1.初步设想

此时假设的是 4 节点上有功/无功需求增加，1,4 节点上的发电机不变（处于最优发电成本状态），3 节点的发电机可以变化（增加成本输出更多的电），观察 5 节点上的负荷怎么进行削减进行最优切负荷。

符号	含义	单位/维度
$\xi=(\xi_1,\xi_2)$	节点 4 上的有功/无功需求随机增量（扰动），一般服从某种分布	p.u.
$\tilde{P}_4^L$	节点 4 变化（增加后）的实际有功需求	p.u.
$\tilde{Q}_4^L$	节点 4 变化（增加后）的实际无功需求	p.u.
$V_i^{\max/\min}$	节点电压幅值上/下界	p.u.
$\delta_i^{\max/\min}$	节点相角上/下界	rad

$$\max_{P_4^G(\xi),P_5^L(\xi),Q_5^L(\xi)} \mathbb{E}[P_5^L(\xi)+Q_5^L(\xi)]$$

s.t.

$$P_i^G(\xi)-P_i^L(\xi)=P_i\big(V(\xi),\delta(\xi)\big),\qquad \forall i\in\{1,2,\cdots,5\}$$

$$P_i\big(V(\xi),\delta(\xi)\big)=V_i(\xi)\sum_{m=1}^{5}V_m(\xi)\big[G_{im}\cos\big(\delta_i(\xi)-\delta_m(\xi)\big)+B_{im}\sin\big(\delta_i(\xi)-\delta_m(\xi)\big)\big],$$
$$\forall i\in\{1,2,\cdots,5\}$$

$$Q_i^G(\xi)-Q_i^L(\xi)=Q_i\big(V(\xi),\delta(\xi)\big),\qquad \forall i\in\{1,2,\cdots,5\}$$

$$\mathrm{Q}_i\big(V(\xi),\delta(\xi)\big)=V_i(\xi)\sum_{m=1}^{5}V_m(\xi)\big[G_{im}\sin\big(\delta_i(\xi)-\delta_m(\xi)\big)-B_{im}\cos\big(\delta_i(\xi)-\delta_m(\xi)\big)\big],$$
$$\forall i\in\{1,2,\cdots,5\}$$

$$P_4^L(\xi)=\tilde{P}_4^L-\xi_1,\qquad Q_4^L(\xi)=\tilde{Q}_4^L-\xi_2,\qquad \xi=(\xi_1;\xi_2)$$

$$V_i^{\min}\le V_i(\xi)\le V_i^{\max},\qquad \forall i\in\{1,2,\cdots,5\}$$

$$\delta_i^{\min}\le \delta_i(\xi)\le \delta_i^{\max},\qquad \forall i\in\{1,2,\cdots,5\}$$

其中，决策变量为

$$u=(P_4^G(\xi),P_5^L(\xi),Q_5^L(\xi))$$

状态变量为

$$x=\big(V_i(\xi),\delta_i(\xi)\big),i\in\{2,\cdots,5\}$$

2.带入数据看看

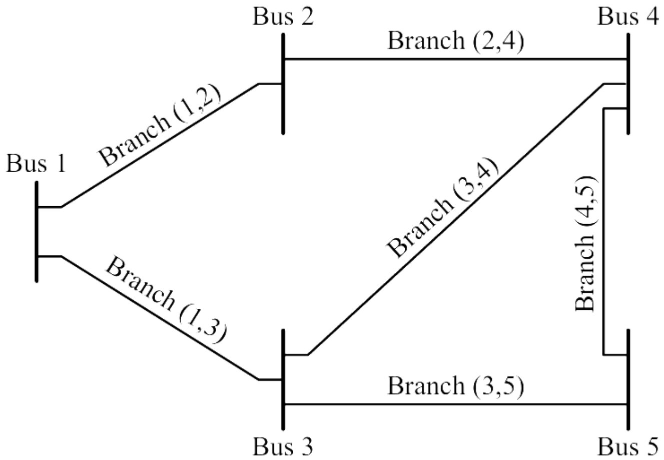

TABLE 5.1
Bus data for Example 5.2. All quantities are given in per-unit. Dots indicates zero values.

