APEC8004: Recitation 3

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A Useful tip:

hitting letter "o" key will give you a panel view of the slides

NOTE:

- Animations are embed in some slides. See the HTML file on the course canvas.

Outline

- Review Assignment 2
- Exercise problem for utility possibilities

Review: Assignment 1, Problem 2

Consider the following

- Consumer 1: $U_2(x_1, y_1) = x_1 y_1$, and $\omega_1 = (4, 0)$
- Consumer 2: $U_1(x_1, y_1) = 2x_2 + y_2$, and $\omega_1 = (4, 8)$

(b) Find the demand functions for the two consumers.

(d) Find the Walrasian equilibrium for this economy.

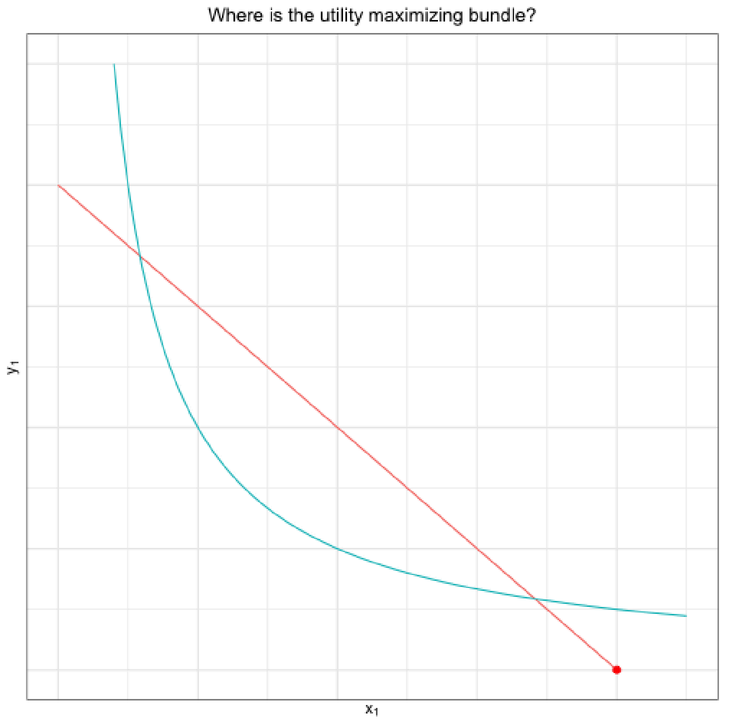
(e) Show that for this economy, using theses consumers' demands, Walras's law is satisfied.

Consumer 1's demand

$$\max_{x_1, y_1} \quad U_1(x_1, y_1) = x_1 y_1$$
$$\text{s.t.} \quad p_x x_1 + p_y y_1 = 4p_x$$

NOTE: U-max problem for a consumer with Cobb–Douglas preferences is always interior solution if prices are strictly positive.

→ A consumer with Cobb–Douglas preferences prefers to consumer a mixture of goods.



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$$\max_{x_1, y_1} \quad U_1(x_1, y_1) = x_1 y_1$$
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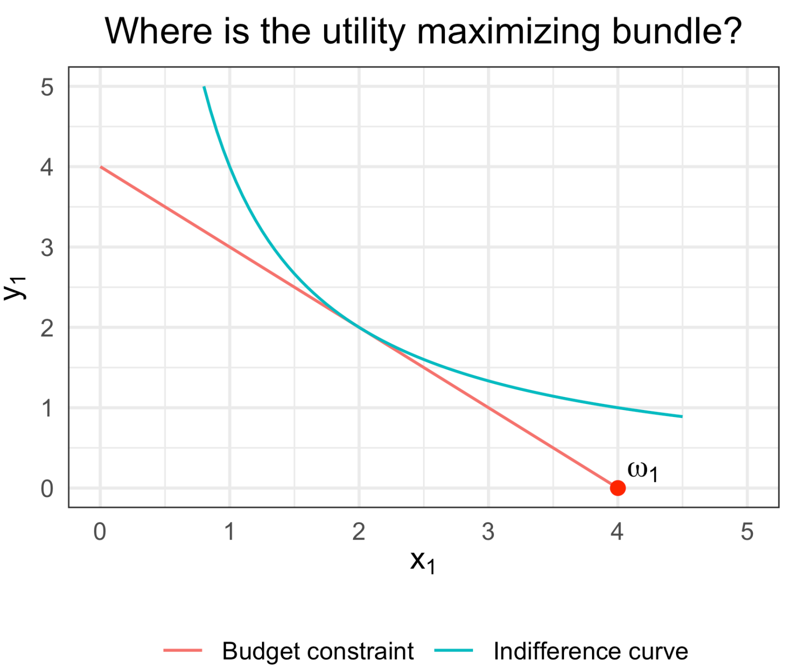
F.O.C.

