On general origamis and Veech groups of flat surfaces

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概　　要

In this century, an origami (a square-tiled translation surface) is intensively studied as an object with special properties of its translation structure and its $SL(2,\mathbb{R})$-orbit embedded in the moduli space, particularly in the context of number theory. We generalize the concept of origamis in the language of flat surfaces arising naturally in the Teichmüller theory. We show that each origami $\mathcal{O}$ defines a system $A_{\mathcal{O}}$ of linear equations and a permutation group $C_{\mathcal{O}}$, for which $\mathrm{Ker}A_{\mathcal{O}}/C_{\mathcal{O}}$ parametrizes the family of flat surfaces with combinatorial structure $\mathcal{O}$. Furthermore, we will present some calculation results on origamis and discuss the Galois conjugacy of the $SL(2,\mathbb{R})$-orbits of origamis.

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東北大情報数理談話会 (2021 年 12 月 16 日) 資料
講演者は日本学術振興会特別研究員（DC2）であり, 本研究は科研費 (課題番号:21J12260) の助成を受けたものである。本研究成果は東北大学サイバーサイエンスセンターの計算機利用による成果を含む。
キーワード：flat surface, Teichmüller disk, origami, dessin d'enfants, Galois action
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