Bus i	Load Real Power P_i^{L}	Load Reactive Power Q_i^{L}	Min. Bus Voltage $V_i^{\min}$	Max. Bus Voltage $V_i^{\max}$
1	·	·	1.00	1.00
2	·	·	0.95	1.05
3	·	·	0.95	1.05
4	0.900	0.400	0.95	1.05
5	0.239	0.129	0.95	1.05

TABLE 5.2
Generator data for Example 5.2. All quantities are given in per-unit.

Bus i	Min. Generator Real Power $P_i^{\mathrm{G,min}}$	Max. Generator Real Power $P_i^{\mathrm{G,max}}$	Min. Generator Reactive Power $Q_i^{\mathrm{G,min}}$	Max. Generator Reactive Power $Q_i^{\mathrm{G,max}}$
1	$-\infty$	∞	$-\infty$	∞
3	0.10	0.40	-0.20	0.30
4	0.05	0.40	-0.20	0.20

导纳矩阵

$$Y=\begin{bmatrix} 1.07-j10.04 & 0.00+j3.33 & -1.07+j6.73 & 0 & 0 \\ 0.00+j3.33 & 5.66-j33.22 & 0 & -5.66+j30.19 & 0 \\ -1.07+j6.73 & 0 & 141-j13.78 & -0.49+j3.83 & 0.00+j3.19 \\ 0 & -5.66+j30.19 & -0.49+j3.80 & 5.95+j36.01 & 0.00+j2.00 \\ 0 & 0 & 0.00+j3.19 & 0.00+j2.00 & 0.00-j5.13 \end{bmatrix}$$

电导矩阵

$$
G=\begin{bmatrix} 1.07 & 0.00 & -1.07 & 0 & 0 \\ 0.00 & 5.66 & 0 & -5.66 & 0 \\ -1.07 & 0 & 141 & -0.49 & 0.00 \\ 0 & -5.66 & -0.49 & 5.95 & 0.00 \\ 0 & 0 & 0.00 & 0.00 & 0.00 \end{bmatrix}
$$

电纳矩阵

$$
B=\begin{bmatrix} -10.04 & 3.33 & 6.73 & 0 & 0 \\ 3.33 & -33.22 & 0 & 30.19 & 0 \\ 6.73 & 0 & -13.78 & 3.83 & 3.19 \\ 0 & 30.19 & 3.80 & 36.01 & 2.00 \\ 0 & 0 & 3.19 & 2.00 & -5.13 \end{bmatrix}
$$

先根据最优发电成本模型算出 1,3,4 节点上发电机的状态，设$P_1^G=a_1, P_3^G=a_2, P_4^G=a_3$为计算结果，此时有$Q_1^G(\xi)=b_1, Q_2^G(\xi)=b_2, Q_4^G(\xi)=b_3$，固定 1,4 节点发电机状态，使 3 节点发电机变化（一般是增加发电），则有

$$
\max_{P_4^G(\xi),P_5^L(\xi),Q_5^L(\xi)} \mathbb{E}[P_5^L(\xi)+Q_5^L(\xi)]
$$

s.t.

$$
a_1-0=1.07-3.33V_2(\xi)\sin(\delta_2(\xi))-1.07V_3(\xi)\cos(\delta_3(\xi))-6.73V_3(\xi)\sin(\delta_3(\xi))
$$

$$
\begin{aligned}
0-0={}&5.66V_2(\xi)^2+3.33V_2(\xi)\sin(\delta_2(\xi))-5.66V_2(\xi)V_4(\xi)\cos(\delta_2(\xi)-\delta_4(\xi))\\
&+30.19V_2(\xi)V_4(\xi)\sin(\delta_2(\xi)-\delta_4(\xi))
\end{aligned}
$$

