Mandatory vs. voluntary disclosure in the dynamic market for lemons*

Cyrus Aghamolla† Shunsuke Matsuno‡

March 10, 2024

Abstract

We consider a dynamic adverse selection setting where a privately informed seller can choose to reveal or withhold past trade information to privately informed buyers. Buyers naturally receive less information when the seller can strategically withhold negative news relative to a setting where current buyers always observe the seller's history of trade, i.e., mandatory disclosure. Despite the informational disadvantage, we find that strategic disclosure by the seller can be *welfare-increasing* relative to mandatory disclosure, under which past trade is always disclosed. This occurs because voluntary disclosure can attenuate the seller's incentive to engage in destructive signaling and can lead to more efficient trade.

Keywords: Dynamic adverse selection, verifiable disclosure, communication, lemons market, social learning, information cascade
JEL classification: C72, D82, D83, G11, G23

*We thank Tim Baldenius, Jeremy Bertomeu, Laura Doval, Jon Glover, Ilan Guttman, and seminar participants at the 2023 AES Winter Retreat for helpful comments.

†Rice University. E-mail: ca58@rice.edu.

‡Columbia Business School. E-mail: SMatsuno26@gsb.columbia.edu.

1 Introduction

In a number of asset or service markets, prior trade information is typically not readily observable by interested buyers. Indeed, few markets mandate that such information be disclosed to current potential buyers. For example, in the used car industry, a buyer who arrives at a dealership typically cannot readily observe whether the dealer has sold or failed to sell cars to previous buyers. Likewise, when hiring a contractor to undertake a project, such as construction or the provision of services, the lack of interest for a given contractor by past buyers is generally not observable. Similarly, employers may not easily discover a potential worker's full employment history. The past sale of illiquid securities in most over-the-counter markets relatedly does not need to be publicly disclosed. Consequently, buyers typically cannot observe the full history of sales, or the lack thereof, for a given seller.

However, a used car dealer or contractor who *has* engaged in past trade can voluntarily disclose such information to present buyers, and can do so credibly since such information is verifiable. Continuing with the previous examples, following the sale of a car, the dealer can present a copy of the bill of sale provided to the previous buyer, indicating the price at which trade took place. Indeed, such a practice is common, with dealers typically announcing recent sales of used vehicles and the price at which they were sold. The contractor or securities dealer can likewise furnish prior deals and their agreed prices to newly interested buyers; the worker can similarly produce documents of past employment. The absence of trade affords no such luxury for the seller—it is difficult to prove a negative that a sale did *not* take place. At the same time, the seller can strategically hide past trade information from buyers, if, for example, such trade reflects poorly on the seller.

Related to this discussion, transparency of past trade and prices in various markets has become increasingly prominent in public policy debates. Indeed, the conventional wisdom has been that greater transparency and a stronger information environment is generally better for market participants. For example, in an effort to "level the playing field," regulators mandated the public disclosure of trades, including prices and volumes, for bonds and other fixed income facilities through the Transactions Reporting and Compliance Engine (TRACE), first implemented in 2002 and since expanded to other asset classes (Bessembinder and Maxwell (2008)). The Securities and Exchange Commission (SEC) has likewise taken steps to increase transparency of asset sales in other markets, such as securities-based swaps.[1] Similarly, in the healthcare industry, regulators imposed that insurers, who engage

[1] See, for example, "Statement on Public Dissemination of Security-Based Swap Transactions," by the SEC, February 16, 2022.

in bargaining with hospitals for the prices of their services, must publicly disclose these negotiated prices, implemented in July 2022 (Kona and Corlette (2022)).

We seek to study the phenomenon of voluntary disclosure of past trade in a dynamic adverse selection model, as well as examine the efficiency and welfare implications of voluntary disclosure relative to a setting where buyers always observe past trade, i.e., mandatory disclosure. The questions we seek to address are thus: (*i*) What are the economic forces that determine trade and disclosure, or the lack thereof, in repeated bargaining?; and (*ii*) How does the presence of voluntary disclosure compare to situations under which buyers always observe past trade information? Given the interest in mandatory disclosure among policymakers and regulators, it is important to better understand the role of voluntary disclosure in asset markets and the welfare and trading volume implications relative to mandatory disclosure.

Our setting is one where a seller can produce and trade one unit of an indivisible asset in each of two periods to short-lived buyers. The seller has private information regarding the quality (which determines the buyers' values and the seller's cost) of the asset, while buyers receive imperfect private signals regarding the asset's quality. Buyers in each period simultaneously make private offers to the seller, who can decide to accept or reject such offers. The seller has discretion over the trade information that buyers observe. Specifically, following trade of the asset in the first period, the seller can disclose to second-period buyers that trade took place in the first period and the trading price. The seller can also choose to withhold such information from buyers. However, in the absence of trade, the seller cannot credibly disclose that trade did not occur. As noted in the examples above, disclosure of trade is verifiable—records are produced during the transaction that can verify to second-period buyers that trade occurred. Conversely, the absence of trade is more difficult to verifiably prove, as such a strategy can be mimicked by sellers who have traded. We refer to this information structure as the *voluntary disclosure regime*.

We find that, under the voluntary regime, sellers disclose early trade that occurred at a high price to future buyers. This signals to later buyers that at least one early buyer privately observed positive information, which positively influences buyer beliefs in the second period. In contrast, if trade occurred at a low price, then the seller chooses to strategically withhold such information from future buyers. At the same time, a seller who has rejected all first-period offers is limited to non-disclosure, as communication of no trade occurring is not verifiable. Consequently, second-period buyers are uncertain as to whether non-disclosure is due to a high-type seller who has rejected all offers or because a low-type seller has accepted a low offer. While the former is positive information, which can improve buyer beliefs, the latter situation is negative information. As a result of this pooling behavior by the low-type seller, we find that second-period buyers revise beliefs downward following non-disclosure.

We compare the voluntary regime to a setting where sellers are required to disclose any trade that occurred along with its price in the first period, thereby allowing second-period buyers to observe the trade history. We refer to this setting as the *mandatory disclosure regime*. Under mandatory disclosure, the low-type seller can no longer conceal that trade at a low price had taken place, allowing the high-type seller to use rejection of first-period offers to signal her type. It would therefore appear that the voluntary disclosure regime should be efficiency-dominated by the mandatory regime. However, we find that the opposite can be true under certain conditions. Indeed, in our main result, we show that, despite the apparent informational disadvantage, *strategic disclosure by the seller can be welfare-improving relative to mandatory disclosure* (Theorem 1).

The reason for this is two-fold. First, as noted above, in the mandatory regime the high-type seller can signal through rejection of early offers in the first period. While the rejection of low offers itself is positive news, the fact that a high offer did not arrive in the first period—which implies that both first-period buyers received the low signal—is negative news. Moreover, the low-type seller can mimic the high-type seller's strategy of early rejection. Consequently, we find that, when initial beliefs are high, a "bad" equilibrium emerges in the mandatory regime, whereby rejection of early offers leads to a *downward* revision of beliefs by second-period buyers. This is because, when initial beliefs are high, the low-type seller's mimicry incentive is strong, resulting in this seller type rejecting low offers too frequently in the first period. Hence, the signaling value of transparency for the high-type seller is diminished, and buyers update more on the fact that trade at a high price did not transpire in the first period (i.e., both first-period buyers received the low signal). As such, second-period buyers more often target the low-type seller, implying that the high-type seller has a *lower* chance of trade in the second period than in the first period. The result therefore entails the worst of both worlds—sellers excessively reject offers in the first period, implying a low likelihood of trade by the low-type seller, but this signaling is not rewarded, and the high-type seller has a low likelihood of trade in the second period.

In contrast to mandatory disclosure, under the voluntary regime the seller is unable to credibly communicate that she has rejected offers. This attenuates the low-type seller's incentive to mimic through offer rejection, and this seller therefore always engages in profitable trade in the first period. Heightened trade in the voluntary regime improves welfare, leading this regime to be welfare-enhancing relative to the mandatory regime when initial beliefs are sufficiently high. We similarly find that trading frequency (or volume) is greater under the voluntary regime compared to mandatory disclosure when the signaling incentive is high. An interesting feature of this analysis is that more information is generally desirable in trade. However, our results suggest that additional information for buyers can be *Pareto destructive* and reduce welfare. This makes voluntary disclosure appealing as it can undo the destructive mimicry incentives under mandatory disclosure. In comparative statics analysis, we additionally find that the welfare advantage of voluntary disclosure is greatest when information asymmetry between the seller and buyers is more severe.

We next expand the analysis to a model with arbitrarily many periods as well as an infinite horizon. As buyers can learn from the behavior of earlier buyers, we document that information cascades can occur, where subsequent buyers ignore their private information when making offers to the seller. We find that a *DOWN* cascade—where buyers only target the low-type seller—is more likely to occur in the voluntary regime than under mandatory disclosure. The reason is that, under the voluntary regime, once market beliefs become sufficiently pessimistic, the seller is unable to raise them again, as non-disclosure is interpreted negatively by the market. Likewise, we find that, in the mandatory regime with infinitely many periods, trade becomes efficient in both states as market beliefs asymptotically converge to the true state, whereas this kind of efficiency occurs only in the low state in the voluntary regime.

These findings would appear to suggest that mandatory disclosure is more efficient than voluntary disclosure in a long horizon. However, we continue to find that voluntary disclosure can be welfare-improving even in the long horizon. This is because the efficiency gains from voluntary disclosure discussed above—reducing the low-type seller's incentive to mimic—are preserved in the long horizon. As such, when the seller's discount factor is sufficiently low, the efficiency gains from voluntary disclosure can outweigh gains from efficient trade in the long-run under mandatory disclosure.

1.1 Related literature

Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) first study voluntary disclosure and establish that, in the absence of disclosure frictions, the sender always reveals her private information, often referred to as the unraveling result. Voluntary disclosure has since been studied in repeated and dynamic settings. Einhorn and Ziv (2008) examine a repeated setting where a manager who withholds disclosure can build a reputation for being uninformed, thereby saving future disclosure costs. Beyer and Dye (2012) likewise consider a repeated game where the manager can be either forthcoming or strategic. They find that a strategic manager will disclose bad news to mimic the forthcoming type. In dynamic settings, Dye and Sridhar (1995), Song Shin (2003), Acharya et al. (2011), Guttman et al. (2014), Ben-Porath et al. (2018), Aghamolla and An (2021), and Bertomeu et al. (2022) examine voluntary disclosure when the sender has intertemporal considerations. Our setting differs from these studies principally in that we consider voluntary disclosure in the context of trade with strategic counter-parties, whereas trade is absent in the above-mentioned papers.

This study also contributes to the recent literature on dynamic adverse selection, such as Hörner and Vieille (2009), Daley and Green (2012, 2016), Kaya and Liu (2015), Fuchs et al. (2016), and Asriyan et al. (2017, 2021). These studies generally feature short-lived buyers who can observe the full trading history of the seller, i.e., whether previous offers were rejected, even if offer values themselves are unobserved. In contrast, we consider an information structure where the seller can strategically disclose past trade information. A few studies in this literature have considered the role of information of past trade. Kim (2017) considers a setting where buyers arrive stochastically and can only observe the amount of time the seller has been on the market, rather than the arrival of past offers. Kaya and Roy (2022) examine a setting where present buyers can observe a partial history of the seller's past trade behavior.[2] Our study varies as we allow the seller to control the trade information that buyers see, including the price at which trade took place.

Furthermore, as our setting includes buyer private information, our study relates to the small but growing stream of literature that embeds buyer private information in dynamic adverse selection models, such as Kaya and Kim (2018) and Aghamolla and Hashimoto (2022). Kaya and Kim (2018) consider stochastic buyer arrival, while Aghamolla and Hashimoto (2022) endogenize the time in which buyers can make offers to the seller. In contrast, we examine the interaction of buyer private information with strategic disclosure by the seller, which is absent in these settings. A recent literature examines strategic disclosure by an informed party to an uninformed party in bargaining. Glode et al. (2018) consider disclosure via Bayesian persuasion in single-period bilateral bargaining. Li et al. (2022) study an auction for takeovers where an informed buyer can disclose hard information to an uninformed competitor. In these settings, agents can strategically disclose information about the asset value specifically. In contrast, the seller is unable to disclose asset value in our setting, but can disclose past trade information. Moreover, both parties have private information in

[2] Kaya and Roy (2023) similarly consider buyer information of trade history in light of the number of buyers in each period (monoposony versus competition).

our setting, while these studies feature an uninformed counter-party. Finally, our model is dynamic, while these papers examine single-period bargaining.

The paper proceeds as follows. In the following section, we outline the model. In Section 3, we present the equilibrium of the voluntary and mandatory disclosure regimes and our main results concerning efficiency comparisons. In Section 4, we explore the model in a long horizon. Section 5 considers extensions and the final section concludes.

2 Model

We consider a long-lived seller who is endowed at the beginning of each period $t \in \{1, 2\}$ with an indivisible asset.[3] In each period, two short-lived buyers arrive and make simultaneous offers.[4] The quality of the asset is determined by the state $\theta \in \{L, H\}$ and is persistent across periods. The prior probability of the high state, $\theta = H$, is denoted by $\pi_0 \in (0, 1)$. The seller's cost of producing an asset of quality θ is c_θ, where $c_H > c_L$, while buyers value the asset at v_θ, where $v_H > v_L$. We assume that there is common knowledge of gains from trade given the state, i.e., $v_\theta > c_\theta$.

The seller is privately and perfectly informed of the true state at the beginning of the game. Each buyer in each period receives a private and imperfect signal of θ, denoted by $s_i \in \{\ell, h\}$, that is correct with probability $q \in (1/2, 1)$, i.e.,

$$\Pr(s_i = h|\theta = H) = \Pr(s_i = \ell|\theta = L) = q.$$

Buyer private signals are independent conditional on the state. For ease of exposition, we often refer to a buyer who privately received signal $s_i \in \{\ell, h\}$ as an s_i-buyer and the seller of type $\theta \in \{L, H\}$ as a θ-seller.

After observing their private signals, each buyer simultaneously makes a private offer, denoted by $x_i \geq 0$, to the seller. In each period, the seller can either accept one offer or reject both offers. If the seller accepts an offer and trade occurs, the winning buyer in period t receives the payoff

$$u_b = v_\theta - p_t, \tag{1}$$

where p_t denotes the winning offer, or trade price, accepted by the seller in period t. Any buyer whose offer is rejected receives a payoff of zero.

[3] In Section 4, we consider an arbitrarily long horizon and the infinite horizon.

[4] The assumption of short-lived buyers is widely used in the dynamic adverse selection literature; see, e.g., Daley and Green (2012, 2016), Kim (2017), Asriyan et al. (2017), among many others.

The seller receives the payoff of $p_t - c_\theta$ upon acceptance of an offer $x_i = p_t$ in period t. In the event that trade does not take place in a given period, the seller does not produce the asset and therefore she does not endure the cost of production without trade (Kaya and Roy (2022)).[5] As such, the seller's payoff is equal to zero in a period without trade.[6] At the end of each period, the seller consumes her payoff for that period.[7] The seller discounts future payoffs by a factor of $\delta \in (0, 1]$. Let $z_t = 1$ denote the event that trade occurs in period t and $z_t = 0$ for no trade. The seller's ex ante payoff is

$$u_s = \begin{cases} p_1 - c_\theta, & \text{if } (z_1, z_2) = (1, 0), \\ \delta(p_2 - c_\theta), & \text{if } (z_1, z_2) = (0, 1), \\ p_1 - c_\theta + \delta(p_2 - c_\theta), & \text{if } (z_1, z_2) = (1, 1), \\ 0, & \text{if } (z_1, z_2) = (0, 0). \end{cases} \tag{2}$$

At the end of the first period, both buyers exit and two new buyers arrive at the beginning of the second period. We consider the following two regimes for the t-period buyers' information of the seller's trading behavior in period $t - 1$.

- *Voluntary disclosure regime.* If trade occurred in the first period, the seller can choose to publicly disclose to buyers the trading price (which implies that trade took place) at the start of the second period. Likewise, the seller can choose to withhold this information and disclose nothing. In the event that trade did *not* occur in $t = 1$, the seller cannot credibly disclose this information. The reason for this difference is that disclosure is meant to be *verifiable*. For example, following trade, the seller can obtain documentation that a trade occurred and the trading price, such as a bill of sale, while she cannot prove the negative that no trade occurred. This feature is common to models of voluntary disclosure, where certain sender types are unable to communicate if their information is unverifiable (e.g., Dye (1985), Acharya et al. (2011), Ben-Porath et al. (2018)).
- *Mandatory disclosure or transparent regime.* Under mandatory disclosure, the seller is

[5] We use the pronoun "she" when referring to the seller and "he" when referring to a buyer.

[6] This assumption allows us to focus on a single asset in each period, which simplifies the analysis while preserving the main economic tensions of the model. We may alternatively assume that the seller has a production cost of zero and receives a payoff equal to her outside option, c_θ, following no trade (e.g., the seller can liquidate the asset or attain her outside option). This alternative assumption is equivalent to the assumption above.

