Automatica 120 (2020) 109122

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Filtering for systems subject to unknown inputs without a priori initial information☆

He Kong [a,*], Mao Shan [a], Daobilige Su [a,b,1], Yongliang Qiao [a], Abdullah Al-Azzawi [a], Salah Sukkarieh [a]

[a] Australian Centre for Field Robotics, The University of Sydney (USYD), NSW, 2006, Australia
[b] College of Engineering, China Agricultural University, Beijing, China

ARTICLE INFO

Article history:
Received 20 October 2019
Received in revised form 12 April 2020
Accepted 28 May 2020
Available online 8 July 2020

Keywords:
Estimation
Arbitrary unknown input
Kalman filter
Internal model principle

ABSTRACT

The last few decades have witnessed much development in filtering of systems with Gaussian noises and arbitrary unknown inputs. Nonetheless, there are still some important design questions that warrant thorough discussions. Especially, the existing literature has shown that for unbiased and minimum variance estimation of the state and the unknown input, the initial guess of the state has to be unbiased. This clearly raises the question of whether and under what conditions one can design an unbiased and minimum variance filter, without making such a stringent assumption. The above-mentioned question will be investigated systematically in this paper, i.e., design of the filter is sought to be independent of *a priori* information about the initial conditions. In particular, for both cases with and without direct feedthrough, we establish necessary and sufficient conditions for unbiased and minimum variance estimation of the state/unknown input, independently of a priori initial conditions, respectively. When the former conditions do not hold, we carry out a thorough analysis of all possible scenarios. For each scenario, we present detailed discussions regarding whether and what can be achieved in terms of unbiased estimation, independently of a priori initial conditions. Extensions to the case with time-delays, conceptually like Kalman smoothing where future measurements are allowed in estimation, will also be presented, amongst others.

1. Introduction

Reliable state/parameter estimation is a prerequisite for closed-loop stability and satisfactory performance of control systems (Goodwin et al., 2008; Söderström, Wang, Pintelon, & Schoukens, 2013; Yuz & Salgado, 2003). Based on the Gaussian noise assumption, the KF with its many variants have become arguably the most versatile estimation framework (Gustafsson, 2000). Robustness issues of the KF and the presence of constraints in practice have impelled the development of other methods such as moving horizon estimation (Kong & Sukkarieh, 2018a, 2018b; Rao, Rawlings, & Lee, 2001). Moreover, to better handle temporary uncertainties in estimation and identification, limited memory techniques such as frequency-sampling filters (see, e.g., Wang & Cluett, 1997) and finite impulse response filters (see, e.g., Nagahara & Yamamoto, 2014; Shmaily, Zhao, & Ahn, 2017) have been developed. Despite existing methods' versatility, their performance might still be impoverished under unmodeled dynamics. If models or statistical properties for the biases/disturbances are available, they can be included into the original system model so that existing methods can be applied for the composite system.

However, in many cases, it might be hard to obtain precise models or statistical properties for the unmodeled dynamics. Typical examples are actuator/sensor faults (see, e.g., Argha, Su, & Celler, 2019; Cristofaro & Johansen, 2014; Duan & Wu, 2006; Duan, 2010, Chap. 11 and the references therein for more thorough discussions), and the case with abrupt disturbances (see the enlightening insights in Ohlsson, Gustafsson, Ljung, & Boyd, 2012). Other relevant examples include soft robotics (Al-Azzawi, Boudali, Kong, Gö ktoğan, & Sukkarieh, 2019) or advanced vehicle applications (Imsland, Johansen, Grip, & Fossen, 2007; Shan, Worrall, & Nebot, 2015) where the applied forces or acceleration are unmeasured and can change arbitrarily due to interactions with the environment, and networked control systems with attacks or data dropouts as considered rigorously in the recent literature (Li, Liu, Zhong, & Ding, 2018; Li, Quevedo, Dey, & Shi, 2017; Li, Shi,

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Juan I. Yuz under the direction of Editor Torsten Söderström.

* Corresponding author.
E-mail addresses: h.kong@acfr.usyd.edu.au (H. Kong), m.shan@acfr.usyd.au (M. Shan), sudao2020@outlook.com (D. Su), y.qiao@acfr.usyd.edu.au (Y. Qiao), a.alazzawi@acfr.usyd.edu.au (A. Al-Azzawi), salah.sukkarieh@sydney.edu.au (S. Sukkarieh).

1 Daobilige Su was with USYD, and he is now with China Agricultural University.

https://doi.org/10.1016/j.automatica.2020.109122

Table 1
The case with direct feedthrough: different scenarios with associated guidelines for unbiased filter design, independent of a priori initial information ($k = 0$ is the initial time step).

Scenario		Real-time estimation using (1)	With time-delays using (30)
(2)–(3), (20) all hold		state/input for $k \geq 0$	–
(2)–(3) hold (20) fails	case (a)	state/input for $k \geq 1$	state/input for $k \geq 1$
	case (b)	possibly state for $k \geq 0$	only state for $k \geq 0$
	case (c)	possibly input for $k \geq 0$	state/input for $k \geq 1$
	case (d)	neither state/input	only state for $k \geq 0$
(2) holds (3) holds for $z \in C$, (20) fails	case (a)	state/input for $k \geq 1$	state/input for $k \geq 0$
	case (c)	only input for $k \geq 0$	

Cheng, Chen, & Quevedo, 2015; Marelli, Sui, Rohr, & Fu, 2019; Wu, Li, Quevedo, & Shi, 2017). However, without precise models/properties for the unmodeled dynamics, accurate estimation is still necessary for control and system monitoring purposes.

Hence, estimation under arbitrary unknown inputs, also called unknown input decoupled estimation, has received much attention in the past decades. The most seminal work in the literature, pioneered by Hautus (Hautus, 1983), amongst others, shows that the strong[2] detectability requirements, i.e., (1) a rank matching condition; (2) the system is minimum phase, are necessary and sufficient for stable estimation of the full state. The work in Hautus (1983) is pivotal to the field of estimation with unknown inputs, and has inspired much of its development ever since.

Especially, works that appeared after Hautus (1983), e.g., Darouach and Zasadzinski (1997), Gillijns and De Moor (2007a), Kitanidis (1987) and Su, Li, and Chen (2015) for the case without feedthrough, and Darouach, Zasadzinski, and Boutayeb (2003), Fang and de Callafon (2012) and Gillijns and De Moor (2007b) for the case with feedthrough, have similar rank matching and system being minimum phase requirements. Generalizations to the cases with rank-deficient shaping matrices have been discussed in Cheng, Ye, Wang, and Zhou (2009), Hsieh (2009a) and Yong, Zhu, and Frazzoli (2016). Note, however, most of the above-mentioned works make certain forms of strong detectability assumptions, which are stringent. As summarized in Kong and Sukkarieh (2019), there exist a few routes to relax the strong detectability conditions. They include building a functional estimator (instead of a full order one), simplifying the problem by assuming the unknown inputs to be only affecting the state, and estimation with time delays by allowing future measurements (this is conceptually like Kalman smoothing, see, e.g., Ansari & Bernstein, 2019; Hsieh, 2009b; Jin, Tank, & Park, 1997).

However, for the general case with feedthrough, when the system model is not strong detectable, whether and under what conditions it is still possible to obtain real-time and asymptotically stable estimation of the full state/unknown input, has remained as a long-open problem. Recently, we have filled the above gap via an internal model approach (IMA) in Kong and Sukkarieh (2019). If we look at estimation from the reference tracking perspective (for the estimator), when the system model is not strong detectable, the estimation/tracking error dynamics cannot be decoupled from the unknown inputs. In this case, to render zero offset, one has to find a way of achieving *integral action*, as a classical result in systems and control (see, e.g., Goodwin, Kong, Mirzaeva, & Seron, 2014; Yuz & Salgado, 2003). The IMA in Kong and Sukkarieh (2019) allows us to do so by introducing an augmented model, incorporating the unknown input sum model (an integrator driven by the unknown input) into the original model, and designing the augmented model to be strong detectable.

Despite the above progress, there are still important questions that warrant thorough discussions. Especially, the existing literature has shown that for real-time unbiased and minimum variance estimation, the initial guess of the state has to be unbiased. This stringent requirement has to hold for most methods mentioned above. Although there exist works on KF without a priori information on initial conditions (see Zhao, Huang, & Liu, 2017 and the references therein), they are only applicable for the case without unknown inputs. This raises the question of whether and under what conditions one can design an unbiased and minimum variance filter for systems with unknown inputs, independently of *a priori* initial information.

The above question will be investigated systematically in this paper. Our contributions are stated as follows. Firstly, for both cases with and without direct feedthrough, we establish necessary and sufficient conditions (see Theorems 2–3) for unbiased and minimum variance estimation of the state/unknown input, independently of a priori initial conditions, respectively. Secondly, when the former conditions do not hold, we carry out a thorough analysis of all possible scenarios. For each scenario, we present detailed discussions regarding whether and what can be achieved in terms of unbiased estimation, independently of a priori initial conditions. To be more specific, we show that for some scenarios, if the system model satisfies certain conditions, it is still possible to partially obtain unbiased state and/or unknown input estimation, independently of a priori initial conditions (see in Lemmas 1–3 and Lemma 4 for detailed discussions for the cases with and without direct feedthrough, respectively.) Extensions to the case with time-delays, similarly as the smoothing scenario with future measurement information, will also be presented (see Corollaries 1–2).

Thirdly, the former results are generalized to the case with additional partially observed information on the unknown input (see Corollary 3). For clarity, most of the above-mentioned results are summarized in Tables 1 and 2, for the case with or without direct feedthrough, respectively. Last but not the least, we discover some further implications of the obtained conditions for the IMA approach. In particular, we obtain a negative result (see Corollary 4), i.e., if the system does not satisfy the above conditions under which an unbiased filter can be designed independently of a priori initial conditions, then it is impossible to use IMA approach of Kong and Sukkarieh (2019) to alleviate the former requirements. Justification and explanations of the above negative results are also given.

