

HAPPY: A novel staking algorithm

shung

<https://twitter.com/shung>
<xmpp:shung@movim.eu>

<https://cryptofrens.xyz/>

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1 Definitions

- (i) A position is a group of staked tokens, with a start time and, optionally, an end time.
- (ii) An interaction is when a position starts or ends.
- (iii) An active position is a position without an end time, or whose end time equals to the reference time.
- (iv) An interval is the time between two consecutive interactions.
- (v) Staking duration is the time since the start of a position until its end time or until the reference time, whichever is shorter.

For each interval, there is a reward amount and a set of active positions. Each active position in the interval has a staking duration at the end of the interval (x) and a stake amount (y). We want a position to receive the following portion of interval's rewards.

$$\frac{xy}{\sum_{i=1}^n x_i y_i}$$

We will use S_n to denote the denominator of the reward proportion at n th interval.

Due to EVM limitations, it is not practical to calculate all active positions' rewards at each interaction. So, we need a formula that can calculate this for a position, based on global variables that have been recorded at its start and end times.

2 Derivation

2.1 Ideal Position

We define an ideal position, whose start time equals to first interaction, and stake amount equals to one. At the end of each interval, its staking duration is the sum of all interval durations upto that reference time, $\sum_{i=1}^n t_i$. This ideal position would receive the following rewards from first interval to the end of m th interval.

$$\frac{t_1}{S_1}r_1 + \frac{t_1 + t_2}{S_2}r_2 + \dots + \frac{\sum_{i=1}^m t_i}{S_m}r_m$$

We will use I_n to denote the ideal position's rewards at n th interval.

2.2 Regular Positions

Regular positions can start at any interaction, as opposed to only the first interaction. They can also have any staking amount greater than zero.

We can express the rewards of a regular position until the end of m th interval, given that the position was started at n th interval.

$$\frac{t_n y}{S_n}r_n + \frac{(t_n + t_{n+1})y}{S_{n+1}}r_{n+1} + \dots + \frac{(\sum_{i=n}^m t_i)y}{S_m}r_m$$

The stake amount of the position, y , is then isolated.

$$\left(\frac{t_n}{S_n}r_n + \frac{t_n + t_{n+1}}{S_{n+1}}r_{n+1} + \dots + \frac{\sum_{i=n}^m t_i}{S_m}r_m \right) y$$

2.3 Regular Positions from the Ideal Position

The ideal position can be used to derive other positions, without having to calculate rewards for all positions at each interaction.

To find a position with stake amount y and start interval n , at the end of interval m , we can break apart the ideal stake position as follows.

$$\begin{aligned}
\sum_{i=1}^m I_i &= \sum_{i=1}^{n-1} I_i + \sum_{i=n}^m I_i \\
&= \sum_{i=1}^{n-1} I_i + \left(\frac{\sum_{i=1}^n t_i}{S_n} r_n + \frac{\sum_{i=1}^{n+1} t_i}{S_{n+1}} r_{n+1} + \dots + \frac{\sum_{i=1}^m t_i}{S_m} r_m \right) \\
&= \sum_{i=1}^{n-1} I_i + \frac{(\sum_{i=1}^{n-1} t_i) + t_n}{S_n} r_n + \frac{(\sum_{i=1}^{n-1} t_i) + t_n + t_{n+1}}{S_{n+1}} r_{n+1} + \\
&\quad \dots + \frac{(\sum_{i=1}^{n-1} t_i) + (\sum_{i=n}^m t_i)}{S_m} r_m \\
&= \sum_{i=1}^{n-1} I_i + \left(\frac{r_n}{S_n} + \frac{r_{n+1}}{S_{n+1}} + \dots + \frac{r_m}{S_m} \right) \sum_{i=1}^{n-1} t_i + \\
&\quad \left(\frac{t_n r_n}{S_n} + \frac{(t_n + t_{n+1}) r_{n+1}}{S_{n+1}} \dots + \frac{(\sum_{i=n}^m t_i) r_m}{S_m} \right) \\
&= \sum_{i=1}^{n-1} I_i + \left(\sum_{i=n}^m \frac{r_i}{S_i} \right) \sum_{i=1}^{n-1} t_i + \\
&\quad \left(\frac{t_n}{S_n} r_n + \frac{\sum_{i=n}^{n+1} t_i}{S_{n+1}} r_{n+1} \dots + \frac{\sum_{i=n}^m t_i}{S_m} r_m \right) \\
&= \sum_{i=1}^{n-1} I_i + \left(\sum_{i=n}^m \frac{r_i}{S_i} \right) \sum_{i=1}^{n-1} t_i + \frac{P}{y} \\
\Rightarrow P_{n \rightarrow m} &= \left(\sum_{i=1}^m I_i - \sum_{i=1}^{n-1} I_i - \left(\sum_{i=n}^m \frac{r_i}{S_i} \right) \sum_{i=1}^{n-1} t_i \right) y
\end{aligned}$$