[7] This assumption simplifies the exposition. The results are not substantively affected if we disallow seller consumption.

required to disclose the trading price at the start of the second period if trade occurred in $t = 1$. That is, the seller can no longer withhold trade information. In the event of no trade, the seller discloses nothing. Second-period buyers in this regime therefore always observe trade and the price if trade had occurred in the previous period.

In either regime, buyers do not observe the offers made in a prior period, unless the seller discloses the winning offer following trade.[8] Moreover, we include a standard assumption on the discount factor δ:

Assumption 1. $(1 + \delta)(v_L - c_L) < \delta(v_H - c_L)$.

This assumption is only relevant in the mandatory regime. If Assumption 1 is violated, then the low-type seller would strictly prefer to trade in the first period with probability one in the mandatory regime, thereby revealing her type with certainty, resulting in perfect separation. We exclude this case to focus the analysis on the more interesting situation where the low-type seller has a mimicry incentive. The sequence of the stage game for a generic period t is summarized as follows:

Stage 1: The seller is endowed with an asset, with value indexed by the state θ. The seller perfectly observes the state.

Stage 2: Two buyers arrive and observe imperfect private signals of the state θ.

Stage 3: In any period $t > 1$, if trade occurred in period $t - 1$ and under the voluntary regime, the seller makes a decision to publicly disclose p_{t-1} or to keep quiet. Under the mandatory regime, the seller must disclose p_{t-1} if trade occurred in period $t - 1$.

Stage 4: Buyers simultaneously make one-time offers to the seller. The seller chooses to accept one offer or to reject both offers. Both buyers exit. The seller consumes her t-period payoff.

The equilibrium concept we employ is perfect Bayesian equilibrium. We consider equilibria in symmetric strategies, whereby all buyers of a certain type, ℓ or h, use the same strategy. Due to the presence of off-equilibrium-path beliefs in our setting, we employ the Grossman-Perry-Farrell refinement of Grossman and Perry (1986) and Farrell (1993), which we discuss further in the following section.

[8] In other words, rejected offers are never observed by future buyers. This assumption is not essential in the two-period case.

3 Equilibrium

3.1 Voluntary regime

We first analyze the equilibrium of the voluntary disclosure regime. Under voluntary disclosure, the seller only discloses the trading price of the first period, p_1 (if trade took place), if this disclosure improves buyer beliefs in the second period. Let $d \in \{0, 1\}$ denote the event of disclosure ($d = 1$) or non-disclosure ($d = 0$). As noted above, if trade does not occur in the first period, the seller is unable to disclose. Moreover, a seller that has traded in the first period can choose to withhold disclosure, thus mimicking the disclosure behavior of a seller for which trade did not occur.

We first conjecture a disclosure strategy by the seller and later confirm that it indeed constitutes an equilibrium disclosure strategy (as well as uniqueness). We conjecture that the seller only discloses p_1, thus indicating that trade took place in period 1, for prices that would be acceptable to the H-seller. Because the seller's cost of production is c_θ in the event of trade, it is natural to conjecture that the type-θ seller does not accept any offer below c_θ. Hence, we conjecture that disclosure only occurs for $p_1 \geq c_H$. Disclosure of $p_1 < c_H$ indicates that $\theta = L$ with probability one and thus period-2 buyers would only target the low-type seller following this disclosure.

The disclosure strategy above also determines the seller's acceptance strategy in period 1. Given that the L-seller can withhold negative information (i.e., trade where $p_1 < c_H$), this seller accepts the highest offer given that one of the offers is at least c_L. Likewise, the high-type seller accepts the highest offer of at least c_H.

Under this conjectured acceptance and disclosure strategy by the seller, we characterize buyer offer strategies for a given belief π in the offer stage game. In the second period, buyers update their beliefs over θ based on the seller's disclosure decision, the information disclosed, and the realization of the private signal s_i. For ease of exposition, we split the updating into two steps. Second-period buyers first update based on the disclosure decision and the information contained therein. We define a mapping $(\pi, d) \mapsto D(\pi, d) := \mathbb{E}[\theta|d; \pi]$ to denote the updating from disclosure and a mapping $\pi \mapsto P(\pi, s_i) := \mathbb{E}[\theta|s_i; \pi]$ to denote the updating from the private signal realization s_i when the prior is $\pi \in [0, 1]$. Following the seller's disclosure decision of $d \in \{0, 1\}$ (and the disclosure of p_1 when $d = 1$), second-period buyers update beliefs from π_0 to $\pi^d := D(\pi_0, d)$. Buyers then update based on their private signal realizations. The posterior is denoted by $\pi_1 = P(\pi^d, s_i)$.

To characterize offer strategies, we consider separately optimistic and pessimistic beliefs.

We consider any posterior belief prior to the second step above, i.e., before buyers update beliefs based on their private signals. In this way, our analysis in this section can apply to both the first and second period offer strategies. Specifically, we consider a generic "prior" belief of $\pi \equiv \pi^d$ for the second period, and the prior belief $\pi \equiv \pi_0$ in the context of the first period.

The offer strategies of the buyers depend critically on the belief level prior to observing private signals. We characterize offer strategies for short-lived buyers and then examine how the seller's acceptance or rejection behavior influences beliefs. As such, while the analysis below applies to both periods, we may interpret it as buyer behavior in the second-period offer stage. We begin with the case where the adverse selection problem is the most severe and then consider progressively more optimistic beliefs. Under severe adverse selection, buyer beliefs satisfy the lemons condition:

$$\mathbb{E}[v_\theta | s_i = h; \pi] \leq c_H. \tag{3}$$

Under condition (3), the buyer's posterior expectation of the asset is below c_H even after observing the high signal and given an "initial" belief π (i.e., the belief prior to signal realization). In this case, neither buyer makes an offer that is acceptable to the H-seller and only the low-type is targeted. As such, Bertrand competition drives both offers to the zero-profit level v_L.

Naturally, as beliefs become more optimistic, buyers begin to target the high-type seller. Consider an improvement of beliefs from the lemons condition to a low-intermediate level:

$$\mathbb{E}[v_\theta | s_i = h; s_j = \ell; \pi] \leq c_H < \mathbb{E}[v_\theta | s_i = h; \pi]. \tag{4}$$

In this case, a buyer who receives the high signal is optimistic enough to target the H-seller, but does not find it profitable to do so if it means winning against a low-signal buyer (i.e., the winner's curse). As such, the equilibrium offer strategy for an h-buyer entails winning against another h-buyer with some probability. In this way, h-buyers find it worthwhile to (probabilistically) target the high-type seller. Hence, the h-buyer's offer strategy must place discrete probability on the offer v_L—the same offer that is made by a buyer who observes the low signal.

Next, because the other buyer may be offering v_L, an h-buyer can win against such a buyer by simply offering c_H, the lowest acceptable offer to the H-seller. However, such a strategy can be beaten by another offer that is, for example, $c_H + \varepsilon$. As such, the h-buyer mixes over offers that also target the H-seller according to an atomless distribution over support $[c_H, \mathbb{E}[v_\theta|s_i = h; \pi]]$. The reason that the h-buyer's offer when targeting the H-seller is not a single point is due to the presence of v_L offers, which leads to mixing in the offer distribution targeting the H-seller. In sum, ℓ-buyers offer v_L with probability one, while h-buyers mix between offering v_L with discrete probability and an offer (also randomized) that also targets the high-type seller over support $[c_H, \mathbb{E}[\theta|s_i = h; \pi]]$.

Similar offer strategies emerge as beliefs become more optimistic. We next consider the following high-intermediate belief level:

$$\mathbb{E}[v_\theta|s_i = \ell; s_j = \ell; \pi] < c_H < \mathbb{E}[v_\theta|s_i = h; s_j = \ell; \pi]. \tag{5}$$

In this case, h-buyers always target the H-seller, while ℓ-buyers continue to only target the low-type seller with offer v_L. As in the previous case, because there is positive probability that the other buyer received signal ℓ, h-buyers can always win against this buyer type and therefore h-buyers continue to mix over offers that target the high-type seller. An interesting property is that ℓ-buyers do not target the H-seller here, even though we can have $\mathbb{E}[\theta|s_i = \ell; \pi] > c_H$ in this case. Because ℓ-buyers always have a lower posterior expectation than buyers who received signal h, an ℓ-buyer that targets the H-seller and wins is likely winning against another ℓ-buyer, resulting in strictly negative profit. As such, ℓ-buyers do not target the H-seller, and Bertrand competition drives their offers to the zero-profit level v_L.

Finally, when beliefs are the most optimistic, i.e.,

$$\mathbb{E}[\theta|s_i = \ell; s_j = \ell; \pi] > c_H, \tag{6}$$

buyers of either type make offers acceptable to the H-seller. Buyers who observe the low signal offer their posterior expectation due to Bertrand competition, while h-buyers mix over offers. These observations are summarized in the following lemma, which establishes equilibrium offer strategies for a given prior π.

Lemma 1. *Suppose that the θ-seller accepts any offer that is greater than or equal to c_θ. Let $\pi \in [0, 1]$ be buyer beliefs before receiving private signals. If π is 0 or 1, the equilibrium offer is v_L or v_H, respectively. Otherwise, the equilibrium offers are characterized as follows.*

Case 1. *(Low beliefs.)* $\mathbb{E}[v_\theta|s_i = h; \pi] \leq c_H$. *Buyers of either type offer v_L with probability one.*

Case 2. *(Low-intermediate beliefs.)* $\mathbb{E}[v_\theta|s_i = h; s_j = \ell; \pi] \leq c_H < \mathbb{E}[v_\theta|s_i = h; \pi]$. *The*

ℓ-buyer offers v_L with probability one, while the h-buyer places positive mass σ on v_L and randomizes over $[c_H, V(\pi_h)]$. That is, the h-buyer's offer x follows the distribution G_2 defined by

$$G_2(x) = \begin{cases} 0 & \text{for } x < v_L \\ \sigma & \text{for } x \in [v_L, c_H), \\ \widetilde{G}_2(x) & \text{for } x \in [c_H, \mathbb{E}[v_\theta | s_i = h; \pi]], \end{cases}$$

where σ and the atomless distribution $\widetilde{G}_2(b)$ are characterized in the Appendix.

Case 3. *(High-intermediate beliefs.) $\mathbb{E}[v_\theta | s_i = \ell; s_j = \ell; \pi] < c_H < \mathbb{E}[v_\theta | s_i = h; s_j = \ell; \pi]$. The ℓ-buyer offers v_L with probability one, while the h-buyer targets the H-seller according to an atomless distribution $\widetilde{G}_3(b)$, characterized in the Appendix.*

Case 4. *(High beliefs.) $\mathbb{E}[v_\theta | s_i = \ell; s_j = \ell; \pi] > c_H$. The ℓ-buyer offers $\mathbb{E}[v_\theta | s_i = \ell; s_j = \ell; \pi]$ with probability one, while the h-buyer uses an atomless distribution $\widetilde{G}_4(b)$, characterized in the Appendix.*[9]

In Lemma 1, we take the seller's strategy as given and characterize optimal buyer offer strategies for a given pre-signal belief π. We see that buyers are more aggressive with their offers when beliefs are higher. An important question therefore is whether the seller is able to influence market beliefs with trading behavior and disclosure in an earlier period. Consider the offer game in period 1 for Cases 1–3, where only h-buyers target the high-type seller. If an offer of c_H or above arrives by at least one buyer, then the seller of either type accepts an offer with certainty in that period. Since trade took place, the seller can choose to disclose or withhold the trading price p_1. In this situation, disclosure of $p_1 \geq c_H$ implies that at least one buyer received signal h, resulting in a positive belief revision following disclosure for period-2 buyers. As such, the seller always discloses the trading price of c_H or above if trade took place.

In contrast, if both period-1 buyers observe signal ℓ, then the highest offer (again, in Cases 1–3) is v_L, for which only the L-seller accepts and the H-seller rejects. As noted previously, the H-seller cannot verifiably disclose that trade did *not* take place. Moreover, disclosure by the L-seller of $p_1 = v_L$ implies that both buyers in period 1 received the low

[9] In the knife-edge case, if $\mathbb{E}[\theta | s_i = \ell; s_j = \ell; \pi] = \mathbb{E}[\theta | s_i = h; \pi]$, which is the boundary of Cases 3 and 4, then the ℓ-buyer is indifferent between targeting the L-seller (offering v_L) and targeting the H-seller (offering c_H), and any randomization of these offers is part of the equilibrium. The h-buyer's strategy is the same as described in Cases 3 and 4.

signal ℓ, resulting in a downward revision in beliefs by period-2 buyers following disclosure. Consequently, the L-seller conceals information regarding her trade in the first period and mimics the non-disclosure of the H-seller.

We therefore have two countervailing effects following non-disclosure by the seller. Since non-disclosure encompasses rejection by the H-seller, the potential lack of trade by this type in the first period can positively improve beliefs of second-period buyers. Conversely, as noted above, non-disclosure implies that both period-1 buyers received the low signal, which negatively impacts market beliefs. Interestingly, we find that, for any pre-signal belief π, the latter effect dominates and non-disclosure always results in a downward revision in beliefs of second-period buyers, thus indicating bad news. This occurs because of mimicry by the L-seller of the H-seller's disclosure strategy; the market cannot determine if non-disclosure is due to lack of trade or due to concealing trade that actually occurred. This weakens the ability of the H-seller to signal through rejection of offers in the first period, resulting in the market placing greater weight on the fact that two negative signals arrived in the first period. The net effect is thus a downward revision in buyer beliefs following non-disclosure in the voluntary regime.

Proposition 1. *For any prior belief π_0 (i.e., for Cases 1–4), there exists a perfect Bayesian equilibrium of the voluntary disclosure regime of the following form:*

(i) *In the first-period offer stage, the θ-seller accepts any offer $x_i \geq c_\theta$. Offer behavior in the first period is as described in Lemma 1.*

(ii) *At the start of $t = 2$ the seller of either type discloses the trade that occurred in the previous period if and only if the winning offer was c_H or higher.*

(iii) *Disclosure always results in upward belief revision and non-disclosure results in downward belief revision by second-period buyers.*

(iv) *Offer behavior in the second period is as described in Lemma 1, where the "initial" belief is the belief following the seller's disclosure decision, π^d.*

Moreover, such an equilibrium is the essentially unique equilibrium characterization that survives the Grossman-Perry-Farrell criterion.

Since voluntary disclosure of the past transaction is a signaling activity, there are other, perhaps less plausible, equilibria, a property not uncommon in sequential games of incomplete information. In the Appendix, we show that the equilibria identified above is an essentially unique equilibrium that survives a natural refinement based on deviation incentives, developed by Grossman and Perry (1986) and Farrell (1993), and employed in Gertner et al.

(1988), Lutz (1989), Maskin and Tirole (1992), Severinov (2008), and Perez-Richet (2014), among others, as well as in the verifiable disclosure studies of Bertomeu and Cianciaruso (2018) and Glode et al. (2018).[10] In this equilibrium, as noted above, the seller discloses trade that occurred at a high price and withholds trade that occurred at a low price. This intuitive acceptance strategy is not necessarily unique. For example, we can construct an equilibrium where the seller of either type withholds trade that occurred at a high price if we assign pessimistic off-path beliefs following disclosure of such trade.[11] The Grossman-Perry-Farrell refinement rules out equilibria where types can benefit from deviating from an on-path action, given that these types are conjectured to deviate. In other words, a given equilibrium fails the Grossman-Perry-Farrell criterion if any deviator selects an off-path action, is "identified" correctly as a type that deviated, and receives a better payoff from this deviation than from the on-path action. In Appendix B, we provide more detail about the Grossman-Perry-Farrell criterion and show that other equilibria do not survive this refinement.[12]

While non-disclosure is always interpreted as bad news, an important, and perhaps efficiency-increasing, feature of the voluntary regime is that the L-seller always trades in the first period. In the following section, we characterize equilibrium in the mandatory regime and then analyze welfare comparisons between the two regimes.

3.2 Mandatory regime

In the mandatory disclosure regime, the seller must disclose whether trade occurred in the first period. Hence, unlike the voluntary regime, the L-seller can no longer conceal if trade took place at a price below c_H. The H-seller can thus use rejection in the first period to signal her type. Conversely, trade by the low-type seller at a price below c_H indicates to second-period buyers that the seller is type L with probability one. We must therefore consider when the L-seller engages in mimicry of the high-type in the first period.

Analogous to the voluntary regime, we let second-period buyer beliefs following the first period, but before private signal realizations, be denoted as π_1. We introduce a mapping

[10] The equilibrium is essentially unique in the sense that any equilibrium that survives the Grossman-Perry-Farrell criterion must take the form described in Proposition 1, but there is a continuum of (uncontroversial) off-path beliefs which support such an equilibrium.

[11] Since disclosure is verifiable, disclosure of trade and the trading price pegs second-period buyer beliefs that trade took place at the disclosed price with probability one. However, buyers' inferences of the type θ are not restricted by the disclosure.