The reminder of the paper is structured as follows. In Section 2, we recall some preliminaries on estimation under unknown inputs. Section 3 contains the first major result of the paper. For the general case with direct feedthrough, we will establish a necessary and sufficient condition for unbiased and minimum variance estimation of the state/unknown input, independently of a priori initial conditions. We also conduct a thorough analysis of all possible scenarios when the former condition fails. Extensions to the case with time-delays will also be presented. A systematic analysis for the case without

[2] The strong* detectability concept was also introduced in Hautus (1983). The two criteria, as discussed in Hautus (1983), are equivalent for discrete-time systems, but differ for continuous systems.

Table 2
The case without direct feedthrough: different scenarios with associated guidelines for unbiased filter design, independent of a priori initial information ($k = 0$ is the initial time step).

Scenario		Real-time estimation using (1)	With time-delays using (37)
(2)–(3), (35) all hold		both state/input	–
(2)–(3) hold (35) fails	case (e) case (f) case (g)	possibly input for $k \geq 0$ (using (31)–(33)) neither state or input (using (31)–((33))) neither state or input (using (31)–((33)))	possibly state using a filter in the form of (4)–(6)
(2)–(3) hold $rank(C) = n$ (38) fails	case (h)	neither state or input (using (31)–(33))	possibly state using a filter in the form of (4)–(6)

direct feedthrough, and extensions to the case with additional partially observed information, will be presented in Section 4. Further implications of the obtained results and discussions will be presented in Section 5. Numerical examples are presented in Section 6. Section 7 concludes the paper.

Notation. We use A^{T} to denote the transpose of matrix A. $\mathbf{R}^n$ stands for the n-dimensional Euclidean space. I_n stands for identity matrices of n dimensions. $\mathbb{C}$, $|z|$ denotes the field of complex numbers, and the absolute value of a given complex number z. $[a_1, \ldots, a_n]$ denotes $[a_1^{\mathrm{T}} \cdots a_n^{\mathrm{T}}]^{\mathrm{T}}$, where $a_1, \ldots, a_n$ are scalars/vectors/matrices with proper dimensions. $\mathscr{E}(w)$ denotes the expectation operation. Vectors/matrices, with dimensions not explicitly stated, are assumed to be algebraically compatible.

2. Preliminaries and problem statement

Consider physical systems represented by the following discrete-time linear time-invariant (LTI) model

$$\begin{cases} x_{k+1} = Ax_k + Bd_k + Gw_k \\ y_k = Cx_k + Dd_k + v_k \end{cases}, \tag{1}$$

where, $x_k \in \mathbf{R}^n$, $d_k \in \mathbf{R}^q$, $y_k \in \mathbf{R}^p$, represent the state, the unknown input, and the output, respectively; $w_k \in \mathbf{R}^g \sim \mathcal{N}(0, Q)$ and $v_k \in \mathbf{R}^p \sim \mathcal{N}(0, R)$ represent zero mean Gaussian process and measurement noises with covariances $Q \geq 0$ and $R > 0$, respectively; A, B, G, C, and D are real and known matrices; the pair (A, C) is assumed to be detectable. Without loss of generality, known control input information has been omitted. For systems with unknown inputs, a fundamental question is the existence condition and design of an estimator with stable estimation error. To address this question, concepts such as strong detectability and strong estimator have been introduced and thoroughly discussed in Hautus (1983). As remarked in Hautus (1983), the term "strong" is to emphasize that stable estimation has to be achieved without any knowledge of the unknown input. As mentioned earlier, in both the original results of Hautus (1983) for the deterministic case and their later extensions to the stochastic case, the system model (1) has to satisfy the strong detectability requirements, which are summarized as follows.

Theorem 1 (*Hautus, 1983*)**.** *The system (1) has a strong estimator if and only if*

$$rank \begin{bmatrix} CB & D \\ D & 0 \end{bmatrix} = rank(D) + rank \begin{bmatrix} B \\ D \end{bmatrix}, \tag{2}$$

and all its invariant zeros are stable, i.e.,

$$rank \underbrace{\begin{bmatrix} zI_n - A & -B \\ C & D \end{bmatrix}}_{\Lambda(z)} = n + rank \begin{bmatrix} B \\ D \end{bmatrix}, \tag{3}$$

for all $z \in \mathbb{C}$ and $|z| \geq 1$.

Conditions (2)–(3) are the so-called rank matching and minimum phase requirements, respectively. The results in Theorem 1 hold for both the deterministic and stochastic cases (this is why we use "estimator" instead of filter/observer therein). In the sequel, wherever appropriate, we will simply refer to (1) as the considered system, while bearing in mind that it is a model intended to represent physical systems of interest.

Problem 1. Given system (1), present conditions under which an unbiased and minimum variance KF can be designed, independently of a priori initial information.

Note that if the filter is designed to be stable, the estimates will become asymptotically unbiased, i.e., the effects of the filter initialization, be it accurate or not, will diminish asymptotically. However, improper initializations can lead to very poor transient performance, which can be avoided by considering Problem 1.

3. Filter design without requiring initial condition information—The general case with direct feedthrough

This section contains the major solutions to Problem 1 for the general case with feedthrough. These results will be extended to the case without feedthrough in Section 4. For the case with feedthrough, we adopt the framework of Gillijns and De Moor (2007b) for the filter design. Other methods in Darouach et al. (2003), Cheng et al. (2009), Hsieh (2009a) and Yong et al. (2016), can be considered similarly and we will not elaborate these extensions further. To be specific, the proposed filter implements the following steps recursively after initialization:

1. Unknown input estimation:
$$\widehat{d}_k = M(y_k - C\widehat{x}_{k|k-1}); \tag{4}$$
2. Measurement update:
$$\widehat{x}_{k|k} = \widehat{x}_{k|k-1} + L(y_k - C\widehat{x}_{k|k-1}); \tag{5}$$
3. Time update:
$$\widehat{x}_{k+1|k} = A\widehat{x}_{k|k} + B\widehat{d}_k. \tag{6}$$

In the above filter, the gain matrices $M \in \mathbf{R}^{q\times p}$ and $L \in \mathbf{R}^{n\times p}$ are to be designed. Define

$$\begin{aligned} &\widetilde{y}_k = y_k - C\widehat{x}_{k|k-1}; \\ &\widetilde{d}_k = d_k - \widehat{d}_k,\ P_k^d = \mathscr{E}(\widetilde{d}_k\widetilde{d}_k^{\mathrm{T}}); \\ &\widetilde{x}_{k|k} = x_k - \widehat{x}_{k|k},\ P_{k|k}^x = \mathscr{E}(\widetilde{x}_{k|k}\widetilde{x}_{k|k}^{\mathrm{T}}); \\ &P_k^{xd} = \mathscr{E}(\widetilde{x}_{k|k}\widetilde{d}_k^{\mathrm{T}});\ \widetilde{x}_{k+1|k} = x_{k+1} - \widehat{x}_{k+1|k}, \\ &P_{k+1|k}^x = \mathscr{E}(\widetilde{x}_{k+1|k}\widetilde{x}_{k+1|k}^{\mathrm{T}}), \end{aligned} \tag{7}$$

as the innovation, the unknown input estimation error, the filtered state error, and the state prediction error, and their covariance/variances, respectively. Assume $x_0 \sim \mathcal{N}(\overrightarrow{x}_0, P_0)$.

3.1. Unbiased and minimum variance estimation without a priori initial state information

We firstly present some preparatory results associated with the filter in (4)–(6). As most existing works, we assume the initial state guess of (1) to be unbiased, and summarize the key properties of the corresponding optimal filter. We choose to do so for completeness and also for comparing with the solutions to Problem 1 later.

Proposition 1. *Suppose system (1) satisfies conditions (2)–(3), and the initial state guess used to initialized the filter (4)–(6) is unbiased, i.e.,* $\widehat{x}_{0|-1} = \mathscr{E}(x_0) = \overrightarrow{x}_0$. *Then the following results hold true:*

(i) the filter in (4)–(6) is unbiased for both state and unknown input estimation if and only if M and L satisfy

$$[M, L]\, D = \left[I_q, 0\right]; \tag{8}$$

(ii) there exist M and L such that (8) holds if and only if

$$rank(D) = q; \tag{9}$$

(iii) assume (9) holds, the unbiased and minimum variance filter gains in (4)–(6) are given by

$$M_* = (D^{\mathrm{T}}\widetilde{R}^{-1}D)^{-1}D^{\mathrm{T}}\widetilde{R}^{-1},\ L_* = K_*(I - DM_*), \tag{10}$$

with $K_* = P^x_{k|k-1}C^{\mathrm{T}}\widetilde{R}^{-1}$ *and*

$$\widetilde{R} = R + CP^x_{k|k-1}C^{\mathrm{T}} > 0; \tag{11}$$

the dynamics of $\widetilde{x}_{k+1|k}$ *is*

$$\widetilde{x}_{k+1|k} = A_c\widetilde{x}_{k|k-1} + \widetilde{G}\underbrace{[w_k, v_k]}_{\widetilde{w}_k}, \tag{12}$$

where,

$$A_c = A - (AL_* + BM_*)C, \tag{13}$$

$\widetilde{G} = \begin{bmatrix} G & -(AL_* + BM_*) \end{bmatrix}$, *with variance satisfying the algebraic Riccati equation (ARE)*

$$P^x_{k+1|k} = GQG^{\mathrm{T}} + \overrightarrow{A}\begin{bmatrix} P^x_{k|k} & P^{xd}_k \\ (P^{xd}_k)^{\mathrm{T}} & P^d_k \end{bmatrix}\underbrace{\begin{bmatrix} A^{\mathrm{T}} \\ B^{\mathrm{T}} \end{bmatrix}}_{\overrightarrow{A}^{\mathrm{T}}}, \tag{14}$$

where,

$$P^d_k = (D^{\mathrm{T}}\widetilde{R}D)^{-1},\ P^{xd}_k = -K_*DP^d_k,$$
$$P^x_{k|k} = P^x_{k|k-1} - K_*(\widetilde{R} - DP^d_kD^{\mathrm{T}})K^{\mathrm{T}}_*,$$

with $\widetilde{R}$ defined in (11);

(iv) assume (9) holds, and the gain matrices M_* and L_* are selected as in (10), we then have A_c in (13) is Schur stable, i.e., the error variance $P^x_{k+1|k}$ in (14) is bounded.

