gg N_d$ 即PN结为 P^+N 结即可

2-9

长PN结二极管处于反偏压状态:

- (1) 解扩散方程求少子分布 $n_p(x)$ 和 $p_n(x)$ 并画出它们的分布示意图。
- (2) 计算扩散区内少子贮存电荷。
- (3) 证明反向电流 $I = -I_0$ 为PN结扩散区内的载流子产生电流。

解:

$$(1) \text{ P侧电子连续方程 } \frac{\partial n_p}{\partial t} = D_n \frac{\partial n_p}{\partial x} - \frac{n_p - n_{p0}}{\tau_n}$$

$$\text{稳态时, 有 } \frac{\partial n_p}{\partial t} = 0, \text{ 即 } D_n \frac{\partial n_p}{\partial x} - \frac{n_p - n_{p0}}{\tau_n} = 0$$

$$\text{令 } L_n^2 = D_n \tau_n, \text{ 代入上式可得: } \frac{d(n_p - n_{p0})}{dx} - \frac{n_p - n_{p0}}{L_n^2} = 0, \text{ 其通解为:}$$

$$n_p = n_{p0} + Ae^{x/L_n} + Be^{-x/L_n}, \quad -W_p \leq x \leq -x_p \quad \text{其中 } A, B \text{ 为常数}$$

设PN结空间电荷区P侧边界为 $-x_p$, 外部接触 $-W_p$, 可得边界条件:

反偏状态时有反向抽取, 即当 $x = -x_p$ 时有 $n_p = 0$;

在外部接触位置即当 $x = -W_p$ 时有 $n_p = n_{p0}$

代入以上边界条件:

$$\begin{cases} n_p(-x_p) = n_{p0} + Ae^{-x_p/L_n} + Be^{x_p/L_n} = 0 \\ n_p(-W_p) = n_{p0} + Ae^{-W_p/L_n} + Be^{W_p/L_n} = n_{p0} \end{cases}$$

长PN结中 $W_n \gg L_n$, 则可得 $A = -n_{p0}e^{x_p/L_n}$, $B = 0$, 代入通解

$$n_p = n_{p0} - n_{p0}e^{(x_p+x)/L_n} = n_{p0}[1 - e^{(x_p+x)/L_n}], \quad -W_p \leq x \leq -x_p$$

同理可解得:

$$p_n = p_{n0}[1 - e^{(x_n-x)/L_p}], \quad x_n \leq x \leq W_n$$

(2) P侧扩散区有 $\Delta n_p = n_p - n_{p0} = -n_{p0}e^{(x_p+x)/L_n}$, 对其从 $-W_p$ 到 $-x_p$ 积分

$$Q_n = -qA \int_{-W_p}^{-x_p} -n_{p0}e^{(x_p+x)/L_n} dx = qAL_n n_{p0} e^{(x_p+x)/L_n} \Big|_{-W_p}^{-x_p} = qAL_n n_{p0} [1 - e^{(x_p-W_p)/L_n}]$$

长PN结有 $W_p \gg L_n$, 则可近似得到 $Q_n = qAL_n n_{p0}$, 符号为正是少子电子被抽取的结果。

N侧扩散区有 $\Delta p_n = p_n - p_{n0} = -p_{n0}e^{(x_n-x)/L_p}$, 对其从 x_n 到 W_n 积分

$$Q_p = qA \int_{x_n}^{W_n} -p_{n0}e^{(x_n-x)/L_p} dx = qAL_p p_{n0} e^{(x_n-x)/L_p} \Big|_{x_n}^{W_n} = qAL_p p_{n0} [e^{(x_n-W_n)/L_p} - 1]$$

长PN结有 $W_n \gg L_p$, 则可近似得到 $Q_p = -qAL_p p_{n0}$, 符号为负是少子空穴被抽取的结果。

(3) 设P、N侧扩散区贮存电荷均匀分布在长为 L_n 、 L_p 的扩散区内。则有

$$\Delta n_p = \frac{Q_n}{-qAL_n} = -n_{p0}, \quad \Delta p_n = \frac{Q_p}{qAL_p} = -p_{n0}$$

P侧扩散区电子产生率 $G_n = -U_n = -\frac{\Delta n_p}{\tau_n} = \frac{n_{p0}}{\tau_n} > 0$, 有产生电流。

N侧扩散区空穴产生率 $G_p = -U_p = -\frac{\Delta p_n}{\tau_p} = \frac{p_{n0}}{\tau_p} > 0$, 有产生电流。

显然和 $I = -I_0 = -qA(\frac{n_{p0}}{\tau_n}L_n + \frac{p_{n0}}{\tau_p}L_p)$ 一致, 即反向电流为扩散区载流子产生电流。