[12] The Grossman-Perry-Farrell equilibrium (GPFE) refinement is closely related to D1 and the intuitive criterion, but is more appropriate in settings with verifiable messages. We discuss the relation of GPFE to other refinements further in the Appendix.

$T : [0, 1] \times \{a, r\} \to [0, 1]$ to indicate the updating from the prior to the posterior using the information in acceptance (denoted by a), which includes both that trade occurred and the trading price, or rejection (denoted by r).

We denote the expected second-period payoff for a θ-seller who has rejected all first-period offers as $Q_\theta(\pi_1)$. If the L-seller accepts an offer of v_L in the first period, her total payoff after period 2 is $(1+\delta)(v_L - c_L)$. If the seller rejects all offers below c_H and does not accept an offer, her period-1 payoff is zero and her continuation payoff is $\delta Q_L(\pi_1)$. For mimicry to occur in the first period, the L-seller must be at least as well off from rejection in the first period as she is from accepting v_L in that period and revealing herself to be of type L (both seller types accept offers of at least c_H). The continuation value must therefore satisfy the following condition in equilibrium:

$$\delta Q_L(\pi_1) \geq (1+\delta)(v_L - c_L) \tag{7}$$

For the L-seller's acceptance strategy, we denote by $\phi \in [0, 1]$ the probability that the L-seller accepts an offer of v_L in the first period. As an initial property, we cannot have an equilibrium in which the L-seller accepts v_L with certainty in the first period, i.e., $\phi = 1$. In this case, upon rejection in the first period, second-period buyers must believe that the seller is a high type with probability one, resulting in offers of v_H. However, this implies that mimicry by rejecting low offers is more valuable for the L-seller in the first period. Hence, we must have $\phi < 1$ in any equilibrium.

In Cases 1–3 discussed in the previous section, there is positive probability of the v_L-offer arriving in the first period, as the prior belief π is sufficiently low in these cases. As in the voluntary disclosure regime, we have countervailing effects following two offers of v_L in the first period. However, as we will see shortly, in contrast to the voluntary regime, lack of trade in the first-period can lead to upward belief revision by second-period buyers. The first effect represents the *signaling value of transparency*. Because the L-seller accepts an offer of v_L with positive probability and the H-seller rejects such offers with probability one in the first period, this effect positively influences beliefs over θ for period-2 buyers. At the same time, lack of trade at a price of c_H or higher implies that both first-period buyers received a low signal. This effect has a negative impact on beliefs. To see which effect dominates, there are two possibilities following two offers of v_L in the first period, based on the initial pre-signal belief level π_0. In what follows, we consider the L-seller's ex ante trade incentive before offers arrive in time 1 and therefore evaluate the continuation value using the prior

π_0.[13]

- $(1+\delta)(v_L - c_L) \in [\delta Q_L(\pi_0), \delta(v_H - c_L))$. In this case, the initial belief π_0 is such that ex ante the L-seller has a stronger incentive for accepting the low offer in the first period than for receiving the continuation payoff $Q_L(\pi_0)$, evaluated at π_0. Consequently, the continuation payoff must be improved through second-period buyers targeting the H-seller with sufficiently high likelihood so that the L-seller begins rejecting offers in the first period. Because the acceptance probability of the L-seller is high, rejection of v_L offers in the first period results in overall positive belief revision by second-period buyers, i.e., $\pi_1 > \pi_0$, and the signaling value dominates the negative information implied by two ℓ-signals in the first period. As such, second-period buyers target the H-seller frequently enough (i.e., making offers of c_H or higher) such that the continuation value following first-period rejection, $Q_L(\pi_1)$, rises to the point where condition (7) holds with equality.
- $(1+\delta)(v_L - c_L) < \delta Q_L(\pi_0)$. Here, the L-seller ex ante prefers the continuation payoff Q_L (evaluated at π_0) over acceptance of v_L in the first-period. The L-seller therefore has a strong incentive for mimicry through rejecting low offers in the first period. Due to the high mimicry, period-2 buyer posterior beliefs are revised *downward* following first-period rejection, $\pi_1 < \pi_0$. That is, second-period buyers target the L-seller more in the second period than first-period buyers did in the first period (i.e., there is a higher weight on v_L in the mixed strategy distribution by period-2 buyers relative to period-1 buyers). Consequently, the negative information of two ℓ signals following first-period rejection overtakes the positive information of rejection of low offers. As second-period buyers target the H-seller less frequently, this lowers the continuation value $Q_L(\pi_1)$ for the L-seller. We have two possibilities. The first is that condition (7) holds with equality in equilibrium. In this case, beliefs are revised downward following rejection but the L-seller still accepts low offers with positive probability in equilibrium, $\phi \in (0,1)$. This occurs when $(1+\delta)(v_L - c_L)$ is not too low relative to $\delta Q_L(\pi_0)$. In the second case, condition (7) holds with strict inequality. Here, the L-seller rejects low offers with probability one and only accepts high offers. While $\phi = 0$ results in maximum downward revision following rejection, this can occur in equilibrium because period-2 buyers who have received the high signal may still target the H-seller with positive probability in the second period. This case occurs when $(1+\delta)(v_L - c_L)$ is low relative to $\delta Q_L(\pi_0)$.

[13]In equilibrium, the L-seller's first-period trade incentive is determined by the second-period buyers' posterior belief π_1 following the acceptance or rejection decision in the first period. In the exposition, we discuss the L-seller's ex ante trading incentives in terms of the prior belief π_0 to present intuition for how the equilibrium posterior π_1 and the L-seller's first-period trading strategy are formed.

This discussion is summarized in the following proposition:

Proposition 2. *The first-period equilibrium in the mandatory regime is as follows. Buyer offer strategies are characterized as in Lemma 1. The seller's behavior depends on the following parameter values.*

(i) *If $(1+\delta)(v_L - c_L) < \delta Q_L(\pi_0)$, then the L-seller accepts the offer v_L with probability $\phi \in [0,1)$, which is uniquely determined by equation (7) if interior,*[14] *and she accepts any offer $x_i \geq c_H$ with probability one (or the highest of such offers). The H-seller accepts the highest offer x_i with probability one if and only if $x_i \geq c_H$. The belief after rejection of the offer v_L is lower than the prior: $\pi_1 := T(\pi_0, r) < \pi_0$.*

(ii) *If $(1+\delta)(v_L - c_L) \in [\delta Q_L(\pi_0), \delta(v_H - c_L))$, then the seller's acceptance behavior is the same as above. The belief after rejection of offer v_L is (weakly) higher than the prior: $\pi_1 := T(\pi_0, r) \geq \pi_0$.*

Such an equilibrium uniquely exists.

3.3 Welfare and efficiency: Mandatory vs. voluntary disclosure

We now compare the welfare and trading efficiency implications of the two regimes. To measure welfare, we consider the total expected payoffs of the seller and both sets of buyers in each regime. We begin with the statement of the welfare result:

Theorem 1. *Welfare between the two regimes takes the following properties:*

(i) *There exists a cutoff π^* such that for $\pi_0 \geq \pi^*$, welfare is equal in each regime.*

(ii) *When $\pi_0 < \pi^*$ and c_L are both high enough, welfare is strictly higher under the voluntary regime than the mandatory regime.*

(iii) *Otherwise, welfare is weakly higher under the mandatory regime than the voluntary regime.*

The cutoff π^* is determined as the boundary between Cases 3 and 4, i.e., $\mathbb{E}[v_\theta | s_i = \ell, s_j = \ell; \pi^*] = c_H$.

Theorem 1 characterizes the welfare ranking between the two regimes based on parameters of the model, primarily through the initial belief π_0. We see in part (ii) that voluntary disclosure is welfare-improving relative to mandatory disclosure under certain conditions. This is perhaps surprising given that mandatory disclosure provides more information to

[14] If $\delta Q_L(\pi_1) > (1+\delta)(v_L - c_L)$ for all $\phi \in [0,1]$, then $\phi = 0$ is the unique equilibrium acceptance probability.

buyers, as they always observe the seller's past trade behavior. Conversely, voluntary disclosure is characterized by strategic information release by the seller, resulting in relevant information being strategically suppressed by the seller.

To understand this result, recall that in the mandatory regime, the seller can use rejection as a signaling mechanism, as buyers can observe whether or not a trade had taken place in the first period. The L-seller can get a higher price in the second period by mimicking the high-type and rejecting low offers, but she forgoes profitable trade at the low price in the first period. However, for a range of initial beliefs, we have a "bad" equilibrium, where the L-seller excessively rejects low offers, leading to a downward revision in beliefs and eroding the H-seller's ability to signal through first-period rejection. As shown in Proposition 2, this situation arises when the L-seller's ex ante payoff from mimicry is high relative to accepting the low offer in the first period. Consequently, while the high type is still targeted in period 2, the probability placed on making an offer acceptable to the H-seller decreases in the second period. Stated differently, the H-seller has a *lower* probability of trade in the second period than in the first.

The bad equilibrium therefore has the worst of both worlds—little trade by the low type in the first period, even though this leads to a downward revision of beliefs, and diminished trade by the high type in the second period. In contrast, under the voluntary regime, the seller cannot use rejection as a signaling mechanism, as she can strategically withhold trade information. While non-disclosure is interpreted negatively by the market due to pooling in non-disclosure, we always have trade by the low-type seller in the first period. Consequently, while the high-type seller is harmed by non-disclosure and sees lower trade in the second period, trade is efficient in the first period in the sense that the L-seller derives no benefit from rejecting low offers. Voluntary disclosure therefore attenuates the low-type seller's incentive for excessive first-period rejection, resulting in a welfare improvement in the first period.

The economic forces driving this result are twofold. First, buyer private signals drive the emergence of the bad equilibrium. As mentioned in the previous section, the lack of trade in $t = 1$ implies that both first-period buyers received ℓ signals. This negative information overtakes the positive, albeit small, inference from observing rejection of two low offers. Without private buyer signals, such an equilibrium would therefore not arise. Second, it is precisely the seller's ability to strategically conceal information that eliminates this bad equilibrium in the voluntary regime, even with buyer private signals. Interestingly, the presence of more information by buyers through private signals is balanced by the loss of information through strategic disclosure.

In part (i) of Theorem 1, initial beliefs are so optimistic that both types of sellers accept offers in the first period with probability one. This implies that welfare is the same in both regimes, as trade always transpires. Finally, in part (iii), when initial beliefs are sufficiently low, the mandatory regime is welfare-improving relative to the voluntary regime. Unlike in part (i), here the transparency allows the H-seller to build her reputation through early rejection, as acceptance occurs sufficiently often by the L-seller in period 1. Consequently, total welfare is higher under the mandatory regime as the H-seller trades more often in this case.

The above discussion implies that the voluntary regime exhibits greater trading frequency or volume when the mimicry incentive is high. We find that this is indeed the case; under voluntary disclosure, trade of the low asset always occurs in the first period, while this occurs with probability less than one in the mandatory regime when initial beliefs are not too high (i.e., Case 4). While the lack of signaling can result in lower trade of the high asset in the voluntary regime, as indicated by part (ii) of Theorem 1, this is more than exceeded by the high trading volume of the low asset, when the mimicry incentive of the L-seller is strong.

Theorem 2. *The voluntary regime exhibits weakly higher trading volume than the mandatory regime for any initial belief π_0 when c_L is sufficiently high.*

3.4 Comparative statics – information asymmetry

A key feature of our setting is that buyers observe imperfect private information. To further explore the implications of private information on welfare, we consider comparative statics in the precision of buyers' private information. Recall that each buyer's private signal is correct with probability $q \in (1/2, 1)$. We interpret this parameter as the degree of information asymmetry between buyers and sellers in the market or as the quality of news or private information.

Under the mandatory regime, an increase in the precision q has two effects. First, when buyers have more precise signals, rejection of two low offers in the first period leads to more pronounced updating by second-period buyers. Specifically, under greater precision, the presence of two low signals in the first period is more indicative that the underlying state is low. (Recall that first-period rejection implies that both buyers in that period received the low signal.) As such, second-period buyers become more pessimistic after observing rejection in the first period. This effect in turn lowers the continuation value $Q_L(\pi_1)$ following first-period rejection for the L-seller. In the case where condition (7) holds with equality, an increase in the precision means that the L-seller must begin accepting low offers with a higher probability $\phi \in (0, 1)$ in the first period. This raises period-2 buyers' beliefs following rejection, resulting in the continuation value being raised to the point where condition (7) binds. Consequently, the belief after first-period rejection stays the same, but the first-period acceptance probability increases in the precision. Hence, in this case, an increase in signal precision improves welfare in the mandatory regime, as efficient trade by the L-seller occurs more frequently.

In contrast, when condition (7) holds with strict inequality, the L-seller rejects low offers with probability one in the first period. As above, as the signal precision increases, second-period buyers update more heavily on the fact that two ℓ-signals were observed by first-period buyers, resulting in a marginal decrease in the L-seller's continuation value. However, the first-period acceptance probability is unchanged—the L-seller continues to reject low offers with probability one. As a result, second-period buyers target the H-seller even less frequently—resulting in less efficient trade for this type—and we have no improvement in the acceptance probability of the low seller. In this case, an increase in signal precision exacerbates the negative features of the bad equilibrium and decreases welfare in the mandatory regime.

Similar to the second case discussed above, under the voluntary regime, an increase in the signal precision leads to a similar, negative effect. As non-disclosure implies that both first-period buyers received the low signal, the downwards updating following non-disclosure becomes more severe, resulting in less frequent trade with the H-seller and lower overall welfare.

The above discussion implies that the welfare gain of voluntary over mandatory disclosure is weakly decreasing in the precision of private signals. In other words, the gap between the voluntary and mandatory regimes narrows in the signal precision. This implies, perhaps surprisingly, that the welfare advantage of voluntary disclosure is greatest when the degree of information asymmetry in the market is *high*. These implications are summarized in the following corollary:

Corollary 1. *Assume that $\pi_0 < \pi^*$ and c_L are both sufficiently high such that welfare is strictly higher under the voluntary regime than under the mandatory regime, where π^* is defined as in Theorem 1. The welfare gain from voluntary over mandatory disclosure is weakly decreasing in the precision of private information q.*

4 Long horizon

Our baseline setting considers a parsimonious two-period model that captures the main economic insights of voluntary disclosure of trade information relative to a mandatory regime. We now extend our baseline setting to more than two periods to examine long-run effects. In particular, we first provide results regarding belief convergence of buyers in the long horizon and then analyze welfare as in Section 3.3.

We consider both the case of $T \geq 2$ periods and the infinite horizon, $T = \infty$. As in the baseline setting, the seller's per-period payoff upon acceptance of an offer is $(p_t - c_\theta)\delta^{t-1}$, where $\delta \in (0, 1]$. For tractability, we assume that buyers in the current period t observe the prior from period $t-2$, π_{t-2}.[15] For ease of exposition, we focus the discussion on initial beliefs π_0 in Cases 2–4 of Lemma 1.[16] As we focus on long-run belief convergence, we define the following scenarios. In an *UP* cascade, remaining buyers make offers that target the high-type seller with probability one, regardless of their private information. Likewise, in a *DOWN* cascade, buyers only target the low-type seller (i.e., the only offer is v_L), again regardless of their private information. We present results separately for each regime and then discuss welfare implications.

In the voluntary regime, buyers do not observe disclosure of trade with the price at v_L, as disclosure of such trade would peg buyer beliefs that the seller is the low type with probability one, inducing a *DOWN* cascade. This would suggest that *DOWN* cascades are unlikely to occur in the voluntary regime. However, once beliefs become sufficiently low, buyers target the low-type seller regardless of their private information. Specifically, once Case 1 is reached (as described in Lemma 1), buyers only offer v_L. As such, Case 1 is an absorbing state—beliefs cannot improve if both buyers only offer v_L in every period. Conditional on $\theta = L$, a *DOWN* cascade occurs with positive probability for any arbitrarily sized finite T and occurs with probability one when $T = \infty$. Since buyer private signals are correlated with the underlying state θ, in the long horizon buyers will more often receive the low signal ℓ. As a result, a seller of type L can less often disclose trade at a price of c_H or higher when T is large. Moreover, in the limit as the time horizon becomes infinitely long, non-disclosure occurs frequently often so that beliefs eventually drop to Case 1, resulting in a *DOWN* cascade with certainty.

[15]This assumption preserves the role of disclosure in the extended setting, and is meant to be a reduced-form representation of the current-period buyers' ability to observe the history of disclosure (under the voluntary regime) or trade (under the mandatory regime) by the seller. As such, this assumption allows us to capture the social learning feature of the long horizon while also preserving tractability of the analysis.

[16]As will be clear, Case 1 is uninteresting in the voluntary regime since it is an absorbing state.