Proof. The proof follows similarly with those in Fang and de Callafon (2012) and Gillijns and De Moor (2007b), and is included here for completeness and later use. (i). From (1), (4), (7), one has

$$\widetilde{d}_k = (I_q - MD)d_k - MC\widetilde{x}_{k|k-1} - Mv_k. \tag{15}$$

When $\widehat{x}_{k|k-1}$ is unbiased, it holds that $\mathscr{E}(\widetilde{x}_{k|k-1}) = 0$. Hence, $\widehat{d}_k$ is unbiased, i.e.,

$$\mathscr{E}(\widetilde{d}_k) = (I_q - MD)\mathscr{E}(d_k) = 0,$$

if and only if M satisfies (8). Similarly, from (1), (5), (7), we have

$$\widetilde{x}_{k|k} = (I_n - LC)\widetilde{x}_{k|k-1} - LDd_k - Lv_k. \tag{16}$$

Thus, given $\widehat{x}_{k|k-1}$ is unbiased, $\widehat{x}_{k|k}$ is unbiased for any d_k, i.e.,

$$\mathscr{E}(\widetilde{x}_{k|k}) = LD\mathscr{E}(d_k) = 0,$$

if and only if L satisfies (8). Therefore, when $\widehat{x}_{0|-1}$ is unbiased and (8) holds, unbiasedness of $\widehat{x}_{k|k-1}$, $\widehat{d}_k$, and $\widehat{x}_{k|k}$ can be obtained simply by induction. (ii). From the solution properties of matrix equations (Laub, 2005, chap. 6), there always exist M and L such that (8) holds if and only if condition (9) holds. (iii). To design the optimal filter, we need to select M and L to minimize the estimation variances, subject to (8). The prediction error dynamics and variances can also be derived. Details on how to do so can be found in Gillijns and De Moor (2007b) and Fang and de Callafon (2012), and are skipped here. (iv). When (9) holds, M and L are selected as in (10), we have (8), which further leads to

$$B = \overrightarrow{A}\left[0, I_q\right] = \overrightarrow{A}\,[L_*, M_*]\,D = (AL_* + BM_*)D, \tag{17}$$

with $\overrightarrow{A}$ defined in (14). When condition (3) holds, based on (17), we have

$$\Lambda(z) \sim \underbrace{\begin{bmatrix} zI_n - A_c & 0 \\ C & D \end{bmatrix}}_{\overline{\Lambda}(z)},$$

with A_c defined in (13). Given $\Lambda(z)$ and $\overline{\Lambda}(z)$ are similar to each other, we have $rank(\overline{\Lambda}(z)) = n + q$, for $z \in \mathbb{C}$ and $|z| \geq 1$. For $rank(\overline{\Lambda}(z))$ to be full column rank, it is necessary that $rank(zI_n - A_c) = n$, for $z \in \mathbb{C}$ and $|z| \geq 1$. In fact, if $rank(zI_n - A_c) < n$, $\overline{\Lambda}(z)$ might lose rank, without making further assumptions on C and D. In other words, A_c in (13) is Schur stable, i.e., the variance $P^x_{k+1|k}$ in (14) is bounded. This completes the proof. ■

Some extra minor requirements can be made to render the optimal filter asymptotic time invariant. We will not elaborate these issues further in the remainder of the paper. Define

$$\widetilde{C} = \begin{bmatrix} C & D \end{bmatrix}, S_1 = \begin{bmatrix} 0_{q\times n} & I \end{bmatrix}, S_2 = \begin{bmatrix} I & 0_{n\times q} \end{bmatrix}. \tag{18}$$

As solutions to Problem 1, we have the following design conditions and properties for the filter (4)–(6), independently of a priori initial information.

Theorem 2. *Given system (1) and the filter in (4)–(6). The following results hold true:*

(i) the filter in (4)–(6) is unbiased for both state and unknown input estimation, independently of a priori initial information, if and only if M and L satisfy

$$[M, L]\,\widetilde{C} = [S_1, S_2]\,, \tag{19}$$

with $\widetilde{C}$, S_1, and S_2 defined above the current theorem;

(ii) there exist M and L such that (19) holds if and only if

$$rank(\widetilde{C}) = n + q; \tag{20}$$

(iii) assume (20) holds, the unbiased and minimum variance filter gains in (4)–(6) are given by

$$M^* = S_1\Delta,\ L^* = S_2\Delta, \tag{21}$$

where,

$$\Delta = (\widetilde{C}^{\mathrm{T}}R^{-1}\widetilde{C})^{-1}\widetilde{C}^{\mathrm{T}}R^{-1};$$

the prediction error dynamics is in the same form with that in (12)–(13) (with expressions of M^* and L^* in (21)), and the variance $\widetilde{P}^x_{k+1|k}$ remains constant and satisfies an ARE in the same form of (14), with

$$\widetilde{P}^d_k = S_1(\widetilde{C}^{\mathrm{T}}R^{-1}\widetilde{C})^{-1}S^{\mathrm{T}}_1,\ \widetilde{P}^{xd}_k = L^*R(M^*)^{\mathrm{T}},$$
$$\widetilde{P}^x_{k|k} = S_2(\widetilde{C}^{\mathrm{T}}R^{-1}\widetilde{C})^{-1}S^{\mathrm{T}}_2;$$

(iv) assume (20) holds, and the gain matrices M_* and L_* are selected as in (21), the variance $\widetilde{P}^x_{k+1|k}$ is bounded.

Proof. The proof follows similarly as that of Proposition 1. (i). From (1), (4), (7), and (15), it can be obtained that $\widehat{d}_k$ is unbiased, independently of whether $\widehat{x}_{k|k-1}$ is unbiased or not (note that $\mathscr{E}(\widetilde{x}_{k|k-1}) = 0$ if and only if $\widehat{x}_{k|k-1}$ is unbiased), if and only if M satisfies (19). Similarly, from (16), $\widehat{x}_{k|k}$ is unbiased, independently of whether $\widehat{x}_{k|k-1}$ is unbiased or not, if and only if L satisfies (19). Parts (ii)–(iv) can be proved as those in Proposition 1. ■

Note that condition (20) is sufficient for (2) –(3), but not versa. This means, there exist systems that satisfy conditions (2)–(3) but not (20). The condition (20) is conceptually equivalent to requiring the error dynamics matrix to be deadbeat and null (i.e., its eigenvalues are all at the origin) in filter/observer design. This makes intuitive sense in that if (20) holds, from y_0, one could derive the unbiased and minimum variance estimation of the true initial state x_0 and the unknown input d_0, which can then be used for future steps of the filter (4)–(6), as if the designed filter is independent of a priori initial information.

From Theorem 2, a consequence of the unbiased estimation requirement, independently of a priori initial information, is that the optimal filter gains M^*, L^* in (21), and the covariance $\widetilde{P}^x_{k+1|k}$ become constant and independent of the initialization error covariance $P^x_{0|-1}$. This is in contrast with Proposition 1, where, if the filter is not initialized with the steady solution to the ARE (14), the optimal gains M_*, L_* in (10), and $P^x_{k+1|k}$ in (14) are time-varying and affected by the initialization error, until convergence to steady state.

3.2. The case when $\widetilde{C}$ is rank deficient

The preceding discussions focus on joint unbiased state/unknown input estimation, without a priori initial information. However, conditions in Theorem 2 are restrictive. When the former conditions do not hold, one naturally wonders whether and under what conditions unbiased state/unknown input estimation can still be archived, without a priori initial information. A similar question of how one can still perform unbiased state estimation, when $rank(D) < q$, has been considered in Hsieh (2009a). While our motivation here is in close spirit to that of Hsieh (2009a), here we are mainly concerned with unbiased estimation, independent of a priori initial information.

Note that when (20) fails, estimation of state/unknown input would both be biased in general. However, as we will show, unbiased estimation, independent of a priori initial information, is still *partially* possible, if non-constant filter gains are designed. For simplicity, we will only discuss existence conditions and design guidelines of unbiased state/unknown input filter, and skip the derivation of optimal filter gains and analysis of the estimation error variance. When (20) fails, there are four potential cases:

$$\begin{aligned}
&\text{(a) } rank(C) = n,\ rank(D) = q, \text{ but (20) fails;}\\
&\text{(b) } rank(C) = n,\ rank(D) < q;\\
&\text{(c) } rank(C) < n,\ rank(D) = q;\\
&\text{(d) } rank(C) < n,\ rank(D) < q.
\end{aligned} \tag{22}$$

For case (a) in (22), when $k = 0$, we choose the gains of the filter (4)–(6) as

$$M_0C = 0,\ L_0C - I_n = 0, \tag{23}$$

such that

$$\mathscr{E}(\widetilde{d}_0) = (I_q - M_0D)\mathscr{E}(d_0),\ \ \mathscr{E}(\widetilde{x}_{0|0}) = -L_0D\mathscr{E}(d_0).$$

For $k \geq 1$, we choose the gains of the filter (4)–(6) to be constant as

$$I_q - M_cD = 0,\ L_cD = 0. \tag{24}$$

Given $rank(C) = n$, $rank(D) = q$, there exist M_0 and L_0 such that conditions (23)–(24) hold.

