

1.初步设想

此时假设的是 4 节点上有功/无功需求增加, 1,4 节点上的发电机不变 (处于最优发电成本状态), 3 节点的发电机可以变化 (增加成本输出更多的电), 观察 5 节点上的负荷怎么进行削减进行最优切负荷。

符号	含义	单位/维度
$\xi = (\xi_1, \xi_2)$	节点 4 上的有功/无功需求随机增量 (扰动), 一般服从某种分布	p.u.
$\tilde{P}_4^L(\xi)$	节点 4 变化 (增加后) 的实际有功需求	p.u.
$\tilde{Q}_4^L(\xi)$	节点 4 变化 (增加后) 的实际无功需求	p.u.
$V_i^{\max/\min}$	节点电压幅值上/下界	p.u.
$\delta_i^{\max/\min}$	节点相角上/下界	rad

$$\max_{P_3^G(\xi), \tilde{P}_4^L(\xi), \tilde{Q}_4^L(\xi), P_5^L(\xi), Q_5^L(\xi)} \mathbb{E}[P_5^L(\xi) + Q_5^L(\xi)]$$

s.t.

$$P_i^G(\xi) - P_i^L(\xi) = P_i(V(\xi), \delta(\xi)), \quad \forall i \in \{1, 2, \dots, 5\}$$

$$P_i(V(\xi), \delta(\xi)) = V_i(\xi) \sum_{m=1}^5 V_m(\xi) [G_{im} \cos(\delta_i(\xi) - \delta_m(\xi)) + B_{im} \sin(\delta_i(\xi) - \delta_m(\xi))], \\ \forall i \in \{1, 2, \dots, 5\}$$

$$Q_i^G(\xi) - Q_i^L(\xi) = Q_i(V(\xi), \delta(\xi)), \quad \forall i \in \{1, 2, \dots, 5\}$$

$$Q_i(V(\xi), \delta(\xi)) = V_i(\xi) \sum_{m=1}^5 V_m(\xi) [G_{im} \sin(\delta_i(\xi) - \delta_m(\xi)) - B_{im} \cos(\delta_i(\xi) - \delta_m(\xi))], \\ \forall i \in \{1, 2, \dots, 5\}$$

$$\tilde{P}_4^L(\xi) = P_4^L + \xi_1, \quad \tilde{Q}_4^L(\xi) = Q_4^L + \xi_2, \quad \xi = (\xi_1; \xi_2)$$

$$V_i^{\min} \leq V_i(\xi) \leq V_i^{\max}, \quad \forall i \in \{1, 2, \dots, 5\}$$

$$\delta_i^{\min} \leq \delta_i(\xi) \leq \delta_i^{\max}, \quad \forall i \in \{1, 2, \dots, 5\}$$

其中, 决策变量为

$$u = (P_3^G(\xi), \tilde{P}_4^L(\xi), \tilde{Q}_4^L(\xi), P_5^L(\xi), Q_5^L(\xi))$$

状态变量为

$$x = (V_i(\xi), \delta_i(\xi)), i \in \{2, \dots, 5\}$$

2. 带入数据看看

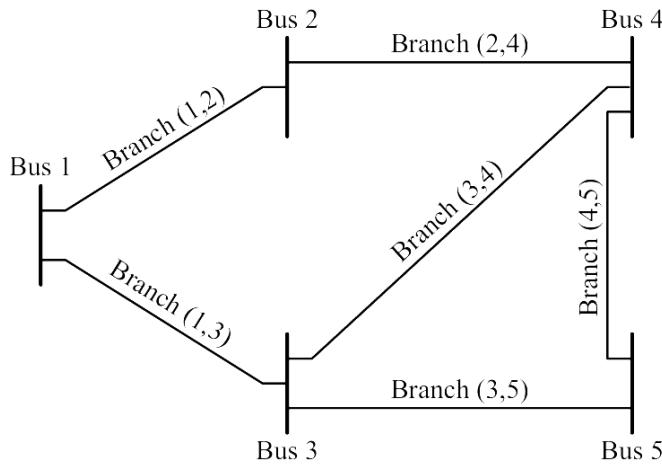


TABLE 5.1
Bus data for Example 5.2. All quantities are given in per-unit. Dots indicates zero values.

