Correlation & Linear models

2022-03-3

Today we'll cover

- Correlation
- Linear Regression

Covariance / Correlation

statistical relationship between two variables

- is there a relationship?
- how strong is it?
- what direction is it in?

Covariance

tumor size	GFP
30	5
35	8
40	8
25	4
35	5

$$\sigma_{xy} = \frac{\sum(x - \bar{x})(y - \bar{y})}{n - 1} \tag{1}$$

the sign of σ_{xy} tells you something about the direction of how two variables covary

Covariance

tumor size	GFP
30	5
35	8
40	8
25	4
35	5

tumor size	GFP	$x - \bar{x}$	y-$\bar{y}$
30	5	-3	-1
35	8	+2	+2
40	8	+7	+2
25	4	-8	-2
35	5	+2	-1

$$\sigma_{xy} = \frac{\sum(x - \bar{x})(y - \bar{y})}{n - 1} \tag{1}$$

the sign of σ_{xy} tells you something about the direction of how two variables covary

Correlation

however covariance is sensitive to the scale of the data (i.e. prove this by multiplying tumor size and GFP by 2?

so we need to scale it ....

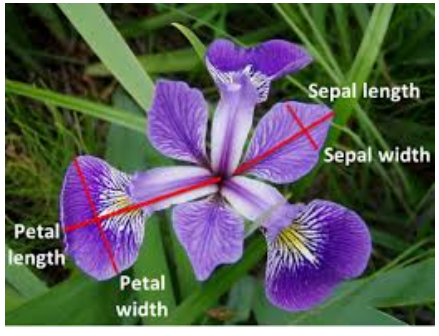
$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma y} \tag{2}$$

ρ_{xy} ranges from -1 to +1

What is the correlation of x and y?

iris

- what is the correlation of Sepal.Width and Sepal.Length?
- what is the correlation between Petal.Length and Petal.Width?
- can you make a correlation matrix?



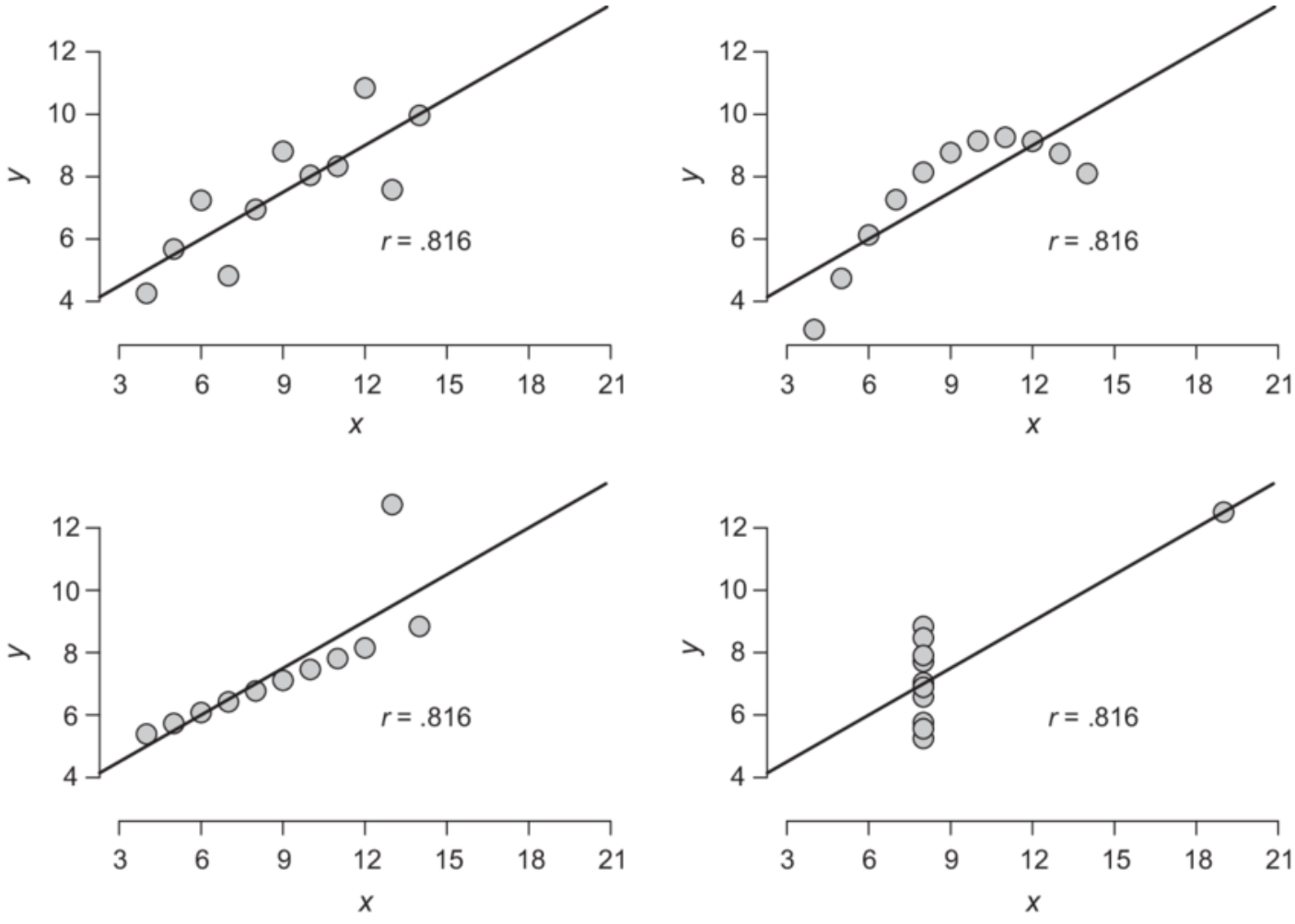
What is the correlation of x and y?

what is the correlation between x and y

- ds1
- ds2
- ds3

What is the correlation of x and y?

