APEC8003: Recitation 4

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Midterm exam, Problem 3

Problem:

- Players: Monocorp (monopoly) and Rocky (monopsony)
- Rocky's utility (measured in monetary terms) is

$$U(H) = \begin{cases} 10H - H^2/2 & \text{if } 0 \leq H \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

- Inverse demand function: $P(H) = 10 - H$
- Monocorp's cost function: $C = 30$ (zero marginal cost)

(a) Find the profit maximizing (linear) price per hour assuming that cable television is provided to Frostbite Falls. How much profit would Monocorp make if it provided cable television to Frostbite Falls? Would Monocorp choose to supply cable television to Frostbite Falls?

Profit maximization problem for Monocorp (monopoly)

$$\max_H \pi = (10 - H)H - 30.$$

Then, $H^* = 5$. Monocorp's profit is $\pi^* = -5 < 0$. So, Monocorp would rather choose not to provide cable TV.

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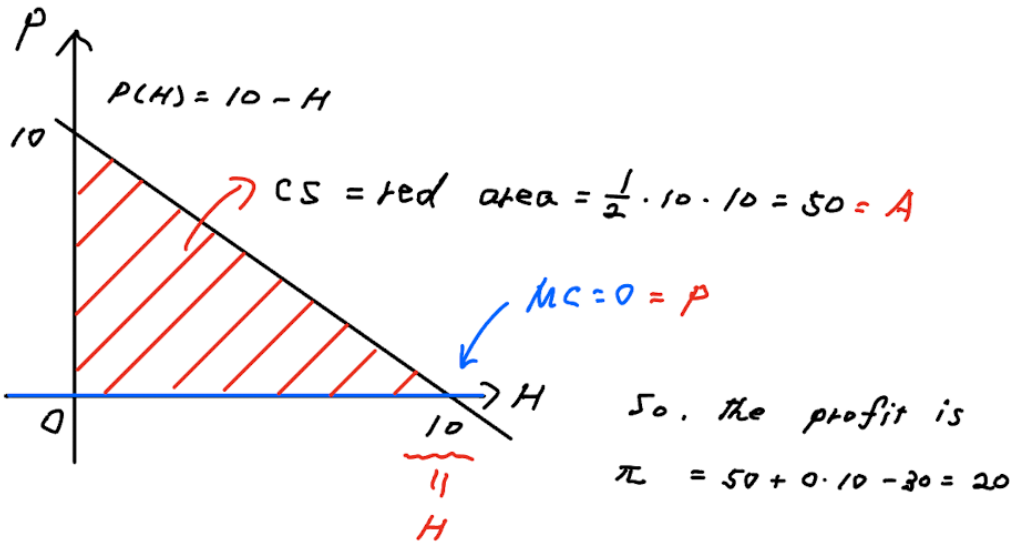
(b) Suppose instead of choosing a simple linear price, Monocorp sets a two-part tariff: $T(H) = A + PH$. Solve for the profit maximizing two-part tariff. How much profit would Monocorp make if it provided cable television? Would Monocorp choose to supply cable television?

Standard solution for Two-part tariff

$$\begin{aligned} \max \quad & A + PH - 30 \\ \text{s.t.} \quad & A + PH \leq \int_0^H (10 - h)dh \end{aligned}$$

Two-part tariff: Visual solution for intuitive understanding



- The monopoly sets per-unit fee P at the competitive price $(\text{i.e., } P = MC = 0)$

→ CS is maximized.

- Then, the monopoly extracts all the CS by setting the lump-sum fee A equal to CS.

What's the consequence for Rocky?

- Since CS is zero, this means Rocky is indifferent between watching cable TV and not.

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(c) Suppose that Rocky offers to pay Monocorp a fixed amount, B, that allows Rocky to watch as much cable television as Rocky wants. How much television should Rocky watch to maximize utility? What is the utility maximizing amount that Rocky should offer to pay Monocorp?

Rocky's utility maximization

$$\begin{aligned} \max \quad & U = 10H - H^2/2 - B \\ \text{s.t.} \quad & B \geq 30 \end{aligned}$$

→ $H = 10$ and $B = 30$, $U = 10 \cdot 10 - 10^2/2 - 30 = 20$.

