APEC8004: Recitation 2

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Reminder

The midterm is on Apr 2nd (Tue).

The topic is general equilibrium.

Outline

- Quick review of Assignment 1
- Exercise problem

Quick review of Assignment 1

"Show that the derived Walrasian equilibrium allocation is PO" (or maybe you could be asked "Show that the first welfare theorem holds for this economy")

For this type of question, you have several approaches:

Approach 1: plug x^* into the contract curve and confirm that it returns y^*

Approach 2: Check $MRS_1 = MRS_2$

- Only valid for interior outcomes. For corner outcomes, this does not hold.

Approach 3: If preferences are all LNS, then you can invoke the 1st welfare theorem.

Approach 4: Argue that any movement away from the equilibrium makes someone worse off.

Exercise problem

Practice makes perfect

Various kinds of utility functions:

$$U(x_1, y_1) = max[x_1, y_1]$$

- 2018 June prelim:

$$U(x_1, y_1) = (x_1 + 2)(y_1 + 2)$$

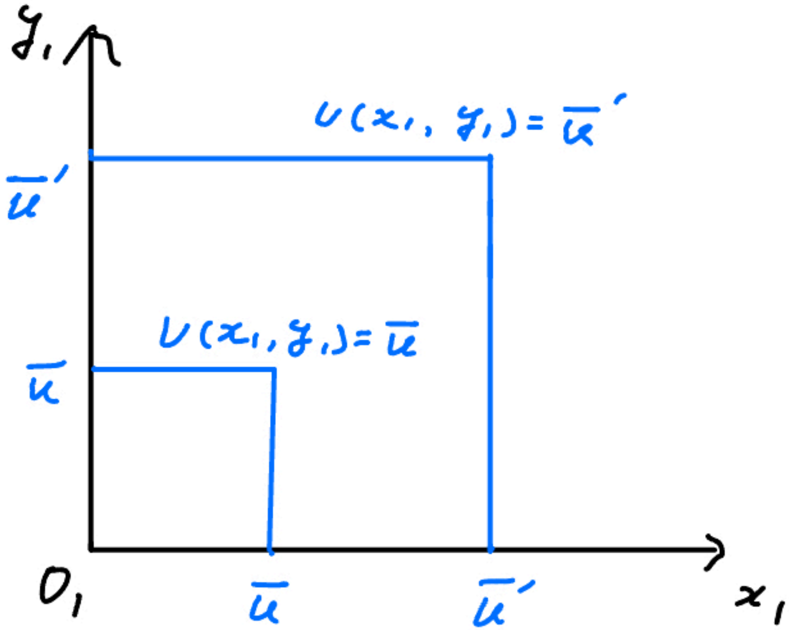
- August 2014prelim and June 2021 prelim:

$$U(x_1, y_1) = \begin{cases} x_1 + 2y_1 & \text{if } x_1 \geq y_2 \\ 2x_1 + y_1 & \text{if } x_1 \leq y_2 \end{cases}$$

Questions

What do their indifference curves look like?