Interestingly, conditional on $\theta = H$, a *DOWN* cascade can occur with positive probability for both $T < \infty$ and $T = \infty$. Because the H-seller cannot use rejection as a signaling mechanism in the voluntary regime, a series of low signals by buyers, resulting in consecutive non-disclosures, can push beliefs downwards to the point where the posterior belief reaches Case 1. As Case 1 is an absorbing state, even if remaining buyers receive signal h, the public belief does not recover since no buyers target the H-seller. However, as $T = \infty$, an *UP* cascade can also occur with positive probability, conditional on $\theta = H$. These findings are summarized in the following proposition.

Proposition 3. *Consider the voluntary regime where initial beliefs π_0 are in any of Cases 2–4.*

(i) *Suppose $\theta = H$. If $T < \infty$, then a DOWN cascade occurs with positive probability, while an UP cascade never occurs. If $T = \infty$, then both a DOWN or UP cascade can arise with positive probability.*

(ii) *Suppose $\theta = L$. If $T < \infty$, then a DOWN cascade occurs with positive probability. If $T = \infty$, then a DOWN cascade occurs with probability one.*

When initial beliefs begin in Case 1 under the voluntary regime, a DOWN cascade ensues for both seller types.

We similarly observe *DOWN* cascades under mandatory disclosure. Upon acceptance of an offer v_L in the mandatory regime, future buyers believe that the seller is of type L with probability one. Because the L-seller accepts v_L with positive probability (when the continuation value becomes low enough) when mimicking the high type, acceptance of the low offer eventually occurs with a higher likelihood as T becomes large, and occurs with probability one as $T \to \infty$ conditional on $\theta = L$.

In contrast to the voluntary regime, conditional on $\theta = H$, a *DOWN* cascade cannot occur. While the H-seller never accepts an offer of v_L, rejection of low offers by the seller is informative for the market, allowing market beliefs to improve when enough periods have passed without trade at the low price. As such, even if beliefs are in Case 1, we do not observe *DOWN* cascades conditional on $\theta = H$. Likewise, with respect to *UP* cascades, even if buyers observe that trade occurred at the high price, they continue to update based on their private signals, given that the posterior belief has not reached one. This posterior convergence does not occur when $T < \infty$, which implies that *UP* cascades additionally cannot occur conditional on $\theta = H$ when $T < \infty$. However, as the time horizon becomes infinitely long, market beliefs converge to one that the seller is of type H, as the H-seller never accepts low offers. Hence, conditional on $\theta = H$, an *UP* cascade occurs with probability one in the mandatory regime.

Proposition 4. *Consider the mandatory regime where initial beliefs π_0 are in any of Cases 1–4.*

(i) *Conditional on $\theta = H$, if $T < \infty$, then neither a DOWN nor UP cascade occurs with positive probability. If $T = \infty$, an UP cascade occurs with probability one.*

(ii) *Conditional on $\theta = L$, if $T < \infty$, then a DOWN cascade occurs with probability strictly between zero and one. If $T = \infty$, a DOWN cascade occurs with probability one.*

Propositions 3 and 4 would suggest that mandatory disclosure should be better in a long horizon, as a *DOWN* cascade can occur with positive probability in the voluntary regime when $\theta = H$, whereas this situation does not arise in the mandatory regime. Moreover, market beliefs converge to the true state in the mandatory regime with probability one when $T = \infty$, resulting in completely efficient trade, even though this only occurs conditional on $\theta = L$ in the voluntary regime. However, as established in Theorem 1, voluntary disclosure can have welfare advantages over mandatory disclosure when initial beliefs and the L-seller's mimicry incentive are high. We observe that these same forces are preserved in the long-horizon; total welfare can be improved in voluntary disclosure as this can mitigate the L-seller's welfare-destroying excessive rejection. Indeed, if the discount factor is sufficiently low, the welfare gains from early efficient trade under voluntary disclosure can outweigh the efficiency gains from belief convergence in the long horizon achieved under the mandatory regime. This is perhaps surprising given that a lower discount factor naturally induces the L-seller to accept early offers of v_L more frequently in the mandatory regime, thereby improving efficiency. In particular, as the L-seller discounts the future more heavily, she places less weight on buyer beliefs in the next period, thus resulting in greater acceptance of v_L in early offers. This additional force results in greater efficiency of trade under the mandatory regime and should minimize the welfare difference between mandatory and voluntary disclosure. However, we find that inefficiency persists under mandatory disclosure, even when δ is low, and does not decrease in the same rate as δ decreases.

Theorem 3. *Let $T \in \{3, 4, \ldots, \} \cup \{\infty\}$. Suppose that $v_L - c_L$ is small enough and $\pi_0 \leq \pi^*$ is high enough.*[17] *Then, there exists a cutoff level of the discount factor, denoted by $\delta^* \in (0, 1)$, such that the voluntary regime gives strictly higher welfare than the mandatory regime for any $\delta \leq \delta^*$.*

[17]Recall that π^* is determined as the boundary belief between Cases 3 and 4 in Lemma 1.

Theorem 3 establishes that voluntary disclosure can be welfare-improving relative to mandatory disclosure under conditions analogous to those in Theorem 1. First, the mimicry incentive for the L-seller must be sufficiently strong; this occurs when the L-seller's profit from accepting offer v_L is low. Second, the initial belief must be sufficiently optimistic. In this case, as shown in the previous section, the unique equilibrium is one where the low-type seller rejects offers but beliefs are revised downward. As such, voluntary disclosure can improve trading efficiency and welfare, which can be significant enough to overtake the benefits of revelation in the long-run under mandatory disclosure.

5 Extensions

In this section, we consider extensions of the model. Specifically, we examine imperfect persistence of seller quality and when buyers arrive probabilistically in each period.

5.1 Imperfect persistence of seller quality

In the baseline setting, we assume that the state θ, and thus the quality of the asset or the seller's type, is persistent across periods, which allows us to cleanly illustrate the underlying economic insights. We now relax this assumption and introduce impersistence in the quality of the asset across the two periods. We denote the t-period state as $\theta_t \in \{L, H\}$ for $t = 1, 2$, which is perfectly observed by the seller. As in the baseline model, buyers in period t observe imperfect private signals of the current-period θ_t. The transition probability from each first-period state θ_1 to the low state is denoted as $\lambda_{\theta_1} \in (0, 1)$, i.e., $\Pr(\theta_2 = L|\theta_1 = L) = \lambda_L$ and analogously $\Pr(\theta_2 = L|\theta_1 = H) = \lambda_H$ (transition from θ_1 to the high state is thus given as $1 - \lambda_{\theta_1}$).[18,19] We focus on the case where asset quality is positively serially correlated in time, i.e., $\lambda_L \geq \lambda_H$.

We first note that equilibrium features of the voluntary disclosure regime are qualitatively similar in this extended setting. That is, the L-seller continues to accept all offers above c_L in the first period and only discloses trade occurring at a price of at least c_H. Likewise,

[18] To avoid additional signaling incentives, we assume that the transition of types occurs after the disclosure of past trade, if the seller chooses to disclose, in the second period.

[19] The Markov transition matrix is thus given as

$$\begin{pmatrix} 1 - \lambda_H & \lambda_H \\ 1 - \lambda_L & \lambda_L \end{pmatrix}.$$

non-disclosure is interpreted negatively by the market, resulting in a downward revision of beliefs by second-period buyers.

With respect to the mandatory regime, the equilibrium forces are also qualitatively similar and indeed both cases described in Proposition 2 continue to hold. However, we find that the introduction of impersistence decreases the incentive for mimicry and the welfare gap between the two regimes. First, in this extended setting, acceptance of a low offer in the first period becomes more attractive for the L-seller relative to the baseline setting. Following acceptance of v_L, second-period buyers have the following belief that the state in period 2 will be of type H:

$$\tilde{T}(\pi_0, v_L) = 1 - \lambda_L,$$

which is strictly higher than the posterior after acceptance of v_L in the baseline setting (where $T(\pi_0, v_L) = 0$). Likewise, first-period mimicry through rejection of low offers becomes less attractive for the L-seller. Because a seller of type H has positive probability of becoming type L in period 2, second-period buyers place less weight on the signal from offer rejection in the first period. (Technical details are included in the Appendix; see the proof of Proposition 5.)

In sum, impersistence strengthens the incentive for acceptance and weakens the incentive for rejection in the mandatory regime. Consequently, the welfare gap between voluntary and mandatory disclosure decreases in the degree of impersistence. Indeed, as the transition probability λ_{θ_1} approaches 1/2, asset quality becomes serially uncorrelated and welfare becomes equal in the two regimes. Nevertheless, as the economic forces are preserved in this setting, our main welfare results also continue to hold in the presence of impersistence, as stated in the following proposition:

Proposition 5. *Assume $\lambda_L \geq \lambda_H$. Theorem 1 continues to hold for sufficiently high positive serial correlation in asset quality, i.e., if $\lambda_L \geq \lambda_L^*$ and $\lambda_H \leq \lambda_H^*$.*

Figure 1 depicts the effect of imperfect persistence on the welfare gap between voluntary and mandatory disclosure. We additionally see in the figure that the minimum level of serial correlation, λ_L^*, that supports the welfare advantage of the voluntary regime is decreasing in c_L. This is due to the strengthened mimicry incentive by the L-seller when c_L is high, resulting in the welfare gap holding for a greater degree of impersistence.

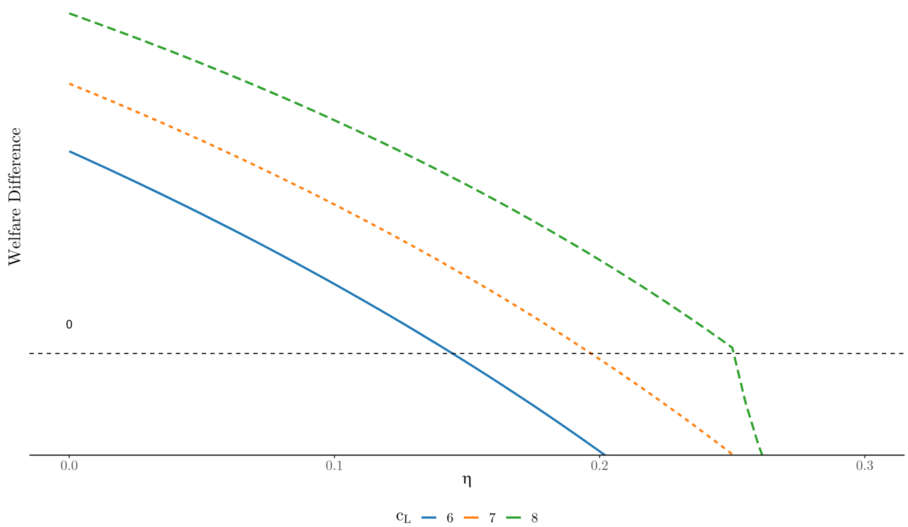

Figure 1: Welfare advantage of the voluntary regime under imperfect persistence

This figure shows the welfare difference of the two regimes under each imperfect persistence parameter η. where $\lambda_H = \eta$ and $\lambda_L = 1 - \eta$. The horizontal line shows the zero line, and the positive value mean that the welfare under the voluntary regime is higher than the mandatory regime. The parameters are set as follows: $q = 0.6, \pi_0 = 0.65, v_L = 15, v_H = 30$ and $c_H = 20$.

5.2 Stochastic buyer arrival

In our baseline model, the only possibility of trade not occurring is due to the seller rejecting offers. While this is reasonable in most situations, there may be factors outside of the seller's control that prevents trade from occurring. In this section, we extend the model to incorporate the possibility that trade cannot happen due to, for example, buyers failing to arrive to the market. Specifically, we assume with probability $s \in (0, 1)$ that buyers arrive in the first period and observe private signals. For parsimony, buyers continue to arrive with probability one in the second period.

In the voluntary regime, the equilibrium in this extended setting is qualitatively similar to the baseline setting. In the mandatory regime, as buyers cannot determine if no trade occurred due to seller rejection of low offers or the lack of trade opportunities, the posterior after observing no trade in the first period becomes

$$\hat{T}(\pi_0, r) = s \cdot T(\pi_0, r) + (1 - s)\pi_0.$$

We see that the possibility of no offers in the first period shifts weight away from the rejection signal. In the case where the L-seller ex ante prefers to reject low offers in the first period (case (i) of Proposition 2), the presence of $(1-s)\pi_0$ in the posterior mitigates the downward updating by second-period buyers following first-period rejection. As such, this strengthens the L-seller's incentive to reject low offers and case (i) occurs for an even greater range of parameter values. The presence of stochastic buyer arrival therefore widens the welfare gap between voluntary and mandatory disclosure in this situation.

Conversely, when the L-seller ex ante prefers to accept low offers in the first (case (ii) of Proposition 2), second-period buyers update less favorably on the lack of trade in the first period, which lowers the L-seller's continuation value $Q_L(\pi_1)$. As condition (7) must hold with equality in this case, the L-seller accepts low offers with a higher probability to raise $Q_L(\pi_1)$ to satisfy indifference. Hence, stochastic buyer arrival improves efficiency in this situation as the L-seller accepts more frequently in the first period.

6 Concluding remarks

In this study, we examine a repeated trade setting where buyers receive private signals and the seller can voluntarily disclose past trade information. Despite the fact that strategic disclosure results in a more opaque information environment for buyers, voluntary disclosure can be welfare-enhancing relative to a mandatory disclosure regime where buyers always observe past trade behavior. This occurs because, when the low-type seller's mimicry incentive is strong, an unfavorable equilibrium emerges in the mandatory regime whereby the low-type seller rejects early low offers, but this early rejection is not met favorably by second-period buyers and instead results in a downward revision of beliefs over the seller's quality. The voluntary disclosure regime improves upon this situation, as it removes the low-type seller's ability to mimic a higher type by rejecting offers. As such, the low-type seller always trades at the low offer in every period, even when this is the highest offer.

The above result depends on two critical economic forces. First, the presence of buyer signals drives the emergence of the bad equilibrium; rejection by the seller in the first period under mandatory disclosure implies that both first-period buyers received the low signal, leading to a downward revision under certain conditions. Moreover, the low-type seller in this situation rejects offers in the first period due to the possibility of receiving a high offer in the second period, which is due to buyer private signals. Second, while this same force should appear in the voluntary disclosure regime as well—the lack of disclosure implies that only low offers arrived—the fact that the seller can strategically withhold trade of low offers largely attenuates the signaling incentives from rejection found in the mandatory regime. We additionally find that these forces are preserved in the long horizon and voluntary disclosure can be welfare-improving over mandatory disclosure even when the horizon is arbitrarily or infinitely long.

References

Acharya, V. V., P. DeMarzo, and I. Kremer (2011): "Endogenous information flows and the clustering of announcements," *American Economic Review*, 101, 2955–2979.

Aghamolla, C. and B.-J. An (2021): "Voluntary disclosure with evolving news," *Journal of Financial Economics*, 140, 21–53.

Aghamolla, C. and T. Hashimoto (2022): "Endogenous offer arrival in the market for lemons," *Available at SSRN 4259615*.

Asriyan, V., W. Fuchs, and B. Green (2017): "Information spillovers in asset markets with correlated values," *American Economic Review*, 107, 2007–40.

——— (2021): "Aggregation and design of information in asset markets with adverse selection," *Journal of Economic Theory*, 191, 105124.

Banks, J. S. and J. Sobel (1987): "Equilibrium selection in signaling games," *Econometrica: Journal of the Econometric Society*, 647–661.

Ben-Porath, E., E. Dekel, and B. L. Lipman (2018): "Disclosure and choice," *The Review of Economic Studies*, 85, 1471–1501.

Bertomeu, J. and D. Cianciaruso (2018): "Verifiable disclosure," *Economic Theory*, 65, 1011–1044.

Bertomeu, J., I. Marinovic, S. J. Terry, and F. Varas (2022): "The dynamics of concealment," *Journal of Financial Economics*, 143, 227–246.

Bessembinder, H. and W. Maxwell (2008): "Markets: Transparency and the corporate bond market," *Journal of economic perspectives*, 22, 217–234.

Beyer, A. and R. A. Dye (2012): "Reputation management and the disclosure of earnings forecasts," *Review of Accounting Studies*, 17, 877–912.

Cho, I.-K. and D. M. Kreps (1987): "Signaling games and stable equilibria," *The Quarterly Journal of Economics*, 102, 179–221.

Daley, B. and B. Green (2012): "Waiting for News in the Market for Lemons," *Econometrica*, 80, 1433–1504.

——— (2016): "An Information-Based Theory of Time-Varying Liquidity," *The Journal of Finance*, 71, 809–870.

Dye, R. A. (1985): "Disclosure of nonproprietary information," *Journal of accounting research*, 123–145.

Dye, R. A. and S. S. Sridhar (1995): "Industry-wide disclosure dynamics," *Journal of Accounting research*, 33, 157–174.

Einhorn, E. and A. Ziv (2008): "Intertemporal dynamics of corporate voluntary disclosures," *Journal of Accounting Research*, 46, 567–589.

Farrell, J. (1993): "Meaning and credibility in cheap-talk games," *Games and Economic Behavior*, 5, 514–531.