Bus i	Load Real Power P_i^L	Load Reactive Power Q_i^L	Min. Bus Voltage V_i^{\min}	Max. Bus Voltage V_i^{\max}
1	.	.	1.00	1.00
2	.	.	0.95	1.05
3	.	.	0.95	1.05
4	0.900	0.400	0.95	1.05
5	0.239	0.129	0.95	1.05

TABLE 5.2
Generator data for Example 5.2. All quantities are given in per-unit.

Bus i	Min. Generator Real Power $P_i^{G,\min}$	Max. Generator Real Power $P_i^{G,\max}$	Min. Generator Reactive Power $Q_i^{G,\min}$	Max. Generator Reactive Power $Q_i^{G,\max}$
1	$-\infty$	∞	$-\infty$	∞
3	0.10	0.40	-0.20	0.30
4	0.05	0.40	-0.20	0.20

导纳矩阵

$$Y = \begin{bmatrix} 1.07 - j10.04 & 0.00 + j3.33 & -1.07 + j6.73 & 0 & 0 \\ 0.00 + j3.33 & 5.66 - j33.22 & 0 & -5.66 + j30.19 & 0 \\ -1.07 + j6.73 & 0 & 141 - j13.78 & -0.49 + j3.83 & 0.00 + j3.19 \\ 0 & -5.66 + j30.19 & -0.49 + j3.80 & 5.95 + j36.01 & 0.00 + j2.00 \\ 0 & 0 & 0.00 + j3.19 & 0.00 + j2.00 & 0.00 - j5.13 \end{bmatrix}$$

电导矩阵

$$G = \begin{bmatrix} 1.07 & 0.00 & -1.07 & 0 & 0 \\ 0.00 & 5.66 & 0 & -5.66 & 0 \\ -1.07 & 0 & 141 & -0.49 & 0.00 \\ 0 & -5.66 & -0.49 & 5.95 & 0.00 \\ 0 & 0 & 0.00 & 0.00 & 0.00 \end{bmatrix}$$

电纳矩阵

$$B = \begin{bmatrix} -10.04 & 3.33 & 6.73 & 0 & 0 \\ 3.33 & -33.22 & 0 & 30.19 & 0 \\ 6.73 & 0 & -13.78 & 3.83 & 3.19 \\ 0 & 30.19 & 3.80 & 36.01 & 2.00 \\ 0 & 0 & 3.19 & 2.00 & -5.13 \end{bmatrix}$$

先根据最优发电成本模型算出 1,3,4 节点上发电机的状态，设 $P_1^G = a_1, P_3^G = a_2, P_4^G = a_3$ 为计算结果，此时有 $Q_1^G(\xi) = b_1, Q_2^G(\xi) = b_2, Q_4^G(\xi) = b_3$ ，固定 1,4 节点发电机状态，4 节点用电需求增加，使 3 节点发电机变化（一般是增加发电），则有

$$\max_{P_3^G(\xi), P_4^L(\xi), Q_4^L(\xi), P_5^L(\xi), Q_5^L(\xi)} \mathbb{E}[P_5^L(\xi) + Q_5^L(\xi)]$$

s.t.