Anscombe's quartet



?cor.test

iris

- is there a significant correlation of Sepal.Width and Sepal.Length?
- is there a significant correlation between Petal.Length and Petal.Width?

what is the null and alternative hypothesis that you're testing?

- whats the correlation between two random normal variables each with mean 0 and sd 1 ?

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what do you think happens if I was to do this?

```
cor(c(rnorm(100, 0, 1), c(10, 20, 30, 40)),
    c(rnorm(100, 0, 1), 12, 22, 33, 45))
```

- whats the correlation between two random normal variables each with mean 0 and sd 1 ?

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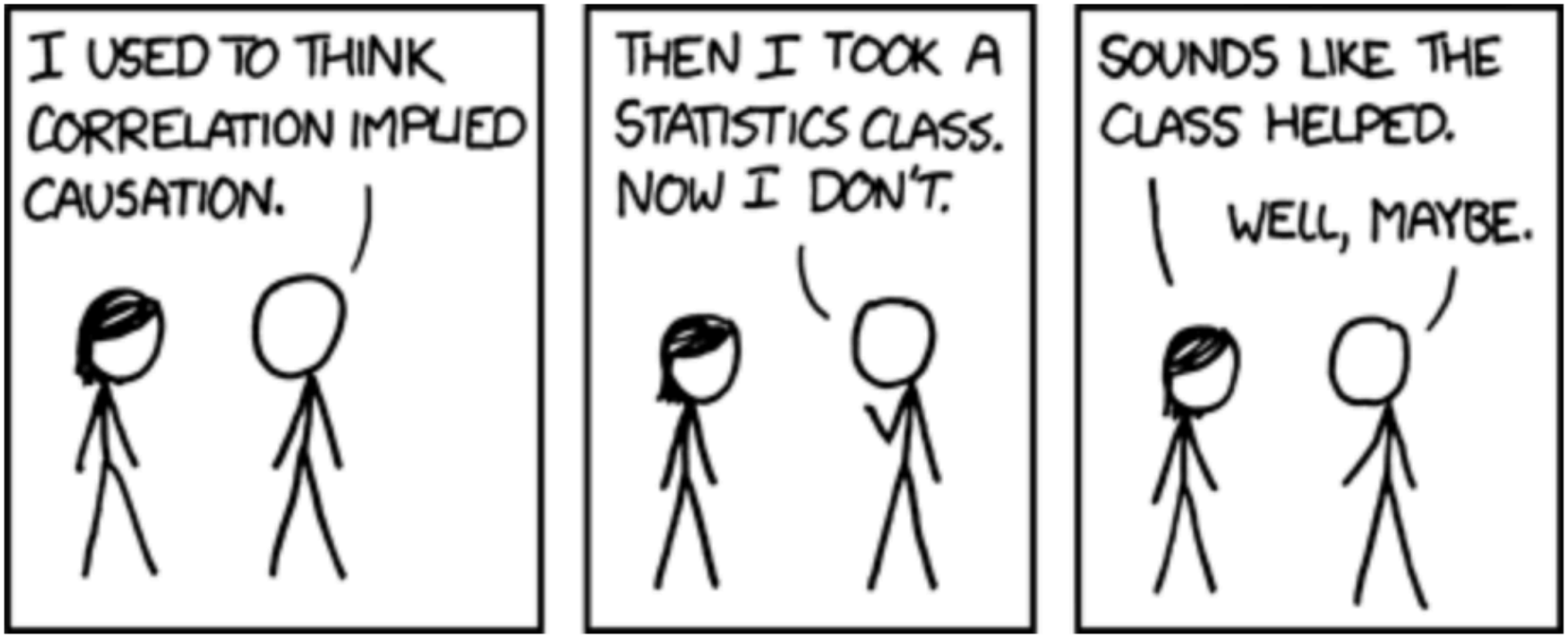
```
cor(c(rnorm(100, 0, 1), c(10, 20, 30, 40)),
    c(rnorm(100, 0, 1), 12, 22, 33, 45))
```