Fuchs, W., A. Öry, and A. Skrzypacz (2016): "Transparency and distressed sales under asymmetric information," *Theoretical Economics*, 11, 1103–1144.

Gertner, R., R. Gibbons, and D. Scharfstein (1988): "Simultaneous signalling to the capital and product markets," *The RAND Journal of Economics*, 173–190.

Glode, V., C. C. Opp, and X. Zhang (2018): "Voluntary disclosure in bilateral transactions," *Journal of Economic Theory*, 175, 652–688.

Grossman, S. J. (1981): "The informational role of warranties and private disclosure about product quality," *The Journal of Law and Economics*, 24, 461–483.

Grossman, S. J. and O. D. Hart (1980): "Takeover bids, the free-rider problem, and the theory of the corporation," *The Bell Journal of Economics*, 42–64.

Grossman, S. J. and M. Perry (1986): "Perfect sequential equilibrium," *Journal of economic theory*, 39, 97–119.

Guttman, I., I. Kremer, and A. Skrzypacz (2014): "Not only what but also when: A theory of dynamic voluntary disclosure," *American Economic Review*, 104, 2400–2420.

Hörner, J. and N. Vieille (2009): "Public vs. private offers in the market for lemons," *Econometrica*, 77, 29–69.

Kaya, A. and K. Kim (2018): "Trading dynamics with private buyer signals in the market for lemons," *The Review of Economic Studies*, 85, 2318–2352.

Kaya, A. and Q. Liu (2015): "Transparency and price formation," *Theoretical Economics*, 10, 341–383.

Kaya, A. and S. Roy (2022): "Market screening with limited records," *Games and Economic Behavior*, 132, 106–132.

——— (2023): "Repeated Trading: Transparency and Market Structure," *Available at SSRN 4336535*.

Kim, K. (2017): "Information about sellers' past behavior in the market for lemons," *Journal of Economic Theory*, 169, 365–399.

Kona, M. and S. Corlette (2022): "Hospital And Insurer Price Transparency Rules Now In Effect But Compliance Is Still Far Away," *Health Affairs Forefront*.

Li, J., T. Liu, and R. Zhao (2022): "Strategic nondisclosure in takeovers," *The Accounting Review*, 97, 345–370.

Lutz, N. A. (1989): "Warranties as signals under consumer moral hazard," *The Rand* *journal of economics*, 239–255.

Maskin, E. and J. Tirole (1992): "The principal-agent relationship with an informed principal, II: Common values," *Econometrica: Journal of the Econometric Society*, 1–42.

Milgrom, P. R. (1981): "Good news and bad news: Representation theorems and applications," *The Bell Journal of Economics*, 380–391.

Perez-Richet, E. (2014): "Interim bayesian persuasion: First steps," *American Economic Review*, 104, 469–474.

Severinov, S. (2008): "An efficient solution to the informed principal problem," *Journal of Economic Theory*, 141, 114–133.

Sobel, J. (2020): "Signaling games," *Complex Social and Behavioral Systems: Game Theory and Agent-Based Models*, 251–268.

Song Shin, H. (2003): "Disclosures and asset returns," *Econometrica*, 71, 105–133.

Appendix

A Proofs

In this section, we use the following notation. Let V be a mapping that denotes the expectation of v_θ as a function of belief π:

$$V(\pi) = \pi v_H + (1-\pi)v_L = \mathbb{E}[v_\theta; \pi].$$

Define $\pi_{s_i} = \mathbb{P}(\theta = H|s_i; \pi) = P(\pi, s_i)$ and $\pi_{s_i s_j} = \mathbb{P}(\theta = H|s_i, s_j; \pi) = P(\pi, (s_i, s_j))$. Moreover, let $\mathbb{P}(s_i|s_j)$ be the probability that buyer i observes s_i given that buyer j observes s_j given that the prior is π.

Under these notations, the conditional expectations in the main text can be simplified so that $\mathbb{E}[v_\theta|s_i; \pi] = V(\pi_{s_i})$ and $\mathbb{E}[v_\theta|s_i, s_j; \pi] = V(\pi_{s_i s_j})$.

Proof of Lemma 1

The case of $\pi \in \{0, 1\}$ is straightforward since this is equivalent to standard Bertrand competition. Now, we consider each case when $\pi \in (0, 1)$. Let $U_b(x_i|s_i)$ be the expected payoff of an s_i-buyer when he bids x_i.

Case 1. (Low beliefs.) $V(\pi_h) \leq c_H$. Suppose that $c_H \geq V(\pi_{hh})$. The h-buyer finds it unprofitable to target the H-seller even when the other buyer received a high signal. Thus, both buyer types bid v_L. Now suppose that $c_H \in [V(\pi_h), V(\pi_{hh}))$. Clearly, the ℓ-buyer optimally targets only the L-seller. Consider next an h-buyer. The best deviation is to overbid c_H,[20] which gives a payoff of

$$\begin{aligned}
U_b(c_H|h) &= \mathbb{P}(\ell \mid h)(V(\pi) - c_H) + \mathbb{P}(h \mid h)(V(\pi_{hh}) - c_H) \\
&\leq \mathbb{P}(\ell \mid h)(V(\pi) - V(\pi_h)) + \mathbb{P}(h \mid h)(V(\pi_{hh}) - V(\pi_h)) \\
&= 0.
\end{aligned}$$

Hence, this deviation is unprofitable.

Case 2. (Low-intermediate beliefs.) $V(\pi) \leq c_H < V(\pi_h)$. We argue that the bidding

[20] We say that a buyer overbids x if he bids $x + \varepsilon$, where $\varepsilon > 0$ is an arbitrary small number. For brevity, we omit ε from the payoff computation.

strategy described in the statement with

$$\sigma = \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{c_H - V(\pi)}{V(\pi_{hh}) - c_H}$$

and

$$\tilde{G}_2(x) = \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{(x - V(\pi))}{(V(\pi_{hh}) - x)}$$

is the equilibrium. Consider an ℓ-buyer. He expects zero profit by making an offer of v_L. If he deviates to $x < v_L$, he gets zero profit as well. If he deviates to $x \in (v_L, c_H)$, his expected payoff is $U_b(x|\ell) = \mathbb{P}(\ell|\ell)(v_L - x) + \mathbb{P}(h|l)\sigma(v_L - x)c < 0$. Since $c_H \geq \pi$, bidding $x \geq c_H$ is also not profitable. Consider next an h-buyer. If he overbids $V(\pi_h) = \mathbb{P}(\ell \mid h)V(\pi) + \mathbb{P}(h \mid h)V(\pi_{hh})$, his expected payoff is

$$\mathbb{P}(\ell \mid h)(V(\pi) - V(\pi_h)) + \mathbb{P}(h \mid h)(V(\pi_{hh}) - V(\pi_h)) = 0.$$

If he overbids v_L, he receives zero profit. The indifference condition combined with $\tilde{G}_2(c_H) = \sigma$ gives $\tilde{G}_2$ and σ.

Case 3. (High-intermediate beliefs.) $V(\pi_{\ell\ell}) < c_H < V(\pi)$. Suppose that an h-buyer randomizes over $[c_H, \mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi_{hh})]$ with the cdf

$$\tilde{G}_3(x) = \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{(x - c_H)}{(V(\pi_{hh}) - x)}.$$

We argue that the bidding strategy described in the statement is the equilibrium. Consider an h-buyer. If he overbids $y := \mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi_{hh})$, his expected payoff is $\mathbb{P}(\ell \mid h)(V(\pi) - x) + \mathbb{P}(h \mid h)(V(\pi_{hh}) - x) = \mathbb{P}(\ell \mid h)(V(\pi) - c_H) > 0$. If he overbids v_L, he again expects $\mathbb{P}(\ell \mid h)(V(\pi) - c_H) > 0$. He cannot do better than this. The indifference condition determines G_3. Consider next an ℓ-buyer. If he bids v_L, he expects zero profit. If he bids $x \in [c_H, \mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi_{hh})]$, he expects

$$U_b(x|\ell) = \mathbb{P}(\ell \mid \ell)(V(\pi_{\ell\ell}) - x) + \mathbb{P}(h \mid \ell)\tilde{G}_3(x)(V(\pi) - x).$$

The first term is always negative, and the second term is negative for $x > V(\pi)$.

Suppose next that $x \in (c_H, V(\pi))$. Observe that

$$\frac{\partial U_b(x \mid \ell)}{\partial x} = -\mathbb{P}(\ell \mid \ell) + \mathbb{P}(h \mid \ell) \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \left[\frac{V(\pi_{hh}) - c_H}{(V(\pi_{hh}) - x)^2} (V(\pi) - x) - \frac{b - c_H}{V(\pi_{hh}) - x} \right]$$

and

$$\frac{\partial^2 U(x \mid \ell)}{\partial b^2} = -\mathbb{P}(h \mid \ell) \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{2(V(\pi_{hh}) - V(\pi))(V(\pi_{hh}) - c_H)}{(x - V(\pi_{hh}))^3} < 0.$$

Since $\frac{\partial U_b(c_H|\ell)}{\partial x} < 0$, we have that $\frac{\partial U_b(x|\ell)}{\partial x} < 0$ for all $x \in (c_H, V(\pi))$. Since

$$U_b(c_H|\ell) = \mathbb{P}(\ell|\ell)(V(\pi_{\ell\ell}) - c_H) < 0,$$

the deviation is not profitable. Finally, if he overbids $\mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi_{hh})$, he expects

$$\begin{aligned}
&\mathbb{P}(\ell \mid \ell)V(\pi_{\ell\ell}) + \mathbb{P}(h \mid \ell)V(\pi) - (\mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi_{hh})) \\
&\quad < \mathbb{P}(\ell \mid \ell)V(\pi_{\ell\ell}) + \mathbb{P}(h \mid \ell)V(\pi) - (\mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi)) \\
&\quad < 0,
\end{aligned}$$

where the last inequality is from $\mathbb{P}(h|h) - \mathbb{P}(h|\ell) = (\pi_h - \pi_\ell)(2q - 1) > 0$. Hence, the deviation is not profitable.

Case 4. (High beliefs.) $V(\pi_{\ell\ell}) > c_H$. Suppose that an h-buyer randomizes over $[V(\pi_{\ell\ell}), \mathbb{P}(\ell \mid h)V(\pi_{\ell\ell}) + \mathbb{P}(h \mid h)V(\pi_{hh})]$ with

$$\tilde{G}_4(x) = \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{(x - V(\pi_{\ell\ell}))}{(V(\pi_{hh}) - x)}.$$

We argue that the offer strategy described in the statement is the equilibrium. By overbidding $V(\pi_{\ell\ell})$, the h-bidder can secure $\mathbb{P}(\ell \mid h)(V(\pi) - V(\pi_{\ell\ell}))$. By bidding in the support of $\tilde{G}_4$, he expects $U_b(x|h) = \mathbb{P}(\ell \mid h)(V(\pi) - x) + \mathbb{P}(h \mid h)\tilde{G}_4(b)(V(\pi_{hh}) - x)$. The indifference condition determines $\tilde{G}_4$. Consider an ℓ-buyer. He expects zero profit by bidding $V(\pi_{\ell\ell})$. As in the previous case, one can show that bidding $x > V(\pi_{\ell\ell})$ is not profitable.

Proof of Proposition 1

First, we show Part (iii) of the statement under the conjectured equilibrium. The disclosure gives a partition of the signal space $\{(\ell,\ell),(\ell,h),(h,\ell),(h,h)\}$. If π_0 is such that Case 3 or 4 of Lemma 1 applies, a low offer (no disclosure) corresponds to (ℓ,ℓ), so $\pi^{d=0} = P(\pi_0,(\ell,\ell)) < \pi_0$. By Bayes's law, $\pi^{d=1} > \pi_0$. If π_0 is such that Case 2 of Lemma 1 applies, then disclosure of an offer implies that either (i) only one of the signals was h and the h-buyer did not bid v_L, or (ii) two of the signals were h and both buyers did not bid v_L, so

$$\pi^{d=1} = \frac{(\mathbb{P}(\ell,h;\pi_0)+\mathbb{P}(h,\ell;\pi_0))(1-\sigma)\pi_0 + \mathbb{P}(h,h;\pi_0)(1-\sigma^2)P(\pi_0,(h,h))}{(\mathbb{P}(\ell,h;\pi_0)+\mathbb{P}(h,\ell;\pi_0))(1-\sigma) + \mathbb{P}(h,h;\pi_0)(1-\sigma^2)},$$

where $\mathbb{P}(s_i,s_j;\pi_0)$ is the probability that the signal realization is (s_i,s_j) given prior π_0, and σ is defined in Lemma 1. Since $\pi^{d=1}$ is a convex combination of π_0 and $P(\pi_0,(h,h))$, it is immediate that $\pi^{d=1} > \pi_0$. By Bayes's law, $\pi^{d=0} < \pi_0$. Finally, if Case 1 of Lemma 1 applies, then (the lack of) disclosure is not informative, so $\pi^{d=0} = \pi_0$.

Given this, Parts (i) and (ii) of the statement can be shown as follows. Suppose that the θ-seller deviates and rejects an offer $x_i \geq c_\theta$. Then, the posterior upon non-disclosure changes to $\pi^{d=0}$, while the period-1 payoff weakly decreases from $x_i - c_\theta \geq 0$ to 0. Since the posterior π^d under the original strategy is at least $\pi^{d=0}$, this is a contradiction. Moreover, if we assign an off-path belief that is less than $\pi^{d=0}$ to disclosure of a low offer ($p_1 = v_L$), both sellers do not deviate to accepting and disclosing the low offer. Finally, given $\pi^{d=1} \geq \pi_0$, both sellers do not deviate to non-disclosure of trade at a high offer ($p_1 \geq c_H$).

Proof of Proposition 2

From Lemma 1, $Q_L(\pi)$ is weakly increasing in π. Since

$$\pi_1 = T(\pi_0, r) = \frac{D(\pi_0, d=0)}{D(\pi_0,d=0) + (1-D(\pi_0,d=0))(1-\phi)},$$

π_1 is monotonically increasing in ϕ. Thus, Q_L is monotonically increasing in ϕ. When $\phi = 0$, $\pi_1 = D(\pi_0, d=0)$, so $Q_L(\pi_1) = Q_L(D(\pi_0,d=0))$. When $\phi = 1$, $\pi_1 = 1$, so $Q_L(\pi_1) = Q_L(1) = v_H - c_L$. Therefore, if $(1+\delta)(v_L - c_L) < Q_L(D(\pi_0,0))$, then $\phi^* = 0$, i.e., the L-seller striclty prefers to reject. Otherwise, by the monotonicity of Q_L, there is a unique solution to (7) as long as $(1+\delta)(v_L - c_L)$ intersects with δQ_L where it is discontinuous. But, in that case, by appropriately specifying the buyer's mixing strategy when the belief is on the boundary of Case 3 and Case 4, there is a unique solution. Furthermore, if

$(1+\delta)(v_L - c_L) < \delta Q_L(\pi_0)$, then by the equilibrium condition, $Q_L(\pi_1) < Q_L(\pi_0)$, so by the monotonicity of Q_L, we have $\pi_1 < \pi_0$. Part (ii) can be shown similarly.

Proof of Theorem 1

First, we formally define welfare. Let $\mathcal{R} \in \{\mathcal{D}, \mathcal{T}\}$ denote each regime, where $\mathcal{D}$ is the voluntary disclosure regime and $\mathcal{T}$ is the transparent/mandatory disclosure regime. Let $W_t(\mathcal{R}|\theta) = \mathbb{I}(\text{trade})(v_\theta - c_\theta)$ be welfare in period t conditional on the seller's type under regime $\mathcal{R}$. Let $W_t(\mathcal{R}) = \pi_0 W_t(\mathcal{R}|H) + (1-\pi_0)W_t(\mathcal{R}|L)$ be period-t welfare under regime $\mathcal{R}$. Overall welfare for regime $\mathcal{R}$ is defined by $W(\mathcal{R}) = W_1(\mathcal{R}) + \delta W_2(\mathcal{R})$.

Suppose that π_0 is such that Case 4 applies ($\pi_0 > \pi^*$). Under the mandatory regime, the L-seller accepts any trade for sure as any offer in equilibrium is no less than c_H. Thus, the acceptance behavior under both regimes are the same, and so is the evolution of beliefs. Hence, both regimes provide the same welfare. This proves part (i) of the statement.

To prove part (ii) and (iii), to simplify notation, we use an equivalent formulation of the model where c_θ is a payoff from the outside option instead of the production cost (see Footnote 6)). This amounts to adding c_θ to payoffs of the seller in each period where trade does not occur. Since welfare is the sum of payoffs for all agents, the change in welfare is that c_θ is added for each term of welfare, i.e., $W(\mathcal{R}|\theta) = \mathbb{I}(\text{trade})v_\theta + (1-\mathbb{I}(\text{trade}))c_\theta$. Since we are interested in the difference in welfare between two regimes, this modification in the assumption does not affect the result.