Lemma 1. *For case (a) in (22), assume the filter gains of (4)–(6) are specified in (23)–(24). Then the following statements hold true:*

(i) if M_0 and L_0 in (23) satisfy

$$B(I_q - M_0D) - AL_0D = 0, \tag{25}$$

then the filter (4)–(6) is an unbiased state/unknown input filter for $k \geq 1$, independently of a priori initial information;

(ii) denote

$$\begin{aligned}
X_1 &= BD^+ + W_1(I_p - DD^+),\\
X_2 &= HC^+ + W_2(I_p - CC^+),
\end{aligned} \tag{26}$$

where, $H = [I_n, 0]$, D^+ and C^+ are Moore–Penrose inverse of D and C, respectively, $W_1 \in \mathbf{R}^{n\times p}$ and $W_2 \in \mathbf{R}^{(n+q)\times p}$ are some free matrix parameters. Then the existence of M_0 and L_0 such that (23) and *(25) hold is equivalent to the existence of W_1 and W_2 such that*

$$\overrightarrow{A}X_2 = X_1.$$

Proof. (i). We prove the first part by induction. When $k = 1$, if (25) holds, from (4)–(6), (23)–(24), we have $\mathscr{E}(\widetilde{x}_{1|0}) = 0$, $\mathscr{E}(\widetilde{d}_1) = 0$, and $\mathscr{E}(\widetilde{x}_{1|1}) = 0$. Similarly, for $k = 2$, we have $\mathscr{E}(\widetilde{x}_{2|1}) = 0$, $\mathscr{E}(\widetilde{d}_2) = 0$, $\mathscr{E}(\widetilde{x}_{2|2}) = \mathscr{E}(\widetilde{x}_{2|1}) = 0$, and so on, for $k \geq 3$. Hence, unbiasedness of the filter is proved. (ii). Denote

$$\overline{M}_0 = [L_0, M_0]\,.$$

The relationships in (23) and (25) can be formulated as

$$\overline{M}_0C = H \text{ and } \overrightarrow{A}\,\overline{M}_0D = B,$$

where, $\overrightarrow{A}$ and H are defined in (14) and (26), respectively. Hence, existence of $\overline{M}_0$ satisfying the above two equations is equivalent to the existence of $X = diag(X_1, X_2)$ and $\overline{M}_0$ such that

$$X\begin{bmatrix} D & 0\\ 0 & C\end{bmatrix} = \begin{bmatrix} B & 0\\ 0 & H\end{bmatrix},\ \ \overrightarrow{A}\,\overline{M}_0 = X_1,\ \ \overline{M}_0 = X_2.$$

where, X_1, and X_2 are defined in (26). The first equation in the above always holds given the fact that both D and C are full column rank. Also, X_1 and X_2 can be parameterized as in (26) (Laub, 2005, chap. 4). The rest of the proof follows naturally. ■

For case (b) in (22), for $k \geq 0$, we choose the gains of the filter (4)–(6) as

$$MC = 0,\ L\widetilde{C} = \begin{bmatrix} I_n & 0_{n\times q}\end{bmatrix}, \tag{27}$$

such that $\mathscr{E}(\widetilde{d}_0) = (I_q - MD)\mathscr{E}(d_0)$; $\mathscr{E}(\widetilde{x}_{0|0}) = 0$.

Lemma 2. *For case (b) in (22), assume the filter gains of (4)–(6) are specified in (27). Then the following statements hold true:*

(i) the filter (4)–(6) is an unbiased state filter (with biased unknown input estimation), independently of a priori initial information;

(ii) there exist M and L such that (27) holds if and only if

$$rank(\widetilde{C}) = n + rank(D). \tag{28}$$

Proof. (i). Unbiasedness for state estimation can be proved similarly as in Lemma 1. The biasedness of unknown input estimation is obvious. (ii). Note that, there always exists M such that $MC = 0$, because C is full column rank. There exists L such that $L\widetilde{C} = \begin{bmatrix} I_n & 0_{n\times q}\end{bmatrix}$ if and only if

$$rank\begin{bmatrix} I_n & 0_{n\times q}\\ C & D\end{bmatrix} = n + rank(D) = rank(\widetilde{C}).$$

This completes the proof. ■

Following the above discussions, for case (c) in (22), for $k \geq 0$, we choose the gains of the filter (4)–(6) as

$$M\widetilde{C} = \begin{bmatrix} 0_{q\times n} & I_q \end{bmatrix}, LD = 0, \tag{29}$$

such that $\mathscr{E}(\widetilde{d}_0) = 0$; $\mathscr{E}(\widetilde{x}_{0|0}) = (I - LC)\mathscr{E}(\widetilde{x}_{0|-1})$.

Lemma 3. *For case (c) in (22), assume the filter gains of (4)–(6) are specified in (29). Then the following statements hold true:*

(i) the filter (4)–(6) is an unbiased unknown input filter (with biased state estimation), independently of a priori initial information;

(ii) there exist M and L such that (29) holds there if and only if

$$rank(\widetilde{C}) = rank(C) + q.$$

Proof. The proof is similar to that of Lemma 2. ■

Lemmas 1–3 reveal the possibility and the associated conditions (there might be alternative routes) for designing the filter (4)–(6) to be partially unbiased (for the state and/or unknown input), independently of a priori initial information. From Lemmas 1–3, our intuition is that it is hard, if possible, to do so, for case (d) in (22).

3.3. The case with time-delays

The discussions in the above focus on filter design conditions guaranteeing real-time unbiased estimation independently of a priori initial information. Corresponding conditions in Theorem 2 are very restricted. Requirements in Lemmas 1–3 might not hold for (1). This raises the question whether the above conditions can be alleviated, guaranteeing unbiased estimation independently of a priori initial information, possibly by allowing time-delayed, instead of, real-time, estimates. Such a question will be investigated next. Denote

$$\mathbf{y}_k^{k+s} = [y_k, \dots, y_{k+s}],\ \mathbf{w}_k^{k+s-1} = [w_k, \dots, w_{k+s-1}],$$
$$\mathbf{d}_k^{k+s} = [d_k, \dots, d_{k+s}],\ \mathbf{v}_k^{k+s} = [v_k, \dots, v_{k+s}].$$

For (1), if future measurement information is allowed in estimation, as Ansari and Bernstein (2019), Hsieh (2009b) and Jin et al. (1997), we introduce the modified system model:

$$\begin{cases} x_{k+1} = Ax_k + \mathbf{B}\mathbf{d}_k^{k+s} + Gw_k \\ \mathbf{y}_k^{k+s} = \mathbf{C}x_k + \mathbf{D}\mathbf{d}_k^{k+s} + \boldsymbol{\Gamma}\mathbf{w}_k^{k+s-1} + \mathbf{v}_k^{k+s} \end{cases}, \tag{30}$$

where,

$$\mathbf{B} = \begin{bmatrix} B & 0 & \cdots & 0 \end{bmatrix},\ \mathbf{C} = [C, CA, CA^2, \dots, CA^s],$$
$$\mathbf{D} = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s-1}B & CA^{s-2}B & \cdots & D \end{bmatrix},$$

and detailed expression of $\boldsymbol{\Gamma}$ is omitted. We then have the following generalizations of Theorem 2, Lemmas 1–3.

Corollary 1. *Assume (1) satisfies conditions (2)–(3). The following statements hold true:*

*(i) for case (a) in (22), there exists a positive integer $s \leq n$ such that both **C** and **D** are full column rank; for $k \geq 1$, it is possible to design an unbiased state/unknown input filter in the form of (4)–(6) for system (30), independently of a priori initial information;*

*(ii) for case (b) in (22), there does not exist a positive integer s such that **D** is full column rank (although **C** can be full column rank), it is possible that one can only design a filter in the form of (4)–(6) to be an unbiased state estimator (with biased unknown input estimation) for system (30), independently of a priori initial information;*

*(iii) for case (c) in (22), there exists a positive integer s such that both **C** and **D** are full column rank; for $k \geq 1$, it is possible that one can design an unbiased state/unknown input filter in the form of (4)–(6) for system (30), independently of a priori initial information;*

*(iv) for case (d) in (22), there does not exist a positive integer s such that **D** is full column rank (although **C** can be full column rank), i.e., one can only design a filter in the form of (4)–(6) to be an unbiased state estimator (with biased unknown input estimation) for system (30), independently of a priori initial information.*

Proof. (i). When conditions (2)–(3) hold, (A, C) is detectable. Hence, there exists a positive integer $s \leq n$ such that **C** is full column rank. Also, when condition in case (a) of (22) holds, **D** is full column rank for any positive integer s, given the full column rankness of D. Part (i) then follows if similar conditions as those in Lemma 1 hold for system (30). (ii). When condition in case (b) of (22) holds, although **C** is always full column rank, **D** cannot be full column rank for any positive integer s, given D is rank deficient. However, as long as condition (28) holds, it can be proved that

$$rank\left(\begin{bmatrix} \mathbf{C} & \mathbf{D} \end{bmatrix}\right) = n + rank(\mathbf{D}).$$

Part (ii) then follows from Lemma 2. Parts (iii) and (iv) follow by similar arguments with the above and Lemmas 1–2. ■

We have the following results if (3) holds for $z \in \mathbb{C}$.

Corollary 2. *Assume system (1) satisfies conditions (2)–(3) (with (3) holding for all $z \in \mathbb{C}$ (instead of only for $|z| \geq 1$). Consider system (30), the following statements hold true: for the cases (a) or (c) in (22), for $k \geq 0$, one can design an unbiased state/unknown input filter in the form of (4)–(6), independently of a priori initial information.*

Proof. Note that (3) holds for all $z \in \mathbb{C}$ is equivalent to system (1) having no invariance zeros. By similar arguments with those in Ansari and Bernstein (2019), one has that there exists a positive integer s such that $\begin{bmatrix} \mathbf{C} & \mathbf{D} \end{bmatrix}$ is full column rank. The rest of the proof then follows from Theorem 2. ■

In reflection of the obtained results of above subsections for different cases, we summarize the similarities and differences among them in Table 1.