beware of outliers

Correlation and Causation



Correlation and Causation

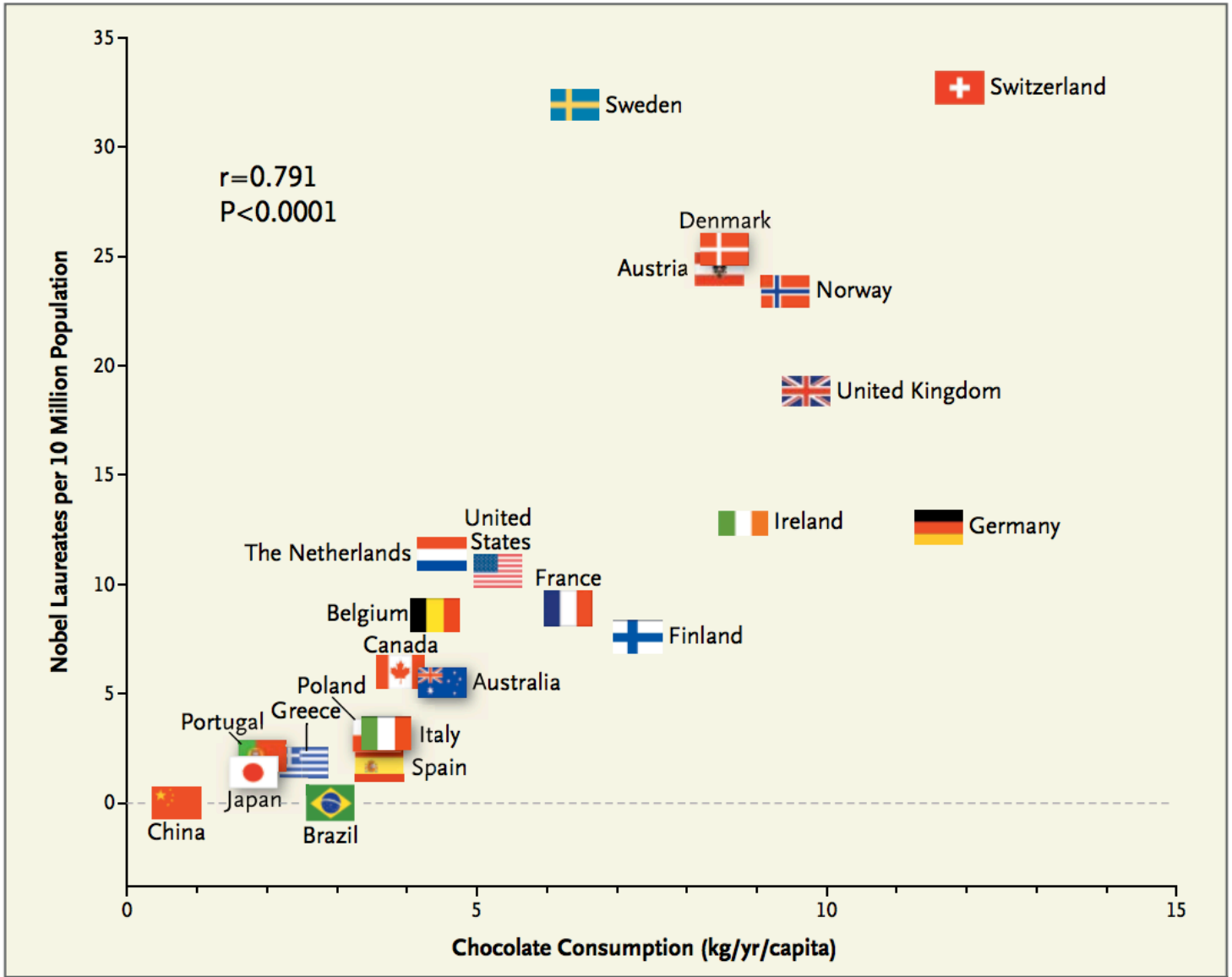
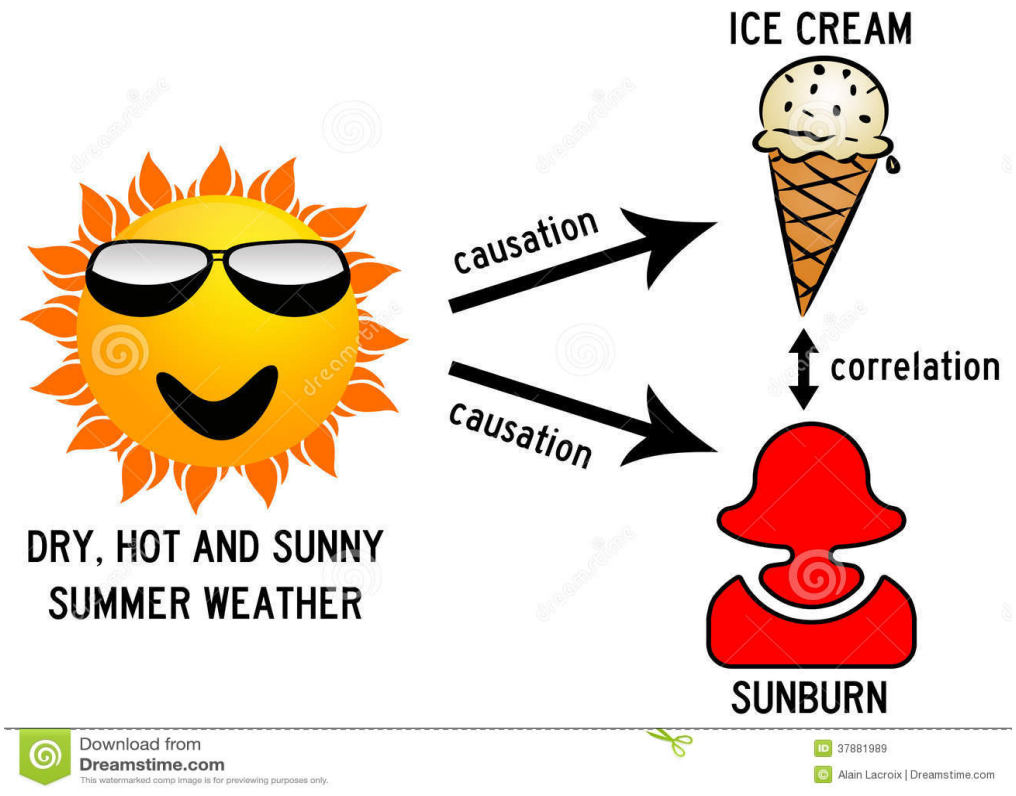


Figure 1. Correlation between Countries' Annual Per Capita Chocolate Consumption and the Number of Nobel Laureates per 10 Million Population.