We compute payoffs of each agent. Let $U_s^{\mathcal{R}}(\pi_0|\theta)$ be the total expected payoff of the θ-seller under regime $\mathcal{R}$ as a function of the initial belief π_0.[21] Moreover, let $\Pi_\theta(\pi)$ be the θ-seller's expected payoff in the static bidding game of Lemma 1 given prior belief π. Similarly, $\Pi_{s_i}(\pi)$ is the s_i-buyer's expected payoff in the static bidding game. From now on, we restrict π_0 to be $\pi_0 < \pi^*$.

To determine payoffs of the seller, let p_θ be the probability that the highest offer is v_L given that the state is θ. Then, the L-seller's expected payoff in each regime is

$$U_s^{\mathcal{D}}(\pi_0|L) = p_L(v_L + \delta\Pi_L(\pi^{d=0})) + (1-p)(\mathbb{E}[x;\pi_0] + \delta\Pi_L(\pi^{d=1})),$$

$$U_s^{\mathcal{T}}(\pi_0|L) = \begin{cases} p_L((1+\delta)v_L) + (1-p_L)(\mathbb{E}[x;\pi_0] + \delta\Pi_L(\pi^{d=1})), & \text{if Eq. (7) holds with equality} \\ p_L(c_L + \delta\Pi_L(\pi^{d=0})) + (1-p_L)(\mathbb{E}[x;\pi_0] + \delta\Pi_L(\pi^{d=1})), & \text{otherwise.} \end{cases}$$

[21] In the two-period model, the L-seller's static payoff $\Pi_L(\pi)$ is equal to the L-seller's continuation payoff $Q_L(\pi)$. These are not the same for the T-period model with $T \geq 3$.

In the first period, if the offer is not v_L, the expected payoff is determined by the expectation of the randomized offer, $\mathbb{E}[x;\pi_0]$. Under the mandatory regime, given that the highest offer is v_L, the L-seller's payoff is pegged at $(1+\delta)v_L$ if the indifference condition holds with equality. If the L-seller strictly prefers to reject the low offer (i.e., $\phi = 0$), then the posterior upon rejection is $\pi^{d=0}$.

The H-seller's expected payoff in each regime is

$$U_s^{\mathcal{D}}(\pi_0|H) = p_H(c_H + \delta\Pi_H(\pi^{d=0})) + (1-p_H)(\mathbb{E}[x;\pi_0] + \delta\Pi_H(\pi^{d=1})),$$

$$U_s^{\mathcal{T}}(\pi_0|H) = p_H(c_H + \delta\Pi_H(T(\pi_0 L, r))) + (1-p_H)(\mathbb{E}[x;\pi_0] + \delta\Pi_H(\pi^{d=1})).$$

Under both regimes, the H-seller rejects the low offer in the first period. The only difference is that, when the first-period offer was low, in the second period, the H-seller has a higher chance of getting a high offer because $T(\pi_0, r) \geq \pi^{d=0}$.

From the proof of Lemma 1, in a static bidding game, the ℓ-buyer always makes zero profit regardless of the initial belief. The h-buyer gets positive payoff only when π_0 is high enough (Cases 3 or 4). In the first period, when both buyers receive the low signal, the offer may be rejected with different probabilities under the two regimes, but buyers get zero payoff anyway. When one of the buyers receives the high signal, a winning buyer's offer is accepted for sure under both regimes, and he potentially obtains a positive payoff. In any case, the first-period buyers obtain the same expected payoff under both regimes. Moreover, in the second period, if the first-period offer was high, the posterior is the same under both regimes and so is the expected payoffs of the buyers. If the first-period offer was low, then in the second period buyers could receive different expected payoffs depending on the regime.

We now analyze the difference in the second-period buyer's expected payoff when the first-period offer was low. Recall that, in this case, π_0 is such that Case 1–3 applies. When π_0 is such that Case 1 or 2 applies in the first period, then following non-disclosure, $\pi^{d=0}$ is such that Case 1 or 2 applies in the second period. When π_0 is such that Case 3 applies in $t=1$, again, following non-disclosure, $\pi^{d=0}$ is such that Case 1 or 2 applies in $t=2$. To see this, let $\bar{\pi}$ be such that $V(\bar{\pi}) = c_H$. Note that when π_0 is the highest possible value in the Case 3 region, i.e., $\pi_{\ell\ell} = \bar{\pi}$, we have $D(\pi_0, 0) = P(\pi_0, ll) = \pi_{\ell\ell} = \bar{\pi}$, so Case 2 applies. Therefore, for any π_0 in Case 3 in $t=1$, $\pi^{d=0}$ is such that Case 1 or 2 applies in $t=2$. This shows that, for any $\pi_0 < \pi^*$, under the voluntary regime, the h-buyer expects zero profit in the second period if the first-period offer was low. On the other hand, under the mandatory regime, the belief after rejection of the low offer could be such that Case 3 or 4 applies.

Thus, the h-buyer expects $\Pi_h(T(\pi_0, r)) \geq 0$ in the second period when the first-period offer was low, and this event occurs with probability $p(1-\phi)$, where $p = \pi_0 p_H + (1-\pi_0)p_L$ is the ex-ante probability that the first-period offer is low.

In summary, the difference in payoffs of each agent between the voluntary and mandatory regime as a function of π_0 and c_L, denoted by $f(\pi_0, c_L)$, is given by

$$f(\pi_0, c_L) = \delta\Big[(1-\pi_0)p_L\{\Pi_L(\pi^{d=0}) - v_L\} \\ - \pi_0 p_H\{\Pi_H(T(\pi_0, r)) - \Pi_H(\pi^{d=0})\}\Big],$$

when Eq. (7) holds with equality, and

$$f(\pi_0, c_L) = (1-\pi_0)p_L(v_L - c_L) \\ - \delta\Big[\pi_0 p_H\{\Pi_H(T(\pi_0, r)) - \Pi_H(\pi^{d=0})\} \\ - p(1-\phi)\Pi_h(T(\pi_0, r))\Big],$$

otherwise.

We prove part (ii) and (iii) as follows. We fix $c_L = v_L - \varepsilon$ and $\pi_0 = \pi^* - \varepsilon'$ where $\varepsilon > 0$ and $\varepsilon' > 0$ are arbitrarily small numbers, and show that the voluntary disclosure regime gives strictly higher welfare. By the continuity of f, this shows that there are open intervals $(\pi^\dagger, \pi^*)$ and $(c_L^\dagger, v_L)$ over which the voluntary regimes gives strictly higher welfare.

When ε' is small enough, $D(\pi_0, 0)$ is such that Case 2 applies, so the seller's continuation payoff satisfies

$$\delta Q_L(\pi_1) \geq \delta Q_L(D(\pi_0, 0)) = \delta\left[\mathbb{P}(\text{high offer})\underbrace{\mathbb{E}[p_2; \text{high offer}]}_{\geq c_H} + (1 - \mathbb{P}(\text{high offer}))\varepsilon\right] > (1+\delta)\varepsilon$$

for ε small enough. That is, there is a nonzero probability that the next-period offer is high when ε' is small enough, so when the cost of waiting is small, the seller prefers to reject for sure. But then, $T(\pi_0, v_L, r) = \pi^{d=0}$, so $f(\pi_0, c_L) = (1-\pi_0)p_L\varepsilon > 0$, meaning that the voluntary disclosure regime gives strictly higher welfare.

Proof of Theorem 2

As in the proof of Theorem 1. we prove the result in the equivalent version of the model, where c_θ is interpreted as a payoff from an outside option. Recall that the equilibrium in the voluntary regime is not affected by the value of c_L. Thus, we consider the mandatory regime. We show that the trading frequency in the mandatory regime is weakly decreasing in c_L.

Consider the L-seller. In the voluntary regime, she trades with probability one in both periods. In the mandatory regime, she rejects a low offer with some probability in the first period.

When the initial belief is high enough such that Case 4 of Lemma 1 applies, the trading frequency under the two regimes is the same. This is because all seller-types trade with probability one in the first period. Thus, we restrict to the case where π_0 is such that Cases 1–3 of Lemma 1 apply.

Consider the L-seller. The trading frequency is always lower in the mandatory regime compared to the voluntary regime, where the L-seller always trades. Moreover, it is straightforward to show that the acceptance probability ϕ is decreasing in c_L. Hence, the L-seller trades less as c_L increases.

Next, consider the H-seller. Since the belief upon rejection of offer v_L, $T(\pi_0, r)$, is weakly greater than $D(\pi_0, 0)$, the trading frequency is weakly higher in the mandatory regime. Moreover, $T(\pi_0, r)$ is increasing in ϕ, so it is decreasing in c_L.

Therefore, the trading frequency in the mandatory regime is monotonically decreasing in c_L. When c_L tends to its upper bound v_L, the L-seller rejects with probability one, so the posterior upon rejection of a low offer, $T(\pi_0, r)$, tends to $D(\pi_0, 0)$. That is, no additional information would be revealed from the rejection. Thus, for the H-seller, the trading frequency tends to the same across two regimes. Hence, when $c_L \to v_L$, the trading frequency is strictly higher under the voluntary regime compared to the mandatory regime. Thus, by the continuity of the trading frequency with respect to c_L, the strict ordering holds for c_L high enough.

Proof of Corollary 1

We consider the difference in welfare between the two regimes for each state, $W(\mathcal{D} \mid \theta) - W(\mathcal{T} \mid \theta)$. Observe that $D(\pi_0, 0)$ is decreasing in q. Therefore, when the equilibrium condition (7) holds with equality, we must have

$$\delta Q_L\left(\underbrace{\frac{D(\pi_0,0)}{D(\pi_0,0)+(1-D(\pi_0,0))(1-\phi)}}_{=\pi_1}\right)=(1+\delta)(v_L-c_L).$$

Since π_1 is increasing in $D(\pi_0, 0)$ and increasing in ϕ, it follows that ϕ is increasing in q. If the equilibrium condition holds with strict inequality, then ϕ does not change for a marginal change in q. The welfare difference given $\theta = L$ is

$$W(\mathcal{D}\mid L)-W(\mathcal{T}\mid L)=\mathbb{P}(p_1=v_L\mid L)(1-\phi)(v_L-c_L),$$

which is decreasing in q.

For $\theta = H$, the welfare difference is proportional to $T(\pi_0, r) \geq D(\pi_0, 0)$. The above analysis of the equilibrium condition suggests that $T(\pi_0, r)$ is constant with respect to a marginal change in q. Hence, the welfare difference given $\theta = H$ is constant with respect to a marginal change in q.

In summary, we have shown that $W(\mathcal{D}) - W(\mathcal{T})$ is weakly decreasing in q.

Proof of Proposition 3

A *DOWN* cascade occurs when the first-period signals are (ℓ, ℓ). This event happens with positive probability regardless of θ, π_0, and T. Thus, a *DOWN* cascade occurs with positive probability for any case.

Now, consider $\theta = H$. Since $D(\pi, 1) > \pi$ for any $\pi \in (0, 1)$, with positive probability there exists $\tau < \infty$ such that π_τ reaches Case 4, i.e., $\mathbb{E}[v_\theta \mid s_i = \ell, s_j = \ell, \pi_\tau] > c_H$ Define the Markov process $\{\tilde{\pi}_t\}_{t=\tau}^{\infty}$, $\tilde{\pi}_\tau = \pi_\tau$, where the transition is taken as if the process is always in Case 1. Then, using the transition probabilities, it is straightforward to show that the log-odds ratio $\log \frac{\tilde{\pi}_t}{1-\tilde{\pi}_t}$ is a random walk with positive drift, so $\{\tilde{\pi}_t\}_{t=\tau}^{\infty} \to 1$ a.s. Since there is positive probability that $\{P(\tilde{\pi}_t, ll)\}_{t=\tau}^{\infty}$ does not down-cross $\overline{\pi}$, it follows that $\mathbb{P}(\{\tilde{\pi}_t\}_{t=\tau}^{\infty} = \{\pi_t\}_{t=\tau}^{\infty}) > 0$. Hence, we conclude that $\{\pi_t\}_{t=1}^{\infty} \to 1$ with positive probability.

Next, consider $\theta = L$. If $T < \infty$, then a *DOWN* cascade occurs with positive probability as we observed. If $T = \infty$, then in a similar manner as above, we can show that the belief drops until Case 1 applies with probability one.

Proof of Theorem 3

Let $c_L = v_L - \varepsilon$ for an arbitrary $\varepsilon > 0$. Let $\pi_0 < \pi^*$. Consider the mandatory regime. In the first period, if v_L is offered, the L-seller rejects with probability one for ε small enough. To see this, note that the equilibrium condition in the first period (t=1) is

$$\varepsilon \leq \frac{\delta(1-\delta)}{1-\delta^T} Q_L(T(\pi_0, r)).$$

The continuation payoff Q_L can be evaluated as

$$\begin{aligned} Q_L(T(\pi_0, r)) &\geq Q_L(D(\pi_0, 0)) \\ &\geq \mathbb{P}(p_2 \geq c_H \mid D(\pi_0, 0))\Pi_L(D(\pi_0, 0)) + \mathbb{P}(p_2 = v_L \mid D(\pi_0, 0))\varepsilon, \end{aligned}$$

where Π_L is the L-seller's expected payoff in a static bidding game. When $\pi_0 < \pi^*$ is high enough, $D(\pi_0, 0)$ is such that Case 2 of Lemma 1 applies. Thus, $\mathbb{P}(\text{offer} \geq c_H \mid D(\pi_0, 0))$ is strictly positive and does not depend on ε. Hence, if we choose

$$\varepsilon < \varepsilon^* := \frac{\frac{\delta(1-\delta)}{1-\delta^T}\mathbb{P}(p_2 \geq c_H \mid D(\pi_0, 0))\Pi_L(D(\pi_0, 0))}{1 - \frac{\delta(1-\delta)}{1-\delta^T}\mathbb{P}(p_2 = v_L \mid D(\pi_0,0))}, \tag{8}$$

then for any $\varepsilon < \varepsilon^*$, there exists an $\eta_\varepsilon > 0$ such that $\varepsilon < \frac{\delta(1-\delta)}{1-\delta^T} Q_L(T(\pi_0, r))$ $\pi_0 = \pi^* - \eta_\varepsilon$ we have $Q_L(T(\pi_0, 0)) > \varepsilon$. In this case, there is no additional information in the rejection in the first period, so $T(\pi_0, r) = D(\pi_0, 0)$.

Now, suppose we take ε and π_0 such that the L-seller rejects w.p. one in the first period as above. We compute the welfare gain (loss) of the voluntary regime compared to the mandatory regime for each seller. Consider the L-seller. The expected welfare gain in the voluntary regime is

$$\mathbb{P}(p_t = v_L \mid \pi_{t-1}; L)(1-\phi^t)(v_L - c_L) = \mathbb{P}(p_t = v_L \mid \pi_{t-1}; L)(1-\phi^t)\varepsilon,$$

where ϕ^t is the probability that the L-seller accepts the v_L-offer in period t given the history up to that period. Thus, the total discounted welfare gain is

$$\sum_{t=1}^{T} \delta^{t-1}\mathbb{E}\left[\mathbb{P}(p_t = v_L \mid \pi_{t-1}; L)(1-\phi^t)\varepsilon\right].$$

Next, consider the H-seller. In period t, the welfare loss in the voluntary regime in period

$t \geq 2$ is

$$\mathbb{P}(p_{t-1} = v_L \mid H, \pi_{t-2})(1 - \phi^{t-1})(w(T(\pi_{t-2}, 0)) - w(D(\pi_{t-2}, 0)),$$

where $w \in \{v_H - c_H, 0\}$ is the static welfare given belief. Thus, the total welfare loss in the voluntary regime is

$$\sum_{t=2}^{T} \delta^{t-1} \mathbb{E}\left[\mathbb{P}(p_{t-1} = v_L \mid H, \pi_{t-2})(1 - \phi^{t-1})(w(T(\pi_{t-2}, 0)) - w(D(\pi_{t-2}, 0))\right].$$

Therefore, the net gain in the voluntary regime is

$$\begin{aligned}
(1-\pi_0)&\Bigg[\sum_{t=1}^{T} \delta^{t-1} \mathbb{E}\left[\mathbb{P}(p_t = v_L \mid \pi_{t-1}; L)(1-\phi^t)\varepsilon\right]\Bigg] \\
&- \pi_0\Bigg[\sum_{t=2}^{T} \delta^{t-1} \mathbb{E}\left[\mathbb{P}(p_{t-1} = v_L \mid H, \pi_{t-2})(1-\phi^{t-1})(w(T(\pi_{t-2}, 0)) - w(D(\pi_{t-2}, 0))\right]\Bigg] \\
&= (1-\pi_0)\Bigg[\mathbb{P}(p_1 = v_L \mid \pi_0) + \delta \mathbb{E}[\mathbb{P}(p_2 = v_L \mid \pi_1)(1-\phi_1)]\Bigg]\varepsilon \\
&\quad + \sum_{t=3}^{T} \delta^{t-1} \Bigg\{\mathbb{E}\big[(1-\pi_0)\mathbb{P}(p_t = v_L \mid \pi_{t-1}; L)(1-\phi^t)\varepsilon \\
&\quad - \pi_0 \mathbb{P}(p_{t-1} = v_L \mid H, \pi_{t-2})(1-\phi^{t-1})(w(T(\pi_{t-2}, 0)) - w(D(\pi_{t-2}, 0)))\big]\Bigg\}.
\end{aligned}$$