$$
\begin{aligned}
P_3^G(\xi)-0={}&141V_3(\xi)^2-1.07V_3(\xi)\cos(\delta_3(\xi))+6.73V_3(\xi)\sin(\delta_3(\xi))-0.09V_3(\xi)V_4(\xi)\cos(\delta_3(\xi)\\
&-\delta_4(\xi))+3.83V_3(\xi)V_4(\xi)\sin(\delta_3(\xi)-\delta_4(\xi))+3.19V_3(\xi)V_5(\xi)\sin(\delta_3(\xi)-\delta_5(\xi))
\end{aligned}
$$

$$
\begin{aligned}
a_3-0.900={}&5.95V_4(\xi)^2-5.66V_4(\xi)V_2(\xi)\cos(\delta_4(\xi)-\delta_2(\xi))+30.19V_4(\xi)V_2(\xi)\sin(\delta_4(\xi)-\delta_2(\xi))\\
&-0.49V_4(\xi)V_3(\xi)\cos(\delta_4(\xi)-\delta_3(\xi))+3.80V_4(\xi)V_3(\xi)\sin(\delta_4(\xi)-\delta_3(\xi))\\
&+2.00V_4(\xi)V_5(\xi)\sin(\delta_4(\xi)-\delta_5(\xi))
\end{aligned}
$$

$$
0-P_5^L(\xi)=3.19V_5(\xi)V_3(\xi)\sin(\delta_5(\xi)-\delta_3(\xi))+2.00V_5(\xi)V_4(\xi)\sin(\delta_5(\xi)-\delta_4(\xi))
$$

$$
b_1-0=10.04-3.33V_2(\xi)\cos(\delta_2(\xi))+1.07V_3(\xi)\sin(\delta_3(\xi))-6.73V_3(\xi)\cos(\delta_3(\xi))
$$

$$
\begin{aligned}
0-0={}&33.22V_2(\xi)^2-3.33V_2(\xi)\cos(\delta_2(\xi))-5.66V_2(\xi)V_4(\xi)\sin(\delta_2(\xi)-\delta_4(\xi))\\
&-30.19V_2(\xi)V_4(\xi)\cos(\delta_2(\xi)-\delta_4(\xi))
\end{aligned}
$$

$$
\begin{aligned}
Q_3^G(\xi)-0={}&13.78V_3(\xi)^2-1.07V_3(\xi)\sin(\delta_3(\xi))-6.73V_3(\xi)\cos(\delta_3(\xi))-0.09V_3(\xi)V_4(\xi)\sin(\delta_3(\xi)\\
&-\delta_4(\xi))-3.83V_3(\xi)V_4(\xi)\cos(\delta_3(\xi)-\delta_4(\xi))-3.19V_3(\xi)V_5(\xi)\cos(\delta_3(\xi)-\delta_5(\xi))
\end{aligned}
$$

$$b_3 - 0.400 = -36.01V_4(\xi)^2 - 5.66V_4(\xi)V_2(\xi)\sin(\delta_4(\xi) - \delta_2(\xi)) - 30.19V_4(\xi)V_2(\xi)\cos(\delta_4(\xi) - \delta_2(\xi)) - 0.49V_4(\xi)V_3(\xi)\sin(\delta_4(\xi) - \delta_3(\xi)) - 3.80V_4(\xi)V_3(\xi)\cos(\delta_4(\xi) - \delta_3(\xi)) - 2.00V_4(\xi)V_5(\xi)\cos(\delta_4(\xi) - \delta_5(\xi))$$

$$0 - Q_5^L(\xi) = 5.13V_5(\xi)^2 - 3.19V_5(\xi)V_3(\xi)\cos(\delta_5(\xi) - \delta_3(\xi)) - 2.00V_5(\xi)V_4(\xi)\cos(\delta_5(\xi) - \delta_4(\xi))$$