4. Filter design without requiring initial condition information—The case without feedthrough

Although the case with $D = 0$ is conceptually similar to the scenarios (b) and (d) considered in (22), in the latter two cases, it is assumed that $rank(D) < q$ but $D \neq 0$. In this section, we extend the results of Section 3 to the case without feedthrough, i.e., $D = 0$ in (1).

4.1. Unbiased estimation without a priori initial information

Assume conditions (2)–(3) hold (with $D = 0$ therein). When $D = 0$, condition (2) reduce to $rank(CB) = rank(B) = q$. In this subsection, we adopt the framework of Gillijns and De Moor (2007a) for the filter design, and implement the following steps recursively after initialization:

1. Time update:

$$\widehat{x}_{k+1|k} = A\widehat{x}_{k|k}; \tag{31}$$

2. Unknown input estimation:

$$\widehat{d}_k = M(y_{k+1} - C\widehat{x}_{k+1|k}); \tag{32}$$

3. Measurement update:

$$\begin{aligned} &\widehat{x}^{*}_{k+1|k+1} = \widehat{x}_{k+1|k} + B\widehat{d}_k;\ \widehat{x}_{k+1|k+1} \\ &= \widehat{x}^{*}_{k+1|k+1} + L(y_{k+1} - C\widehat{x}^{*}_{k+1|k+1}). \end{aligned} \tag{33}$$

In the above filter, the gain matrices $M \in \mathbf{R}^{q\times p}$ and $L \in \mathbf{R}^{n\times p}$ are to be designed. Assume $x_0 \sim \mathcal{N}(\overrightarrow{x}_0, P_0)$. We define $\widetilde{d}_k, \widetilde{x}_{k|k}$ similarly with (7). As proved in Gillijns and De Moor (2007a), the unbiasedness and optimality of the state/unknown input estimates $\widehat{x}_{k+1|k+1}$ and $\widehat{d}_k$ in (31)–(33) rely on the unbiasedness of $\widehat{x}_{k|k}$, and therefore, of the initial guess $\widehat{x}_{0|0}$. However, as shown in the next, under certain conditions, it might still possible to have unbiased state/unknown input estimation, regardless of whether $\widehat{x}_{0|0}$ is unbiased or not.

Theorem 3. *Given (1) with $D = 0$ satisfying conditions (2)–(3) (with $D = 0$ therein). Then the following results hold true:*

(i) the filter in (31)–(33) is unbiased for both state and unknown input estimation, independently of a priori initial information, if and only if M and L satisfy

$$M\widehat{C} = S_1,\ \ LCA = A, \tag{34}$$

where, S_1 is defined in (18), and

$$\widehat{C} = \begin{bmatrix} CA & CB \end{bmatrix} = C\overrightarrow{A},$$

with $\overrightarrow{A}$ defined in (14);

(ii) there exist M and L such that equations in (34) hold if and only if

$$rank(\widehat{C}) = q + rank(CA),\ rank(A) = rank(CA). \tag{35}$$

Proof. The proof follows similarly from the analysis in Gillijns and De Moor (2007a) and Theorem 2 in the current paper. (i). From (1), (31)–(32), for unknown input estimation, we have

$$\mathscr{E}(\widetilde{d}_k) = (I_q - MCB)\mathscr{E}(d_k) - MCA\mathscr{E}(\widetilde{x}_{k|k}),$$

which leads to the fact that $\widehat{d}_k$ is unbiased, independently of whether $\widehat{x}_{k|k}$ is unbiased or not, if and only there exists M such that the first equation in (34) holds. Now suppose the first equation in (34) holds, i.e., $\mathscr{E}(\widetilde{d}_k) = 0$. Then, from (1), (31)–(33), we have

$$\mathscr{E}(\widetilde{x}_{k+1|k+1}) = (I_n - LC)A\mathscr{E}(\widetilde{x}_{k|k}) + (I_n - LC)B\mathscr{E}(\widetilde{d}_k),$$

which leads to the fact that $\widehat{x}_{k+1|k+1}$ is unbiased, independently of whether $\widehat{x}_{k|k}$ is unbiased or not, if and only L is designed such that the second equation in (34) holds. (ii). The existence of M such that the first equation in (34) holds is equivalent to

$$\begin{aligned} &rank\begin{bmatrix} I_q & 0_{q\times n} \\ CB & CA \end{bmatrix} = rank(\widehat{C}) \Leftrightarrow rank\begin{bmatrix} I_q & \\ & CA \end{bmatrix} \\ &= q + rank(CA) = rank(\widehat{C}). \end{aligned}$$

Similarly, the existence of L such that the second equation in (34) holds is equivalent to

$$rank[A, CA] = rank(CA) \Leftrightarrow rank(A) = rank(CA).$$

The proof is completed. ∎

Conditions in (35) turn out to be restrictive, and when they do not hold, we have the following potential cases:

$$\begin{aligned} &(e)\ rank(\widehat{C}) = q + rank(CA),\ rank(CA) < rank(A); \\ &(f)\ rank(\widehat{C}) < q + rank(CA),\ rank(CA) = rank(A); \\ &(g)\ rank(\widehat{C}) < q + rank(CA),\ rank(CA) < rank(A). \end{aligned} \tag{36}$$

Lemma 4. *For case (e) in (36), if the filter gain M in (31)–(33) is specified such that $M\widehat{C} = \begin{bmatrix} 0_{q\times n} & I_q \end{bmatrix}$, then the filter is an unbiased unknown input filter (with biased state estimation, since we cannot find L such that $LCA = A$), independently of a priori initial information.*

Proof. The proof follows from that of Theorem 3. ∎

For case (f)–(g), it appears that the filter in (31)–(33) will generally be biased for state/unknown input estimation. To illustrate, for case (f), if the filter gains M and L in (31)–(33) are specified so that

$$MCB = I_q,\ L\widehat{C} = \overrightarrow{A},$$

with $\overrightarrow{A}$ defined in (14), the filter (31)–(33) is an unbiased state filter (with biased unknown input estimation), independently of a priori initial information. However, there exists L such that $L\widehat{C} = \overrightarrow{A}$ holds if and only if

$$\begin{aligned} &rank\begin{bmatrix} C\overrightarrow{A} \\ \overrightarrow{A} \end{bmatrix} = rank(\widehat{C}) \Leftrightarrow rank(\overrightarrow{A}) = rank(C\overrightarrow{A}) \\ &\Leftrightarrow q + rank(A) < q + rank(CA) = q + rank(A) \end{aligned}$$

where, we have used the structure of A, B, and obtained a contradiction for case (f) being considered. If future measurement information is allowed in estimation, as in Section 3, we introduce the modified system model:

$$\begin{cases} x_{k+1} = Ax_k + \mathbf{B}\mathbf{d}_k^{k+s} + Gw_k \\ \mathbf{y}_k^{k+s} = \mathbf{C}x_k + \overline{\mathbf{D}}\mathbf{d}_k^{k+s} + \boldsymbol{\Gamma}\mathbf{w}_k^{k+s-1} + \mathbf{v}_k^{k+s} \end{cases}, \tag{37}$$

where, $\mathbf{y}_k^{k+s}$, $\mathbf{w}_k^{k+s-1}$, $\mathbf{d}_k^{k+s}$, $\mathbf{v}_k^{k+s}$, $\mathbf{B}$, $\mathbf{C}$ are as those in (30), $\overline{\mathbf{D}}$ has the same structure with $\mathbf{D}$ with $D = 0$ therein. Given $\overline{\mathbf{D}}$ is not full column rank, when s is chosen such that $\mathbf{C}$ is full column rank, system (37) is case (b) in (22), for which part (ii) of Corollary 1 becomes applicable. Due to limited space, we do not formally state these results here.

When an additional assumption that $rank(C) = n$ is made, one always holds that $rank(A) = rank(CA)$ and $rank(\widehat{C}) = rank(C\overrightarrow{A}) = rank(\overrightarrow{A})$. Hence, condition (35) reduces to

$$rank(\overrightarrow{A}) = q + rank(A). \tag{38}$$

When $rank(C) = n$ but condition (38) does not hold, there is only one possible scenario

$$(h)\ rank(\overrightarrow{A}) < q + rank(A), \tag{39}$$

for which part (ii) of Corollary 1 becomes applicable. We summarize the obtained results of for the case when $D = 0$ in Table 2.

4.2. The case with additionally partial information

As in Su et al. (2015), this subsection discusses the situation without direct feedthrough but some aggregate information of the unknown input is available as

$$Ed_k = r, \tag{40}$$

where, without loss of generality, $E \in \mathbf{R}^{\bar{q}\times q}$ is assumed to be full row rank. As in the former reference, we assume $p \geq q$ and $n \geq q$. Denote $F_0 \in \mathbf{R}^{\bar{q}\times q}$ as an orthogonal complement of E^{T} such that $EF_0 = 0_{q\times(q-\bar{q})}$ and $F_0^{\mathrm{T}}F_0 = I_{q-\bar{q}}$. Via a system transformation using the previous relationships, the model in (1) (with $D = 0$) can be reformulated as

$$\begin{cases} x_{k+1} = Ax_k + u_k + F\delta_k + Gw_k \\ y_k = Cx_k + v_k \end{cases}, \tag{41}$$

where, u_k is a transformed but known input term, $F = BF_0$, and $\delta_k \in \mathbf{R}^{\bar{q}}$ is the newly defined unknown input. Following Su et al. (2015), we introduce a full order state filter of the following form for (41):

$$\widehat{x}_{k+1} = N\widehat{x}_k + Ju_k + Hy_{k+1}, \tag{42}$$

with state estimation error dynamics

$$\begin{aligned} e_{k+1} &= x_{k+1} - \widehat{x}_{k+1} = Ne_k + (J - I + HC)u_k - Hv_{k+1} \\ &+ (A - HCA - N)x_k - (HCF - F)\delta_k + (I - HC)Gw_k. \end{aligned}$$

The following conditions are sufficient conditions for unbiasedness of the above filter, independent of initial condition information (i.e., whether $e_k = 0$ or not):

$$N = A - HCA = 0,\ HCF = F,\ J + HC = I. \tag{43}$$

The last equation in the above condition can always be satisfied by proper selection of J, once H is designed. Denote

$$\Pi = \begin{bmatrix} 0_{\bar{q}\times n} & E \\ CA & CB \end{bmatrix}.$$

We then have the following result, which is a generalization of those in Su et al. (2015), for the filter (42) to be unbiased, independent of initial condition information.

