APEC 8003 2024: Solution for Exercise Problem 2

Shunkei Kakimoto

Example Problem 2 (Continuous strategies):

Two players are deciding how to split \$1. They simultaneously put in a bid for how much of the dollar they will receive, s_i, $i = 1, 2$. If the sum of the bids is less than or equal to \$1 then each player receives their bid. If the sum of the bids is more than \$1 then both players receive a payoff of 0.

(1) Write down the payoff function for each player as a function of strategies.

(2) Find the best response function for each player.

(3) Find all pure strategy Nash equilibria for this game.

Solutions

A set of strategies for Player i ($i = \{1, 2\}$) is $S_i \in [0, 1]$. Let $s_i \in S_i$.

Part (1)

Let v_i denote player i's payoff function ($i = \{1, 2\}$). Then,

$$v_i = \begin{cases} s_i & \text{if } s_i + s_{-i} \leq 1 \\ 0 & \text{if } s_i + s_{-i} > 1 \end{cases}$$

Part (2)

Let $br_i(s_{-i})$ denote player i's best response correspondence to the opponent's strategy s_{-i} ($i = \{1, 2\}$). If Player i-1 picks s_{-i}, player i's best response is $1 - s_{-i}$. Specifically if player i-1 picks s_{-i}=1, player i's best response is $1 - s_{-i} = 1 - 1 = 0$, which makes player 1's choice indifferent among $s_i \in [0, 1]$. Thus, i's player's ($i = \{1, 2\}$) best response correspondence can be summarized as follows:

$$br_i(s_{-i}) = \begin{cases} 1 - s_{-i} & \text{if } s_{-i} < 1 \\ [0, 1] & \text{if } s_{-i} = 1 \end{cases}$$

Part (3)

The graph below shows the best response correspondences for player 1 and player 2. The Nash equilibria are the intersections of the player 1's and player 2's best response correspondences, which are the points on the line $s_1 + s_2 = 1$ and the point $(1, 1)$. Thus, the Nash equilibria for this game are (s_1, s_2) satisfying $s_1 + s_2 = 1$ $(s_1, s_2 \in [0, 1])$ and $(s_1, s_2) = (1, 1)$.