Summary of Bertomeu, Cheynel, Xuejun, and Liang (2020, WP)

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Setting

- The manager privately observes a signal about the value of the firm.
  - x: the signal of the fundamental value.
    - full support on $\mathbb{R}$
    - p.d.f. $f(\cdot)$
    - c.d.f $F(\cdot)$
- The manager can manage report with cost.
  - θ^{-1}: marginal manipulation cost
- Investors value the firm as a function of reported earnings.
  - $x \mapsto R(x) \in \mathbb{R}$: the firm's reporting strategy
  - $\gamma : \mathbb{R} \ni r \to \gamma(r) \in \mathbb{R}$: investors' pricing strategy
  - Investors respond to earnings with

$$\bar{\gamma}(r) = \mathbb{E}\left[\alpha(\tilde{x}) \mid \bar{R}(\tilde{x}) = r\right].$$

    - $\bar{R}(\cdot)$: investors conjecture about $R(\cdot)$
    - $\alpha(\cdot)$: the mapping between unmanaged earnings and value.
    - $\alpha(\cdot)$ is assumed to be increasing and differentiable.
- The manager chooses the reporting strategy to maximize her utility

$$R(x) \in \arg\max_{r} \bar{\gamma}(r) - \frac{1}{\theta}\psi(r - x). \tag{1}$$

  - $\psi(\cdot)$: cost function
    - twice-differentiable, convex, and $\psi(0) = \psi'(0) = 0$.
- We focus on the following fully separating equilibrium (which is perfect Bayesian Nash equilibrium):

Definition. A fully separating equilibrium is the pair of $R(\cdot)$ and $\gamma(\cdot)$ satisfying the following conditions.
(i) R and γ are increasing.
(ii) Beliefs are correct:

$$\bar{R}(\cdot) = R(\cdot), \qquad \bar{\gamma}(\cdot) = \gamma(\cdot).$$

(iii) Optimization: the manager solves Equation (1).

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Estimation

- The FOC of the manage's problem (1) is
$$\gamma'(R(x)) = \frac{1}{\theta}\psi'(R(x) - x). \tag{2}$$
- We now investigate the distribution of r.
  - Since $r = R(x)$ and x has pdf f, by change of variables we have
$$g(r) = \frac{1}{R'(R^{-1}(r))} f(R^{-1}(r)). \tag{3}$$
  - Note that R is increasing in the equilibrium.
- Apply the implicit function theorem on Equation (2).
  - Let
$$h(x, R(x)) \coloneqq \gamma'(R(x)) - \frac{1}{\theta}\psi'(R(x) - x).$$
  - Since $h(x, R(x)) = 0$ by the FOC, we have
$$R'(x) = -\frac{\theta^{-1}\psi''(R(x) - x)}{\gamma''(R(x)) - \theta^{-1}\psi''(R(x) - x)}. \tag{4}$$
- Using (4), we have
$$R'(R^{-1}(r)) = -\frac{\theta^{-1}\psi''(r - R^{-1}(r))}{\gamma''(r) - \theta^{-1}\psi''(r - R^{-1}(r))}$$
- Substituting this expression into (3), we obtain
$$g(r; \theta) = -\frac{\gamma''(r) - \theta^{-1}\psi''(r - R^{-1}(r))}{\theta^{-1}\psi''(r - R^{-1}(r))} f(R^{-1}(r)). \tag{5}$$
- Therefore, we can estimate θ by MLE:
$$\hat{\theta} = \arg\max_{\theta} \frac{1}{n}\sum_i \ln g(r_i; \theta).$$
- However, we cannot compute $g(r; \theta)$ since we do not know the following components:
  - ψ: cost function
    $\to$ Specify as quadratic cost for any bias $b(r) \coloneqq r - R^{-1}(r)$
$$\psi(b) = b^2.$$
  - f: distribution of unmanaged earnings x
    $\to$ Assume that $x \sim \mathcal{N}(m_x, \sigma_x)$
  - γ: pricing strategy
    $\to$ Can non-parametrically estimate by regressing observed price on reported earnings, r.
  - b: bias
    $\to$ By Equation (2), the bias satisfies
$$\begin{aligned} b(r) &= (\psi')^{-1}(\theta\gamma'(r)) \\ &= \frac{1}{2}\theta\gamma'(r). \qquad \because \text{quadratic cost} \end{aligned}$$
- Under the above specifications, we can compute an estimated likelihood by
$$\hat{g}(r; \theta, m_x, \sigma_x) \coloneqq (1 - \hat{\gamma}''(r)\frac{\theta}{2})\frac{1}{\sigma_x}\phi\left(\frac{r - \frac{\theta}{2}\hat{\gamma}'(r) - m_x}{\sigma_x}\right). \tag{6}$$
  - ϕ is the p.d.f. of standard normal distribution.
  - To see this, note that $f(R^{-1}(r)) = f(r - b(r))$ and $x \equiv r - b(r)$ can be written $x = \sigma_x z - m_x$, where $z \sim \mathcal{N}(0, 1)$.

Step-by-Step

1. Estimate $\gamma(\cdot)$

   - Regress observed stock prices, $\{p_i\}_{i=1}^n$, on earnings surprises, $\{r_i\}_{i=1}^n$ with third order polynomial:
$$p_i = a_0 + a_1 r_i + a_2 r_i^2 + a_3 r_i^3 + \varepsilon_{i.}$$
   - Then, we get
$$\begin{aligned}\hat{\gamma}(r_i) &= \hat{a}_0 + \hat{a}_1 x_i + \hat{a}_2 x_i^2 + \hat{a}_i x_i^3,\\ \hat{\gamma}'(r_i) &= \hat{a}_1 + 2\hat{a}_2 x_i + 3\hat{a}_3 x_i^2,\\ \hat{\gamma}''(r_i) &= 2\hat{a}_2 + 6\hat{a}_3 x_i.\end{aligned}$$
   - Alternatively, we can use non-parametric method and numerical differentiation.
     - In fact, the original paper uses a cubic splines.
     - I suspect that the polynomial specification may not work well when (i) the empirical support of r_i is wide and/or (ii) there is a significant discontinuity around zero.

2. Compute likelihood function

   - Using the estimated likelihood function (6), compute the likelihood of observing reported earnings $\{r_i\}_{i=1}^n$:
$$\mathcal{L}(\theta, m_x, \sigma_x \mid r) \coloneqq \frac{n}{\sigma_x} + \sum_{i=1}^n (1 - \hat{\gamma}''(r_i)\frac{\theta}{2}) + \sum_{i=1}^n \phi\left(\frac{r - \frac{\theta}{2}\hat{\gamma}'(r) - m_x}{\sigma_x}\right).$$
   - Maximize $\mathcal{L}$ w.r.t. (θ, m_x, σ_x).