APEC 8003 2024: Solution for Exercise Problem 1

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The Mega-Meta Corporation is a monopolist in the production of game theory software. There are N consumers of this software, $i = 1, 2, ..., N$. Each consumer i has an inverse demand curve: $P_i(Q_i) = a - Q_i$, where P_i is the price to consumer i, and Q_i is quantity sold to consumer i. Assume there is a fixed cost, F, to develop the software, but no cost for distributing the software (zero marginal cost). The cost function for Mega-Meta is: $C(Q) = F$, where $Q = \sum_{i=1}^{N} Q_i$ (note: C does not depend on Q because marginal cost is zero).

Part (c)

Suppose that Mega-Meta faces potential competition from Twenty-Cent Inc. Assume that Mega-Meta moves first and chooses a quantity to produce for each consumer (Q_{i1}). Next, Twenty-Cent decides whether or not to enter. If it enters, it pays a fixed cost of F, and then it chooses a quantity to sell to each consumer (Q_{i2}). If it doesn't enter, then Twenty-Cent earns a 0 payoff and Mega-Meta remains a monopolist.

(i) Suppose that Mega-Meta anticipates entry and produces as a Stackelberg leader and Twenty-Cent plays as a Stackelberg follower. What is the Stackelberg equilibrium?

(ii) How much would Mega-Meta need to sell to each consumer in order to convince Twenty-Cent not to enter?

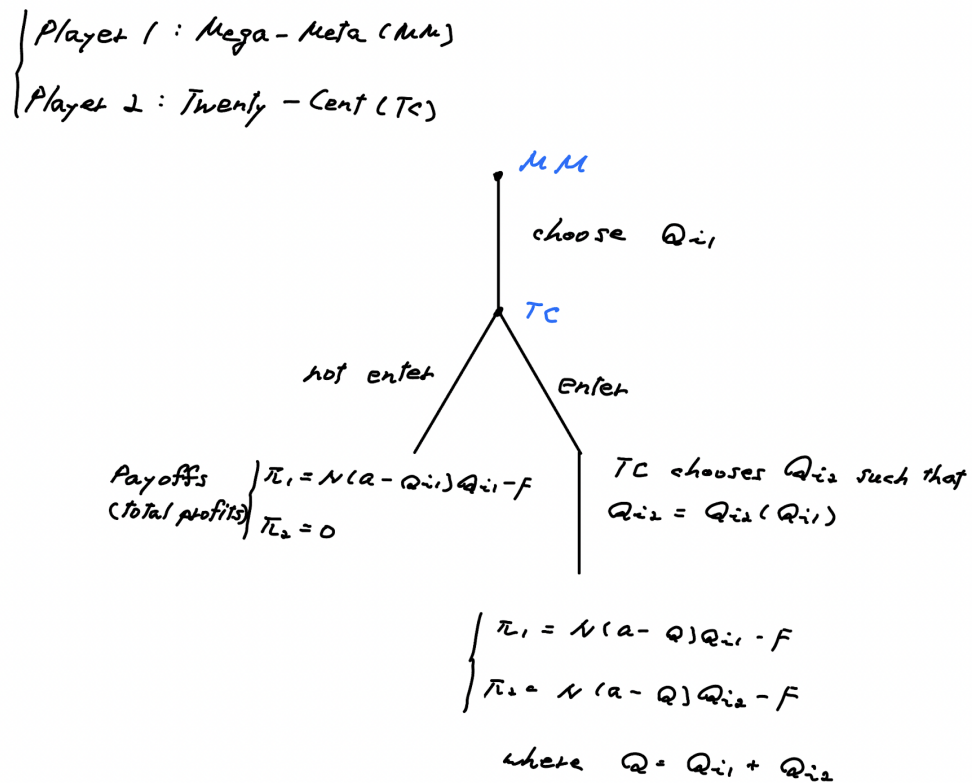(iii) Suppose that a = 12, N = 100 , and F = 400. What is the subgame perfect equilibrium in this case?

Solution is the next page

Solutions

Part (c)

The (informal) extensive form for this game looks like this:

$$\begin{cases} Player\ 1 : Mega - Meta\ (MM) \\ Player\ 2 : Twenty - Cent\ (TC) \end{cases}$$

MM

choose Q_{i1}

TC

not enter

Payoffs (Total profits) $\begin{cases} \pi_1 = N(a - Q_{i1})Q_{i1} - F \\ \pi_2 = 0 \end{cases}$

enter

TC chooses Q_{i2} such that $Q_{i2} = Q_{i2}(Q_{i1})$

$$\begin{cases} \pi_1 = N(a - Q)Q_{i1} - F \\ \pi_2 = N(a - Q)Q_{i2} - F \end{cases}$$

where $Q = Q_{i1} + Q_{i2}$

Part (c)-i

First, consider the TC's problem:

$$\max_{Q_{i2}} \quad \pi_{TC,i} = (a - Q_{i1} - Q_{i2})Q_{i2} - F$$

F.O.C.

