Recitation 5: Exercise Problem for General Equilibrium

1. Consider the following 2×2 competitive exchange economy, with consumers $j = 1, 2$ and goods x and y. The consumers' preferences are given by:

$$U_1(x_1, y_1) = \ln x_1 + 2 \ln y_1 \quad \text{and} \quad U_2(x_2, y_2) = \min\{2 \ln x_2, \ln y_2\}.$$

Endowments are $\omega_1 = (3/4, 1/4)$ and $\omega_2 = (1/4, 3/4)$.

a. Derive the contract curve for this economy. Construct a carefully labeled Edgeworth-box diagram depicting the economy, including the endowment, an indifference curve for each consumer, and the contract curve.

b. Derive the two consumers' offer curves (demands). Find a Walrasian equilibrium for the economy. Show that the equilibrium allocation is also Pareto optimal. Add the equilibrium allocation and equilibrium price line to your diagram.

Solution.

a. The interior of the contract curve must be at the kinks of 1's Leontief-style indifference curves. The kinks, as always, are at the point where the two terms inside the max operator are equal. Here, that means we must have

$$2 \ln x_2 = \ln y_2,$$

which gives an inverted square function, as in Figure 1. From the perspective of 0_1, the expression for the contract curve is

$$1 - y_1 = (1 - x_1)^2 \quad \text{or} \quad y_1 = 1 - (1 - x_1)^2. \tag{1}$$

Because the curve starts and ends at the corners, the entire PO set is interior, except for the two corners of the box.

b. Combine the FONCs for 1's interior optimization problem to obtain $y = 2x(p^x/p^y)$. Insert this into the constraint to get demands

$$x_1(p, \omega_1) = \frac{1}{4} + \frac{1}{12}\frac{p^y}{p^x} \qquad \text{and} \qquad y_1(p, \omega_1) = \frac{1}{6} + \frac{1}{2}\frac{p^y}{p^x}.$$

We can write demands so that y depends on x, which is the offer curve:

$$y_1 = \frac{2x_1}{12x_1 - 3}. \tag{2}$$

Consumer 2's offer curve is a portion of the contract curve. It's the set of kinks between the values $x_2 = .25$ (bundle a in the figure, which would be demanded if $p^y = 0$) and $x_2 = \sqrt{.75}$ (bundle b in the figure, which would be demanded if $p^x = 0$).

The equilibrium is where these two cross. Combine equations (1) and (2) to get

$$\frac{2x_1}{12x_1 - 3} = 1 - (1 - x_1)^2 = 2x_1 - x_1^2,$$

which may be written $2 = (2 - x_1)(12x_1 - 3)$. Complete the square to find

$$x_1^* = \frac{9}{8} - \sqrt{\frac{81}{64} - \frac{2}{3}} \approx .351.$$

From this we can compute

$$y_1^* = \frac{2x_1^*}{12x_1^* - 3} \approx .579.$$

Equilibrium relative prices are

$$p^* = \frac{p^{x*}}{p^{y*}} \approx .824.$$

Finally, 2's allocation is

$$x_2^* \approx .649 \qquad \text{and} \qquad y_2^* \approx .421.$$

Being on the contract curve, we know the allocation is PO.

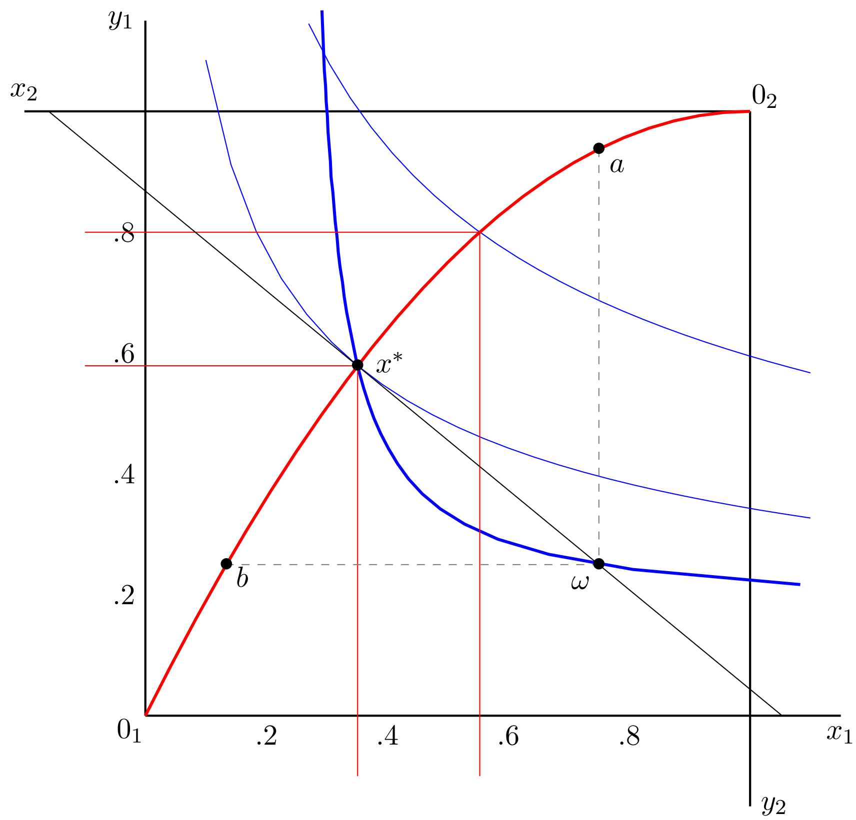

Figure 1: Edgeworth box for problem 1.