2-10

若 PN 结边界条件为 $x = W_n$ 处 $p = P_{n0}$, $x = -W_p$ 处 $n = n_{p0}$ 。其中 W_p 和 W_n 分别与 L_p 与 L_n 具有相同的数量级, 求 $n_p(x)$ 、 $p_n(x)$ 以及 $I_n(x)$ 、 $I_p(x)$ 的表达式。

解:

$$\text{P侧电子连续方程 } \frac{\partial n_p}{\partial t} = D_n \frac{\partial n_p}{\partial x} - \frac{n_p - n_{p0}}{\tau_n}$$

$$\text{稳态时, 有 } \frac{\partial n_p}{\partial t} = 0, \text{ 即 } D_n \frac{\partial n_p}{\partial x} - \frac{n_p - n_{p0}}{\tau_n} = 0$$

$$\text{令 } L_n^2 = D_n \tau_n, \text{ 代入上式可得: } \frac{d(n_p - n_{p0})}{dx} - \frac{n_p - n_{p0}}{L_n^2} = 0, \text{ 其通解为:}$$

$$n_p = n_{p0} + Ae^{x/L_n} + Be^{-x/L_n}, \quad -W_p \leq x \leq -x_p \quad \text{其中 } A, B \text{ 为常数}$$

边界条件:

正偏时有正向注入, 即当 $x = -x_p$ 时有 $n_p = n_{p0}e^{V/V_T}$;

在外部接触位置即当 $x = -W_p$ 时有 $n_p = n_{p0}$

代入以上边界条件:

$$\begin{cases} n_p(-x_p) = n_{p0} + Ae^{-x_p/L_n} + Be^{x_p/L_n} = n_{p0}e^{V/V_T} \\ n_p(-W_p) = n_{p0} + Ae^{-W_p/L_n} + Be^{W_p/L_n} = n_{p0} \end{cases}$$

$$\text{联立可解得 } A = \frac{e^{W_p/L_n} n_{p0} (e^{V/V_T} - 1)}{2 \sinh \frac{W_p - x_p}{L_n}}, \quad B = \frac{e^{-W_p/L_n} n_{p0} (e^{V/V_T} - 1)}{2 \sinh \frac{x_p - W_p}{L_n}}, \text{ 代入通解:}$$

$$\begin{aligned} n_p &= n_{p0} + \frac{e^{W_p/L_n} n_{p0} (e^{V/V_T} - 1)}{2 \sinh \frac{W_p - x_p}{L_n}} e^{x/L_n} + \frac{e^{-W_p/L_n} n_{p0} (e^{V/V_T} - 1)}{2 \sinh \frac{x_p - W_p}{L_n}} e^{-x/L_n} \\ &= n_{p0} + n_{p0} (e^{V/V_T} - 1) \frac{\sinh \frac{W_p + x}{L_n}}{\sinh \frac{W_p - x_p}{L_n}}, \quad -W_p \leq x \leq -x_p \end{aligned}$$

同理可解得

$$p_n = p_{n0} + p_{n0} (e^{V/V_T} - 1) \frac{\sinh \frac{W_n - x}{L_p}}{\sinh \frac{W_n - x_n}{L_p}}, \quad x_n \leq x \leq W_n$$

电流强度分别是

$$I_n = qAD_n \frac{dn_p}{dx} = \frac{qAn_{p0}}{L_n} (e^{V/V_T} - 1) \frac{\cosh \frac{W_p + x}{L_n}}{\sinh \frac{W_p - x_p}{L_n}}, \quad -W_p \leq x \leq -x_p$$

$$I_p = -qAD_n \frac{dp_n}{dx} = \frac{qAp_{n0}}{L_p} (e^{V/V_T} - 1) \frac{\cosh \frac{W_n - x}{L_p}}{\sinh \frac{W_n - x_n}{L_p}}, \quad x_n \leq x \leq W_n$$

2-11

在 P^+N 结二极管中，N区的宽度 W_n 远小于 L_p ，用 $I_p|_{x=W_n} = qS\Delta p_n A$ (S 为表面复合速度)作为N侧末端的少数载流子电流，并以此为边界条件之一，推导出载流子和电流分布。绘出在 $S = 0$ 和 $S = \infty$ 时N侧少数载流子的分布形状(计算机解)。