Correlation is not Causation



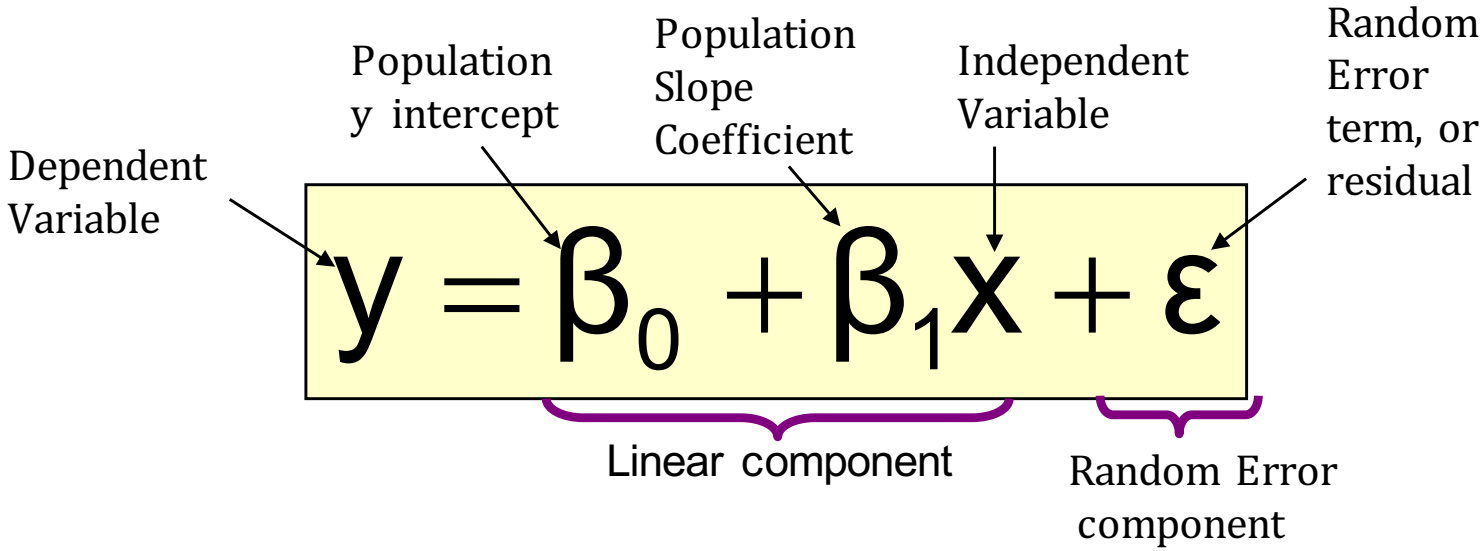
What is Linear Regression

Linear regression attempts to model the relationship between two variables by fitting a linear equation (a straight line) to the observed data. One variable is considered to be an independent variable or predictor and the other is considered to be a dependent variable, or response.

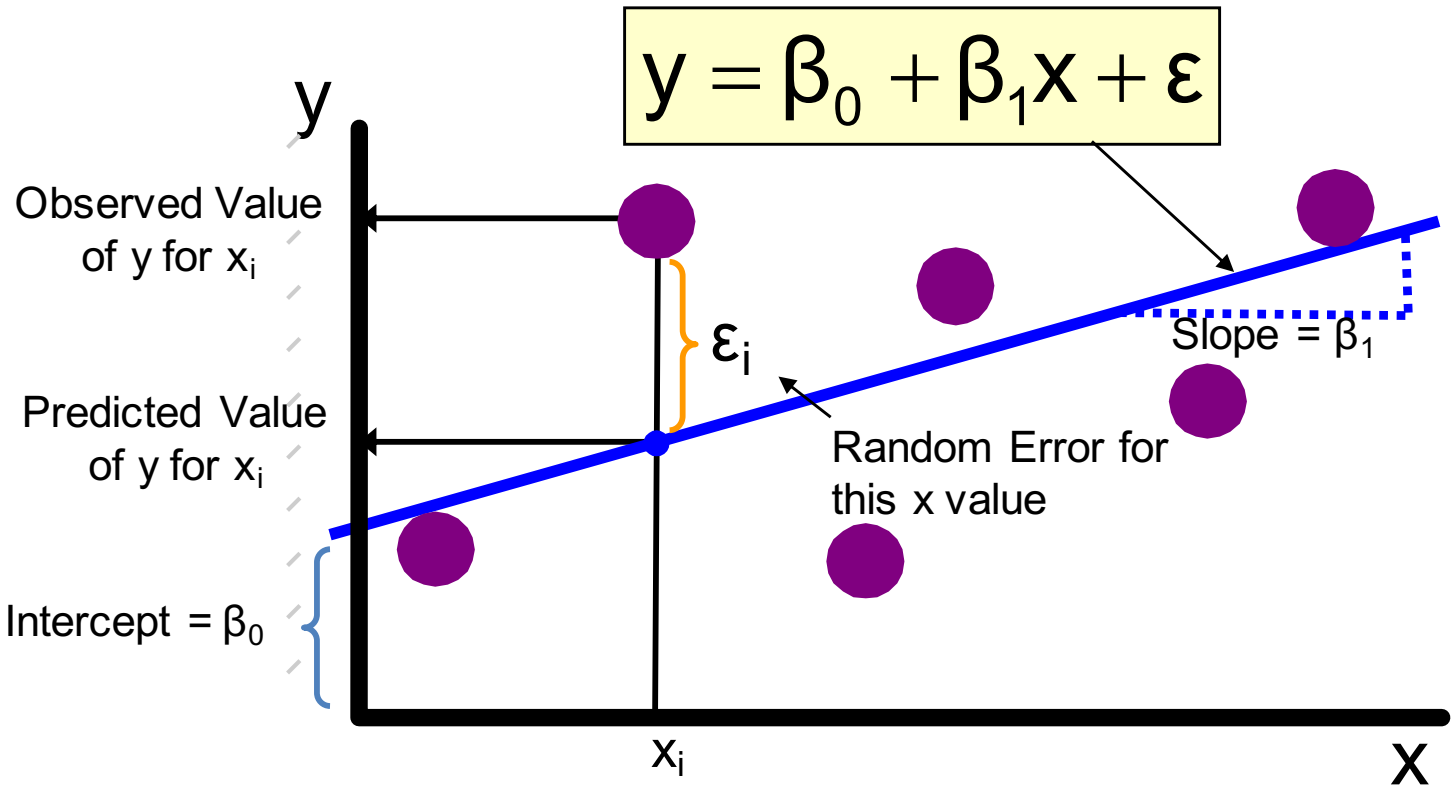
The dependent variable in linear regression should be continuous and normally distributed. The independent variable can be continuous or categorical.