$$0.10 \le P_3^G \le 0.40$$

$$0.05 \le P_4^G \le 0.40$$

$$-0.20 \le Q_3^G \le 0.30$$

$$-0.20 \le Q_4^G \le 0.20$$

$$0.95 \le V_i \le 1.05, \quad i \in \{2,3,4,5\}$$

$$-180.0^\circ \le \delta_i \le 180.0^\circ, \quad i \in \{2,3,4,5\}$$

符号	含义	单位/维度
$\xi = (\xi_1, \xi_2)$	节点 4 上的有功/无功需求随机增量（扰动），一般服从某种分布	p.u.
$\tilde{P}_4^L$	节点 4 变化（增加后）的实际有功需求	p.u.
$\tilde{Q}_4^L$	节点 4 变化（增加后）的实际无功需求	p.u.
$V_i^{\max/\min}$	节点电压幅值上/下界	p.u.
$\delta_i^{\max/\min}$	节点相角上/下界	rad
$\Delta P_5^L(\xi)$	节点 5 上的有功需求削减量	p.u.
$\Delta Q_5^L(\xi)$	节点 5 上的无功需求削减量	p.u.
$\tilde{P}_5^L$	节点 5 上变化（削减后）的实际有功需求	p.u.
$\tilde{Q}_5^L$	节点 5 上变化（削减后）的实际无功需求	p.u.
$P_i^{G,max/\min}(\xi)$	发电有功上下界（随机或确定）	p.u
$Q_i^{G,max/\min}(\xi)$	发电无功上下界（随机或确定）	p.u
$\epsilon_P^{G,max/min}$	发电有功上下界机会约束允许违约概率上限	—
$\epsilon_Q^{G,max/min}$	发电无功上下界机会约束允许违约概率上限	—
$\epsilon_V^{max/min}$	电压幅值上下界机会约束允许违约概率上限	—
$\epsilon_\delta^{max/min}$	相角上下界机会约束允许违约概率上限	—

3.思考与优化

怎样往模型中加入机会约束？

能不能把 max 的形式改成 min 的形式？

$$\min_{P_4^G(\xi),\Delta P_5^L(\xi),\Delta Q_5^L(\xi)} \mathbb{E}[\Delta P_5^L(\xi)+\Delta Q_5^L(\xi)]$$

s.t.

$$P_i^G(\xi)-P_i^L(\xi)=P_i\big(V(\xi),\delta(\xi)\big),\qquad \forall i\in\{1,2,\cdots,5\}$$

$$P_i\big(V(\xi),\delta(\xi)\big)=V_i(\xi)\sum_{m=1}^{5}V_m(\xi)\big[G_{im}\cos\big(\delta_i(\xi)-\delta_m(\xi)\big)+B_{im}\sin\big(\delta_i(\xi)-\delta_m(\xi)\big)\big],$$
$$\forall i\in\{1,2,\cdots,5\}$$

$$Q_i^G(\xi)-Q_i^L(\xi)=Q_i\big(V(\xi),\delta(\xi)\big),\qquad \forall i\in\{1,2,\cdots,5\}$$

$$\mathrm{Q}_i\big(V(\xi),\delta(\xi)\big)=V_i(\xi)\sum_{m=1}^{5}V_m(\xi)\big[G_{im}\sin\big(\delta_i(\xi)-\delta_m(\xi)\big)-B_{im}\cos\big(\delta_i(\xi)-\delta_m(\xi)\big)\big],$$
$$\forall i\in\{1,2,\cdots,5\}$$

$$P_4^L(\xi)=\tilde{P}_4^L-\xi_1,\qquad Q_4^L(\xi)=\tilde{Q}_4^L-\xi_2,\qquad \xi=(\xi_1;\xi_2)$$
$$P_5^L(\xi)=\tilde{P}_5^L+\Delta P_5^L(\xi),\qquad Q_5^L(\xi)=\tilde{Q}_5^L+\Delta Q_5^L(\xi)$$

$$\mathbb{P}\big(P_i^G(\xi)-P_i^{G,\min}(\xi)\ge 0\big)\ge 1-\epsilon_P^{G,min},\qquad \forall i\in\{1,3,4\}$$

$$\mathbb{P}(P_i^{G,\max}(\xi)-P_i^G(\xi)\ge 0)\ge 1-\epsilon_P^{G,max},\qquad \forall i\in\{1,3,4\}$$

$$\mathbb{P}(Q_i^G(\xi)-Q_i^{G,\min}(\xi)\ge 0)\ge 1-\epsilon_Q^{G,min},\qquad \forall i\in\{1,3,4\}$$

