

# APEC8004: Recitation 4

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# Outline

- Review of the last class: What's the main point?
- Exercise problem

## Background of public good

In the previous class of general equilibrium (before the midterm exam), we focused on competitive markets (the markets clear, all goods are private and no externalities, ...).

Under competitive markets, we have a very nice theorem:

### First welfare theorem

**Competitive markets yield Pareto efficient outcomes.**

Put another way: In a competitive market, we let each individual consumer solely seek to maximize their own utility. The outcomes always achieve social efficiency (the size of the pie is maximized).

### Questions

Is this true under the existence of public good?

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No.

## Goals

- What is the problem preventing us to achieve the socially efficient outcome?
  - How do public goods distort the
- Can we overcome the problem? Is there any good way (mechanism design or intervention scheme) to correct the problem?

# What is the Optimal Provision of Public Goods?

## Samuelson condition

Interior PO allocation for public goods satisfies

$$\sum_j \left( \frac{\partial U_j}{\partial y} / \frac{\partial U_j}{\partial x_j} \right) = \frac{1}{g'}$$

where,  $g(\cdot)$  is a production function.

## NOTE

- The Samuelson condition is necessary but not sufficient for an interior Pareto optimum.

Okay, we understand the optimal level of public goods (i.e.  $y^{PO}$ ). The question is whether this outcome can be realized through market activity.

## Voluntary-contribution equilibrium (VCE)

In the market, each person  $j$  solely seek to maximize their utility, taking other's contributions as given.

$$\begin{aligned} \max_{x_j} \quad & U_j(x_j, y) \\ \text{s.t.} \quad & x_j = \omega_j - z_j \\ & y \leq g \left( z_j + \sum_{k \neq j} z_k \right) \end{aligned}$$

### Summary:

- The VCE is not necessarily PO.
- Generally,  $y^{PO} > y^{VCE}$  (VCE yields underprovision of public goods)

## Message from VCE

The private provision of public goods generates a special type of externality: If one individual provides a unit of a public good, all individuals benefit.

→ Private provision of public goods is typically Pareto inefficient.

### Free-rider problem

Each consumer has an incentive to enjoy the benefits of the public good provided by others while providing it insufficiently herself.



Can we achieve socially efficient outcomes in the existence of public good?

Theoretically, there is a market institution (e.g., super smart government) that can achieve optimality.

→ [Lindahl equilibrium](#)

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### Critical assumption for Lindahl equilibrium

The institution knows each individual's willingness to pay for unit of public good (i.e.,  $P_j$ ).

→ How can we get that information?

- Mechanism design
  - Vickrey-Clark-Gloves (VCG) mechanism

to be continued....

## Summer 2013 environment prelim

Two consumers are the only members of an island economy. They have identical preferences over two goods, a private numeraire good  $x$  and a pure public good  $q$ . Preferences are given by

$$U_i(x_i, q) = \ln x_i + 2 \ln q.$$

Each consumer is endowed with  $\omega_i = 10$  of the private good, of which  $x_i$  is consumed directly and the remainder  $z_i = \omega_i - x_i$  is contributed to the provision of the public good.  $q$  is produced according to the simple production function  $q = z_1 + z_2$ .

- Determine the outcome  $(x_1, x_2, z_1, z_2)$ , that a benevolent social planner would choose so as to maximize the unweighted sum of preferences,  $W = U_1(x_1, q) + U_2(x_2, q)$ .
- Find the voluntary-contribution equilibrium for this economy. Show that the VCE level of the public good,  $\hat{q}$  is less than  $q^*$ .
- Find the Lindahl price at which the consumers, taking this price as given when selecting their contribution  $z_i$ , will choose the socially optimal level of contribution to the public good.