What is Linear Regression

Simple linear regression model



The Line of Best Fit



Clinical Example

Ovarian Cancer

This dataset from a cohort of ovarian cancer patients. It contains clinical information, including age, treatment group, presence of residual disease, quality of life, blood pressure* and cholesterol levels*. The patients were followed up for $\sim$ 3.5 years after treatment.

Age - age at treatment
Resid Disease - was there residual disease after treatment
Rx - which drug were they put on, A or B
ECOG - quality of life
BP - blood pressure at start of treatment
Chol - cholesterol levels at start of treatment

Ovarian Cancer

	Age	Resid Disease	Rx	ECOG	BP	Chol
1	72.33	yes	A	good	117.83	13.58
2	74.49	yes	A	good	114.00	7.78
3	66.47	yes	A	bad	117.55	10.95
4	74.50	yes	A	bad	113.50	22.50
5	43.14	yes	A	good	139.19	22.11
6	63.22	no	B	bad	124.80	8.46
7	64.42	yes	B	good	118.09	23.19
8	58.31	no	B	good	130.09	26.51
.	.	.	.	.	.	.
.	.	.	.	.	.	.
25	44.21	yes	B	good	138.34	26.25
26	59.59	no	B	bad	129.18	22.93

Ovarian Cancer

	Age	Resid Disease	Rx	ECOG	BP	Chol
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5	43.14	yes	A	good	139.19	22.11
6	63.22	no	B	bad	124.80	8.46
7	64.42	yes	B	good	118.09	23.19
8	58.31	no	B	good	130.09	26.51
.	.	.	.	.	.	.
.	.	.	.	.	.	.
25	44.21	yes	B	good	138.34	26.25
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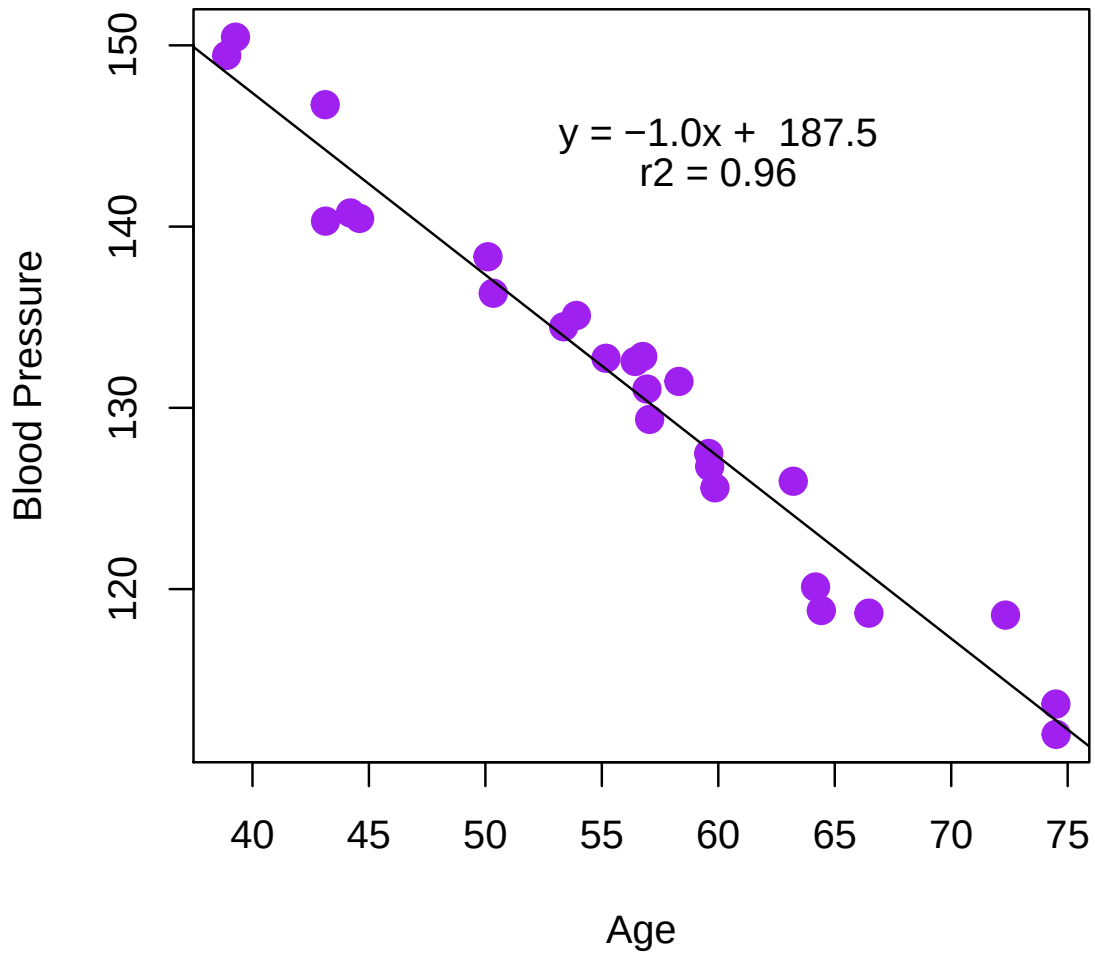
Linear Regression - Ovarian Cancer