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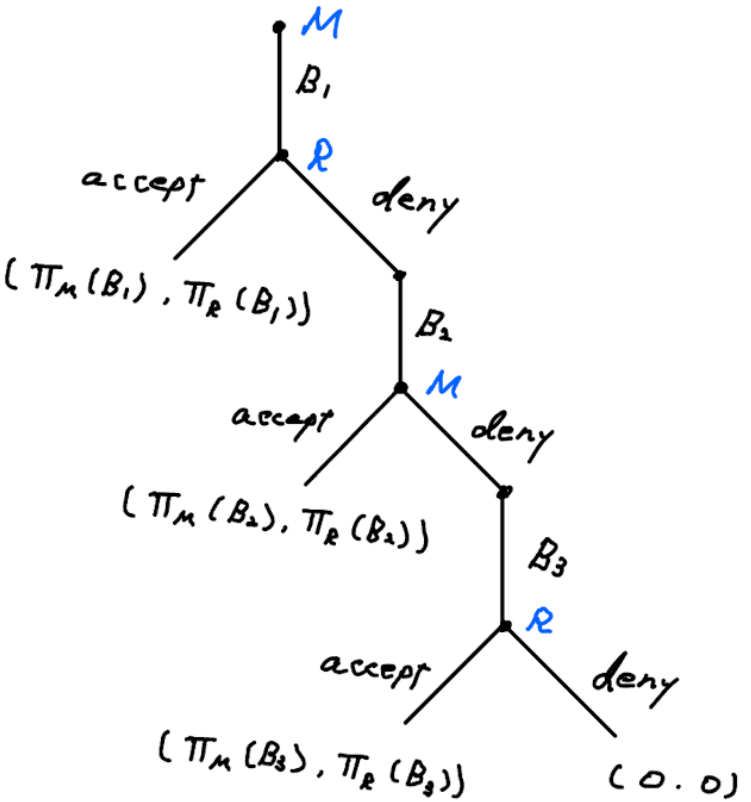
(d) Monocorp and Rocky engage in a bargaining game. They both agree to bargain on the size of a fixed payment from Rocky to Monocorp that lets Rocky watch as much cable television as he wants. Suppose that Monocorp makes the first offer, which Rqcky accepts or rejects. If Rocky rejects the offer, then in the next period Rocky makes an offer, which Monocorp accepts or rejects. If Monocorp rejects the offer, then in the next period Monocorp makes an offer. If Rocky rejects this offer, both players get a payoff of 0. The discount factor between periods is $\delta = 0.5$. Solve for the subgame perfect Nash equilibrium for this bargaining game.

Bargaining game

Rocky watches TV for $H = 10$ (by question (c)).

- Rocky's payoff: $\Pi_M(B) = (10 - 10^2/2) = 50 - B$
- Monocorp's payoff:
  $\Pi_R(B) = (10 - 10)10 - 30 + B = B - 30$

Using backward induction, let's start from the 3rd period (final period).



3rd period:

(1) Ricky's best response to Rocky's offer B

Ricky compares the payoffs from accepting $(\Pi_M(B))$ and rejecting (0).

$$br_R(B_1) = \begin{cases} \text{accept} & \text{if } B_1 < 50 \\ \text{indifferent between accepting and rejecting} & \text{if } B_1 = 50 \\ \text{reject} & \text{if } B_1 > 50 \end{cases}$$

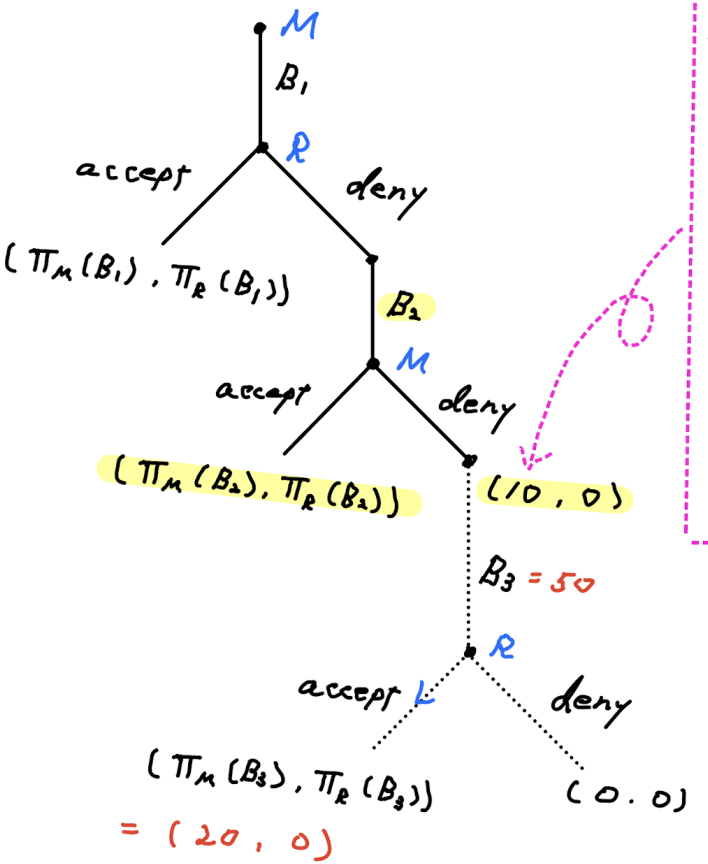
(2) Monocorp's best response to Ricky's strategy