$$\frac{\partial \pi_{TC,i}}{\partial Q_{i2}} = 0 \iff Q_{i2} = \frac{a - Q_{i1}}{2}$$

That is, TC's best response function for MM's Q_{i1} is

$$Q_{i2}(Q_{i1}) = \frac{a - Q_{i1}}{2} \quad \cdots (*)$$

Given the TC's best response function, MM's profit maximization problem is

$$\max_{Q_{i1}} \quad \pi_{MM,i} = \Big(a - Q_{i1} - Q_{i2}(Q_{i1})\Big)Q_{i2} - F$$

$$= \Big(a - Q_{i1} - \frac{a - Q_{i1}}{2}\Big) - F$$

$$= \frac{a - Q_{i1}}{2} - F$$

F.O.C.

$$\frac{\partial \pi_{MM,i}}{\partial Q_{i1}} = 0 \iff Q_{i1}^* = \frac{a}{2}$$

Substituting $Q_{i1}^* = \frac{a}{2}$ into the TC's best response function, we get

$$Q_{i2}^* = \frac{a - Q_{i1}^*}{2} = \frac{a}{4}$$

.

The Stackelberg equilibrium is:

$$Q_{i1}^* = \frac{a}{2}, \quad Q_{i2}^* = \frac{a}{4}$$

. At this equilibirium, MM earns $\Pi_1 = N(a - \frac{a}{2} - \frac{a}{4})\frac{a}{2} - F = N \cdot \frac{a^2}{8} - F$, and TC earns $\Pi_2 = N(a - \frac{a}{2} - \frac{a}{4})\frac{a}{4} - F = N \cdot \frac{a^2}{16} - F$.

Part (c)-ii

When MM chooses Q_{i1}, TC chooses $Q_{i2} = Q_{i2}(Q_{i1}) = \frac{a - Q_{i1}}{2}$ (see Equation $(*)$). Then, TC's total profit is

$$\Pi_{TC} = N(a - Q_{i1} - Q_{i2}(Q_{i1})) - F$$

$$= N(a - Q_{i1} - \frac{a - Q_{i1}}{2}) - F$$

$$= N\Big(\frac{a - Q_{i1}}{2}\Big)^2 - F$$

If TC enters, TC earns $\Pi_{TC} = N\Big(\frac{a - Q_{i1}}{2}\Big)^2 - F$. If TC does not enter, TC's profit is zero. In order to make deter TC from entering, MM nees to choose Q_{it} such that

$$N\Big(\frac{a - Q_{i1}}{2}\Big)^2 - F \le 0$$

or,

$$Q_{i1} \le a - 2\sqrt{\frac{F}{N}}$$

Part (c)-iii

Depending on MM's choice of Q_{1t}, MM can let TC enter or deter TC from entering into the industry. If MM lets TC enter the industry, MM and TC play Stackelberg competition. Meanwhile, MM can deter TC from entering by choosing $Q_{i1} \le a - 2\sqrt{\frac{F}{N}}$ as we found in part (ii). Ultimately, the choice of Q_{i1} by MM depends on the profits MM can make for each case.

(case I) Suppose that MM let YC enter the industry, and they play Stackelberg competion. From part (i), MM's profit is

$$\Pi_1 = N \cdot \frac{a^2}{8} - F = 100 \cdot \frac{12^2}{8} - 400 = 1400$$

.

(case II) Suppose that MM deter TC from entering by choosing $Q_{i1} \le a - 2\sqrt{\frac{F}{N}} = 12 - 2\sqrt{400/100} = 8$. MM's profit is

$$\Pi_1 = N(a - Q_{i1})Q_{i1} - F$$

. Π_1 is decreasing in Q_{i1}. Thus, when $Q_{i1} = 8$, MM can get the highest payoff while detering firm 2 from entering into the industry. MM profit at $Q_{i1} = 8$ is

$$\Pi_1 = 100 \cdot (12 - 8) \cdot 8 - 400 = 2800$$

, which is greater than the profit from playing Stackelberg competition.

The SPNE is such that $\{Q_{i1} = 8; not entering, Q_{i2}(Q_{i1}) = \frac{a - Q_{i1}}{2}$ with entry$\}$.

(NOTE: Be careful in answering SPNE! You need to specify players' strategy at all nodes, not just nodes on the equilibrium path. (e.g., What should TC do when entering? What should TC do when not entering?))