Recitation 2: Solutions for the exercise problems

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Prove that if $X \perp\!\!\!\perp e$, then $E[e|X] = 0$

Answer: Simply, $E[e|X] = E[e] = 0$ if $X \perp\!\!\!\perp e$.

or

$$
\begin{aligned}
E[e|X=x] &= \int_{-\infty}^{\infty} e\, f(e|X=x)\, de \\
&= \int_{-\infty}^{\infty} e\, \frac{f(e,x)}{f_X(x)}\, de \\
&= \int_{-\infty}^{\infty} e\, \frac{f_e(e) f_X(x)}{f_X(x)}\, de && \text{(becuase if } X \perp\!\!\!\perp e, f(e,x) = f_X(x) f_e(e)\text{)} \\
&= \int_{-\infty}^{\infty} e\, f_e(e)\, de \\
&= E[e] && = 0
\end{aligned}
$$

This holds for any x. So, E[e|X]=0

Review:
+ Conditional density function (PSE 4.8 P83-84)
+ Independence (PSE 4.11, P87)
+ Conditional expectation (PSE 4.14, P93)

Prove that if $E[e|X] = 0$, then $E[Xe] = 0$

Answer:

$$
\begin{aligned}
E[Xe] &= E[E[Xe|X]] && \text{(LIE)} \\
&= E[X E[e|X]] && \text{(Conditioning theorem)} \\
&= E[X \cdot 0] && \text{(because } E[e|X] = 0\text{)} \\
&= 0
\end{aligned}
$$

Review:
+ Conditioning theorem: Theorem 2.3 (E P23)
+ Law of iterated expectations (PSE P95)