- To offer $B_1 = 50$ (profit maximizing fixed payment).
  - or $50 - \varepsilon$

The NE for 3rd period: Monocorp offers $B_1 = 50$ and Rocky accepts.

- Monocorp's payoff: $\Pi_M(50) = 20$
- Rocky's payoff: $\Pi_R = 0$

2nd period



Why 10 ?

※ Note: M knows that if M rejects R's offer, he can gets 20 in the next period (t=3)

↳ At the current period (t=2), the value of 20 in the next period (t=3) amounts to $20 \times \delta = 20 \cdot 0.5 = 10$

By the same way as we did in 3rd period,

(1) Monocorp's best response to Rocky's offer B

- accept if the payoff $\Pi_M(B_2) \geq 20\delta = 10$, reject otherwise.

(2) Rocky's best response to Monocorp's strategy

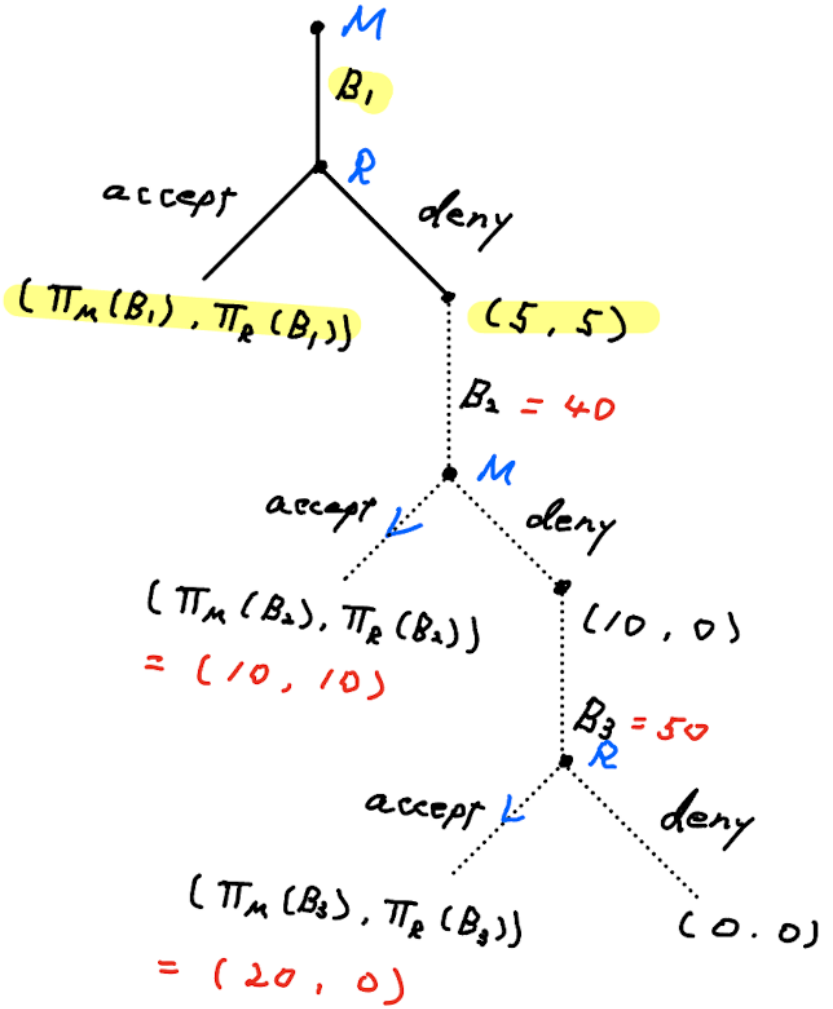
- Offer B_2 such that making Monocorp indifferent.

$\Pi_M(B_2) = 10 \rightarrow B_2 - 30 = 10 \rightarrow B_2 = 40$

The NE for 2nd period: Rocky offers $B_2 = 40$ and Monocorp accepts.