解：

设空间电荷区与N侧中性区交界为 $x = 0$ 的点，此时N侧外部接触点 $x = W'_n = W_n - x_n$ 。

$$\text{N侧少子连续方程为 } \frac{\partial p_n}{\partial t} = D_p \frac{\partial p_n}{\partial x} - \frac{p_n - p_{n0}}{\tau_p} \implies \frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial \Delta p_n}{\partial x} - \frac{\Delta p_n}{\tau_p}.$$

$$\text{稳态时有 } \frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial \Delta p_n}{\partial x} - \frac{\Delta p_n}{\tau_p} = 0, \text{ 通解为:}$$

$$\Delta p_n = C_1 e^{x/L_p} + C_2 e^{-x/L_p}, \quad \text{其中 } C_1, C_2 \text{ 为常数}$$

边界条件：

$$\text{当 } x = 0 \text{ 时有 } \Delta p_n|_{x=0} = C_1 + C_2 = p_{n0}(e^{V/V_T} - 1);$$

$$\text{当 } x = W'_n \text{ 时 } I_p|_{x=W'_n} = -qAD_p \frac{dp_n}{dx} = -qAD_p \frac{d\Delta p_n}{dx} q = S\Delta p_n A|_{x=W'} \text{ 即:}$$

$$-\frac{qAD_p}{L_p} (C_1 e^{W'_n/L_p} - C_2 e^{-W'_n/L_p}) = qSA(C_1 e^{W'_n/L_p} + C_2 e^{-W'_n/L_p})$$

联立边界条件可得

$$C_1 = \frac{(D_p - L_p S) e^{-W'_n/L_p}}{2[D_p \cosh(\frac{W'_n}{L_p}) + L_p S \sinh(\frac{W'_n}{L_p})]} p_{n0}(e^{V/V_T} - 1)$$

$$C_2 = \frac{(D_p + L_p S) e^{W'_n/L_p}}{2[D_p \cosh(\frac{W'_n}{L_p}) + L_p S \sinh(\frac{W'_n}{L_p})]} p_{n0}(e^{V/V_T} - 1)$$

代入通解：

$$\Delta p_n = \frac{D_p \cosh(\frac{W'_n - x}{L_p}) + L_p S \sinh(\frac{W'_n - x}{L_p})}{D_p \cosh(\frac{W'_n}{L_p}) + L_p S \sinh(\frac{W'_n}{L_p})} p_{n0} (e^{V/V_T} - 1)$$

$$I_p = \frac{qAD_p}{L_p} \frac{D_p \sinh(\frac{W'_n - x}{L_p}) + L_p S \cosh(\frac{W'_n - x}{L_p})}{D_p \cosh(\frac{W'_n}{L_p}) + L_p S \sinh(\frac{W'_n}{L_p})} p_{n0} (e^{V/V_T} - 1)$$

当有 $W_n \ll L_p$ 时, $W'_n \ll L_p$,

$$\cosh(\frac{W'_n - x}{L_p}) \approx 1 \quad \sinh(\frac{W'_n - x}{L_p}) \approx \frac{W'_n - x}{L_p}$$

$$\cosh(\frac{W'_n}{L_p}) \approx 1 \quad \sinh(\frac{W'_n}{L_p}) \approx \frac{W'_n}{L_p}$$

则通解化为

$$\Delta p_n = \frac{D_p + S(W'_n - x)}{D_p + SW'_n} p_{n0} (e^{V/V_T} - 1)$$

$$I_p = \frac{qAD_p}{L_p^2} \frac{D_p(W'_n - x) + L_p^2 S}{D_p + SW'_n} p_{n0} (e^{V/V_T} - 1)$$

当 $S = 0$ 时

$$\Delta p_n = p_{n0} (e^{V/V_T} - 1)$$

$$I_p = \frac{qAD_p}{L_p^2} p_{n0} (e^{V/V_T} - 1) (W'_n - x)$$

当 $S = \infty$ 时

$$\Delta p_n = \frac{(W'_n - x)}{W'_n} p_{n0} (e^{V/V_T} - 1)$$

$$I_p = \frac{qAD_p p_{n0}}{W'_n} (e^{V/V_T} - 1)$$

2-12

推导公式(2-6-7) 和(2-6-8)。

正偏压下, 给定电流, 电压随温度线性地减小:

$$\frac{dV}{dT} = \frac{V - E_{g0}/q}{T} \quad (2-6-7)$$

给定电压, 电流随温度升高而迅速增加:

$$\frac{1}{I} \frac{dI}{dT} = \frac{E_{g0} - qV}{KT^2} \quad (2-6-8)$$

解:

(2-6-7)有 $I_0 = qA(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a}) n_i^2$, 其中括号内参量对温度变化不敏感, 则可得到

$$I_0 \propto n_i^2 \propto T^3 e^{-E_{g0}/KT}$$

对温度 T 求导并除以 I_0 可得

$$\frac{1}{I_0} \frac{dI_0}{dT} = \frac{3}{T} + \frac{E_{g0}}{KT^2} \approx \frac{E_{g0}}{KT^2}$$