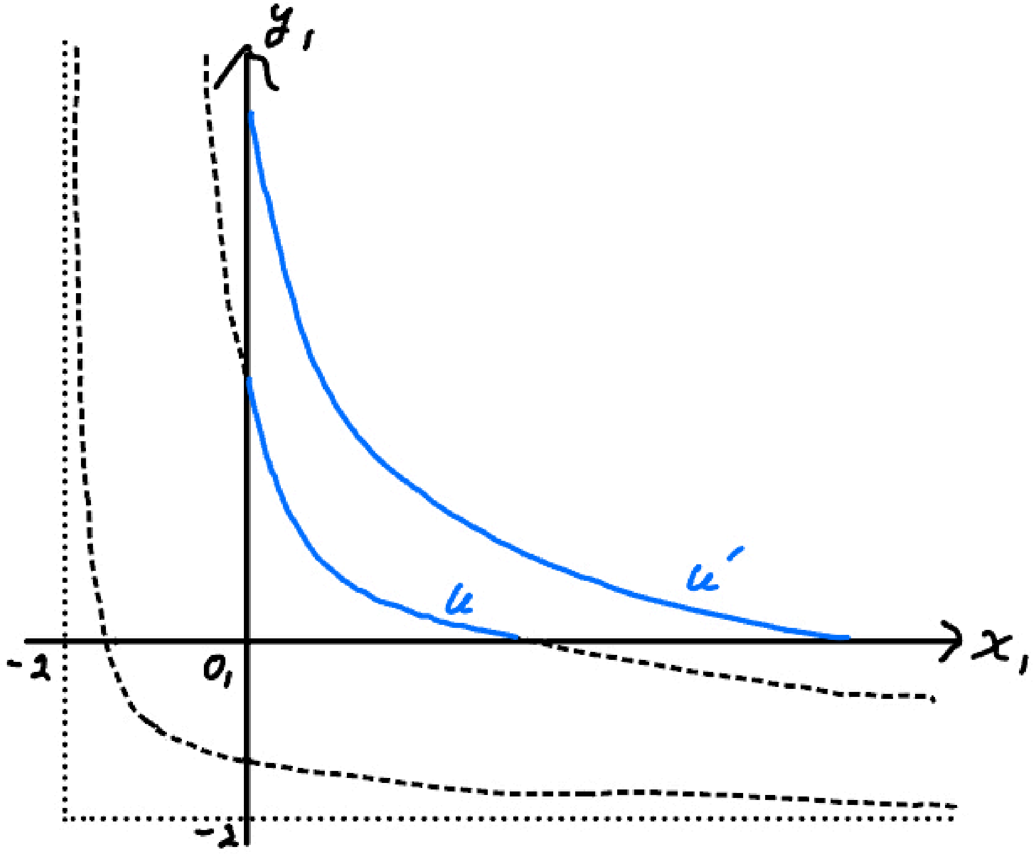
$$U(x_1, y_1) = max[x_1, y_1]$$



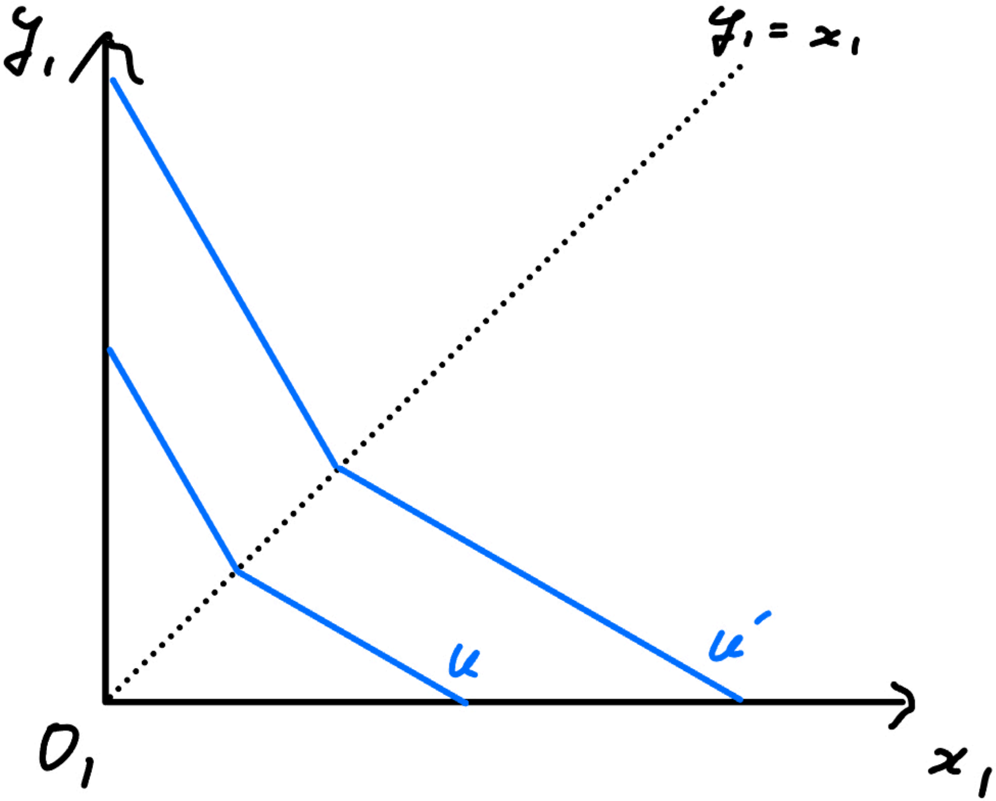
$$max[x_1, y_1] = \bar{u}$$

$$\Rightarrow \begin{cases} x_1 = \bar{u} \text{ and } y_1 \leq \bar{u} \\ y_1 = \bar{u} \text{ and } x_1 \leq \bar{u} \end{cases}$$

$$U(x_1, y_1) = (x_1 + 2)(y_1 + 2)$$



$$U(x_1, y_1) = \begin{cases} x_1 + 2y_1 & \text{if } x_1 \geq y_1 \\ 2x_1 + y_1 & \text{if } x_1 \leq y_1 \end{cases}$$



Exercise problem

Consider the following specific 2×2 competitive exchange economy, with consumers $j = 1, 2$ and goods x and y. The consumers' preferences and initial endowments are given as follows:

- Consumer 1: $U_1(x_1, y_1) = x_1 y_1 \quad \text{and} \quad \omega_1 = (12, 4)$
- Consumer 2: $U_2(x_2, y_2) = max[x_2, y_2] \quad \text{and} \quad \omega_2 = (4, 12)$

1. Find the set of Pareto-optimal allocations for this economy. You do not need to provide a mathematical derivation. Construct a carefully labeled Edgeworth-box diagram depicting the economy, including the endowment and at least one indifference curve for each consumer. Indicate the contract curve in your diagram.