$$\text{MRS} = \text{ slope of IC}$$
$$\frac{y}{x} = \frac{p_x}{p_y}$$
$$y_1 = \frac{p_x}{p_y} x_1$$

By substituting this into the budget constraint, and solving for x_1 and y_1, you'll get

$$x_1^* = 2,\ y_1^* = \frac{2p_x}{p_y}$$



Consumer 2's demand

$$\max_{x_2,y_2} \quad U_2(x_2,y_2) = 2x_2 + y_2$$
$$\text{s.t.} \quad p_x x_2 + p_y y_2 = 4p_x + 8p_y$$

Linear utility functions represent perfect substitute preferences.

→ the consumer is willing to substitute one good for the other at a constant rate

→ Depending on the slope of the indifference curve (i.e., price ratio), utility maximizing bundle can be corner solutions.

One point advice:

If you see indifference curves which intersect with x or/and y axis, carefully think about in what condition (the price ratio) U-max solution occurs at the boundary.

$$\max_{x_2, y_2} \quad U_2(x_2, y_2) = 2x_2 + y_2$$
$$\text{s.t.} \quad p_x x_2 + p_y y_2 = 4p_x + 8p_y$$

Case I: If $-\frac{p_x}{p_y} = -2$, or $\frac{p_x}{p_y} = 2$.

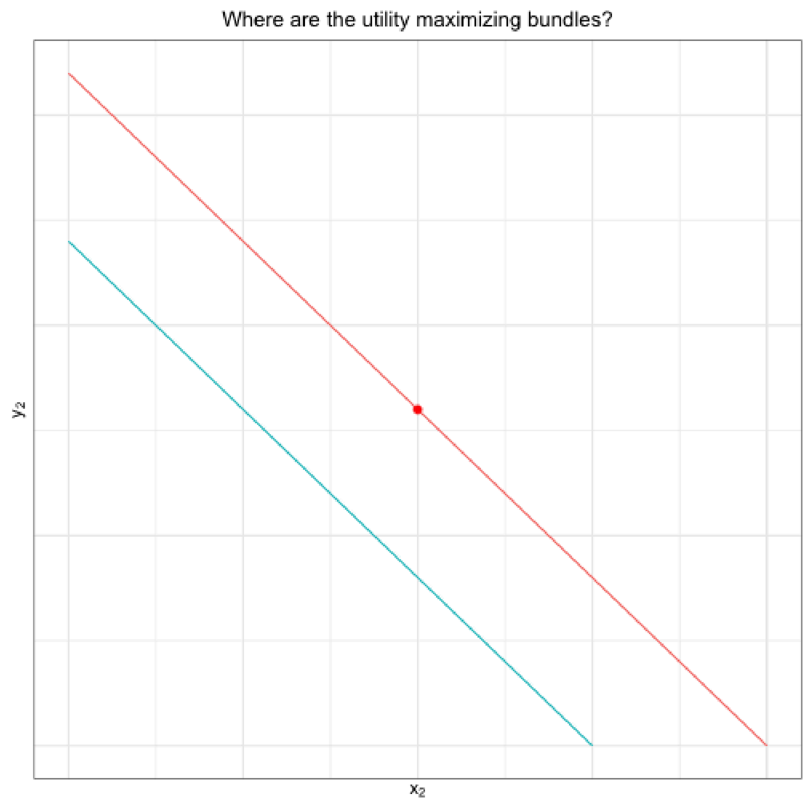
Now, the budget constraint is

$$\begin{aligned} x_2 + p_y x_2 &= 4p_x + 8p_y \\ \frac{p_x}{p_y} x_2 + y_2 &= 4\frac{p_x}{p_y} + 8 \\ 2x_2 + y_2 &= 16 \end{aligned}$$

U_2 is maximized when his IC coincides with his budget constraint.

