APEC8004: Recitation 1

Shunkei Kakimoto

- Jay's office hours
  - Tuesday: 10:30 - 12:00
  - Wednesday: 3:30 - 4:00
- My office hours
  - at Waite Library
  - Monday 12 - 1 pm
  - Thursday 12 - 1 pm
  - Or feel free to shoot me an email to set up a time for a meeting (Zoom or in person)
- Assignment
  - Due on Friday 3 pm
  - Submit on Canvas by Friday 3 pm

Outline

- Quickreview of this week's material
- Exercise problem

Pareto optimality

- Prices have nothing to do with this.
- Aggregate amount of endowments is important. (Who owns what does not matter)
- Contract curve: the trajectory allocations that are feasible and PO.**

Walrasian equilibrium (General equilibrium)

- Prices matters.
- In terms of endowment, who owns what matters.
- Offer curve for consumer j: the trajectory of consumer j's demand at various price ratios. The intersection of offer curves of each consumer is the Warlasian equilibrium allocation.

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How they are related?

- **The first welfare theorem**: If preferences satisfy LNS, then a Walrasian equilibrium is PO.
- **The second welfare theorem**: If preferences are (i) continuous, (ii) convex, and (iii) strongly monotonic, then a PO allocation can be a Walrasian equilibrium.

Note for prelim

Some proofs have been asked in the past prelims:

Walras's law: 2021 June Question IV.2

First welfare theorem: 2021 June Question IV.2

Existence of equilibrium (strict convexity case): 2017 June Question IV.2, 2018 June Question IV.2, 2021 January Question IV.1

Quasilinear preferences:

For the definition, see Mas-Colell P45.

Quasilinear utility function (in two commodity case):

$$u(x, y) = \underbrace{x}_{\text{numeraire}} + \underbrace{f(y)}_{\substack{f(\cdot) \text{ is strictly concave} \\ y \text{ does not depend on income (wealth)}}}$$

- In this case, we say preferences are quasilinear with respect to commodity x.
- x is preferable for consumers (e.g., x is a private good and y is a public good).
- Commodity x is called numeraire.

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Characteristics:

- In the two commodity case, the indifference curves are parallel along the axis of x.
  - The numeraire goods shift the indifference curves outward as consumption increases, without changing their slope.
- For any set of prices, the consumers' preferred level of y (e.g., public goods) will not depend on income.

$$u(x, y) = x + f(y)$$

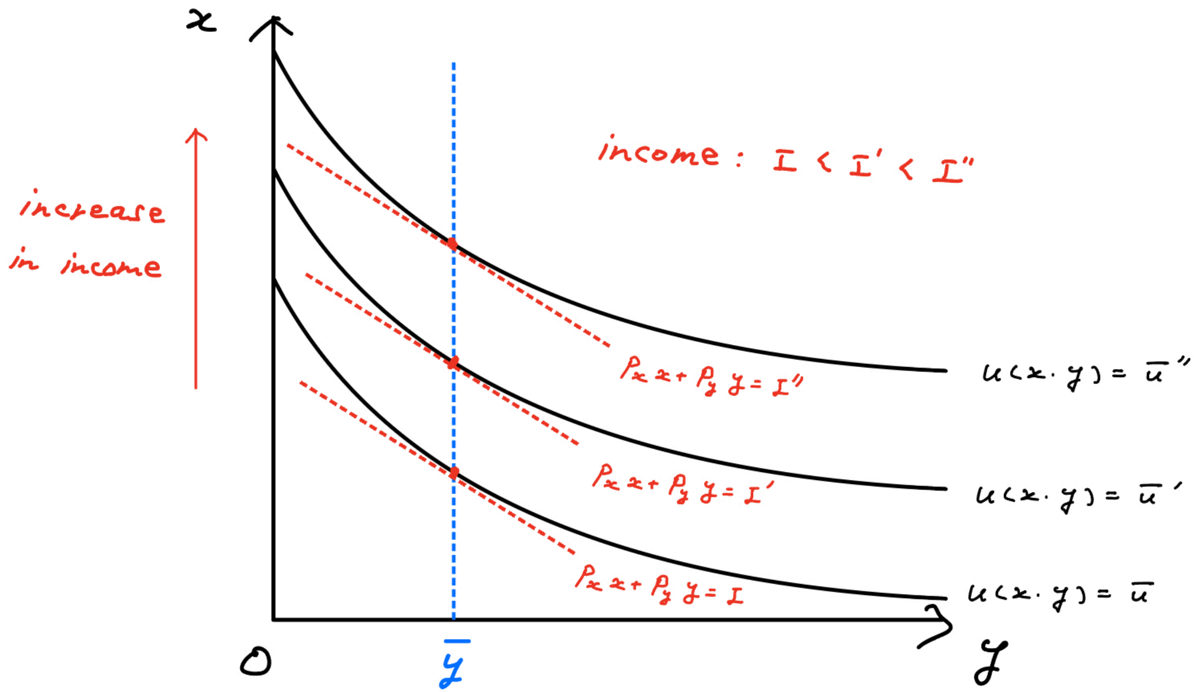
Why commodity y is free from income effect?

$$\begin{aligned} MRS \text{ of } y \text{ for } x &= -\left.\frac{dx}{dy}\right|_{u=\bar{u}} \\ &= -\frac{\frac{\partial u}{\partial y}}{\frac{\partial u}{\partial x}} \\ &= -\frac{f'(y)}{1} \\ &= -f'(y) \end{aligned}$$

That is, at a given level of y, every indifference curve will have the same slope regardless of the level of x.

$$MRS \text{ of } y \text{ for } x = f'(y)$$

Graphically,



For any set of prices, the consumer's preferred level of y will not depend on income I.

→ y is not affected by income effect.

Exercise problem

Consider an exchange economy economy with two consumers, $j = 1, 2$, two goods, x and y. Consumer 1 has an endowment of ω_1 has an endowment of $\omega_1 = (12, 0)$ and consumer 2 has an endowment of $\omega_2 = (0, 8)$. Utility functions are given by

$$U_1(x_1, y_1) = x_1 + lny_1 \text{ and } U_2(x_2, y_2) = logx_2 + y_2$$

a. Derive the contract curve for this economy.

b. Derive the demands (offer curves) for the two consumers. Solve for a Walrasian equilibrium allocation and prices (x^*, p^*). You may include a carefully labeled diagram as part of your answer if you wish, but this is not required. Show that the Walrasian equilibrium allocation is Pareto optimal

c. Consider the allocation at which consumer 1 gets the entire resources endowment: $x_1 = (12, 8)$ and $x_2 = (0, 0)$. Can this be the allocation for a Walrasian equilibrium? If not, explain why. If so, find the price vector and a redistributed endowment vector $\tilde{\omega}$ that supports it as an equilibrium.

Tricky utility functions:

For example,

- $U(x, y) = max[x, y]$
  - like Leontief utility function, but $max[x, y]$ instead of $min[x, y]$
- $U(x, y) = (x + 2)(y_1 + 2)$
  - a type of the Stone–Geary utility function

Exercise problem (maybe next time)

$$U_1(x_1, y_1) = x_1 y_1, \quad \omega_1 = (12, 4)$$
$$U_2(x_2, y_2) = max[x_2, y_2], \quad \omega_2 = (4, 12)$$

1. What is the set of PO allocations?
2. What is the Walrasian equilibrium if any.