正偏情况下 $I = I_0 e^{V/V_T}$ ，当 I_0 为常数时，对 T 求导

$$\left. \frac{dV}{dT} \right|_{I_0=\text{常数}} = \frac{dV_T}{dT} \ln \frac{I}{I_0} - V_T \left(\frac{1}{I_0} \frac{dI_0}{dT} \right) = \frac{V}{T} - \frac{V_T E_{g0}}{KT^2} = \frac{V - E_{g0}/q}{T}$$

(2-6-8)当 V 为常数时，对 T 求导并除以 I

$$\frac{1}{I} \frac{dI}{dT} = \frac{1}{I} \left(\frac{dI_0}{dT} e^{V/V_T} - \frac{I_0 V}{V_T^2} e^{V/V_T} \frac{dV_T}{dT} \right) = \frac{E_{g0}}{KT^2} - \frac{V}{V_T T} = \frac{E_{g0} - qV}{KT^2}$$

2-13

把一个硅二极管用做变容二极管。在结的两边掺杂浓度分别为 $N_a = 10^{19} \text{ cm}^{-3}$ 以及 $N_d = 10^{15} \text{ cm}^{-3}$ 。二极管的面积为100平方密尔。

(1)求在 $V_R = 1$ 和 $5V$ 时的二极管的电容。

(2)计算用此变容二极管及 $L = 2mH$ 的储能电路的共振频率。

(注:密耳(mil)为长度单位, $1mil = 10^{-3}in(\text{英寸}) = 2.54 \times 10^{-5}m$)

解:

$$(1) A = 100mil^2 = 100 \times (2.54 \times 10^{-5})^2 m^2 = 6.4516 \times 10^{-8} m^2 = 6.4516 \times 10^{-4} cm^2$$

内建电势差:

$$\psi_0 = V_T \ln \frac{N_a N_d}{n_i^2} = 0.026V \times \ln \frac{10^{19} \times 10^{15}}{(1.5 \times 10^{10})^2} \approx 0.817V$$

将势垒区宽度 $W = \sqrt{\frac{2\epsilon(\psi_0 + V_R)}{qN_d}}$ 代入变容二极管电容公式

$$C_T = \frac{A\epsilon}{W} = A \sqrt{\frac{q\epsilon_r\epsilon_0 N_d}{2(\psi_0 + V_R)}}$$

当 $V_R = 1V$ 时

$$C_T = 6.4516 \times 10^{-4} cm^2 \times \sqrt{\frac{1.6 \times 10^{-19} C \times 11.9 \times 8.85 \times 10^{-14} F/cm \times 10^{15} cm^{-3}}{2 \times (0.817V + 1V)}} \approx 4.39 \times 10^{-12} F = 4.39pF$$

当 $V_R = 5V$ 时

$$C_T = 6.4516 \times 10^{-4} cm^2 \times \sqrt{\frac{1.6 \times 10^{-19} C \times 11.9 \times 8.85 \times 10^{-14} F/cm \times 10^{15} cm^{-3}}{2 \times (0.817V + 5V)}} \approx 2.46 \times 10^{-12} F = 2.46pF$$

(2) 有 $\omega_r = \frac{1}{\sqrt{LC}}$, 分别代入 $V_R = 1V$ 和 $V_R = 5V$ 时的电容可得:

$$\omega_r|_{V_R=1V} = \frac{1}{\sqrt{2mH \times 4.39pF}} \approx 1.07 \times 10^7 rad/s$$

$$\omega_r|_{V_R=5V} = \frac{1}{\sqrt{2mH \times 2.46pF}} \approx 1.43 \times 10^7 rad/s$$

2-14

P^+N 结杂质分布 $N_a = \text{常数}$, $N_d = N_{d0}e^{-x/L}$, 导出 $C-V$ 特性表达式。

解:

在 P^+N 结中, 有 $x_n \approx W$, 势垒大部分降落在 N 侧。

设 PN 结交界 $x = 0$ 为内建电场零点。

在 N 侧有泊松方程

$$\frac{d^2\psi}{dx^2} = -\frac{\rho}{\epsilon} = -\frac{qN_{d0}}{\epsilon}e^{-x/L} \quad (0 \leq x \leq x_n)$$

通解:

$$\psi = -\frac{qN_{d0}L^2}{\epsilon}e^{-x/L} + C_1x + C_2 \quad (0 \leq x \leq x_n) \quad \text{其中 } C_1, C_2 \text{ 为常数}$$

边界条件:

在 $x = 0$ 处定义为内建电场零点: $\psi(0) = 0$

在 $x = x_n$ 处电场连续, 即 $E|_{x=x_n} = -\frac{d\psi}{dx}|_{x=x_n} = 0$

代入边界条件可解得

$$C_1 = -\frac{qN_{d0}L}{\epsilon}e^{-x_n/L} \quad C_2 = \frac{qN_{d0}L^2}{\epsilon}$$

可得电势分布方程

$$\psi = -\frac{qN_{d0}L^2}{\epsilon}e^{-x/L} - \frac{qN_{d0}L}{\epsilon}xe^{-x_n/L} + \frac{qN_{d0}L^2}{\epsilon} \quad (0 \leq x \leq x_n)$$

内建电势差:

$$\psi_0 = \psi(x_n) - \psi(0) = -\frac{qN_{d0}L^2}{\epsilon}e^{-x_n/L} - \frac{qN_{d0}L}{\epsilon}x_ne^{-x_n/L} + \frac{qN_{d0}L^2}{\epsilon}$$

当 $x_n \ll L$ 时有 $e^{-x_n/L} \approx 1 - \frac{x_n}{L}$ 代入上式

$$\begin{aligned} \psi_0 &= -\frac{qN_{d0}L^2}{\epsilon}\left(1 - \frac{x_n}{L}\right) - \frac{qN_{d0}L}{\epsilon}x_n\left(1 - \frac{x_n}{L}\right) + \frac{qN_{d0}L^2}{\epsilon} \\ &= -\frac{qN_{d0}L^2}{\epsilon} + \frac{qN_{d0}Lx_n}{\epsilon} - \frac{qN_{d0}Lx_n}{\epsilon} + \frac{qN_{d0}x_n^2}{\epsilon} + \frac{qN_{d0}L^2}{\epsilon} \\ &= \frac{qN_{d0}x_n^2}{\epsilon} \end{aligned}$$

在 P^+N 结中, 有 $x_n \approx W$, 代入可得 $W = \sqrt{\frac{\psi_0 \epsilon}{qN_{d0}}}$

在偏压 V_R 下, $W = \sqrt{\frac{\epsilon(\psi_0 + V_R)}{qN_{d0}}}$

空间电荷区N侧电荷

$$\begin{aligned} Q &= qA \int_0^{x_n} N_{d0} e^{-x/L} dx = -qALN_{d0} e^{-x/L} \Big|_0^{x_n} = -qALN_{d0} e^{-x_n/L} + qALN_{d0} \\ &= qALN_{d0} (1 - e^{-x_n/L}) = qALN_{d0} \frac{x_n}{L} = qAN_{d0} W \\ &= A \sqrt{\epsilon(\psi_0 + V_R) q N_{d0}} \end{aligned}$$

有 $C = \frac{dQ}{dV_R}$ 可得

$$C = \frac{dQ}{dV_R} = \frac{A \sqrt{\epsilon q N_{d0}}}{2 \sqrt{\psi_0 + V_R}} = \frac{A \epsilon}{2} \sqrt{\frac{q N_{d0}}{\epsilon(\psi_0 + V_R)}} = \frac{A \epsilon}{2W}$$

2-15

若 P^+N 二极管N区宽度 W_n 是和扩散长度同一数量级, 推导小信号交流空穴分布和二极管导纳, 假设在 $x = W_n$ 处表面复合速度无限大。

解:

设外加偏压 $v = V + v_a e^{i\omega t}$, 电流为 $i = I + i_a e^{i\omega t}$, N侧少子分布 $p_n(x, t) = P_n(x, t) + p_a e^{i\omega t}$

则连续方程有:

$$\left. \begin{aligned} \frac{\partial p_n}{\partial t} &= D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} \\ 0 &= D_p \frac{\partial^2 P_n}{\partial x^2} - \frac{P_n - p_{n0}}{\tau_p} \end{aligned} \right\} \Rightarrow i\omega p_a e^{i\omega t} = D_p \frac{\partial^2 p_a}{\partial x^2} e^{i\omega t} - \frac{p_a e^{i\omega t}}{\tau_p} \Rightarrow \frac{\partial^2 p_a}{\partial x^2} - p_a \frac{i\omega \tau_p + 1}{D_p \tau_p} = 0$$

令 $L_p'^2 = \frac{L_p^2}{i\omega \tau_p + 1} = \frac{D_p \tau_p}{i\omega \tau_p + 1}$ 则连续方程可化为

$$\frac{\partial^2 p_a}{\partial x^2} - \frac{p_a}{L_p'^2} = 0$$

可得通解为

$$p_a = C_1 e^{x/L_p'} + C_2 e^{-x/L_p'} \quad (0 \leq x \leq x_n), \text{ 其中 } C_1, C_2 \text{ 为常数}$$