Thus, U-max bundles are

$$(x_2^*, y_2^*) = \{(x_2, y, 2); 2x_2 + y_2 = 16, \\ 0 \leq x_2 \leq 8\}$$

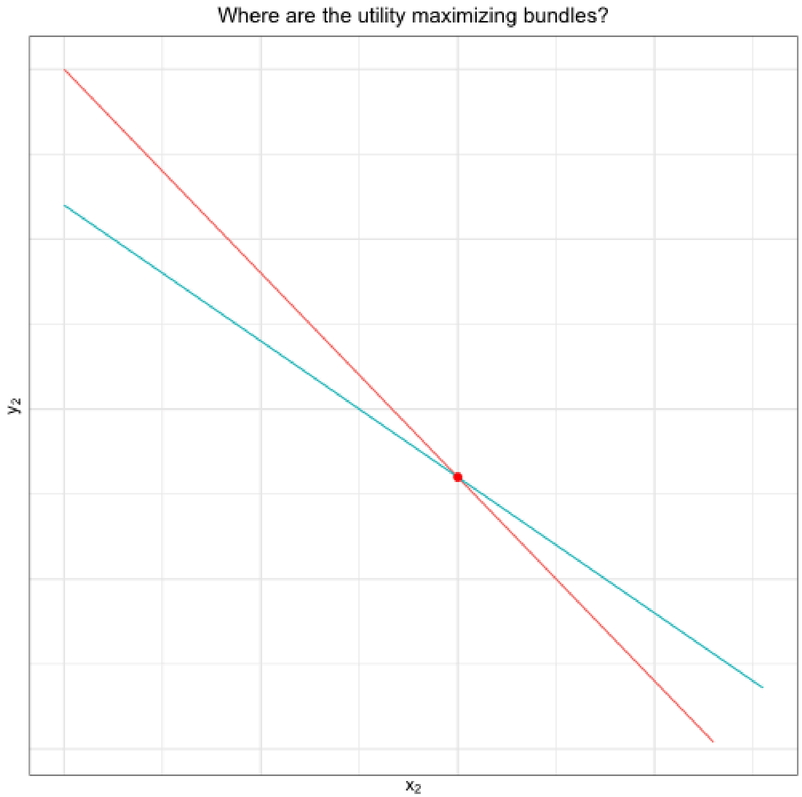


$$\max_{x_2, y_2} \quad U_2(x_2, y_2) = 2x_2 + y_2$$
$$\text{s.t.} \quad p_x x_2 + p_y y_2 = 4p_x + 8p_y$$

Case II: If $-\frac{p_x}{p_y} < -2$, or $\frac{p_x}{p_y} > 2$

The utility maximizing bundle is the y-intercept of the budget line.