$$\begin{aligned} a_1 - 0 &= 1.07 - 3.33V_2(\xi)\sin(\delta_2(\xi)) - 1.07V_3(\xi)\cos(\delta_3(\xi)) - 6.73V_3(\xi)\sin(\delta_3(\xi)) \\ 0 - 0 &= 5.66V_2(\xi)^2 + 3.33V_2(\xi)\sin(\delta_2(\xi)) - 5.66V_2(\xi)V_4(\xi)\cos(\delta_2(\xi) - \delta_4(\xi)) \\ &\quad + 30.19V_2(\xi)V_4(\xi)\sin(\delta_2(\xi) - \delta_4(\xi)) \\ P_3^G(\xi) - 0 &= 141V_3(\xi)^2 - 1.07V_3(\xi)\cos(\delta_3(\xi)) + 6.73V_3(\xi)\sin(\delta_3(\xi)) - 0.09V_3(\xi)V_4(\xi)\cos(\delta_3(\xi)) \\ &\quad - \delta_4(\xi) + 3.83V_3(\xi)V_4(\xi)\sin(\delta_3(\xi) - \delta_4(\xi)) + 3.19V_3(\xi)V_5(\xi)\sin(\delta_3(\xi) - \delta_5(\xi)) \\ a_3 - \tilde{P}_4^L(\xi) &= 5.95V_4(\xi)^2 - 5.66V_4(\xi)V_2(\xi)\cos(\delta_4(\xi) - \delta_2(\xi)) + 30.19V_4(\xi)V_2(\xi)\sin(\delta_4(\xi) - \delta_2(\xi)) \\ &\quad - 0.49V_4(\xi)V_3(\xi)\cos(\delta_4(\xi) - \delta_3(\xi)) + 3.80V_4(\xi)V_3(\xi)\sin(\delta_4(\xi) - \delta_3(\xi)) \\ &\quad + 2.00V_4(\xi)V_5(\xi)\sin(\delta_4(\xi) - \delta_5(\xi)) \\ 0 - P_5^L(\xi) &= 3.19V_5(\xi)V_3(\xi)\sin(\delta_5(\xi) - \delta_3(\xi)) + 2.00V_5(\xi)V_4(\xi)\sin(\delta_5(\xi) - \delta_4(\xi)) \\ b_1 - 0 &= 10.04 - 3.33V_2(\xi)\cos(\delta_2(\xi)) + 1.07V_3(\xi)\sin(\delta_3(\xi)) - 6.73V_3(\xi)\cos(\delta_3(\xi)) \\ 0 - 0 &= 33.22V_2(\xi)^2 - 3.33V_2(\xi)\cos(\delta_2(\xi)) - 5.66V_2(\xi)V_4(\xi)\sin(\delta_2(\xi) - \delta_4(\xi)) \\ &\quad - 30.19V_2(\xi)V_4(\xi)\cos(\delta_2(\xi) - \delta_4(\xi)) \\ Q_3^G(\xi) - 0 &= 13.78V_3(\xi)^2 - 1.07V_3(\xi)\sin(\delta_3(\xi)) - 6.73V_3(\xi)\cos(\delta_3(\xi)) - 0.09V_3(\xi)V_4(\xi)\sin(\delta_3(\xi)) \\ &\quad - \delta_4(\xi) - 3.83V_3(\xi)V_4(\xi)\cos(\delta_3(\xi) - \delta_4(\xi)) - 3.19V_3(\xi)V_5(\xi)\cos(\delta_3(\xi) - \delta_5(\xi)) \end{aligned}$$

$$\begin{aligned}
b_3 - \tilde{Q}_4^L(\xi) = & -36.01V_4(\xi)^2 - 5.66V_4(\xi)V_2(\xi)\sin(\delta_4(\xi) - \delta_2(\xi)) - 30.19V_4(\xi)V_2(\xi)\cos(\delta_4(\xi) \\
& - \delta_2(\xi)) - 0.49V_4(\xi)V_3(\xi)\sin(\delta_4(\xi) - \delta_3(\xi)) - 3.80V_4(\xi)V_3(\xi)\cos(\delta_4(\xi) - \delta_3(\xi)) \\
& - 2.00V_4(\xi)V_5(\xi)\cos(\delta_4(\xi) - \delta_5(\xi))
\end{aligned}$$

$$\begin{aligned}
0 - Q_5^L(\xi) = & 5.13V_5(\xi)^2 - 3.19V_5(\xi)V_3(\xi)\cos(\delta_5(\xi) - \delta_3(\xi)) - 2.00V_5(\xi)V_4(\xi)\cos(\delta_5(\xi) - \delta_4(\xi)) \\
\tilde{P}_4^L(\xi) = & 0.900 + \xi_1, \quad \tilde{Q}_4^L(\xi) = 0.400 + \xi_2, \quad \xi = (\xi_1; \xi_2)
\end{aligned}$$

$$0.10 \leq P_3^G \leq 0.40$$

$$-0.20 \leq Q_3^G \leq 0.30$$

$$0.95 \leq V_i \leq 1.05, \quad i \in \{2,3,4,5\}$$

$$-180.0^\circ \leq \delta_i \leq 180.0^\circ, \quad i \in \{2,3,4,5\}$$

3.思考与优化

怎样往模型中加入机会约束?