- $\Pi_M = 10$ and $\Pi_R(40) = 50 - 40 = 10$

1st period


M
B_1
R
accept
deny
$(\pi_M(B_1), \pi_R(B_1))$
(5, 5)
$B_2 = 40$
M
accept
deny
$(\pi_M(B_2), \pi_R(B_2))$
$= (10, 10)$
(10, 0)
$B_3 = 50$
R
accept
deny
$(\pi_M(B_3), \pi_R(B_3))$
$= (20, 0)$
(0, 0)

(1) Rocky's best response to Rocky's offer B

- accept if the payoff $\Pi_M(B_1) \geq 10\delta = 5$, reject otherwise.

(2) Monocorp's best response to Rocky's strategy

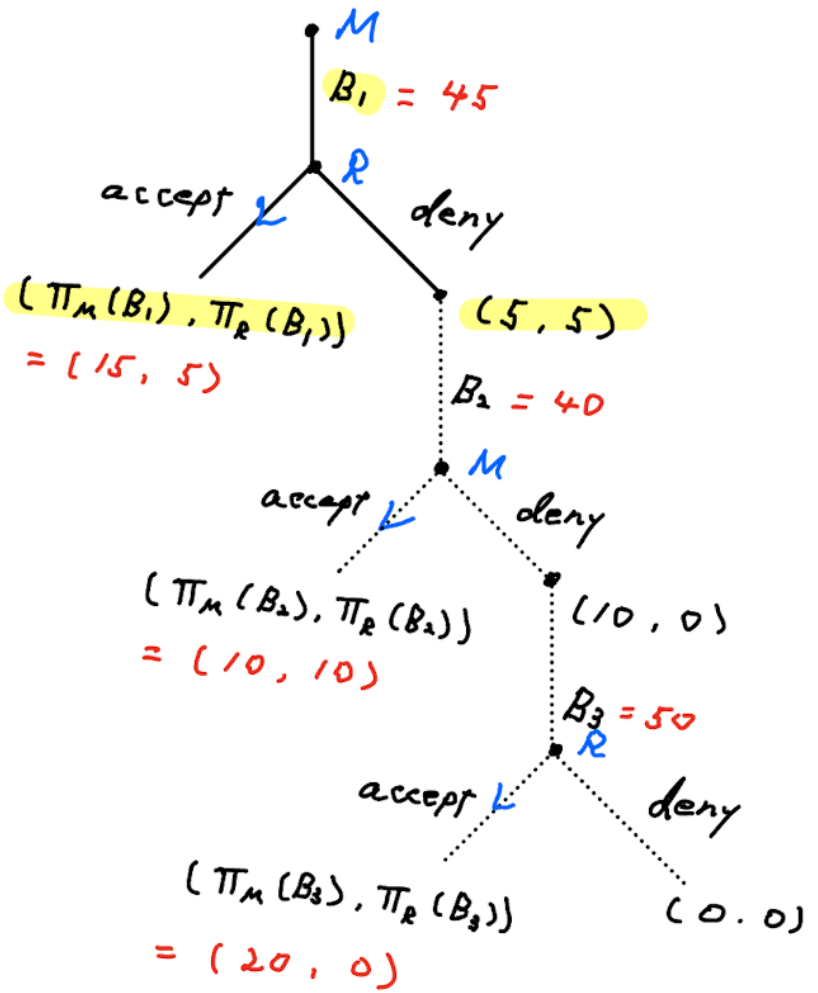
- Offer B_1 such that making Rocky indifferent.

$\Pi_R(B_1) = 5 \rightarrow 40 - B_1 = 5 \rightarrow B_1 = 45$

The NE for 2nd period: Rocky offers $B_1 = 45$ and Rocky accepts.

- $\Pi_M(45) = 45 - 30 = 15$ and $\Pi_R = 5$

1st period


M
$B_1 = 45$
R
accept
deny
$(\pi_M(B_1), \pi_R(B_1))$
$= (15, 5)$
$(5, 5)$
$B_2 = 40$
M
accept
deny
$(\pi_M(B_2), \pi_R(B_2))$
$= (10, 10)$
$(10, 0)$
$B_3 = 50$
R
accept
deny
$(\pi_M(B_3), \pi_R(B_3))$
$= (20, 0)$
$(0, 0)$

SPNE is:

- Period 1:Monocorp offers 45 and Rocky accepts.
- Period 2:Rocky offers 40 and Monocorp accepts.
- Period 3:Monocorp offers 50 and Rocky accepts.