In this cohort of ovarian cancer patients,

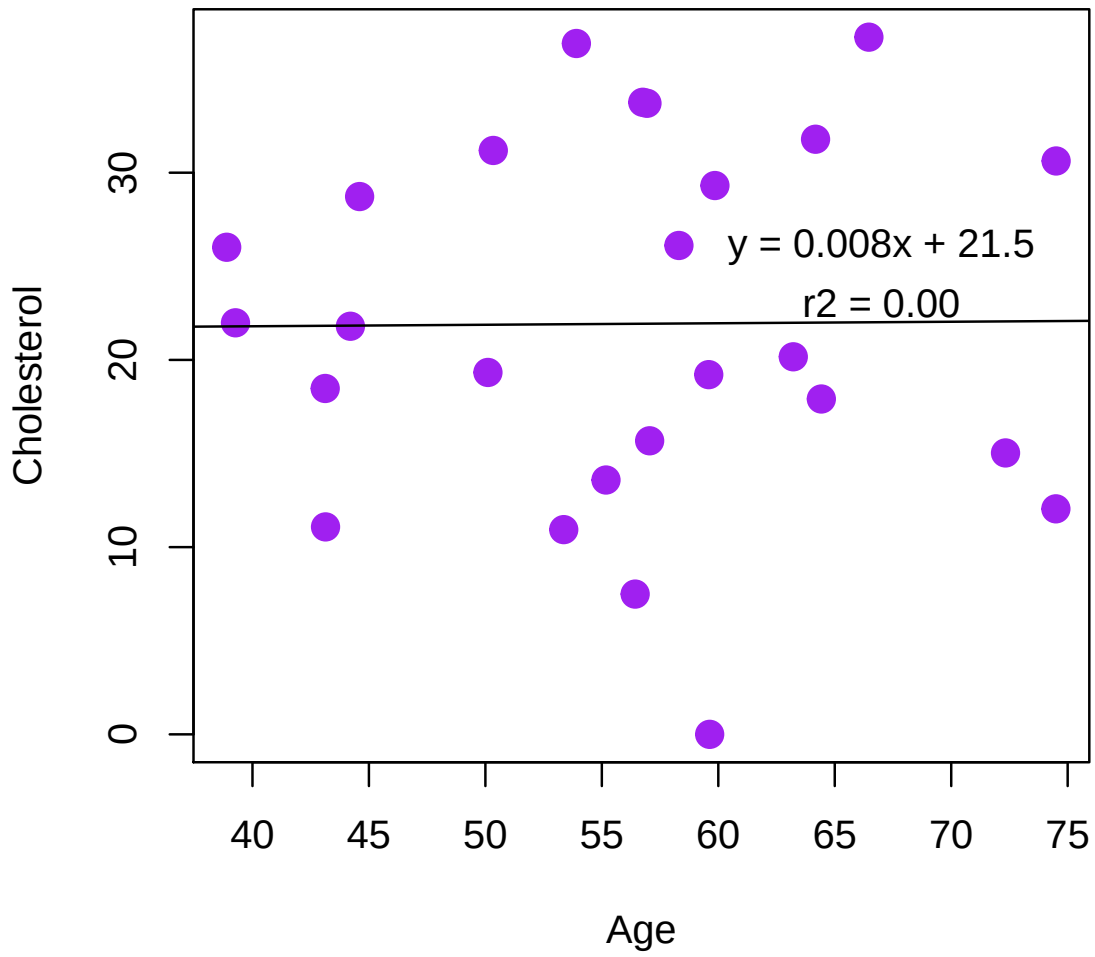
- is there a linear relationship between age and blood pressure?
- is there a linear relationship between age and cholesterol levels?

In these instances, age is the independent variable. Age can influence blood pressure and cholesterol, but blood pressure and cholesterol are not going to change a subjects age.

Ovarian Cancer - Age $\sim$ Blood Pressure



Ovarian Cancer - Age $\sim$ Cholesterol



r^2

coefficient of determination - proportion of variance in the dependent variable that can be explained by the independent variable.