In the above expression, the net welfare gain (loss) from period $t = 3$ onward is of order δ^2. Therefore, if we can take δ to be small enough while satisfying (8), then the expression is strictly positive, meaning that the voluntary regime gives strictly higher welfare. To see that this is possible, note that the right hand side of (8) goes to zero as $\delta \to 0$ at a rate slower than δ^2:

$$\begin{aligned}
\delta^{-2}\frac{\delta(1-\delta)\mathbb{P}(p_2 \geq c_H \mid D(\pi_0, 0))\Pi_L(D(\pi_0, 0))}{1 - \delta^T - \delta(1-\delta)\mathbb{P}(p_2 = v_L \mid D(\pi_0, 0))} &= \frac{(\delta^{-1} - 1)\mathbb{P}(p_2 \geq c_H \mid D(\pi_0, 0))\Pi_L(D(\pi_0, 0))}{1 - \delta^T - \delta(1-\delta)\mathbb{P}(p_2 = v_L \mid D(\pi_0, 0))} \\
&= \frac{\mathbb{P}(p_2 \geq c_H \mid D(\pi_0, 0))\Pi_L(D(\pi_0, 0))}{\underbrace{\delta\frac{1 - \delta^T - \delta(1-\delta)}{(1-\delta)}}_{\to 0 \text{ as } \delta \to 0} \mathbb{P}(p_2 = v_L \mid D(\pi_0, 0))}.
\end{aligned}$$

Hence, there exists $\delta^* \in (0, 1)$ and $\varepsilon^*_{\delta^*}$ such that the voluntary regime gives strictly higher welfare for any $\delta \leq \delta^*$ and $\varepsilon \leq \varepsilon^*_{\delta^*}$

Proof of Proposition 5

First, consider the voluntary disclosure regime. Observe that the L-seller's disclosure strategy is the same as the perfect-persistence case: she withholds trade if and only if the price is v_L. That she discloses a high offer ($p_1 \geq c_H$) is straightforward. To see this, suppose she receives offer v_L and discloses it. The second-period buyers' posterior belief that $\theta_2 = L$ is with probability λ_L. Alternatively, if she withholds the offer, the posterior belief that $\theta_2 = L$ is

$$\bar{D}(\pi_0, 0)\lambda_H + (1 - \bar{D}(\pi_0, 0))\lambda_L,$$

where $\bar{D}(\pi_0, 0) \in (0, 1)$ is the posterior belief that $\theta_1 = H$ given non-disclosure. By the assumption $\lambda_H \leq \lambda_L$, the above expression is weakly less than λ_L. Hence, withholding disclosure of the offer is weakly dominating. But then, the posterior about the first-type is given by the same signal-partition as the perfect-persistence case, i.e., $\bar{D}(\pi_0, 0) = D(\pi_0, 0)$.

Given the equilibrium acceptance behavior, we can compute the posterior belief given non-disclosure. Let $\tilde{D} : [0, 1] \times \{0, 1\} \to [0, 1]$ be the mapping that takes the belief and disclosure decision and gives the posterior probability that $\theta_2 = H$. Then,

$$\tilde{D}(\pi_0, d) = D(\pi_0, d)(1 - \lambda_H) + (1 - D(\pi_0, d))(1 - \lambda_L), \quad d \in \{0, 1\}.$$

The second-period bidding behavior is determined by this belief.

Next, consider the mandatory regime. Clearly, given a high offer ($p_1 \geq c_H$), both seller types accept the offer. Suppose that the offer is v_L. The H-seller rejects this w.p. one as before. Consider the L-seller. We use notation $\tilde{T}(\pi_0, v_L)$ to denote the posterior given acceptance of v_L and $\tilde{T}(\pi_0, r)$ to denote the posterior given rejection when the prior is π_0. If she accepts the offer, the second-period buyers believe that $\theta_2 = L$ w.p. λ_L, i.e., $\tilde{T}(\pi_0, v_L) = 1 - \lambda_L$. If she rejects the offer, then the posterior is

$$\tilde{T}(\pi_0, r) = T(\pi_0, r)(1 - \lambda_H) + (1 - T(\pi_0, r))(1 - \lambda_L).$$

Notice that, given $\lambda_H \leq \lambda_L$ and $T(\pi_0, r) \geq D(\pi_0, 0)$, we have the analogous relationship between the posterior upon non-disclosure in two regimes: $\tilde{T}(\pi_0, r) \geq \tilde{D}(\pi_0, 0)$.

To derive the L-seller's equilibrium acceptance strategy of the v_L-offer, let $\tilde{Q}_L(\pi_1)$ be the continuation payoff of the (period-1) L-seller when the belief is π_1. (Recall that the transition of types occurs after the disclosure stage.) This continuation payoff can be expressed using the continuation payoffs under perfect persistence as follows:

$$\tilde{Q}_L(\pi_1) = (1 - \lambda_L)Q_H(\pi_1) + \lambda_L Q_L(\pi_1).$$

Thus, the equilibrium condition analogous to (7) is

$$v_L - c_L + \delta\tilde{Q}_L(\tilde{T}(\pi_0, v_L)) \leq \delta\tilde{Q}_L(\tilde{T}(\pi_0, r)).$$

From the above discussion, we can see that the posterior under two regimes change continuously with respect to the impersistence parameter λ_{θ_1} and that the model goes back to the perfect persistence case by letting $\lambda_H \to 0$ and $\lambda_L \to 1$. Since the welfare difference is continuous in the posterior belief, Theorem 1 implies that, under the situation where the voluntary regime gives strictly higher welfare, a small level of imperfect persistence does not change the welfare ranking. That is, there exists an open neighborhood (relative to $[0, 1]$) $U_L := (\lambda_L^*, 1]$ and $U_H := [0, \lambda_H^*)$ such that Case (ii) of Theorem 3 holds for all $\lambda_L \in U_L$ and $\lambda_H \in U_H$.

B Refinement and Uniqueness of the Voluntary Disclosure Equilibrium

In Section 3.1, we state that the voluntary disclosure equilibrium in Proposition 1 is a unique PBE that survives a particular refinement. In this appendix, we describe the refinement and show that the refinement indeed selects the equilibrium in Proposition 1 as a unique equilibrium (up to equivalent classes).

The overview of this section is as follows. We identify that there are three types of equilibria: the seller discloses all trade, with the off-path belief that punishes no disclosure (Equilibria E1 and E2 in Table 2); the seller discloses only the high-offer (Equilibrium E3 in Table 2); and the seller does not disclose any trade, with the off-path belief that punishes disclosure (Equilibrium E4 in Table 2). Then, we show that the equilibrium we identified in the main text—the seller discloses only the high-offer—is the only equilibrium that survives the Grossman-Perry-Farrel equilibrium (GPFE) criterion. We also discuss why other commonly employed refinements do not help with equilibrium selection in our setting.

B.1 Tie-breaking

We assume the following tie-breaking rule:

Assumption 2. *When the seller is indifferent between disclosing and not disclosing, she chooses not to disclose, and this is common knowledge among the seller and buyers.*

This tie-breaking assumption is innocuous, and it is solely to simplify the exposition. When the seller is indifferent between disclosing and not disclosing, the posterior belief upon disclosure is the same regardless of disclosure. By assuming that the seller does not disclose in such a case, we can avoid some notational clutter. This assumption can be justified by assuming that there is a small disclosure cost, and the equilibrium is the limit when the disclosure cost tends to zero.

B.2 Multiple Equilibria

In Proposition 1, we conjectured that the seller follows a straightforward acceptance and disclosure strategy and showed that it is indeed an equilibrium. This in turn guaranteed that the buyers' bidding behavior is as if the bidding game is static. When the seller follows other acceptance strategies, buyers may follow different bidding strategies as well. To describe all PBE, we start with the following observation about the seller's acceptance strategy.

Lemma B.1. *The seller's acceptance strategy is a cutoff rule: the θ-seller accepts an offer if and only if it is equal to or greater than a cutoff r_θ. Furthermore, $r_L \leq r_H$.*

Proof. Consider an offer x. Suppose that the θ-seller accepts the offer in equilibrium. Let $d^* \in \{0, 1\}$ be the equilibrium disclosure decision. Let π_a be the posterior belief given the acceptance and the disclosure decision d^*. Let π_r be the posterior given rejection (and non-disclosure) of the offer. Given a posterior π, the seller can compute the distribution over the next-period offer, denoted by F_π. In the second period, the seller follows a straightforward acceptance strategy, so

$$(x - c_\theta) + \delta \int_0^\infty \max\{x' - c_\theta, 0\} dF_{\pi_a}(x') \geq \delta \int_0^\infty \max\{x' - c_\theta, 0\} dF_{\pi_r}(x').$$

Hence, the seller accepts any offer greater than x as well. If instead the seller rejects x in equilibrium, we have

$$\delta \int_0^\infty \max\{x' - c_\theta, 0\} dF_{\pi_r}(x') \geq (x - c_\theta) + \delta \int_0^\infty \max\{x' - c_\theta, 0\} dF_{\pi_a}(x'),$$

so the seller rejects any offer smaller than x as well.

To see that $r_L \le r_H$, suppose toward a contradiction that $r_L > r_H$. Consider an offer $x \in (r_H, r_L)$. The L-seller's expected payoff is

$$\delta \int_0^\infty \max\{x' - c_\theta, 0\} dF_{\pi_r}(x'),$$

while the H-seller's expected payoff is

$$(x - c_H) + \delta \int_0^\infty \max\{x' - c_\theta, 0\} dF_{\pi_a}(x') \ge \delta \int_0^\infty \max\{x' - c_\theta, 0\} dF_{\pi_r}(x').$$

If the L-seller deviates and accepts the offer and follows the disclosure strategy of the H-seller, she gets

$$(x - c_L) + \delta \int_0^\infty \max\{x' - c_\theta, 0\} dF_{\pi_a}(x') > (x - c_H) + \delta \int_0^\infty \max\{x' - c_\theta, 0\} dF_{\pi_a}(x').$$

Therefore, the L-seller profitably deviates. $\square$

Given this lemma, if the seller follows an r_θ-cutoff strategy, then the buyers' bidding behavior is described by Lemma 1, except that c_θ is replaced by r_θ. Moreover, the equilibrium offers can be classified as either low or high offer, denoted by p_{low} and p_{high}, respectively. Specifically, we have

$$p_{\text{low}} = \begin{cases} V(\pi_{\ell\ell}), & c_H \le V(\pi_{\ell\ell}), \\ v_L, & c_H > V(\pi_{\ell\ell}), \end{cases}$$

$$p_{\text{high}} \in \begin{cases} [V(\pi_{\ell\ell}), \mathbb{P}(l \mid h)V(\pi_{\ell\ell}) + \mathbb{P}(h \mid h)V(\pi_{hh})], & c_H \le V(\pi_{\ell\ell}), \\ [c_H, \mathbb{P}(l \mid h)c_H + \mathbb{P}(h \mid h)V(\pi_{hh})], & V(\pi_{\ell\ell}) < c_H < V(\pi_0), \\ [c_H, V(\pi_h)], & V(\pi_0) \le c_H < V(\pi_h), \end{cases}$$

where $V(\cdot)$ is defined as in Appendix A. This classification is based on the seller's true production cost, because given the offer the seller's payoff is determined by c_θ, not r_θ. Notice that when $c_H \le V(\pi_{\ell\ell})$, the low offer is higher than c_H. Moreover, when $c_H < V(\pi_h)$, the equilibrium offer is v_L for sure, so there is no high offer. Therefore, we can describe the seller's action at the disclosure stage by a message $m \in \{p_{\text{low}}, p_{\text{high}}, N\}$, where $m = p$ means disclosure of the offer p and $m = N$ means non-disclosure.

From Lemma 1 and B.1, in any equilibrium, the L-seller accepts both low and high offers.

Thus, without loss, $r_L = c_L$.[22] To classify the H-seller's equilibrium acceptance strategies, we consider all the acceptance *outcomes* of the H-seller: (A) accept both low and high offers ($r_H \leq V(\pi_{\ell\ell})$), (B) reject the low offer and accept the high offer ($r_H \in (V(\pi_{\ell\ell}), V(\pi_h))$). If $c_H \geq V(\pi_h)$, then there is only the low offer, and we separate the case where the H-seller rejects this low offer as (C). (The case where she accepts the low offer is covered in A.)

We identify all disclosure equilibria in each type of equilibria and show that they can be supported by an appropriate specification of off-path beliefs. Let $\pi^o(m) \in [0, 1]$ be an off-path belief for an off-path message m. The following lemma is useful to identify all disclosure equilibria.

Lemma B.2. *Fix a transaction price p. Let $\theta, \theta' \in \{L, H\}$ with $\theta \neq \theta'$. If the θ-seller discloses p, then so does the θ'-seller. Similarly, if the θ-seller withholds p, then so does the θ'-seller.*

Proof. If the θ-seller discloses p but the θ'-seller withholds p, then disclosure reveals that the type is θ and non-disclosure reveals that the type is θ'. Thus, either type has an incentive to deviate. $\square$

Remark B.1. *By this lemma, it is without loss to say that "the seller discloses p" without specifying the quality type, because the H-seller and L-seller both disclose p.*

Consider the equilibrium of type (A). Clearly, the first-period buyers bid v_L regardless of the signal realization. By Lemma B.2, there are four possible disclosure equilibria. First, suppose the seller discloses all transactions. Suppose that $c_H \leq V(\pi_{\ell\ell})$. The seller does not deviate if $\pi^o(N) < D(\pi_0, 0)$. In this case, it is without loss to set $r_H = c_H$. If $c_H > V(\pi_{\ell\ell})$, then the H-seller is making a loss by accepting the low offer, so this equilibrium can be supported if the H-seller prefers to stay in the equilibrium even when the buyers assign the most pessimistic belief to non-disclosure. In particular, the H-seller does not deviate if

$$v_L - c_H + \delta \mathbb{E}\left[\max\{p - c_H, 0\}; \pi^o(N)\right] \geq 0, \tag{9}$$

where p is the next-period highest offer under the off-path belief $\pi^o(N)$. Therefore, with the off-path belief (9), the H-seller does not deviate for $\pi^o(N) = 0$, and it is without loss to set $r_H = v_L$.[23]

[22] Any reservation price $r_L \in [c_L, v_L]$ is optimal for the L-seller, so the seller's acceptance strategy is unique up to this indifference.

[23] Since the H-seller has to accept the v_L-offer, she is indifferent between $r_H \in [0, v_L]$. Note that $r_H = v_L \leq V(\pi_{\ell\ell})$ is automatically satisfied.

Second, suppose the seller discloses only the high offer. Since the low offer is withheld but accepted by the H-seller, this can be supported if $V(\pi_{\ell\ell}) \geq c_H$ and $\pi^o(p_{\text{low}}) \leq D(\pi_0, 0)$.[24] In this case, it is without loss to set $r_H = c_H$.[25] Third, suppose the seller discloses only the low offer. Upon non-disclosure, the belief is updated to $D(\pi_0, 1)$, so the low type deviates to non-disclosure. Thus, this cannot be an equilibrium. Fourth, suppose the seller withholds disclosing all transactions. This can be supported if $V(\pi_{\ell\ell}) \geq c_H$ and $\pi^o(p) \leq \pi_0$. In this case, it is without loss to set $r_H = c_H$.[26]

In the equilibrium of type (B), by Lemma B.2, there are four possible disclosure equilibria. First, suppose the seller discloses all transactions. Since the H-seller rejects the low offer, the L-seller is identified as the low type by disclosing the low offer. Thus, this cannot be an equilibrium. Second, suppose the seller discloses only the high offer. This can be supported if $V(\pi_{\ell\ell}) < c_H < V(\pi_h)$ and $\pi^o(p_{\text{low}}) \leq D(\pi_0, 0)$. It is without loss to set $r_H = c_H$.[27] Third, suppose the seller discloses only the low offer. Again, this cannot be an equilibrium. Fourth, suppose the seller withholds disclosing all transactions. This can be supported if $V(\pi_{\ell\ell}) < c_H$ and $\pi^o(p) \leq \pi_0$. The H-seller's optimal strategy is $r_H = c_H$.

In the equilibrium of type (C), every offer is withheld from disclosure by construction. The posterior in equilibrium is π_0, so deviation is prevented by an off-path belief with $\pi^o(p) \leq \pi_0$. It is without loss to set $r_H = c_H$.[28] In this case, the disclosure strategy of p_{high} does not matter, because it is off-path.