2. True or False: This economy has a Walrasian equilibrium? If true, find the equilibrium price vector and allocation. If false, explain why the equilibrium does not exist.

Memo:

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- Consumer 1: $U_1(x_1, y_1) = x_1 y_1 \quad \text{and} \quad \omega_1 = (12, 4)$
- Consumer 2: $U_2(x_2, y_2) = max[x_2, y_2] \quad \text{and} \quad \omega_2 = (4, 12)$

1. Find the set of PO allocations

The problem is formally written as,

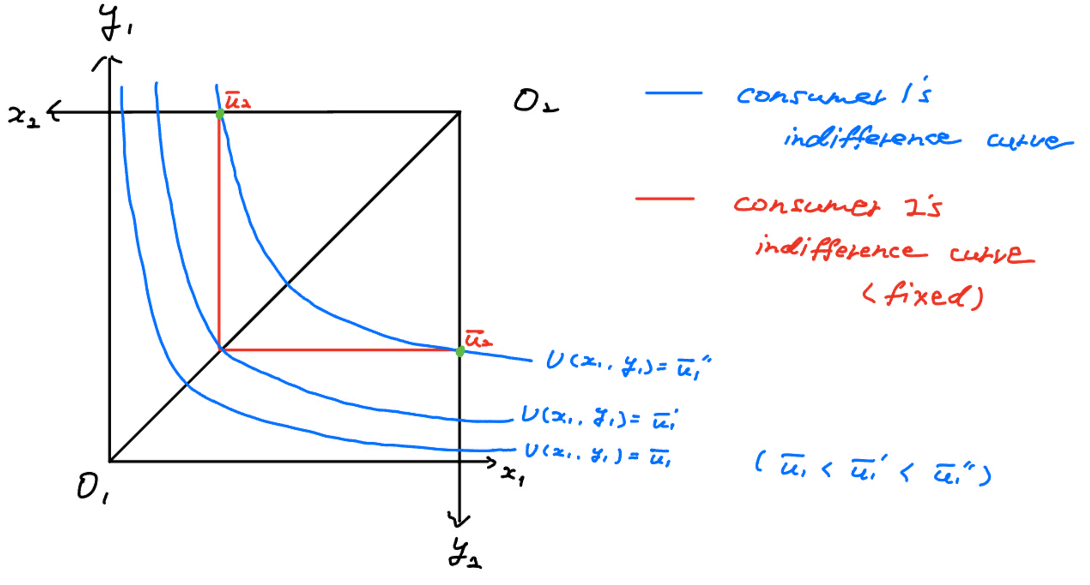
$$\begin{aligned} \max_{x_1, y_1} \quad & U_1(x_1, y_1) = x_1 y_1 \\ \text{s.t.} \quad & max[x_2, y_2] = \overline{u_2} \quad \text{(where, } \overline{u_2} \text{ is some fixed level of utility)} \\ & x_1 + x_2 = 16 \\ & y_1 + y_2 = 16 \end{aligned}$$

In other words, given the feasibility constraints and consumer 2's utility at some level, what is the set of bundles to maximize consumer 1's utility?

