Recitation 3: Solutions for the exercise problems

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Prove that if $X \perp\!\!\!\perp e$, then $E[e|X] = 0$

Answer: Simply, $E[e|X] = E[e] = 0$ if $X \perp\!\!\!\perp e$.

or

$$\begin{aligned} E[e|X=x] &= \int_{-\infty}^{\infty} e\, f(e|X=x)\, de \\ &= \int_{-\infty}^{\infty} e\, \frac{f(e,x)}{f_X(x)}\, de \\ &= \int_{-\infty}^{\infty} e\, \frac{f_e(e) f_X(x)}{f_X(x)}\, de && \text{(becuase if } X \perp\!\!\!\perp e, f(e,x) = f_X(x) f_e(e)) \\ &= \int_{-\infty}^{\infty} e\, f_e(e)\, de \\ &= E[e] \\ &= 0 \end{aligned}$$

This holds for any x. So, E[e|X]=0

Review:
+ Conditional density function (PSE 4.8 P83-84)
+ Independence (PSE 4.11, P87)
+ Conditional expectation (PSE 4.14, P93)

Prove that if $E[e|X] = 0$, then $E[Xe] = 0$

Answer:

$$\begin{aligned} E[Xe] &= E[E[Xe|X]] && \text{(LIE)} \\ &= E[XE[e|X]] && \text{(Conditioning theorem)} \\ &= E[X \cdot 0] && \text{(because } E[e|X] = 0) \\ &= 0 \end{aligned}$$

Review:
+ Conditioning theorem: Theorem 2.3 (E P23)
+ Law of iterated expectations (PSE P95)