Sunshine and Rainbows: A novel staking algorithm

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Abstract

Sunshine and Rainbows is a new staking algorithm that incentivizes long-term staking. In this paper, we define the distribution rule, and derive a formula to use this incentive scheme in smart contracts.

1 Definitions

(i) A position is a group of staked tokens, with a start time and, optionally, an end time.

(ii) An interaction is when a position starts or ends.

(iii) An active position is a position without an end time, or whose end time equals to the reference time.

(iv) An interval is the time between two consecutive interactions.

(v) Staking duration is the time since the start of a position until its end time or until the refererence time, whichever is shorter.

For each interval, there is a reward amount and a set of active positions. At the end of the interval, each of its active positions has a staking duration,

x, and a stake amount, y. We want a position to receive the following portion of interval's rewards.

$$\frac{xy}{\sum_{i=1}^{n} x_i y_i}$$

The denominator is the sum of all active positions' xy. We will use S_n to denote the denominator of the reward proportion at nth interval.

Due to EVM limitations, it is not practical to calculate all active positions' rewards at each interaction. So, we need a formula that can calculate this for a position, based on global variables that have been recorded at its start and end times.

2 Derivations

2.1 Ideal Position

We define an ideal position, whose start time equals to first interaction, and stake amount equals to one. At the end of each interval, its staking duration is the sum of all interval durations upto that reference time, $\sum_{i=1}^{n} t_i$. This ideal position would receive the following rewards from first interval to the end of mth interval.

$$I_{1\to m} = \frac{t_1}{S_1} r_1 + \frac{t_1 + t_2}{S_2} r_2 + \ldots + \frac{\sum_{i=1}^{m} t_i}{S_m} r_m$$

We will use I_n to denote the ideal position's rewards at nth interval.

2.2 Regular Positions

Regular positions can start at any interaction, as opposed to only the first interaction. They can also have any staking amount greater than zero.

We can express the rewards of a regular position P until the end of mth interval, given that the position was started at nth interval.

$$\begin{aligned} P_{n\to m} &= \frac{t_n y}{S_n} r_n + \frac{(t_n + t_{n+1}) y}{S_{n+1}} r_{n+1} + \ldots + \frac{(\sum_{i=n}^{m} t_i) y}{S_m} r_m \\ &= \left(\frac{t_n}{S_n} r_n + \frac{t_n + t_{n+1}}{S_{n+1}} r_{n+1} + \ldots + \frac{\sum_{i=n}^{m} t_i}{S_m} r_m \right) y \end{aligned}$$

2.3 Regular Positions from the Ideal Position

The ideal position can be used to derive other positions, without having to calculate rewards for all positions at each interaction.

To find a position with stake amount y and start interval n, at the end of interval m, we can break apart the ideal stake position $I_{1\to m}$ as follows.

$$
\begin{aligned}
I_{1\to m} &= \sum_{i=1}^{n-1} I_i + \sum_{i=n}^{m} I_i \\
&= \sum_{i=1}^{n-1} I_i + \left(\frac{\sum_{i=1}^{n} t_i}{S_n} r_n + \frac{\sum_{i=1}^{n+1} t_i}{S_{n+1}} r_{n+1} + \ldots + \frac{\sum_{i=1}^{m} t_i}{S_m} r_m \right) \\
&= \sum_{i=1}^{n-1} I_i + \frac{\left(\sum_{i=1}^{n-1} t_i\right) + t_n}{S_n} r_n + \frac{\left(\sum_{i=1}^{n-1} t_i\right) + t_n + t_{n+1}}{S_{n+1}} r_{n+1} + \\
&\quad \ldots + \frac{\left(\sum_{i=1}^{n-1} t_i\right) + \left(\sum_{i=n}^{m} t_i\right)}{S_m} r_m \\
&= \sum_{i=1}^{n-1} I_i + \left(\frac{r_n}{S_n} + \frac{r_{n+1}}{S_{n+1}} + \ldots + \frac{r_m}{S_m} \right) \sum_{i=1}^{n-1} t_i + \\
&\quad \left(\frac{t_n r_n}{S_n} + \frac{(t_n + t_{n+1})\, r_{n+1}}{S_{n+1}} \ldots + \frac{\left(\sum_{i=n}^{m} t_i\right) r_m}{S_m} \right) \\
&= \sum_{i=1}^{n-1} I_i + \left(\sum_{i=n}^{m} \frac{r_i}{S_i} \right) \sum_{i=1}^{n-1} t_i + \\
&\quad \left(\frac{t_n}{S_n} r_n + \frac{\sum_{i=n}^{n+1} t_i}{S_{n+1}} r_{n+1} \ldots + \frac{\sum_{i=n}^{m} t_i}{S_m} r_m \right) \\
&= \sum_{i=1}^{n-1} I_i + \left(\sum_{i=n}^{m} \frac{r_i}{S_i} \right) \sum_{i=1}^{n-1} t_i + \frac{P_{n\to m}}{y} \\
\implies P_{n\to m} &= \left(\sum_{i=n}^{m} I_i - \left(\sum_{i=n}^{m} \frac{r_i}{S_i} \right) \sum_{i=1}^{n-1} t_i \right) y
\end{aligned}
$$

With the resulting expression, now it is possible to calculate a position's rewards, by just using these global variables. Then for each position, we only need to record $\sum I$, $\sum t$, and $\sum (r/S)$ when the position starts.

2.4 The Denominator of the Reward Proportion, S

We had defined S to be $\sum_{i=1}^{n} x_i y_i$ for each interval. However, calculating every position's xy is not practical. So, we will derive another formula to calculate S. t will refer to to the end time of the interval, and e_i will refer to the start time of the ith position. Hence, we can write x_i in terms of e_i and t.

$$
\begin{aligned}
S &= \sum_{i=1}^{n} (x_i y_i) \\
&= \sum_{i=1}^{n} ((t - e_i) y_i) \\
&= \sum_{i=1}^{n} (t y_i - e_i y_i) \\
&= \sum_{i=1}^{n} (t y_i) - \sum_{i=1}^{n} (e_i y_i) \\
&= t \sum_{i=1}^{n} (y_i) - \sum_{i=1}^{n} (e_i y_i)
\end{aligned}
$$

We can now easily keep track of $\sum_{i=1}^{n} (y_i)$ (total staked) and $\sum_{i=1}^{n} (e_i y_i)$ ("sum of entry times") at each interval.