能不能把 max 的形式改成 min 的形式?

符号	含义	单位/维度
$\xi = (\xi_1, \xi_2)$	节点 4 上的有功/无功需求随机增量 (扰动), 一般服从某种分布	p.u.
$\tilde{P}_4^L(\xi)$	节点 4 变化 (增加后) 的实际有功需求	p.u.
$\tilde{Q}_4^L(\xi)$	节点 4 变化 (增加后) 的实际无功需求	p.u.
$V_i^{\max/\min}$	节点电压幅值上/下界	p.u.
$\delta_i^{\max/\min}$	节点相角上/下界	rad
$\Delta P_5^L(\xi)$	节点 5 上的有功需求削减量	p.u.
$\Delta Q_5^L(\xi)$	节点 5 上的无功需求削减量	p.u.
$\tilde{P}_5^L(\xi)$	节点 5 上变化 (削减后) 的实际有功需求	p.u.
$\tilde{Q}_5^L(\xi)$	节点 5 上变化 (削减后) 的实际无功需求	p.u.
$P_i^{G,\max/\min}(\xi)$	发电有功上下界 (随机或确定)	p.u
$Q_i^{G,\max/\min}(\xi)$	发电无功上下界 (随机或确定)	p.u
$\epsilon_p^{G,\max/\min}$	发电有功上下界机会约束允许违约概率上限	—
$\epsilon_q^{G,\max/\min}$	发电无功上下界机会约束允许违约概率上限	—
$\epsilon_v^{\max/\min}$	电压幅值上下界机会约束允许违约概率上限	—
$\epsilon_\delta^{\max/\min}$	相角上下界机会约束允许违约概率上限	—

$$\min_{P_3^G(\xi), \tilde{P}_4^L(\xi), \tilde{Q}_4^L(\xi), \tilde{P}_5^L(\xi), \tilde{Q}_5^L(\xi)} \mathbb{E}[\Delta P_5^L(\xi) + \Delta Q_5^L(\xi)]$$

s.t.

$$P_i^G(\xi) - P_i^L(\xi) = P_i(V(\xi), \delta(\xi)), \quad \forall i \in \{1, 2, \dots, 5\}$$

$$P_i(V(\xi), \delta(\xi)) = V_i(\xi) \sum_{m=1}^5 V_m(\xi) [G_{im} \cos(\delta_i(\xi) - \delta_m(\xi)) + B_{im} \sin(\delta_i(\xi) - \delta_m(\xi))], \\ \forall i \in \{1, 2, \dots, 5\}$$

$$Q_i^G(\xi) - Q_i^L(\xi) = Q_i(V(\xi), \delta(\xi)), \quad \forall i \in \{1, 2, \dots, 5\}$$

$$Q_i(V(\xi), \delta(\xi)) = V_i(\xi) \sum_{m=1}^5 V_m(\xi) [G_{im} \sin(\delta_i(\xi) - \delta_m(\xi)) - B_{im} \cos(\delta_i(\xi) - \delta_m(\xi))], \\ \forall i \in \{1, 2, \dots, 5\}$$

$$\tilde{P}_4^L(\xi) = P_4^L + \xi_1, \quad \tilde{Q}_4^L(\xi) = Q_4^L + \xi_2, \quad \xi = (\xi_1; \xi_2)$$

$$\tilde{P}_5^L(\xi) = P_5^L - \Delta P_5^L(\xi), \quad \tilde{Q}_5^L(\xi) = Q_5^L - \Delta Q_5^L(\xi)$$

$$\mathbb{P}(P_i^G(\xi) - P_i^{G,\min}(\xi) \geq 0) \geq 1 - \epsilon_p^{G,\min}, \quad \forall i \in \{1,3,4\}$$

$$\mathbb{P}(P_i^{G,\max}(\xi) - P_i^G(\xi) \geq 0) \geq 1 - \epsilon_p^{G,\max}, \quad \forall i \in \{1,3,4\}$$

$$\mathbb{P}(Q_i^G(\xi) - Q_i^{G,\min}(\xi) \geq 0) \geq 1 - \epsilon_q^{G,\min}, \quad \forall i \in \{1,3,4\}$$