Corollary 3. *For system (1) with $D = 0$, assume some aggregate information of the unknown input is available as in (40), and assume E and B are full row and column rank, respectively. Then the filter in (42)–(43) is an unbiased state estimator for system (1) if*

$$\mathrm{rank}(\Pi) = n + q. \tag{44}$$

Proof. The relationships in the first two equalities of (42) are equivalent to the existence of H such that $H\begin{bmatrix} CA & CF \end{bmatrix} = \begin{bmatrix} A & F \end{bmatrix}$, which can be guaranteed if $\begin{bmatrix} CA & CF \end{bmatrix}$ is full column rank. We also have that

$$\Pi \begin{bmatrix} I_n & 0 \\ 0 & \begin{bmatrix} F_0 & E^{\mathrm{T}} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 & 0 & EE^{\mathrm{T}} \\ CA & CF & CBE^{\mathrm{T}} \end{bmatrix},$$

where we have used the relationship $F = BF_0$ in (41). Note that $\begin{bmatrix} F_0 & E^{\mathrm{T}} \end{bmatrix}$ is a square and nonsingular matrix. If Π is full column rank, then one must have that $\begin{bmatrix} 0 & 0 \\ CA & CF \end{bmatrix}$ is full column rank, i.e., $\begin{bmatrix} CA & CF \end{bmatrix}$ is full column rank. This completes the proof. ■

A necessary condition for (44) to hold is $p + \bar{q} \geq n + q$. Also, the optimal choice of H in the minimum variance sense can also be derived. Due to limited space, we omit such discussions.

5. Further implications and discussions

The previous sections discuss the existence conditions for designing an unbiased and minimum variance filter, independent of a priori initial condition information, for the cases with and without feedthrough, respectively. We next present some further discussions and implications of the requirement of unbiased estimation independently of a priori initial information.

On the one hand, a major advantage of the designed filters in this paper, as will be illustrated via numerical examples later, is that they render improved performance during the transient process than existing methods. However, a consequence of the requirement of unbiased estimation independently of a priori initial information is that the filters need to have some deadbeat properties, and the covariances of the state/unknown input estimates become constant values while in existing frameworks (that require unbiasedness on the initial condition), these are updated recursively until convergence to steady-state values. As a result, the designed filters in this paper might have worse steady-state performance than existing techniques, and we suggest that one should use the former during transient, and adopt the latter afterwards.

On the other hand, as remarked earlier, the conditions in (20) and (35) (also (44)) tend to be restrictive, and one naturally wonder if the IMA approach of Kong and Sukkarieh (2019) can be utilized to alleviate the former requirements. It turns out the answer to the above question is negative. In the following, we present some discussions to elaborate and justify the above finding. We next briefly recall the IMA framework of Kong and Sukkarieh (2019). Denote the sum of d_k over time as

$$\bar{d}_k = \sum_{i=0}^{k-1} d_k \in \mathbf{R}^q,$$

taking time 0 as the initial sampling instant. A closer inspection then reveals that a model does exist for $\bar{d}_k$:

$$\bar{d}_{k+1} = \bar{d}_k + d_k, \tag{45}$$

which is an integrator driven by d_k. When (1) does not satisfy (2)–(3), we can incorporate (45) into (1) to form the following augmented model

$$\begin{cases} \begin{bmatrix} x_{k+1} \\ \bar{d}_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A & B_d \\ 0 & I_q \end{bmatrix}}_{\bar{A}} \underbrace{\begin{bmatrix} x_k \\ \bar{d}_k \end{bmatrix}}_{\bar{x}_k} + \underbrace{\begin{bmatrix} B \\ I_q \end{bmatrix}}_{\bar{B}} d_k \\ + \begin{bmatrix} Gw_k \\ 0 \end{bmatrix} \\ y_k = \underbrace{\begin{bmatrix} C & C_d \end{bmatrix}}_{\bar{C}} \bar{x}_k + Dd_k + v_k \end{cases}, \tag{46}$$

where, $\bar{n} = n + q$, matrices $B_d \in \mathbf{R}^{n\times q}$ and $C_d \in \mathbf{R}^{p\times q}$ are design choices for the user. With slight abuse of notation, in (46), we use the same symbols with the state and output of system (1).

In Kong and Sukkarieh (2019), we have shown that a necessary and sufficient condition for the strong detectability requirements to hold for (46) is that the original system model is detectable plus some requirements on B_d and C_d, implying that one can estimate the augmented state variable, comprised of the original system state and the unknown input sum, with asymptotically stable error. On the one hand, by introducing the augmented model and its associated strong detectability conditions, the IMA is useful in allowing us to still perform stable estimation, when the original system model (1) is not strong detectable. On the other hand, the IMA is not as helpful in relaxing the requirement of unbiased estimation independently of a priori initial information. We formally state the results in the following corollary.

Corollary 4. *The following results hold true:*

(i) For condition (20) and *(35) to hold, one must have*

$$p \geq n + q,\ \mathrm{rank}(C) = n,\ \mathrm{rank}(D) = q;\ \ \mathrm{rank}(CB) = q,$$

respectively;

(ii) Assume that for the system model (1), we have rank $\begin{bmatrix} C & D \end{bmatrix} < n + q$ or rank$(CB) < q$, for the cases with or without direct feedthrough, respectively. Then one cannot design an unbiased state/unknown input filter, independently of a priori initial information using (1); it is impossible to design an unbiased filter, independently of a priori initial information, using the IMA and the augmented model (46), either.

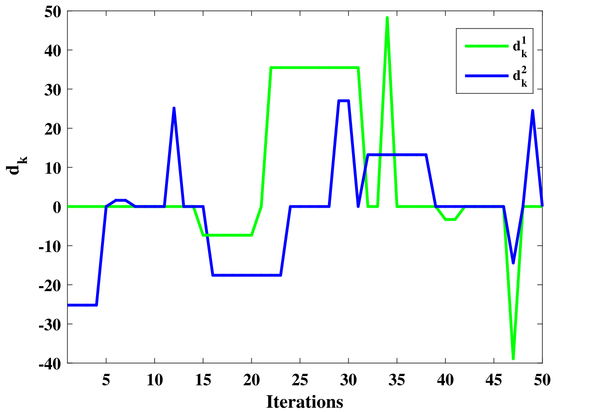


Fig. 1. The evolution of one scenario of the unknown input vector during simulation.

Proof. (i). The statement for the case with direct feedthrough is apparent. For the case without direct feedthrough, for condition (35) to hold, one must have that $rank(CB) = q$. This can be seen by the fact that

$$q + rank(CA) = rank(\widehat{C}) \leq rank(CA) + rank(CB)$$
$$\Rightarrow q \geq rank(CB) \geq q \Rightarrow rank(CB) = q.$$

(ii). For the case with direct feedthrough, even though the original system model (1) is strong detectable; and if further more, $rank\begin{bmatrix} C & D \end{bmatrix} < n+q$, then it is impossible to design an unbiased filter, independently of a priori initial information, using the IMA and (46). Similarly, for the case without direct feedthrough, if the original system model (1) (with $D = 0$) is strong detectable and in addition $rank(CB) < q$, it is impossible to design an unbiased filter, independently of a priori initial information, using the IMA and (46). This is because, with the IMA and (46) (with $D = 0$), from Theorem 3, one can design an unbiased filter, independently of a priori initial information, only if $q + rank(\overline{CA}) = rank\left(\begin{bmatrix} \overline{CA} & \overline{CB} \end{bmatrix}\right) \Rightarrow rank\left(\overline{CB}\right) = rank\left(CB\right) = q$, which cannot hold if $rank(CB) < q$. This completes the proof. ∎

Despite the negative results in Corollary 4, we remark that, they are a consequence of the stringent requirement of unbiased estimation independently of a priori initial information, and should not be interpreted as a limitation of the IMA method. This is because, from Theorems 2 and 3, when $rank\begin{bmatrix} C & D \end{bmatrix} < n+q$ or $rank(CB) < q$, to achieve real-time unbiased state/unknown input estimation, independently of a priori initial information, arguably the only viable choice left for the designer is to install more sensors and remodel the system.

6. Illustrative examples

We use some numerical examples of the case with direct feedthrough to illustrate the results (in all the examples below, the first iteration happens at time step 0).

