

Summary of Bird, Karolyi, and Ruchti (2019, JAE)

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Model

- e : latent earnings surprise
 - an interim signal about earnings w.r.t. the market's expectation
 - $e = 0$ means that the latent earnings just meet the market's expectation.
 - After observing e , the manager can choose whether to manipulate earnings.

- Reported earnings:

$$R = e + m + \varepsilon.$$

- m : desired manipulation
 - ε : noise
 - $e, m, \varepsilon \in \mathbb{Z}$: in terms of cent
 - ε makes it possible that the desired manipulation cannot be achieved.
- $R \mapsto \mathcal{B}(R)$: the capital market benefit function
 - In the main analysis, \mathcal{B} is defined by

$$\mathcal{B}(R) = \mathcal{B}\mathbb{1}_{R \geq 0} + f(R)\mathbb{1}_{R < 0} + g(R)\mathbb{1}_{R \geq 0}$$

- The firms' take \mathcal{B} as given.
 - See the original paper for its justification.

- The cost of manipulation is

$$c(m) = \beta m^\gamma.$$

- $\beta \sim \mathcal{U}[0, 2\eta]$.
- ε : the potential uncertainty of manipulation

$$\varepsilon \sim \mathcal{N}^{\text{discrete}}(0, \mathbb{I}_{m \neq 0} \cdot (1 + \zeta(m-1))\psi^2).$$

- pdf: $\phi_{m,\theta}(\varepsilon)$.
 - ζ : more manipulation, more uncertainty

- The utility of the manager is given by

$$\begin{aligned} u(e, m; \theta) &= \mathbb{E}_\varepsilon[\mathcal{B}(R)] - C(m) \\ &= \int_{-\infty}^{\infty} \phi_{m,\theta}(\varepsilon) \mathcal{B}(e + m + \varepsilon) d\varepsilon - \beta m^\gamma. \end{aligned}$$

- e : state variable
 - m : choice variable
 - $\theta := (\eta, \gamma, \psi^2, \zeta)$

- The manager chooses

$$m_e^* = \arg \max_m u(e, m, \theta).$$

- Given R, m_e^* , and ε , we can back out e .

Estimation

Model Inputs

- Inputs
 - observed market reactions to earnings announcements
 - * 3-day cumulative market-adjusted returns (CMAR)
 - observed earnings surprise distribution
 - * w.r.t. analyst forecast
- Benefit function is estimated by

$$CMAR_{it} = \alpha + \mathcal{B} \cdot MBE_{it} + f^k(Surprise_{it}) + g^j(MBE \times Surprise_{it}) + e_{it}.$$

- $\mathcal{B} \simeq 1.5\%$
- Earnings surprise distribution is estimated semiparametrically by

$$Frequency_b = a + \Delta \cdot MBE_b + f^k(Surprise_b) + g^j(MBE \times Surprise_b) + e_b$$

- Δ measures manipulation
- $\Delta \simeq 2.5\%$

SMM

Overview

1. Choose candidate $\theta = (\eta, \gamma, \psi^2, \zeta)$.
2. Simulate optimal firm behavior by $m_e^* = \arg \max_m u(e, m; \theta)$
 - For each bin, use 10,000 simulated firms.
 - Then, we have for each bin the proportion of firms that (i) moves to a new bin and (ii) remain in the same bin.
3. Invert $Px = \pi$ to get $x = P^{-1}\pi$.
 - P : transition (by manipulation) matrix
 - x : latent earnings surprise
 - π : empirical earnings surprise
4. Evaluate the moments of x to optimize θ .
 - The criterion function incorporates two characteristics of x :
 - Smoothness (the sum of the squared differences in the frequencies of adjacent bins).
 - The sum of the squared differences in the frequencies between π and candidate x .

Step-by-Step

1. Choose candidate $\theta = (\eta, \gamma, \psi^2, \zeta)$.
2. Simulate optimal firm behavior by $m_e^* = \arg \max_m u(e, m; \theta)$
 - Let S be the number of simulations.
 - For each simulation s and for each bin b , draw $\beta_{b,s} \sim \mathcal{U}[0, 2\eta]$.

- Compute the utilities for manipulation $m \in \{0, 1, 2, \dots, 20 - b\}$:

$$u_{s,\text{Discrete}}(b, m; \theta) = \sum_{\varepsilon=-20}^{20} \phi_{m,\theta}(\varepsilon) \mathcal{B}(b + m + \varepsilon) d\varepsilon - \beta_{b,s} m^\gamma.$$

- The optimal level of earnings management is

$$m_{b,s}^*(\theta) := \arg \max_{m \in \{0, \dots, 20-b\}} u(b, m; \theta)$$

- Calculate the manipulation strategy. For each bin b , the probability of a firm moving to bin $j \geq b$.

$$p_{b,j,S}^{\text{strategy}}(\theta) = \begin{cases} \frac{\sum_{s=1}^S \mathbb{1}_{m_{b,s}^*(\theta)=j-b}}{S} & \forall b, \forall j \geq b \\ 0 & \text{otherwise.} \end{cases}$$

- Incorporate the effect of noise term to get actual transition probability.
 - A firm moves from b to j if the strategy is to move to $j + k$, and the shock is $-k$.

$$p_{b,j,S}(\theta) = \begin{cases} \frac{\sum_{s=1}^S \sum_{k=0}^{41-j} \mathbb{1}_{m_{b,s}^*(\theta)=j+k-b} \cdot \mathbb{1}_{\varepsilon_s(m_{b,s}^*(\theta), \theta) = -k}}{S}, & \forall b, j \end{cases}$$

- Obtain

$$P(\theta) = \begin{pmatrix} p_{1,1,S}(\theta) & p_{2,1,S}(\theta) & p_{3,1,S}(\theta) & \dots & p_{41,1,S}(\theta) \\ p_{1,2,S}(\theta) & p_{2,2,S}(\theta) & p_{3,2,S}(\theta) & \dots & p_{41,2,S}(\theta) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{1,41,S}(\theta) & p_{2,41,S}(\theta) & p_{3,41,S}(\theta) & \dots & p_{41,41,S}(\theta) \end{pmatrix}.$$

3. Invert $Px = \pi$ to get $x = P^{-1}\pi$.

- Recover the latent earnings surprise vector, $x_S(\theta)$, by the empirical surprise vector, π , using $P(\theta)$:

$$x_S(\theta) := P_S(\theta)^{-1}\pi.$$

4. Evaluate the moments of x to optimize θ .

- Obtain θ by GMM:

$$\theta_T^S := \arg \min_{\theta \in \Theta} \left[\begin{pmatrix} \pi \\ \mathbf{x}(\theta)_2 \\ \vdots \\ \mathbf{x}(\theta)_T \end{pmatrix} - \begin{pmatrix} \mathbf{x}(\theta) \\ \mathbf{x}(\theta)_1 \\ \vdots \\ \mathbf{x}(\theta)_{T-1} \end{pmatrix} \right]' \Omega \left[\begin{pmatrix} \pi \\ \mathbf{x}(\theta)_2 \\ \vdots \\ \mathbf{x}(\theta)_T \end{pmatrix} - \begin{pmatrix} \mathbf{x}(\theta) \\ \mathbf{x}(\theta)_1 \\ \vdots \\ \mathbf{x}(\theta)_{T-1} \end{pmatrix} \right].$$

- Basically, this objective function evaluates the smoothness of $x_S(\theta)$ and its similarity to π .
 - For the formal argument, see the original paper.