在 $x = x_n$ 处

$$p_n(x_n) = p_{n0} e^{v/V_T} = p_{n0} e^{V/V_T} \exp\left(\frac{v_a e^{i\omega t}}{V_T}\right) \approx p_{n0} e^{V/V_T} \left(1 + \frac{v_a e^{i\omega t}}{V_T}\right) = P(x_n) + p_{a1} e^{i\omega t}$$

其中 $P(x_n) = p_{n0} e^{V/V_T}$, $p_{a1} = \frac{v_a p_{n0}}{V_T} e^{V/V_T}$ 。

可得边界条件

$$p_a = \begin{cases} p_{a1} & x = x_n \\ 0 & x = W_n \end{cases}$$

代入边界条件可得

$$C_1 = \frac{p_{a1} e^{-W_n/L'_p}}{2 \sinh(\frac{x_n - W_n}{L'_p})}, \quad C_2 = \frac{p_{a1} e^{W_n/L'_p}}{2 \sinh(\frac{W_n - x_n}{L'_p})}$$

代入通解

$$p_a = p_{a1} \frac{\sinh(\frac{W_n - x}{L'_p})}{\sinh(\frac{W_n - x_n}{L'_p})} \quad (0 \leq x \leq x_n)$$

可得N侧交流电流分布

$$i_a = -qAD_p \frac{dp_a}{dx} = \frac{qAD_p p_{a1}}{L'_p} \frac{\cosh(\frac{W_n - x}{L'_p})}{\sinh(\frac{W_n - x_n}{L'_p})}$$

交流少子电流

$$i \approx i_{pmax} = i_p(x_n) = \frac{qAD_p p_{a1}}{L'_p} \coth(\frac{W_n - x}{L'_p}) = \frac{qAD_p v_a p_{n0}}{V_T L'_p} e^{V/V_T} \coth(\frac{W_n - x}{L'_p})$$

可得交流导纳:

$$Y = \frac{i}{v} = \frac{qAD_p p_{n0}}{V_T L'_p} e^{V/V_T} \coth(\frac{W_n - x}{L'_p})$$

2-16

一个硅二极管工作在0.5V的正向电压下，当温度从25°C上升到150°C时，计算电流增加的倍数。假设 $I \approx I_0 e^{V/2V_T}$ ， I_0 每10°C增加一倍。

解:

法一: 有 $V_T(27^\circ C) = 0.026V$ 可得

$$V_T(25^\circ C) = \frac{298}{300} \times 0.026V \quad V_T(150^\circ C) = \frac{423}{300} \times 0.026V$$

I_0 每10°C增加一倍:

$$\frac{I_0(150^\circ C)}{I_0(25^\circ C)} = 2^{\frac{150-25}{10}} = 2^{12.5}$$

前后电流比

$$\begin{aligned} \gamma &= \frac{I(150^\circ C)}{I(25^\circ C)} = \frac{I_0(150^\circ C)}{I_0(25^\circ C)} \exp\left[\frac{V}{2} \left(\frac{1}{V_T(150^\circ C)} - \frac{1}{V_T(25^\circ C)}\right)\right] \\ &= 2^{12.5} \times \exp\left[\frac{0.5 \times 300}{2 \times 0.026} \left(\frac{1}{423} - \frac{1}{298}\right)\right] \\ &\approx 331 \end{aligned}$$

则增加了 $(331 - 1) = 330$ 倍

法二: 对 I 微分并除以 I

$$\frac{1}{I} \frac{dI}{dT} = \frac{1}{I} \left(\frac{dI_0}{dT} e^{V/2V_T} - I_0 e^{V/2V_T} \frac{V}{2} \frac{1}{V_T^2} \frac{dV_T}{dT} \right) = \frac{1}{I_0} \frac{dI_0}{dT} - \frac{V}{2V_T T}$$

两边同乘 dV 并对其从 $25^\circ C$ 到 $150^\circ C$ 进行积分

$$\begin{aligned} \int_{25^\circ C}^{150^\circ C} \frac{1}{I} dI &= \int_{25^\circ C}^{150^\circ C} \frac{1}{I_0} dI_0 - \frac{V}{2V_T T} dT \\ \Rightarrow \ln \frac{I(150^\circ C)}{I(25^\circ C)} &= \ln \frac{I_0(150^\circ C)}{I_0(25^\circ C)} + \frac{Vq}{2kT} \Big|_{298K}^{423K} \approx 5.8 \\ \Rightarrow \frac{I(150^\circ C)}{I(25^\circ C)} &= e^{5.8} \approx 331 \end{aligned}$$