6.1. Filtering without initial condition information

Consider the system model (1) with

$$A = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix},\ G = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix},$$
$$C = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}^{\mathrm{T}},\ D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}},$$

with $w_k \sim \mathcal{N}(0, 25)$ and $v_k \sim \mathcal{N}(0, 4I_4)$. It can be verified that the above model satisfies condition (20). We then follow the guidelines specified in Theorem 2 to design an unbiased and optimal filter, independently of a priori initial information. We next compare its performance with the filter specified in Proposition 1, which assumes to have unbiased but wrong initial conditions. As such, we run both filters with the same initialization draw randomly from the distribution $\widehat{x}_{0|-1} \sim \mathcal{N}(0, 9I_2)$ while the true initial state is $x_0 \sim \mathcal{N}(0, 10000I_2)$. The two entries of the unknown input vector at each time step are randomly piecewise constant. See in Fig. 1 for a realization of the unknown input. Based on the above setup, we run 1000 Monte Carlo simulation scenarios. We firstly illustrate performance of the two filtering frameworks for one scenario in Fig. 2, which shows the evolutions of state and unknown input estimation error for two different filtering frameworks for one simulation scenario (for simplicity, only the first entry of the error vectors is shown). From Fig. 2, it can be observed that during transient process, the filter designed to be independently of a priori initial information renders much improved performance than the one that assumes untruly unbiased initial information. To have a quantitative comparison, we compute the ratio of the state RMSE of the filter in Theorem 2 against that of Proposition 1, and show the results in Fig. 3. From Fig. 3, we can see that during the transient process, the proposed filter in this paper offers better performance than the filter with untruly unbiased initialization. However, as the filters reach steady-state, the former renders poorer performance than the latter. This is because the requirement of unbiased estimation independently of a priori initial information renders the state/unknown input estimation error covariances to be constant values which can be worse than those of recursively updated schemes in existing methods.

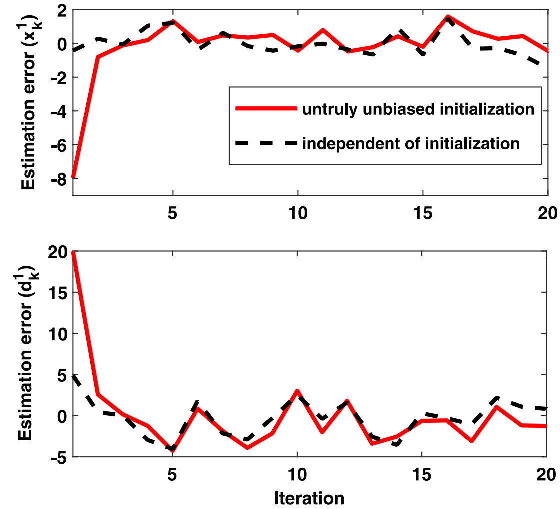


Fig. 2. Evolution of state/unknown input estimation errors.

6.2. The case when $\widetilde{C}$ is rank deficient

For the case with $\widetilde{C}$ rank deficient, we use the same system setup as the previous example with $B = \begin{bmatrix} 0.25 & 0.1 \\ 0 & 0.5 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$. It can be checked that the model does not meet condition (20) but satisfies the condition specified in Lemma 1. We follow the guidelines specified in Lemma 1 to design the filter. Using the same simulation setup as the example in the previous subsection, we compare its performance with the designed filter with the one specified using Proposition 1, which assumes unbiased but wrong initial conditions. The evolutions of

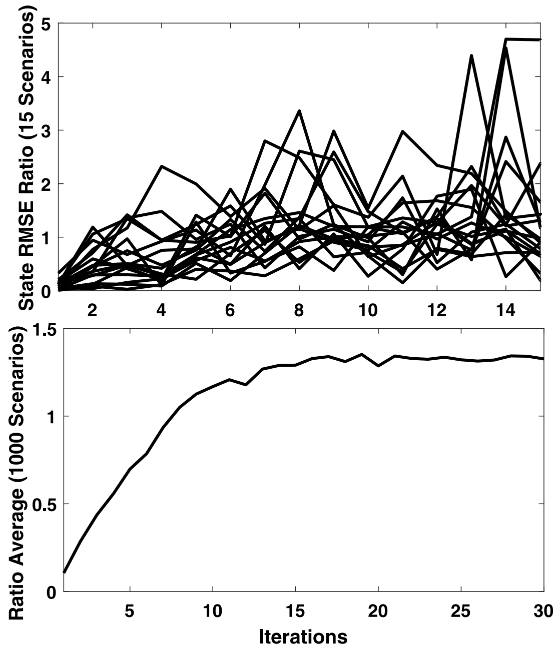


Fig. 3. Evolutions of state RMSE ratio (the filter in Theorem 2 against that of Proposition 1) for 15 scenarios (top) and average of state RMSE ratio for 1000 scenarios (bottom).

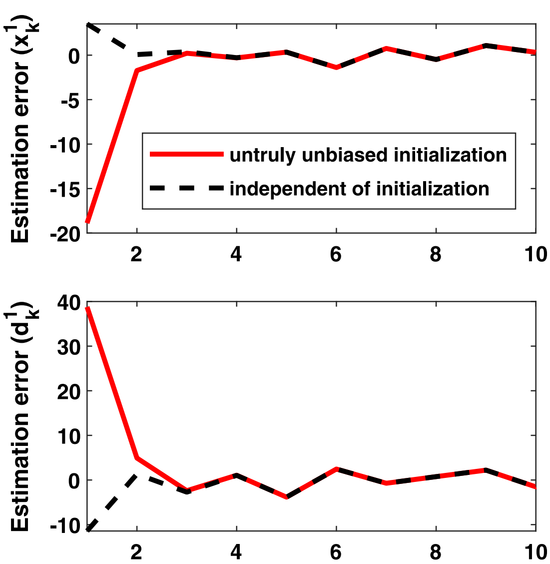


Fig. 4. Evolutions of state/unknown input estimation errors for two different filtering frameworks ($\widetilde{C}$ rank deficient).

state/unknown input estimation errors for two different filtering frameworks are shown in Fig. 4, where we notice similar patterns with the example in the previous subsection. It can also be seen from Fig. 4 that the errors of the two filters resemble after a few iterations. This is expected for the following two major reasons. On the one hand, the designed filter using Lemma 1 from the second iteration step, has the same gains as those of the filter specified in Proposition 1. On the other hand, when the conditions in Lemma 1 hold, it is necessary that $A = BD^{+}C$ (this can be seen by setting the free parameters W_1 and W_2 in (26) to be zero, and noting both C and D are full column rank). When $A = BD^{+}C$, for the filter specified by Proposition 1, we have that $A_c = (AL_* + BM_*)(DD^{+} - I)C$, whose eigenvalues can be very small (to see this more clearly, with the above system information, A_c has its eigenvalues at 0 and 0.0913). In other words, when conditions in Lemma 1 hold, the filter in Proposition 1 is quasi-deadbeat so that the effects of the wrong initial conditions diminish quickly. Under such conditions, the improvement of the designed filter using Lemma 1 over existing methods, during transient process, becomes less significant. Based on the above simulation results, we suggest that one should use the designed filters in this paper during transient to improve performance, and adopt existing techniques afterwards for steady-state performance.

7. Conclusion

For systems with Gaussian noises and arbitrary unknown inputs, the existing literature has shown that for unbiased and minimum variance estimation of the state and the unknown input, the initial guess of the state has to be unbiased. This paper has systematically addressed the question of whether and under what conditions one can design an unbiased and minimum variance filter, without making such a stringent assumption. In other words, design of the optimal filter is sought to be independent of a priori initial condition information. For both cases with and without direct feedthrough, we establish necessary and sufficient conditions under which the above design goal can be achieved. Moreover, when the above-mentioned design conditions do not hold, we carry out a thorough analysis of all possible scenarios. For each scenario, we present detailed discussions regarding whether and what can be achieved in terms of unbiased estimation, independently of a priori initial conditions. Extensions to the case with additional partially observed information, and to the case when future measurement information is allowed in estimation, have also been carried out. Numerical simulations have been presented to illustrate the theoretical findings. A major advantage of the designed filters in this paper is that they render improved performance during the transient process than existing methods. However, a feature of the designed filters is that the covariances of the state/unknown input estimates become constant values, while in existing frameworks (that require unbiasedness on the initial condition), these are updated recursively until convergence to steady-state values. For desirable performance in practice, we suggest that one should use the former during transient, and adopt the latter afterwards. In future work, we will seek alternative conditions for designing unbiased and minimum variance filters, independent of a priori information about the initial conditions. We also intend to adopt the concept proposed in this paper for investigating secure estimation of cyber–physical systems under attacks on initial conditions.

Acknowledgments

The authors appreciate the reviewers and Editors' constructive suggestions which have helped to improve the quality and presentation of this paper significantly. He Kong is grateful to Prof. Graham Goodwin and A/P Maria Seron at University of Newcastle, Australia, and Prof. Guang-Ren Duan at Harbin Institute of Technology, China, for their kind encouragement and longlasting inspirations over the years. He is also indebted to Dr. Ian Manchester, Dr. Guodong Shi, and Dr. Ruigang Wang at University of Sydney, for thoughtful discussions on estimation.

References

Al-Azzawi, A., Boudali, A. M., Kong, H., Gö ktoğan, A. H., & Sukkarieh, S. (2019). Modelling of uniaxial EGaIn-based strain sensors for proprioceptive sensing of soft robots. In *Proc. of the IEEE/RSJ IROS* (pp. 7474–7480).

Ansari, A., & Bernstein, D. S. (2019). Deadbeat unknown-input state estimation and input reconstruction for linear discrete-time systems. *Automatica*, *103*, 11–19.

Argha, A., Su, S. W., & Celler, B. G. (2019). Control allocation-based fault tolerant control. *Automatica*, *103*, 408–417.

Cheng, Y., Ye, H., Wang, Y., & Zhou, D. (2009). Unbiased minimum-variance state estimation for linear systems with unknown input. *Automatica*, *45*(2), 485–491.

Cristofaro, A., & Johansen, T. A. (2014). Fault tolerant control allocation using unknown input observers. *Automatica*, *50*(7), 1891–1897.

Darouach, M., & Zasadzinski, M. (1997). Unbiased minimum variance estimation for systems with unknown exogenous inputs. *Automatica*, *33*(4), 717–719.

Darouach, M., Zasadzinski, M., & Boutayeb, M. (2003). Extension of minimum variance estimation for systems with unknown inputs. *Automatica*, *39*(5), 867–876.

Duan, G. R. (2010). *Advances in mechanics and mathematics*, *Analysis and design of descriptor linear systems*. Springer.