H_0: horizontal line is correct

H_1: the slope of the line differs from 0

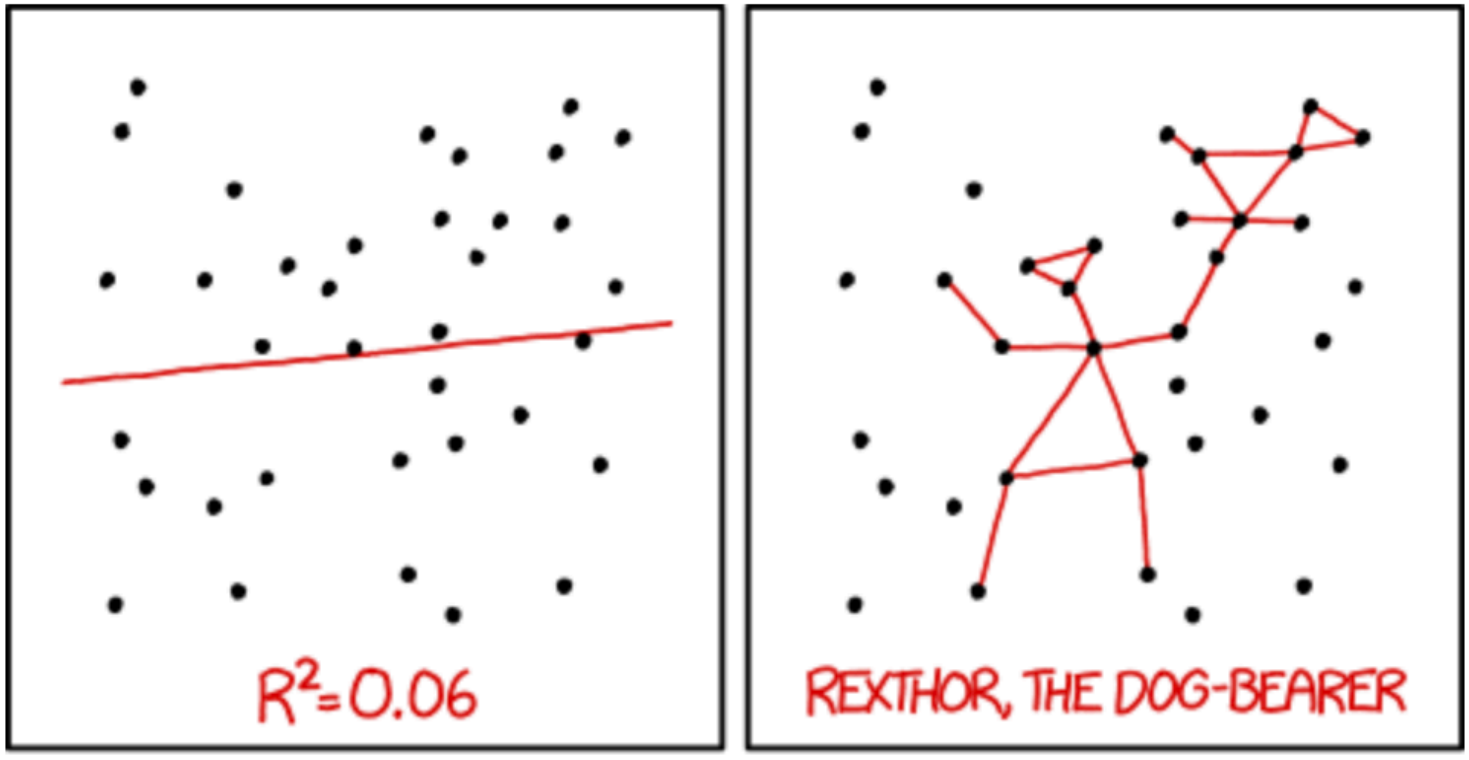
Model fitting

we're actually fitting two models and comparing them against each other - does a horizontal line fit the data better than the regression line?

lots of ways of comparing which model is the best?

R^2

Linear Regression - Look at the data!



I DON'T TRUST LINEAR REGRESSIONS WHEN IT'S HARDER TO GUESS THE DIRECTION OF THE CORRELATION FROM THE SCATTER PLOT THAN TO FIND NEW CONSTELLATIONS ON IT.