$$\mathbb{P}(Q_i^{G,\max}(\xi) - Q_i^G(\xi) \geq 0) \geq 1 - \epsilon_q^{G,\max}, \quad \forall i \in \{1,3,4\}$$

$$\mathbb{P}(V_i(\xi) - V_i^{\min} \geq 0) \geq 1 - \epsilon_V^{\min}, \quad \forall i \in \{1,2,\dots,5\}$$

$$\mathbb{P}(V_i^{\max} - V_i(\xi) \geq 0) \geq 1 - \epsilon_V^{\max}, \quad \forall i \in \{1,2,\dots,5\}$$

$$\mathbb{P}(\delta_i(\xi) - \delta_i^{\min} \geq 0) \geq 1 - \epsilon_{\delta}^{\min}, \quad \forall i \in \{1,2,\dots,5\}$$

$$\mathbb{P}(\delta_i^{\max} - \delta_i(\xi) \geq 0) \geq 1 - \epsilon_{\delta}^{\max}, \quad \forall i \in \{1,2,\dots,5\}$$

其中，决策变量为

$$u = (P_3^G(\xi), \tilde{P}_4^L(\xi), \tilde{Q}_4^L(\xi), \tilde{P}_5^L(\xi), \tilde{Q}_5^L(\xi))$$

状态变量为

$$x = (V_i(\xi), \delta_i(\xi)), i \in \{2, \dots, 5\}$$

4. 带入数据

先根据最优发电成本模型算出 1,3,4 节点上发电机的状态，设 $P_1^G = a_1, P_3^G = a_2, P_4^G = a_3$ 为计算结果，此时有 $Q_1^G(\xi) = b_1, Q_2^G(\xi) = b_2, Q_4^G(\xi) = b_3$ ，固定 1,4 节点发电机状态，4 节点用电需求增加，使 3 节点发电机变化（一般是增加发电），则有

$$\min_{P_3^G(\xi), \tilde{P}_4^L(\xi), \tilde{Q}_4^L(\xi), \tilde{P}_5^L(\xi), \tilde{Q}_5^L(\xi)} \mathbb{E}[\Delta P_5^L(\xi) + \Delta Q_5^L(\xi)]$$

s.t.

$$a_1 - 0 = 1.07 - 3.33V_2(\xi)\sin(\delta_2(\xi)) - 1.07V_3(\xi)\cos(\delta_3(\xi)) - 6.73V_3(\xi)\sin(\delta_3(\xi))$$

$$0 - 0 = 5.66V_2(\xi)^2 + 3.33V_2(\xi)\sin(\delta_2(\xi)) - 5.66V_2(\xi)V_4(\xi)\cos(\delta_2(\xi) - \delta_4(\xi)) \\ + 30.19V_2(\xi)V_4(\xi)\sin(\delta_2(\xi) - \delta_4(\xi))$$

$$P_3^G(\xi) - 0 = 141V_3(\xi)^2 - 1.07V_3(\xi)\cos(\delta_3(\xi)) + 6.73V_3(\xi)\sin(\delta_3(\xi)) - 0.09V_3(\xi)V_4(\xi)\cos(\delta_3(\xi)) \\ - \delta_4(\xi)) + 3.83V_3(\xi)V_4(\xi)\sin(\delta_3(\xi) - \delta_4(\xi)) + 3.19V_3(\xi)V_5(\xi)\sin(\delta_3(\xi) - \delta_5(\xi))$$

$$a_3 - \tilde{P}_4^L(\xi) = 5.95V_4(\xi)^2 - 5.66V_4(\xi)V_2(\xi)\cos(\delta_4(\xi) - \delta_2(\xi)) + 30.19V_4(\xi)V_2(\xi)\sin(\delta_4(\xi) - \delta_2(\xi)) \\ - 0.49V_4(\xi)V_3(\xi)\cos(\delta_4(\xi) - \delta_3(\xi)) + 3.80V_4(\xi)V_3(\xi)\sin(\delta_4(\xi) - \delta_3(\xi)) \\ + 2.00V_4(\xi)V_5(\xi)\sin(\delta_4(\xi) - \delta_5(\xi))$$