$$(x_2^*, y_2^*) = (0, 4\frac{p_x}{p_y} + 8)$$

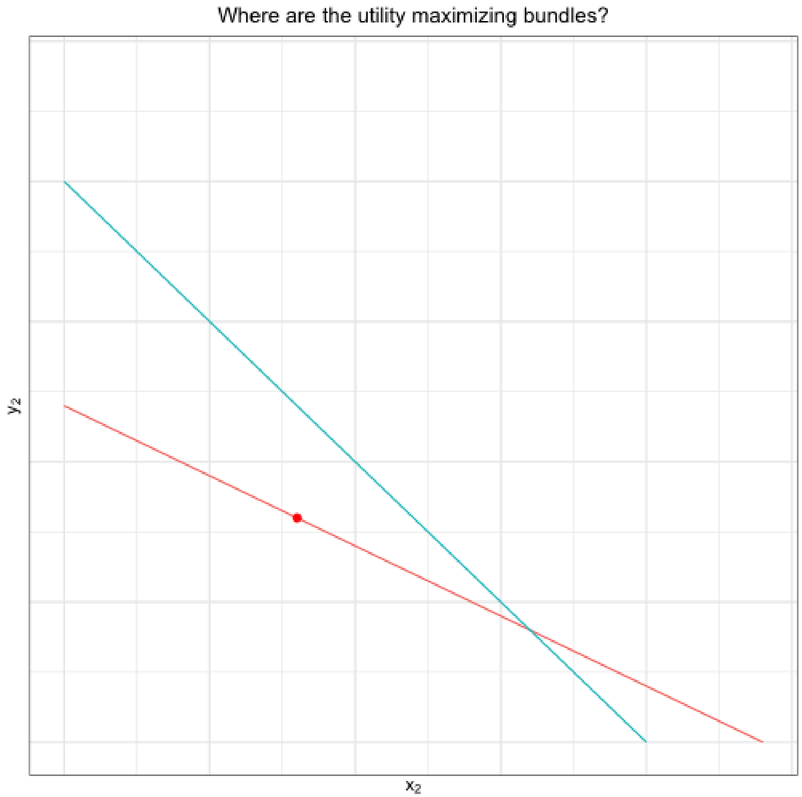


$$\max_{x_2, y_2} \quad U_2(x_2, y_2) = 2x_2 + y_2$$
$$\text{s.t.} \quad p_x x_2 + p_y y_2 = 4p_x + 8p_y$$

Case III: If $-\frac{p_x}{p_y} > -2$, or $\frac{p_x}{p_y} < 2$

The utility maximizing bundle is the x-intercept of the budget line.

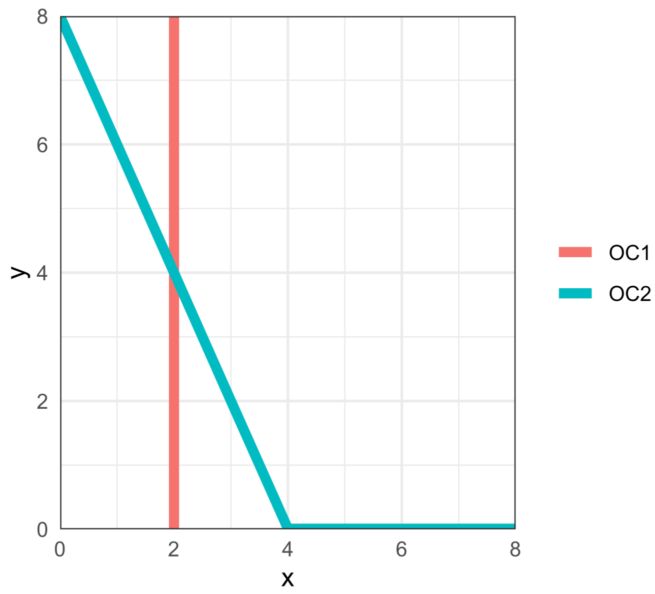
$$(x_2^*, y_2^*) = (4 + 8\frac{p_y}{p_x}, 0)$$



Walrasian equilibrium

$$(x_1^*, y_1^*) = (2, 2\frac{p_x}{p_y})$$

$$(x_2^*, y_2^*) = \begin{cases} (4 + 8\frac{p_y}{p_x}, 0) & \text{if } \frac{p_x}{p_y} < 2 \\ \{(x_2, y_2); 2x_2 + y_2 = 16, 0 \leq x_2 \leq 8\} & \text{if } \frac{p_x}{p_y} = 2 \\ (0, 4\frac{p_x}{p_y} + 8) & \text{if } \frac{p_x}{p_y} > 2 \end{cases}$$



Walras's law (important!)

Walras's law (strong form)

If all the preferences satisfy local nonsatiation, for any price vector p, it must be that $p \cdot z(p) = 0$

NOTE:

- If you are asked to show whether Walras's law is satisfied, you need to show $p \cdot z(p) = 0$ for all price vector p.
- How?

$$\begin{bmatrix} p_x \\ p_y \end{bmatrix} \begin{bmatrix} x_1^*(p_x, p_y) + x_2^*(p_x, p_y) - \omega_{1x} - \omega_{2x} & y_1^*(p_x, p_y) + y_2^*(p_x, p_y) - \omega_{1y} - \omega_{2y} \end{bmatrix} = 0$$

Utility possibilities and social welfare.

An economy consists of two people and two goods, x and y. The aggregate endowment is $\Omega = (4, 9)$. Utility functions are

$$U_1(x_1, y_1) = \sqrt{x_1 y_1}$$

$$U_2(x_2, y_2) = 2\sqrt{x_2 y_2}$$

a. Derive the contract curve for this economy.

b. Now obtain the utility possibilities set for the economy:

$$v = \{(U_1(\mathbf{x_1}), U_2(\mathbf{x_2})) \in \mathbb{R}^2 | \sum_j \mathbf{x}_j^i \geq \sum_j \omega_j^i \text{ for each } i = 1, 2\}$$

Determine whether v is a convex set in $(U1, U2)$-space.