Duan, G. R., & Wu, A. G. (2006). Robust fault detection in linear systems based on PI observers. *International Journal of Systems Science*, *37*(12), 809–816.

Fang, H., & de Callafon, R. A. (2012). On the asymptotic stability of minimum-variance unbiased input and state estimation. *Automatica*, *48*(12), 3183–3186.

Gillijns, S., & De Moor, B. (2007a). Unbiased minimum-variance input and state estimation for linear discrete-time systems. *Automatica*, *43*(1), 111–116.

Gillijns, S., & De Moor, B. (2007b). Unbiased minimum-variance input and state estimation for linear discrete-time systems with direct feedthrough. *Automatica*, *43*(5), 934–937.

Goodwin, G. C., Agüero, J. C., Welsh, J. S., Yuz, J. I., Adams, G. J., & Rojas, C. R. (2008). Robust identification of process models from plant data. *Journal of Process Control*, *18*(9), 810–820.

Goodwin, G. C., Kong, H., Mirzaeva, G., & Seron, M. M. (2014). Robust model predictive control: reflections and opportunities. *Journal of Control and Decision*, *1*(2), 115–148.

Gustafsson, F. (2000). *Adaptive filtering and change detection*. John Wiley & Sons.

Hautus, M. L. J. (1983). Strong detectability and observers. *Linear Algebra and its Applications*, *50*, 353–368.

Hsieh, C. (2009a). Extension of unbiased minimum-variance input and state estimation for systems with unknown inputs. *Automatica*, *45*(9), 2149–2153.

Hsieh, C. (2009b). Optimal time-delayed joint input and state estimation for systems with unknown inputs. In *Proc. of the joint IEEE CDC–CCC* (pp. 4426–4431).

Imsland, L., Johansen, T. A., Grip, H. F., & Fossen, T. I. (2007). On non-linear unknown input observers–applied to lateral vehicle velocity estimation on banked roads. *International Journal of Control*, *80*(11), 1741–1750.

Jin, J., Tank, M. J., & Park, C. (1997). Time-delayed state and unknown input observation. *International Journal of Control*, *66*(5), 733–745.

Kitanidis, P. K. (1987). Unbiased minimum-variance linear state estimation. *Automatica*, *23*(6), 775–778.

Kong, H., & Sukkarieh, S. (2018a). Metamorphic moving horizon estimation. *Automatica*, *97*, 167–171.

Kong, H., & Sukkarieh, S. (2018b). Sub-optimal receding horizon estimation via noise blocking. *Automatica*, *98*, 66–75.

Kong, H., & Sukkarieh, S. (2019). An internal model approach to estimation of systems with arbitrary unknown inputs. *Automatica*, *108*.

Laub, A. J. (2005). *Matrix analysis for scientists and engineers*. SIAM.

Li, Y., Liu, S., Zhong, M., & Ding, S. X. (2018). State estimation for stochastic discrete-time systems with multiplicative noises and unknown inputs over fading channels. *Applied Mathematics and Computation*, *320*, 116–130.

Li, Y., Quevedo, D. E., Dey, S., & Shi, L. (2017). SINR-Based DoS attack on remote state estimation: A game-theoretic approach. *IEEE Transactions on Control of Network Systems*, *4*(3), 632–642.

Li, Y., Shi, L., Cheng, P., Chen, J., & Quevedo, D. E. (2015). Jamming attacks on remote state estimation in cyber-physical systems: A game-theoretic approach. *IEEE Transactions on Automatic Control*, *60*(10), 2831–2836.

Marelli, D. E., Sui, T., Rohr, E. R., & Fu, M. (2019). Stability of kalman filtering with a random measurement equation: application to sensor scheduling with intermittent observations. *Automatica*, *99*, 390–402.

Nagahara, M., & Yamamoto, Y. (2014). FIR digital filter design by sampled-data H_∞ discretization. In *Proc. of the 19th IFAC world congress* (pp. 3110–3115).

Ohlsson, H., Gustafsson, F., Ljung, L., & Boyd, S. (2012). Smoothed state estimates under abrupt changes using sum-of-norms regularization. *Automatica*, *48*(4), 595–605.

Rao, C. V., Rawlings, J. B., & Lee, J. H. (2001). Constrained linear state estimation–a moving horizon approach. *Automatica*, *37*(10), 1619–1628.

Shan, M., Worrall, S., & Nebot, E. (2015). Delayed-state nonparametric filtering in cooperative tracking. *IEEE Transactions on Robotics*, *31*(4), 962–977.

Shmaily, Y. S., Zhao, S., & Ahn, C. K. (2017). Unbiased finite impluse response filtering: An iterative alternative to Kalman filtering ignoring noise and initial conditions. *IEEE Control Systems Magazine*, *37*(5), 70–89.

Söderström, T., Wang, L., Pintelon, R., & Schoukens, J. (2013). Can errors-in-variables systems be identified from closed-loop experiments. *Automatica*, *49*(2), 681–684.

Su, J., Li, B., & Chen, W. H. (2015). On existence optimality and asymptotic stability of the Kalman filter with partially observed inputs. *Automatica*, *53*, 149–154.

Wang, L., & Cluett, W. R. (1997). Frequency-sampling filters: an improved model structure for step-response identification. *Automatica*, *33*(5), 939–944.

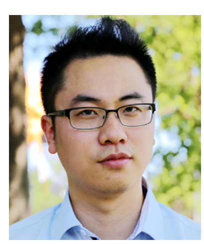
Wu, J., Li, Y., Quevedo, D. E., & Shi, L. (2017). Improved results on transmission power control for remote state estimation. *Systems & Control Letters*, *107*, 44–48.

Yong, S. Z., Zhu, M., & Frazzoli, E. (2016). A unified filter for simultaneous input and state estimation of linear discrete-time stochastic systems. *Automatica*, *63*, 321–329.

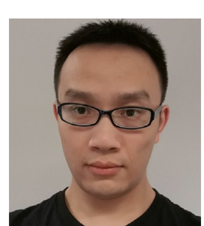
Yuz, J. I., & Salgado, M. E. (2003). From classical to state-feedback-based controllers. *IEEE Control Systems Magazine*, *23*(4), 58–67.

Zhao, S., Huang, B., & Liu, F. (2017). Linear optimal unbiased filter for time-variant systems without apriori information on initial conditions. *IEEE Transactions on Automatic Control*, *62*(2), 882–887.

He Kong received the Bachelor's degree in Electrical Engineering from China University of Mining and Technology and Master's degree in Control Science and Engineering from Harbin Institute of Technology (Centre for Control Theory and Guidance Technology), China. He then undertook doctoral studies at the Centre for Complex Dynamic Systems and Control, the University of Newcastle, Australia, and received the Ph.D. degree in Electrical Engineering. He is currently a research fellow at the Australian Centre for Field Robotics, the University of Sydney, Australia. His research interests include estimation and inference of cyber–physical systems, moving horizon estimation/control, path planning and motion control of field robots, robot audition, machine learning applications in agriculture, etc.



Mao Shan received the B.S. degree in electrical engineering from the Shaanxi University of Science and Technology, Xi'an, China, in 2006, and the M.S. degree in automation and manufacturing systems and Ph.D. degree from the University of Sydney, Australia, in 2009 and 2014, respectively. He is currently a Research Fellow with the Australian Centre for Field Robotics, the University of Sydney, Australia. His research interests include autonomous systems, localization and tracking algorithms and applications.



Daobilige Su received his B.Eng. in Mechatronic Engineering from Zhejiang University, China in 2010, M. Eng. in Automation and Robotics from Warsaw University of Technology, Poland and M.Eng. in Automation from University of Genova, Italy through European Master on Advanced Robotics (EMARO) program in 2012, and Ph.D. in robotics at Centre for Autonomous System (CAS), University of Technology Sydney (UTS), Australia in 2017. He was a post-doctoral research associate at Australian Centre for Filed Robotics (ACFR), the University of Sydney from 2017 to 2020. He is currently an Associate Professor at College of Engineering, China Agricultural University, China. His research areas include field robotics, SLAM, robot audition, computer vision, and machine learning.

Yongliang Qiao received M.S. degree from Northwest A&F University, Yangling, China, and the Ph.D. degree in computer science from the University of Technology of Belfort-Montbéliard, France. He is currently a postdoctoral researcher at the Australian Centre for Field Robotics, the University of Sydney, Australia. His research interests include agricultural robots, deep learning, multi-sensor fusion, and pattern recognition.

Abdullah Al-Alazzawi received his BSc and MSc in mechanical engineering from the University of Technology, Baghdad, Iraq, in 2002 and 2005, respectively. Abdullah is currently pursuing the Ph.D. degree in soft robotics at the Australian Centre for Field Robotics (ACFR), the University of Sydney, Australia. His research interests include proprioceptive sensing, deformation reconstruction, and sensing optimization.

Salah Sukkarieh received a Bachelor degree in mechanical (mechatronics) engineering and the Ph.D. degree from the University of Sydney, Australia. He is currently Professor of Robotics and Intelligent Systems at the University of Sydney. Salah is also the CEO of Agerris, a new AgTech startup company from the Australian Centre for Field Robotics (ACFR), developing autonomous robotic solutions to improve agricultural productivity and environmental sustainability. He was the Director of Research and Innovation at the ACFR from 2007 to 2018, where he led the strategic research and industry engagement program. He is an international expert in the research, development and commercialization of field robotic systems and has led a number of robotics and intelligent systems R&D projects in logistics, commercial aviation, aerospace, education, environment monitoring, agriculture and mining. Salah was awarded the NSW Science and Engineering Award for Excellence in Engineering and Information and Communications Technologies in 2014, and the 2017 CSIRO Eureka Prize for Leadership in Innovation and Science, and the 2019 NSW Australian of the Year nominee. He is a Fellow of the Australian Academy of Technological Sciences and Engineering, and has served/is serving on the editorial board for the Journal of Field Robotics, Journal of Autonomous Robots, amongst others.