$$\mathbb{P}(Q_i^{G,\max}(\xi)-Q_i^G(\xi)\ge 0)\ge 1-\epsilon_Q^{G,max},\qquad \forall i\in\{1,3,4\}$$

$$\mathbb{P}(V_i(\xi)-V_i^{\min}\ge 0)\ge 1-\epsilon_V^{min},\qquad \forall i\in\{1,2,\cdots,5\}$$

$$\mathbb{P}(V_i^{\max}-V_i(\xi)\ge 0)\ge 1-\epsilon_V^{max},\qquad \forall i\in\{1,2,\cdots,5\}$$

$$\mathbb{P}(\delta_i(\xi)-\delta_i^{\min}\ge 0)\ge 1-\epsilon_\delta^{min},\qquad \forall i\in\{1,2,\cdots,5\}$$

$$\mathbb{P}(\delta_i^{\max}-\delta_i(\xi)\ge 0)\ge 1-\epsilon_\delta^{max},\qquad \forall i\in\{1,2,\cdots,5\}$$

其中，决策变量为

$$u=(P_4^G(\xi),\Delta P_5^L(\xi),\Delta Q_5^L(\xi))$$

状态变量为

$$x=\big(V_i(\xi),\delta_i(\xi)\big),i\in\{2,\cdots,5\}$$

4.带入数据

先根据最优发电成本模型算出 1,3,4 节点上发电机的状态，设$P_1^G=a_1,P_3^G=a_2,P_4^G=a_3$为计算结果，此时有$Q_1^G(\xi)=b_1,Q_2^G(\xi)=b_2,Q_4^G(\xi)=b_3$，固定 1,4 节点发电机状态，使 3 节点发电机变化（一般是增加发电），则有

$$\min_{P_4^G(\xi),\Delta P_5^L(\xi),\Delta Q_5^L(\xi)} \mathbb{E}[\Delta P_5^L(\xi)+\Delta Q_5^L(\xi)]$$

s.t.

$$a_1-0=1.07-3.33V_2(\xi)\sin(\delta_2(\xi))-1.07V_3(\xi)\cos(\delta_3(\xi))-6.73V_3(\xi)\sin(\delta_3(\xi))$$

$$\begin{aligned}0-0=&5.66V_2(\xi)^2+3.33V_2(\xi)\sin(\delta_2(\xi))-5.66V_2(\xi)V_4(\xi)\cos(\delta_2(\xi)-\delta_4(\xi))\\&+30.19V_2(\xi)V_4(\xi)\sin(\delta_2(\xi)-\delta_4(\xi))\end{aligned}$$

$$\begin{aligned}P_3^G(\xi)-0=&141V_3(\xi)^2-1.07V_3(\xi)\cos(\delta_3(\xi))+6.73V_3(\xi)\sin(\delta_3(\xi))-0.09V_3(\xi)V_4(\xi)\cos(\delta_3(\xi)\\&-\delta_4(\xi))+3.83V_3(\xi)V_4(\xi)\sin(\delta_3(\xi)-\delta_4(\xi))+3.19V_3(\xi)V_5(\xi)\sin(\delta_3(\xi)-\delta_5(\xi))\end{aligned}$$

$$\begin{aligned}a_3-0.900=&5.95V_4(\xi)^2-5.66V_4(\xi)V_2(\xi)\cos(\delta_4(\xi)-\delta_2(\xi))+30.19V_4(\xi)V_2(\xi)\sin(\delta_4(\xi)-\delta_2(\xi))\\&-0.49V_4(\xi)V_3(\xi)\cos(\delta_4(\xi)-\delta_3(\xi))+3.80V_4(\xi)V_3(\xi)\sin(\delta_4(\xi)-\delta_3(\xi))\\&+2.00V_4(\xi)V_5(\xi)\sin(\delta_4(\xi)-\delta_5(\xi))\end{aligned}$$

