Sunshine and Rainbows: A novel staking algorithm

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Abstract

Sunshine and Rainbows is a novel staking algorithm that incentivizes long-term staking. In this paper, we define the distribution rule, and derive an algorithm to use this incentive scheme in smart contracts.

1 Definition

For a given interval, rewards are distributed based on the formula below.

$$reward_{position} = reward_{total} \times \frac{staked\ balance_{position}}{staked\ balance_{total}} \times \frac{staking\ duration_{position}}{staking\ duration_{average}}$$

Staking duration of a position starts with `stake()` and ends with `withdraw()`. Optionally, it can restart with `harvest()`.

To simplify the notation, we will use r for total rewards distributed at an interval, R for the rewards earned by a position of interest, B for the *staked balance* of the position, b for the total *staked balance* in the contract, S for the *staking duration* of the position, and s for the average *staking duration* in the contract. Now we can rewrite the formula above for a given interval n.

$$R_n = r_n \frac{B_n}{b_n} \frac{S_n}{s_n}$$

To further simplify, we might refer to *staked balance* $\times$ *staking duration* as *unit value*. We will use V for $B \times S$ and v for $b \times s$.

$$R_n = r_n \frac{V_n}{v_n}$$

2 Position Notation

We will use $R_{a:b \to c}(d)$ notation when describing rewards earned by positions. a is the interval which the staking duration of the position starts (from the beginning of the interval, not middle, not end), d is the balance of the position, $b \to c$ is the inclusive interval range for the accumulated rewards. If d is omitted, balance is one. If a is omitted, start interval is one. So we read $R_{1:1 \to m}(1)$ as the rewards accumulated by a position, whose staking duration starts at the beginning of the first interval and balance equals one, from the beginning of the first interval to the end of m^{th} interval. Using the omission rules, we can also write the same position as $R_{1 \to m}$.

3 Derivations

3.1 Ideal Position

We define an ideal position, whose staking duration starts at the beginning of the first interval, and its staked balance equals to one. At the end of each interval, its staking duration is the sum of all interval durations, $t_{1 \to n}$. The rewards accumulated by the ideal position is simply derived from the original formula.

$$R_{1 \to m} = r_1 \frac{t_1}{v_1} + r_2 \frac{t_{1 \to 2}}{v_2} + \ldots + r_m \frac{t_{1 \to m}}{v_m}$$

3.2 Regular Positions

Unlike the ideal position, regular positions can start at any interval. They can also have any staked balance greater than zero. For a whose staking duration starts at the beginning of interval m, we can describes its accumulated rewards at the end of interval n as shown below.

$$
\begin{aligned}
R_{m:m\to n}(B) &= r_m \frac{BS_m}{v_m} + r_{m+1}\frac{BS_{m+1}}{v_{m+1}} + \ldots + r_n \frac{BS_n}{v_n} \\
&= \left(r_m \frac{t_m}{v_m} + r_{m+1}\frac{t_{m\to m+1}}{v_{m+1}} + \ldots + r_n \frac{t_{m\to n}}{v_n} \right) B
\end{aligned}
$$

We assume the staked balance of the position does not change, hence it is always B.

3.3 Regular Positions from Ideal Position

The ideal position can be used to derive other positions in constant time.

Below, we derive such formula to find the rewards of a position A starting at interval m. We check the rewards of the position at the end of interval n.

$$
\begin{aligned}
R_{1\to n} &= R_{1\to m-1} + R_{m\to n} \\
&= R_{1\to m-1} + r_m \frac{t_{1\to m}}{v_m} + r_{m+1}\frac{t_{1\to m+1}}{v_{m+1}} + \ldots + r_n \frac{t_{1\to n}}{v_n} \\
&= R_{1\to m-1} + r_m \frac{t_{1\to m-1} + t_m}{v_m} + r_{m+1}\frac{t_{1\to m-1} + t_{m\to m+1}}{v_{m+1}} + \\
&\quad \ldots + r_n \frac{t_{1\to m-1} + t_{m\to n}}{v_n} \\
&= R_{1\to m-1} + t_{1\to m-1}\left(\frac{r_m}{v_m} + \frac{r_{m+1}}{v_{m+1}} + \ldots + \frac{r_n}{v_n} \right) + \\
&\quad \left(r_m \frac{t_m}{v_m} + r_{m+1}\frac{t_{m\to m+1}}{v_{m+1}} + \ldots + r_n \frac{t_{m\to n}}{v_n} \right) \\
&= R_{1\to m-1} + t_{1\to m-1} \sum_{i=m}^{n} \frac{r_i}{v_i} + \frac{R_{m:m\to n}(B)}{B} \\
\implies R_{m:m\to n}(B) &= \left(R_{m\to n} - t_{1\to m-1} \sum_{i=m}^{n} \frac{r_i}{v_i} \right) B
\end{aligned}
$$

It is now possible to calculate any position's rewards by using these common variables that form the resulting expression. We can simply record

$\sum I$, $\sum t$, and $\sum(r/v)$ when a position begins, and calculate the difference in $\sum I$ and $\sum(r/v)$ values when a position ends. Then one can simply plug the values into the resulting formula.

3.4 Combined Position

It is also possible to combine multiple positions that have different staking durations.

To combine two positions, $R_{m:m\to p}(B_A)$ and $R_{n:n\to p}(B_B)$ we first stash the rewards of the former position until the end of interval $n-1$, which is $R_{m:m\to n-1}(B_A)$. So that we are left with $R_{m:n\to p}(B_A)$ and $R_{n:n\to p}(B_B)$ to combine.

$$\begin{aligned}
R_{m:n\to p}(B_A) + R_{n:n\to p}(B_B) = R_{n:n\to p}(B_B) + \Bigg(& r_n \frac{t_{m\to n-1} + t_n}{v_n} + \\
& r_{n+1} \frac{t_{m\to n-1} + t_{n\to n+1}}{v_{n+1}} + \\
& \ldots + r_p \frac{t_{m\to n-1} + t_{n\to p}}{v_p} \Bigg) B_A \\
= R_{n:n\to p}(B_A + B_B) + t_{m\to n-1}(B_A) \sum_{i=n}^{p} \frac{r_i}{v_i} &
\end{aligned}$$

In this fashion, we can continue combining positions at constant time.