$$0 - \tilde{P}_5^L(\xi) = 3.19V_5(\xi)V_3(\xi)\sin(\delta_5(\xi) - \delta_3(\xi)) + 2.00V_5(\xi)V_4(\xi)\sin(\delta_5(\xi) - \delta_4(\xi))$$

$$b_1 - 0 = 10.04 - 3.33V_2(\xi)\cos(\delta_2(\xi)) + 1.07V_3(\xi)\sin(\delta_3(\xi)) - 6.73V_3(\xi)\cos(\delta_3(\xi))$$

$$0 - 0 = 33.22V_2(\xi)^2 - 3.33V_2(\xi)\cos(\delta_2(\xi)) - 5.66V_2(\xi)V_4(\xi)\sin(\delta_2(\xi) - \delta_4(\xi)) \\ - 30.19V_2(\xi)V_4(\xi)\cos(\delta_2(\xi) - \delta_4(\xi))$$

$$Q_3^G(\xi) - 0 = 13.78V_3(\xi)^2 - 1.07V_3(\xi)\sin(\delta_3(\xi)) - 6.73V_3(\xi)\cos(\delta_3(\xi)) - 0.09V_3(\xi)V_4(\xi)\sin(\delta_3(\xi)) \\ - \delta_4(\xi)) - 3.83V_3(\xi)V_4(\xi)\cos(\delta_3(\xi) - \delta_4(\xi)) - 3.19V_3(\xi)V_5(\xi)\cos(\delta_3(\xi) - \delta_5(\xi))$$

$$b_3 - \tilde{Q}_4^L(\xi) = -36.01V_4(\xi)^2 - 5.66V_4(\xi)V_2(\xi)\sin(\delta_4(\xi) - \delta_2(\xi)) - 30.19V_4(\xi)V_2(\xi)\cos(\delta_4(\xi) \\ - \delta_2(\xi)) - 0.49V_4(\xi)V_3(\xi)\sin(\delta_4(\xi) - \delta_3(\xi)) - 3.80V_4(\xi)V_3(\xi)\cos(\delta_4(\xi) - \delta_3(\xi)) \\ - 2.00V_4(\xi)V_5(\xi)\cos(\delta_4(\xi) - \delta_5(\xi))$$

$$0 - \tilde{Q}_5^L(\xi) = 5.13V_5(\xi)^2 - 3.19V_5(\xi)V_3(\xi)\cos(\delta_5(\xi) - \delta_3(\xi)) - 2.00V_5(\xi)V_4(\xi)\cos(\delta_5(\xi) - \delta_4(\xi))$$

$$\tilde{P}_4^L(\xi) = 0.900 + \xi_1, \quad \tilde{Q}_4^L(\xi) = 0.400 + \xi_2, \quad \xi = (\xi_1; \xi_2)$$

$$\tilde{P}_5^L(\xi) = 0.239 - \Delta P_5^L(\xi), \quad \tilde{Q}_5^L(\xi) = 0.129 - \Delta Q_5^L(\xi)$$

$$\mathbb{P}(P_3^G(\xi) - 0.10 \geq 0) \geq 1 - \epsilon_P^{G,min}, \quad \mathbb{P}(0.40 - P_3^G(\xi) \geq 0) \geq 1 - \epsilon_P^{G,max}$$

$$\mathbb{P}(Q_3^G(\xi) - (-0.20) \geq 0) \geq 1 - \epsilon_Q^{G,min}, \quad \mathbb{P}(0.30 - Q_3^G(\xi) \geq 0) \geq 1 - \epsilon_Q^{G,max}$$

$$\mathbb{P}(V_i(\xi) - 0.95 \geq 0) \geq 1 - \epsilon_V^{min}, \quad \mathbb{P}(1.05 - V_i(\xi) \geq 0) \geq 1 - \epsilon_V^{max}, i \in \{2,3,4,5\}$$

$$\mathbb{P}(\delta_i(\xi) - (-\pi) \geq 0) \geq 1 - \epsilon_\delta^{min}, \quad \mathbb{P}(\pi - \delta_i(\xi) \geq 0) \geq 1 - \epsilon_\delta^{max}, i \in \{2,3,4,5\}$$