则增加了 $(331 - 1) = 330$ 倍

2-17

采用电容测试仪在 $1MHz$ 测量GaAs P^+N 结二极管的电容反偏压关系。下面是从 $0-5V$ 每次间隔 $0.5V$ 测得的电容数据，以微微法为单位：19.9, 17.3, 15.6, 14.3, 13.3, 12.4, 11.6, 11.1, 10.5, 10.1, 9.8。计算 ψ_0 和 N_d 。二极管的面积为 $4 \times 10^{-4} cm^2$ 。

解： P^+N 结中有

$$\frac{1}{C_T^2} = \frac{2}{A^2 \epsilon q N_d} (\psi_0 + V_R)$$

令

$$\frac{1}{C_T^2} = KV_R + B, \quad \text{其中 } K = \frac{2}{A^2 \epsilon q N_d}, B = \frac{2}{A^2 \epsilon q N_d} \psi_0$$

代入题目数据并进行线性拟合可得

$$K = 0.0016 V^{-1} pF^{-2} \quad B = 0.0025 pF^{-2}$$

可得

$$\begin{aligned} N_d &= \frac{2}{A^2 \epsilon q K} \\ &= \frac{2}{(4 \times 10^{-4})^2 \times 13.2 \times 8.85 \times 10^{-14} \times 1.6 \times 10^{-19} \times 0.0016 \times 10^{24}} \\ &\approx 4.18 \times 10^{16} cm^{-3} \\ \psi_0 &= \frac{A^2 \epsilon q N_d B}{2} \\ &= \frac{(4 \times 10^{-4})^2 \times 13.2 \times 8.85 \times 10^{-14} \times 1.6 \times 10^{-19} \times 4.18 \times 10^{16} \times 0.0025 \times 10^{24}}{2} \\ &\approx 1.56V \end{aligned}$$

2-18

在 $I_f = 0.5mA$, $I_r = 1.0mA$ 条件下测量 P^+N 长二极管恢复特性。得到的结果是 $t_s = 350ns$ 。用严格解和近似公式两种方法计算 τ_p 。

解：

近似解：由 $t_s = \tau_p \ln(1 + \frac{I_f}{I_r})$ 可得

$$\tau_p = \frac{t_s}{\ln(1 + \frac{I_f}{I_r})} = \frac{350ns}{\ln(1 + \frac{0.5}{1.0})} \approx 863.2ns$$

严格解：有 $erf\sqrt{\frac{t_s}{\tau_p}} = \frac{I_f}{I_f + I_r} = \frac{0.5}{0.5 + 1} = \frac{1}{3}$ ，查表可得 $\sqrt{\frac{t_s}{\tau_p}} \approx 0.3046$ ，可得

$$\tau_p = \frac{350ns}{0.3046^2} \approx 3.77\mu s$$

2-19

用二极管恢复法测量 P^+N 二极管空穴寿命。

(1) 对于 $I_f = 1mA$ ， $I_r = 2mA$ ，在具有 $0.1ns$ 上升时间的示波器上测得 $t_s = 3ns$ ，求 τ_p 。

(2) 若(1)中快速示波器无法得到，只得采用一只具有 $10ns$ 上升时间较慢的示波器，问怎样才能使测量精确？叙述你的结果。

解：

(1) 由 $t_s = \tau_p \ln(1 + \frac{I_f}{I_r})$ 可得

$$\tau_p = \frac{t_s}{\ln(1 + \frac{I_f}{I_r})} = \frac{3ns}{\ln(1 + \frac{1}{2})} \approx 7.40ns$$

(2) $10ns$ 上升时间的示波器只可测 $t_s \gg 10ns$ 的 t_s ，有 $t_s = \tau_p \ln(1 + \frac{I_f}{I_r})$ ，只需增大 I_f 或减小 I_r 即可

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在硅中当最大电场接近 $10^6 V/cm$ 时发生击穿。假设在 P 侧 $N_a = 10^{20} cm^{-3}$ ，为要得到 $2V$ 的击穿电压，采用单边突变近似，求 N 侧的施主浓度。

解：当雪崩击穿发生时

$$1 = \frac{AW|\epsilon_m|}{B} \exp \frac{-B}{|\epsilon_m|} \left[1 - \exp \frac{-B}{|\epsilon_m|} \right]$$

可解得

$$\begin{aligned} W &= \frac{B}{A|\epsilon_m|} \exp \frac{B}{|\epsilon_m|} / \left[1 - \exp \frac{-B}{|\epsilon_m|} \right] \\ &= \frac{1.8 \times 10^6 V/cm}{9 \times 10^5 cm^{-1} \times 10^6 V/cm} \times \exp \frac{1.8 \times 10^6}{10^6} / \left[1 - \exp \frac{-1.8 \times 10^6}{10^6} \right] \\ &\approx 1.45 \times 10^{-5} cm \end{aligned}$$