$$0-P_5^L(\xi)=3.19V_5(\xi)V_3(\xi)\sin(\delta_5(\xi)-\delta_3(\xi))+2.00V_5(\xi)V_4(\xi)\sin(\delta_5(\xi)-\delta_4(\xi))$$

$$b_1-0=10.04-3.33V_2(\xi)\cos(\delta_2(\xi))+1.07V_3(\xi)\sin(\delta_3(\xi))-6.73V_3(\xi)\cos(\delta_3(\xi))$$

$$\begin{aligned}0-0=&33.22V_2(\xi)^2-3.33V_2(\xi)\cos(\delta_2(\xi))-5.66V_2(\xi)V_4(\xi)\sin(\delta_2(\xi)-\delta_4(\xi))\\&-30.19V_2(\xi)V_4(\xi)\cos(\delta_2(\xi)-\delta_4(\xi))\end{aligned}$$

$$\begin{aligned}Q_3^G(\xi)-0=&13.78V_3(\xi)^2-1.07V_3(\xi)\sin(\delta_3(\xi))-6.73V_3(\xi)\cos(\delta_3(\xi))-0.09V_3(\xi)V_4(\xi)\sin(\delta_3(\xi)\\&-\delta_4(\xi))-3.83V_3(\xi)V_4(\xi)\cos(\delta_3(\xi)-\delta_4(\xi))-3.19V_3(\xi)V_5(\xi)\cos(\delta_3(\xi)-\delta_5(\xi))\end{aligned}$$

$$\begin{aligned}b_3-0.400=&-36.01V_4(\xi)^2-5.66V_4(\xi)V_2(\xi)\sin(\delta_4(\xi)-\delta_2(\xi))-30.19V_4(\xi)V_2(\xi)\cos(\delta_4(\xi)\\&-\delta_2(\xi))-0.49V_4(\xi)V_3(\xi)\sin(\delta_4(\xi)-\delta_3(\xi))-3.80V_4(\xi)V_3(\xi)\cos(\delta_4(\xi)-\delta_3(\xi))\\&-2.00V_4(\xi)V_5(\xi)\cos(\delta_4(\xi)-\delta_5(\xi))\end{aligned}$$

$$0-Q_5^L(\xi)=5.13V_5(\xi)^2-3.19V_5(\xi)V_3(\xi)\cos(\delta_5(\xi)-\delta_3(\xi))-2.00V_5(\xi)V_4(\xi)\cos(\delta_5(\xi)-\delta_4(\xi))$$

$$\mathbb{P}(P_3^G(\xi)-0.10\geq 0)\geq 1-\epsilon_P^{G,min},\qquad \mathbb{P}(0.40-P_3^G(\xi)\geq 0)\geq 1-\epsilon_P^{G,max}$$

$$\mathbb{P}(P_4^G(\xi)-0.05\geq 0)\geq 1-\epsilon_P^{G,min},\qquad \mathbb{P}(0.40-P_4^G(\xi)\geq 0)\geq 1-\epsilon_P^{G,max}$$

$$\mathbb{P}(Q_3^G(\xi)-(-0.20)\geq 0)\geq 1-\epsilon_Q^{G,min},\qquad \mathbb{P}(0.30-Q_3^G(\xi)\geq 0)\geq 1-\epsilon_Q^{G,max}$$

$$\mathbb{P}(Q_4^G(\xi)-(-0.20)\geq 0)\geq 1-\epsilon_Q^{G,min},\qquad \mathbb{P}(0.20-Q_4^G(\xi)\geq 0)\geq 1-\epsilon_Q^{G,max}$$

$$\mathbb{P}(V_i(\xi) - 0.95 \geq 0) \geq 1 - \epsilon_V^{min}, \qquad \mathbb{P}(1.05 - V_i(\xi) \geq 0) \geq 1 - \epsilon_V^{max}, i \in \{2,3,4,5\}$$

$$\mathbb{P}(\delta_i(\xi) - (-\pi) \geq 0) \geq 1 - \epsilon_\delta^{min}, \qquad \mathbb{P}(\pi - \delta_i(\xi) \geq 0) \geq 1 - \epsilon_\delta^{max}, i \in \{2,3,4,5\}$$