Table 1 summarizes all possible equilibria. Rows (1) to (4) correspond to type (A), rows (5) and (6) to type (B), and row (7) to type (C). The classification of types (A), (B), and (C) is obtained by the H-seller's acceptance outcome. Note, however, that specification of the acceptance strategy (r_L, r_H) determines the equilibrium. For example, row (2) and row (4) represent the same equilibrium. The disclosure strategy is the same, and the difference is that when $c_H \leq V(\pi_{\ell\ell})$, the low offer is greater than $r_H = c_H$, so the H-seller accepts it, while when $c_H \in (V(\pi_{\ell\ell}), V(\pi_h))$, the low offer is less than $r_H = c_H$, so the H-seller rejects it. Moreover, in the case of $c_H \geq V(\pi_h)$, we can specify the disclosure strategy of p_{high} as (d, d) or (n, n). That is, both specification yields an equivalent outcome. Thus we set the disclosure strategy to be (d, d) in that case. Hence, the equilibria in Table 1 can be reduced to the ones Table 2. In equilibrium (E1), the belief is high enough so the low offer is profitable to the H-seller, and she discloses this transaction. In equilibrium (E2),

[24]If $V(\pi_{\ell\ell}) < c_H$, then the H-seller rejects the low offer and withholds disclosure.
[25]The H-seller is indifferent between $r_H \in [c_H, V(\pi_{\ell\ell})]$.
[26]The H-seller is indifferent between $r_H \in [c_H, V(\pi_{\ell\ell})]$.
[27]The H-seller is indifferent between $r_H \in (v_L, c_H]$.
[28]The H-seller is indifferent between $r_H \geq v_L$.

the H-seller is forced to accept even the offer that is lower than her production cost c_H, because the buyers assign the most pessimistic belief to non-disclosure. Equilibrium (E3) is the one we identified in the main text and we show that this is the unique GPFE. The seller follows a straightforward acceptance rule and she discloses the transaction if and only if the offer was high. In equilibrium (E4), the seller withholds disclosing all trades, because the buyers assign the most pessimistic belief to any disclosure. In equilibrium (E4) we require that there are low and high offers on the equilibrium path. If instead $c_H \geq V(\pi_h)$, then the disclosure strategy for the high offer does not matter, so the equilibrium is equivalent to (E3). We show that (E2) uniquely survives a refinement that we explain further below.

Table 1: Equilibria of the Voluntary Disclosure Regime

This table shows all possible perfect Bayesian equilibria of the voluntary regime.

	Acceptance Strategy	Acceptance outcome of p_{low}	Disclosure of p_{low}	Acceptance outcome of p_{high}	Disclosure of p_{high}	Parameter Restriction
(1)	(c_L, c_H)	(a, a)	(d, d)	(a, a)	(d, d)	$c_H \leq V(\pi_{\ell\ell})$
(2)	(c_L, v_L)	(a, a)	(d, d)	(a, a)	(d, d)	$c_H > V(\pi_{\ell\ell})$
(3)	(c_L, c_H)	(a, a)	(n, n)	(a, a)	(d, d)	$c_H \leq V(\pi_{\ell\ell})$
(4)	(c_L, c_H)	(a, a)	(n, n)	(a, a)	(n, n)	$c_H \leq V(\pi_{\ell\ell})$
(5)	(c_L, c_H)	(a, r)	(n, n)	(a, a)	(d, d)	$V(\pi_{\ell\ell}) < c_H < V(\pi_h)$
(6)	(c_L, c_H)	(a, r)	(n, n)	(a, a)	(n, n)	$V(\pi_{\ell\ell}) < c_H < V(\pi_h)$
(7)	(c_L, c_H)	(a, r)	(n, n)			$c_H \geq V(\pi_h)$

Table 2: Reduced Equilibria of the Voluntary Disclosure Regime

This table shows PBE that give all possible PBE outcomes.

	Acceptance Strategy	Disclosure of p_{low}	Disclosure of p_{high}	Parameter Restriction
(E1)	(c_L, c_H)	(d, d)	(d, d)	$c_H \leq V(\pi_{\ell\ell})$
(E2)	(c_L, v_L)	(d, d)	(d, d)	$c_H > V(\pi_{\ell\ell})$ and (9)
(E3)	(c_L, c_H)	(n, n)	(d, d)	
(E4)	(c_L, c_H)	(n, n)	(n, n)	$c_H < V(\pi_h)$.

B.3 Grossman-Perry-Farrell Equilibrium (GPFE)

To select an equilibrium, we use the Grossman-Perry-Farrell equilibrium (GPFE) refinement, developed by Grossman and Perry (1986) and Farrell (1993). Roughly speaking, an equilibrium fails the GPFE criterion if there is a set of types who wish to deviate, given that the off-path belief is formed by Bayes's rule under the correct conjecture of deviators.

This refinement is more appropriate in disclosure games, where common refinements such as the intuitive criterion and the D1 criterion fail to eliminate unnatural equilibria. Indeed, Bertomeu and Cianciaruso (2018) show that the GPFE criterion selects a unique equilibrium for generic disclosure games, while the intuitive criterion does not.[29] We discuss why the intuitive criterion and the D1 criterion fail to select a unique equilibrium in Appendix B.5.

To formally state the solution concept, note that there are four "types" of the seller from the perspective of second-period buyers, depending on the quality and the offer type. Let $\Omega := \{L, H\} \times \{p_{\text{low}}, p_{\text{high}}\}$ be the enriched type space and let $\omega \in \Omega$ be a typical element and ω_i be the i-th coordinate of ω. Each type ω determines whether to accept the offer and whether to disclose it. Note that a message $m = p$ can be sent by a type $\omega \in \{L, H\} \times p$ who has accepted the offer, while a message $m = N$ can be sent by any type.

To define GPFE, we follow Bertomeu and Cianciaruso (2018) and Sobel (2020) to define a *self-signaling set* Ξ by

$$\Xi(m) = \{\omega \in \Omega \mid \Pi_S(\omega, m^*; \pi(m^*)) < \Pi_S(\omega, m; \pi_\Xi(m))\},$$

where $\Pi_S(\omega, m; \pi)$ is the type-ω seller's expected payoff given message m and posterior belief π over $\{L, H\}$ after the disclosure decision. A message m^* is the equilibrium message and $\pi(m^*)$ is the equilibrium posterior belief after the disclosure decision. The belief $\pi_\Xi(m)$ is the posterior belief over $\{L, H\}$ conditional on Ξ and m. Here, π_Ξ depends on m because if $m = p$, then this is informative about the first-period signal realizations. Notice that π_Ξ is a posterior over the quality-type of the seller, because only the belief over the quality-type is payoff-relevant. We call a PBE that does not admit a self-signaling set Ξ a GPFE. We argue that there is a unique GPFE (up to off-path beliefs).

We say that $(m, \Xi(m))$ is *fully revealing* (FR) if only one quality type is included in $\Xi(m)$. If $(m, \Xi(m))$ is FR, then the quality type is revealed by the message m or a message $m' \neq m$, i.e., $\pi_\Xi(m) \in \{0, 1\}$.

We compute the posterior belief for each message $m \in \{p_{\text{low}}, p_{\text{high}}, N\}$. First, when $m \in \{p_{\text{low}}, p_{\text{high}}\}$, then

$$\pi_\Xi(m) = \begin{cases} \mathbb{I}(m = p_{\text{low}}) D(\pi_0, 0) + (1 - \mathbb{I}(m = p_{\text{low}})) D(\pi_0, 1), & (m, \Xi(m)) \text{ is not FR}, \\ \mathbb{I}(\omega_1 = \{\theta_H\}), & (m, \Xi(m)) \text{ is FR}. \end{cases} \tag{10}$$

[29] Our disclosure subgame is not necessarily covered in Bertomeu et al. (2022) because the seller's payoff depends on the price as well.

When $m = N$, the computation of the posterior could be nontrivial when, for example, $\Xi(m) = \{(L, p_{\text{low}}), (L, p_{\text{high}}), (H, p_{\text{low}})\}$, as it requires the computation of the distribution of ω. However, it turns out that $\Xi(m)$ cannot contain two distinct offer types. To see this, suppose that $\Xi(N) = \{(L, p_{\text{low}}), (L, p_{\text{high}}), (H, p_{\text{high}})\}$. If this is a self-signaling set, then (H, p_{low}) also deviates because type (H, p_{low}) receives the same belief (and possibly makes a loss) on the equilibrium path as (L, p_{low}) by disclosing the low offer. By a similar reasoning, any self-signaling set with $|\Xi(N)| = 3$ is impossible. Therefore, the following computation of off-path beliefs is enough:

$$\pi_{\Xi}(N) = \begin{cases} \mathbb{I}(p = p_{\text{low}})D(\pi_0, 0) + (1 - \mathbb{I}(p = p_{\text{low}}))D(\pi_0, 1), & |\Xi| = 2 \text{ and } \Xi \text{ is not FR,} \\ \mathbb{I}(\omega_1 = H), & \Xi \text{ is FR,} \\ \pi_0, & \Xi = \Omega. \end{cases} \tag{11}$$

In the next section, we examine if the equilibria we identified above admit a self-signaling set. To do so, the following lemma is useful.

Lemma B.3. *Unless $\pi_0 \in \{0, 1\}$, $(m, \Xi(m))$ cannot be FR.*

Proof. If $(m, \Xi(m))$ is FR, then either the low-quality types does not want to deviate ($\pi_{\Xi}(m) = 0$) or both types want to deviate ($\pi_{\Xi}(m) = 1$). □

B.4 Uniqueness of Equilibrium

We now apply the refinement to the equilibria in Table 2. Following Bertomeu and Cianciaruso (2018), we say that two PBEs are equivalent if each seller-type obtains the same payoff and the disclosure strategies convey the same information. The uniqueness is up to this equivalence, as the seller's acceptance strategy is not unique and there is a continuum of beliefs that support a PBE.

Equilibrium E1 The only off-path message is $m = N$. Observe that $\Xi(N) = \{(L, p_{\text{low}}), (H, p_{\text{low}})\}$ constitutes a self-signaling set. For this set, the posterior is $\pi_{\Xi}(N) = D(\pi_0, 0)$, and both (L, p_{low}) and (H, p_{low}) strictly prefer to deviate by Assumption 2. Since $D(\pi_0, 0) < D(\pi_0, 1)$, clearly high-offer types do not wish to deviate. Hence, this equilibrium is not a GPFE.

Equilibrium E2 The only off-path message is $m = N$. As we have mentioned, the on-path posterior belief upon disclosure is π_0 for sure. Observe that

$$\Xi(N) = \{(L, p_{\text{low}}), (H, p_{\text{low}}), (L, p_{\text{high}}), (H, p_{\text{high}})\}$$

constitutes a self-signaling set. For this set, the posterior is $\pi_\Xi(N) = \pi_0$. The L-seller strictly prefers to deviate by Assumption 2, and the H-seller strictly prefers to deviate because she is making a loss by accepting the low offer. Hence, this equilibrium is not a GPFE.

Equilibrium E3 We show that there is no self-signaling set, note that the only off-path message is $m = p_{\text{low}}$. From Lemma B.3 and equation (10), it suffices to consider $\Xi(p_{\text{low}}) = \{(L, p_{\text{low}}), (H, p_{\text{low}})\}$ and $\pi_\Xi(p_{\text{low}}) = D(\pi_0, 0)$. High-offer types do not wish to deviate as they receive posterior $D(\pi_0, 1)$ on-path, and low-offer types do not wish to deviate by Assumption 2.[30]

Equilibrium E4 Consider an off-path message $m = p_{\text{high}}$, i.e., disclosure of a high offer. Observe that $\Xi(p_{\text{high}}) = \{(L, p_{\text{high}}), (H, p_{\text{high}})\}$ constitutes a self-signaling set. To see this, note that $\pi_\Xi(p_{\text{high}}) = D(\pi_0, 1) > \pi_0$. Furthermore, low offer types cannot send $m = p_{\text{high}}$.

Summary In sum, regardless of the level of the prior belief, we obtain the following unique GPFE (up to equivalence). The seller's acceptance strategy is $(r_L, r_H) = (c_L, c_H)$, and both quality types of the seller disclose the high offer and withhold the low offer.[31] The off-path belief is $\pi^o(p_{\text{low}}) \leq D(\pi_0, 0)$. This equilibrium corresponds to the equilibrium we identify in Proposition 1.[32]

B.5 Other Refinements

There are many forms of refinements proposed in the literature. Among them are the intuitive criterion and the D1 criterion proposed by Cho and Kreps (1987) and Banks and Sobel (1987). In our setting, these refinements do not select a unique equilibrium. In this section, we focus on the disclosure equilibrium where the sender withholds disclosure of trade for any price (Equilibrium E4 in Table 2) to show that these refinements fail to eliminate

[30] Assumption 2 is unnecessary if the low offer is lower than c_H, i.e., $c_H < V(\pi_{\ell\ell})$, in which case the H-seller has to make a loss to disclose p_{low}.

[31] The level of prior belief determines if the low offer is accepted by the H-seller in equilibrium. Specifically, when $c_H < (\geq) V(\pi_{\ell\ell})$, then the equilibrium low offer is accepted (rejected) by the H-seller.

[32] Note that when $c_H \geq V(\pi_{\ell\ell})$, the low offer is $V(\pi_{\ell\ell})$.

this unnatural equilibrium. Similar arguments can be applied to show that other PBE are also not excluded by these refinements.

B.5.1 Intuitive Criterion

To formally state the intuitive criterion, let $T(m)$ be the set of quality-price types who can send the message $m \in \{N, p_{\text{high}}, p_{\text{low}}\}$. For example, $\Omega(p_{\text{high}}) = \{(L, p_{\text{high}}), (H, p_{\text{high}})\}$. Define the set of types who will not benefit from deviating to m by

$$S(m) = \{\omega \in \Omega(m) \mid \Pi_S(\omega, m^*; \pi(m^*)) > \max_{\pi \in \text{marg}_1 \Omega(m)} \Pi_S(\omega, m; \pi)\},$$

where m^* is the equilibrium message and $\text{marg}_1 \Omega(m) \subset [0, 1]$ is the set of marginal distributions of the quality types. Given this, if there is some type $\omega \in \Omega$ such that

$$\Pi_S(\omega, m^*; \pi(m^*)) < \min_{\pi \in \text{marg}_1 \Omega(m) \setminus S(m)} \Pi_S(\omega, m; \pi), \tag{12}$$

then we say that the equilibrium fails the intuitive criterion. That is, among the set of types who can send m and might benefit from m, if the seller benefits from deviation even if buyers assign the worst beliefs, then the type-message pair is eliminated. Here, the deviation should be considered for *any* possible beliefs that are consistent with the message. Bayes's law from the price information does not apply here, because we cannot specify the distribution on the possible deviating types among $\Omega(m)$ (or $\Omega(m) \setminus S(m)$).

Now, consider Equilibrium E4. We focus on the off-path message $m = p_{\text{high}}$. Since $\Omega(p_{\text{high}}) = \{(L, p_{\text{high}}), (H, p_{\text{high}})\}$, we have $\text{marg}_1 \Omega(m) = [0, 1]$. That is, depending on buyers' belief as to which quality type is disclosing, any belief is a best response to the disclosure. Consequently, $S(m) = \varnothing$ and $\text{marg}_1 \Omega(m) \setminus S(m) = [0, 1]$. Thus, the inequality (12) cannot hold for any type ω. That is, the intuitive criterion does not affect the PBE at all. The same argument applies for the off-path message $m = p_{\text{low}}$.

The issue is that buyers consider all possibilities of the seller's type for the deviation p_{high}. Thus, the Bayesian updating from the price information has no role. If we fix the buyers' belief on Ωp_{high}, then we can apply Bayes's law to incorporate price information from the seller's voluntary disclosure. The GPFE achieves this: we fix a possible deviating set and apply Bayes's law.

B.5.2 D1 Criterion

Similarly, the D1 criterion does not affect the equilibrium. To define the D1 criterion, let $D_\omega(m) \subset [0,1]$ ($D^0_\omega(m) \subset [0,1]$) be the set of beliefs that type ω weakly (strictly) prefers to deviate to the off-path message m in an equilibrium. If there are types $\omega \neq \omega'$ in $\Omega(m)$ such that

$$D_\omega(m) \cup D^0(\omega) \subsetneq D_{\omega'}, \tag{13}$$

then the D1 criterion requires that the off-path belief assigns probability one to type ω'. Intuitively, type ω' is "more likely" to deviate than ω.

Consider Equilibrium E4 and $m = p_{\text{high}}$. Since the on-path belief is π_0, we have $D_{(\theta,p_{\text{high}})}(p_{\text{high}}) = [\pi_0, 1]$ and $D^0_{(\theta,p_{\text{high}})}(p_{\text{high}}) = (\pi_0, 1]$ for both $\theta \in \{L, H\}$. Hence, condition (13) cannot possibly hold. A similar argument applies to the other deviation as well.

The issue here is the same as the intuitive criterion. For the off-path disclosure of p_{high}, the buyers consider all the possibilities of types, so the information of the price from the disclosure is not incorporated. The GPFE fixes this by defining a self-signaling set as a fixed point.