Summary of Bird, Karolyi, and Ruchti (2019, JAE)

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Model

- e: latent earnings surprise
  - an interim signal about earnings w.r.t. the market's expectation
  - $e = 0$ means that the latent earnings just meet the market's expectation.
  - After observing e, the manager can choose whether to manipulate earnings.
- Reported earnings:

$$R = e + m + \varepsilon.$$

  - m: desired manipulation
  - ε: noise
  - $e, m, \varepsilon \in \mathbb{Z}$: in terms of cent
  - ε makes it possible that the desired manipulation cannot be achieved.
- $R \mapsto \mathcal{B}(R)$: the capital market benefit function
  - In the main analysis, $\mathcal{B}$ is defined by

$$\mathcal{B}(R) = \begin{cases} \text{const.} & \text{if } R \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

  - The firms' take $\mathcal{B}$ as given.
  - See the original paper for its justification.
- The cost of manipulation is

$$c(m) = \beta m^{\gamma}.$$

  - $\beta \sim \mathcal{U}[0, 2\eta]$.
- ε: the potential uncertainty of manipulation

$$\varepsilon \sim \mathcal{N}^{\text{discrete}}(0, \mathbb{I}_{m \neq 0} \cdot (1 + \zeta(m-1))\psi^2).$$

  - pdf: $\phi_{m,\theta}(\varepsilon)$.
  - ζ: more manipulation, more uncertainty
- The utility of the manager is given by

$$\begin{aligned} u(e, m; \theta) &= \mathbb{E}_{\varepsilon}[\mathcal{B}(R)] - C(m) \\ &= \int_{-\infty}^{\infty} \phi_{m,\theta}(\varepsilon)\mathcal{B}(e + m + \varepsilon) d\varepsilon - \beta m^{\gamma}. \end{aligned}$$

  - e: state variable

- m: choice variable
- $\theta := (\eta, \gamma, \psi^2, \zeta)$

- The manager chooses
$$m_e^* = \arg\max_m u(e, m, \theta).$$

- Given R, m_e^*, and ε, we can back out e.

Estimation

Model Inputs

- Inputs
  - observed market reactions to earnings announcements
    - 3-day cumulative market-adjusted returns (CMAR)
  - observed earnings surprise distribution
    - w.r.t. analyst forecast

- Benefit is estimated by
$$\mathit{CMAR}_{it} = \alpha + \mathcal{B} \cdot \mathit{MBE}_{it} + f^k(\mathit{Surprise}_{it}) + g^j(\mathit{MBE} \times \mathit{Surprise}_{it}) + e_{it}.$$
  - $\mathcal{B} \simeq 1.5\%$

- Earnings surprise distribution is estimated semiparametrically by
$$\mathit{Frequency}_b = a + \Delta \cdot \mathit{MBE}_b + f^k\left(\mathit{Surprise}_b\right) + g^j\left(\mathit{MBE} \times \mathit{Surprise}_b\right) + e_b$$
  - Δ measures manipulation
  - $\Delta \simeq 2.5\%$

SMM

Overview

1. Choose candidate $\theta = (\eta, \gamma, \psi^2, \zeta)$.
2. Simulate optimal firm behavior by $m_e^* = \arg\max_m u(e, m; \theta)$
   - For each bin, use 10,000 simulated firms.
   - Then, we have for each bin the proportion of firms that (i) moves to a new bin and (ii) remain in the same bin.
3. Invert $Px = \pi$ to get $x = P^{-1}\pi$.
   - P: transition (by manipulation) matrix
   - x: latent earnings surprise
   - π: empirical earnings surprise
4. Evaluate the moments of x to optimize θ.
   - The criterion function incorporates two characteristics of x:
     - Smoothness (the sum of the squared differences in the frequencies of adjacent bins).
     - The sum of the squared differences in the frequencies between π and candidate x.