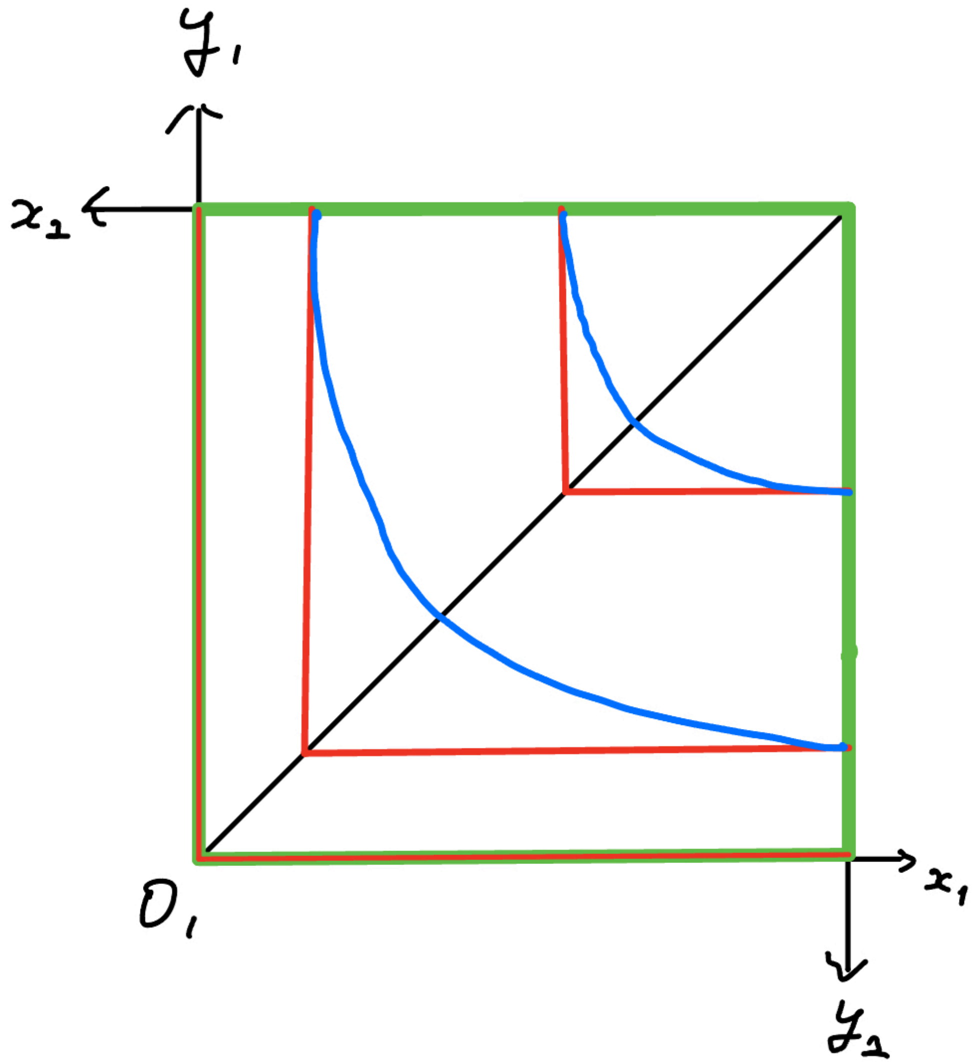
Visual inspection of the conditions of PO allocations with Edgeworth box is useful.

- Given consumer 2's indifference curve at some fixed utility level, move consumer 1's indifference curve to find consumer 1's utility maximizing bundles.

For example,



The contract curve for this economy is the entire outline of the Edgeworth box (green line).



- Consumer 1: $U_1(x_1, y_1) = x_1 y_1 \quad \text{and} \quad \omega_1 = (12, 4)$
- Consumer 2: $U_2(x_2, y_2) = max[x_2, y_2] \quad \text{and} \quad \omega_2 = (4, 12)$

2. Is there any Walrasian equilibrium for this economy?

(i) Consumer 1's problem:

$$\begin{aligned} \max_{x_1, y_1} \quad & U_1(x_1, y_1) = x_1 y_1 \\ \text{s.t.} \quad & p_x x_1 + p_y y_1 = 12p_x + 4p_y \end{aligned}$$

(ii) Consumer 2's problem

$$\begin{aligned}\max_{x_2,y_2} \quad & U_2(x_2,y_2)=max[x_2,y_2]\\ \text{s.t.} \quad & p_x x_2+p_y y_2=4p_x+12p_y\end{aligned}$$

June 2022 Prelim

Consider a 2×2 competitive exchange economy, where subscript $j = 1, 2$ indexes consumers and goods are x and y. Preferences are represented by the utility functions $U_2(x_1, y_1) = x_1 y_1$ and $U_2(x_2, y_2) = x_2 + 2y_2$. The initial endowment vectors are $\omega_1 = (6, 10)$ and $\omega_2 = (6, 0)$.

1. In an Edgeworth box diagram, indicate the endowment ω and draw indifference curves for each consumer. Also draw the contract curve, the set of Pareto-optimal allocations. Label your diagram carefully.
2. Derive the demands $x_j(p, \omega_j)$ for the two consumers. Be sure to include demand behavior when prices are zero.
3. Derive the Walrasian equilibrium price-allocation vector (p^*, x^*). Indicate and label the equilibrium, both x^* and p^*, in your Edgeworth box from part (a).
4. Consider the PO allocation $x_1' = (6, 3)$ and $x_2' = (6, 7)$. Find a price vector, and also an alternative endowment vector ω', that together will support this allocation as a Walrasian equilibrium.