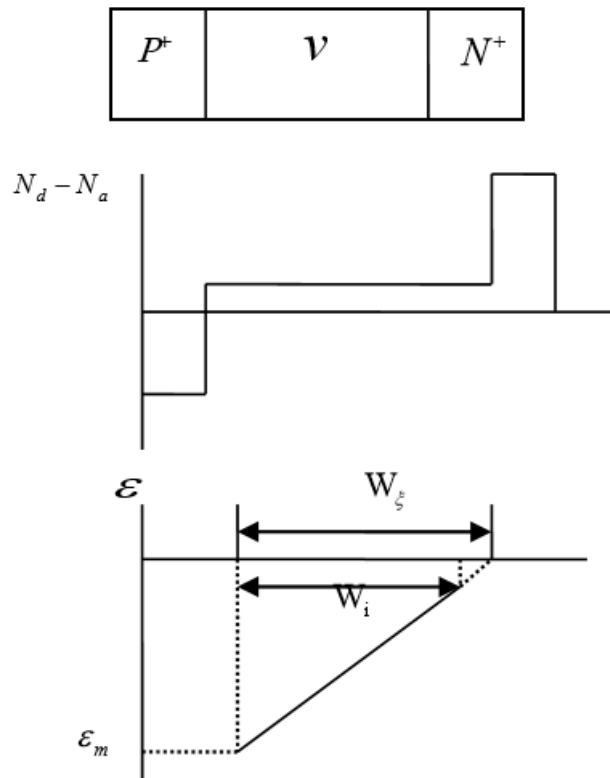
有 $|\epsilon_m| = \frac{qN_d W}{\epsilon}$ 可得

$$N_d = \frac{\epsilon |\epsilon_m|}{qW} = \frac{11.8 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times 1.45 \times 10^{-5}} \text{cm}^{-3} \approx 4.50 \times 10^{17} \text{cm}^{-3}$$

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对于下图中的 $P^+ - v - N^+$ 二极管，假设 P^+ 和 N^+ 区不承受任何外加电压，证明雪崩击穿的条件可表示为：

$$\frac{Ak\epsilon_0\epsilon_m^2}{qN_v B} \exp\left(-\frac{B}{|\epsilon_m|}\right) \left[1 - \exp\left(-\frac{qBN_v W_i}{k\epsilon_0\epsilon_m^2}\right)\right] = 1$$



解： P^+vN^+ 二极管的雪崩击穿临界电场 $|\epsilon_m|$ 与 P^+N 结相当。

设标准 P^+N 结的SCR宽度为 W_ξ ，则有：

$$\epsilon(x) = \epsilon_m \left(1 - \frac{x}{W_\xi}\right), \quad \text{其中 } \epsilon_m = \frac{qN_v W_\xi}{\epsilon}$$

电离系数：

$$\alpha(x) = A \exp \left[-\frac{B}{|\epsilon_m| \left(1 - \frac{x}{W_\xi}\right)} \right]$$

当 $x \rightarrow 0$ 时，取一阶泰勒展开

$$\alpha(x) \approx A \exp \left[-\frac{B}{|\epsilon_m|} \left(1 + \frac{x}{W_\xi}\right) \right]$$

对其从0到 W_i 积分

$$\begin{aligned}
 \int_0^{W_i} \alpha(x) dx &= A \exp\left(-\frac{B}{|\epsilon_m|}\right) \int_0^{W_i} \exp\left(-\frac{Bx}{|\epsilon_m|W_\xi}\right) dx \\
 &= -\frac{A|\epsilon_m|W_\xi}{B} \exp\left(-\frac{B}{|\epsilon_m|}\right) \exp\left(-\frac{Bx}{|\epsilon_m|W_\xi}\right) \Big|_0^{W_i} \\
 &= -\frac{A|\epsilon_m|W_\xi}{B} \exp\left(-\frac{B}{|\epsilon_m|}\right) \left[\exp\left(-\frac{BW_i}{|\epsilon_m|W_\xi}\right) - 1 \right]
 \end{aligned}$$

代入 $W_\xi = \frac{|\epsilon_m|\epsilon}{qN_v}$, 并代入雪崩击穿条件 $\int_0^{W_i} \alpha(x) dx = 1$, 可得

$$\frac{A\epsilon_m^2\epsilon}{qN_vB} \exp\left(-\frac{B}{|\epsilon_m|}\right) \left[1 - \exp\left(-\frac{qBN_vW_i}{\epsilon_m^2\epsilon}\right) \right] = 1$$