Step-by-Step

1. Choose candidate $\theta = (\eta, \gamma, \psi^2, \zeta)$.

2. Simulate optimal firm behavior by $m_e^* = \arg\max_m u(e, m; \theta)$

   - Let S be the number of simulations.
   - For each simulation s and for each bin b, draw $\beta_{b,s} \sim \mathcal{U}[0, 2\eta]$.
     - Compute the utilities for manipulation $m \in \{0, 1, 2, \dots, 20 - b\}$:

$$u_{s,\text{Discrete}}(b, m; \theta) = \sum_{-20}^{20} \phi_{m,\theta}(\varepsilon)\mathcal{B}(b + m + \varepsilon)d\varepsilon - \beta_{b,s}m^{\gamma}.$$

     - The optimal level of earnings management is

$$m_{b,s}^*(\theta) := \underset{m\in\{0,\dots,20-b\}}{\arg\max} \; u(b, m; \theta)$$

   - Calculate the manipulation strategy. For each bin b, the probability of a firm moving to bin $j \geq b$.

$$p_{b,j,S}^{\text{strategy}}(\theta) = \begin{cases} \frac{\sum_{s=1}^{S} \mathbb{1}_{m_{b,s}^*(\theta)=j-b}}{S} & \forall b, \forall j \geq b \\ 0 & \text{otherwise.} \end{cases}$$

   - Incorporate the effect of noise term to get actual transition probability.
     - A firm moves from b to j if the strategy is to move to $j + k$, and the shock is $-k$.

$$p_{b,j,S}(\theta) = \begin{cases} \frac{\sum_{s=1}^{S} \sum_{k=0}^{20-j} \mathbb{1}_{m_{b,s}^*(\theta)=j+k-b} \cdot \mathbb{1}_{\varepsilon_s(m_{b,s}^*(\theta),\theta)} = -k}{S}, \forall b, j \end{cases}$$

     - (Why is $\varepsilon \in \{0, \dots, 20 - j\}$, rather than $\varepsilon \in \{1, \dots, 20\}$???)
     - Note that only firms with state $e < 0$ considers manipulation.
     - Obtain

$$P(\theta) = \begin{pmatrix} p_{1,1,S}(\theta) & p_{2,1,S}(\theta) & p_{3,1,S}(\theta) & \dots & p_{41,1,S}(\theta) \\ p_{1,2,S}(\theta) & p_{2,2,S}(\theta) & p_{3,2,S}(\theta) & \dots & p_{41,2,S}(\theta) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{1,41,S}(\theta) & p_{2,41,S}(\theta) & p_{3,41,S}(\theta) & \dots & p_{41,41,S}(\theta) \end{pmatrix}.$$

3. Invert $Px = \pi$ to get $x = P^{-1}\pi$.

   - Recover the latent earnings surprise vector, $x_S(\theta)$, by the empirical surprise vector, π, using $P(\theta)$:

$$x_S(\theta) := P_S(\theta)^{-1}\pi.$$

4. Evaluate the moments of x to optimize θ.

   - Obtain θ by GMM:

$$\theta_T^S := \underset{\theta\in\Theta}{\arg\min} \left[\begin{pmatrix} \pi \\ \boldsymbol{x}_S(\theta)_{(2,\dots T)} \end{pmatrix} - \begin{pmatrix} \boldsymbol{x}_S(\theta) \\ \boldsymbol{x}_S(\theta)_{(1,\dots T-1)} \end{pmatrix} \right]' \Omega \left[\begin{pmatrix} \pi \\ \boldsymbol{x}_S(\theta)_{(2,\dots T)} \end{pmatrix} - \begin{pmatrix} \boldsymbol{x}_S(\theta) \\ \boldsymbol{x}_S(\theta)_{(1,\dots T-1)} \end{pmatrix} \right].$$

   - Basically, this objective function evaluates the smoothness of $x_S(\theta)$ and its similarity to π.
     - For the formal argument, see the original paper.