

On general origamis and Veech groups of flat surfaces

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概 要

In this century, an origami (a square-tiled translation surface) is intensively studied as an object with special properties of its translation structure and its $SL(2, \mathbb{R})$ -orbit embedded in the moduli space, particularly in the context of number theory. We generalize the concept of origamis in the language of flat surfaces arising naturally in the Teichmüller theory. We show that each origami \mathcal{O} defines a system $A_{\mathcal{O}}$ of linear equations and a permutation group $C_{\mathcal{O}}$, for which $\text{Ker} A_{\mathcal{O}}/C_{\mathcal{O}}$ parametrizes the family of flat surfaces with combinatorial structure \mathcal{O} . Furthermore, we will present some calculation results on origamis and discuss the Galois conjugacy of the $SL(2, \mathbb{R})$ -orbits of origamis.

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キーワード: flat surface, Teichmüller disk, origami, dessin d’enfants, Galois action

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