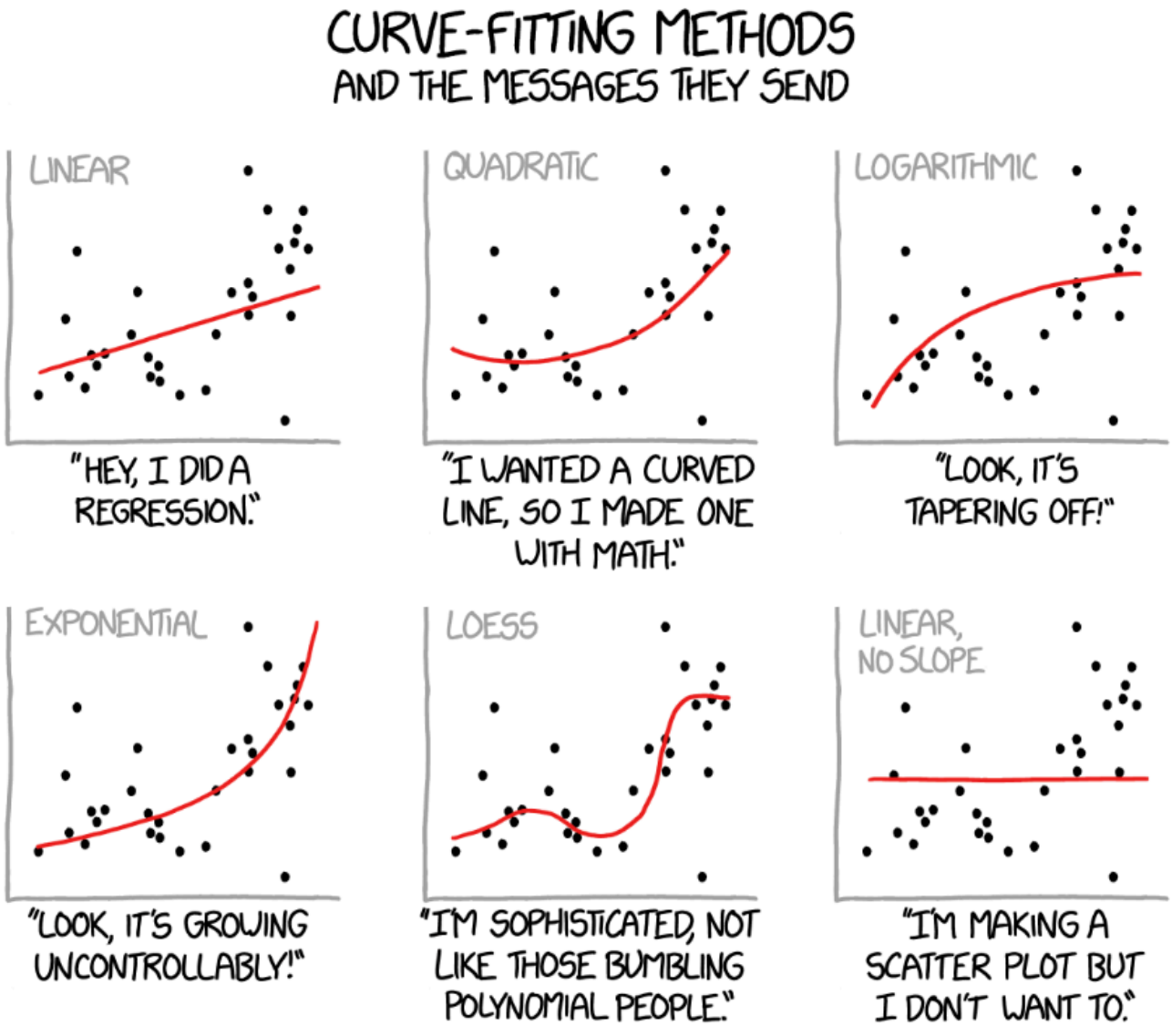
Assumptions of Linear Regression

- The sample must be representative of the population to which the inference will be made
- The data (residuals) should be normally distributed
- There should be no multicollinearity
- The the error terms are independently and identically normally distributed

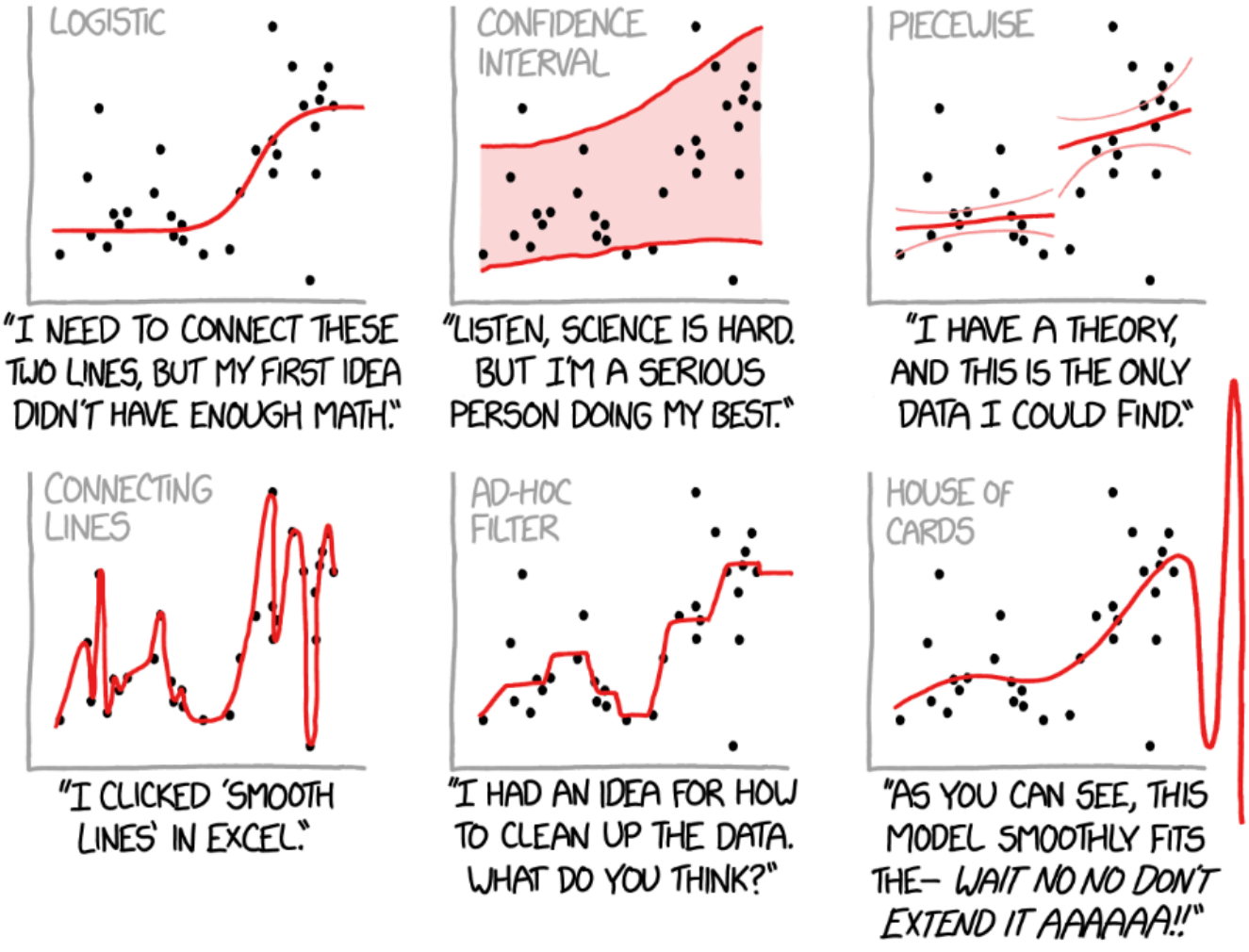
Summary - Linear Regression

- Linear regression attempts to model the relationship between two variables by fitting a linear equation $y = \beta_0 + \beta_1 x + \epsilon$, to the observed data.
- The intercept, β_0 and slope, β_1, from the observed data can be used to predict the relationship between variables.

Linear Regression and Other Models



Linear Regression and Other Models



Questions?

- Next lecture: Multiple regression