Disclosure, Signaling, and First-Mover (Dis)advantage

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Abstract

This paper studies voluntary disclosure in a leader-follower game in a product market. The leader is privately informed about the demand prospect of the market. The leader chooses a production level and decides whether to disclose it. On the one hand, such disclosure is beneficial, as the leader can enjoy the first-mover advantage. On the other hand, the follower learns the leader's private information through disclosed information, so the leader firm has an incentive to contract production to signal low demand. This is costly to the leader, as the leader may end up producing and earning less than the follower (first-mover disadvantage). To avoid such signaling costs, the leader can conceal production information. In equilibrium, when the leader is long-term oriented, the leader discloses the production plan only when the private demand signal is low. More competition leads to less disclosure. When the leader firm is short-term oriented, an interval disclosure equilibrium can emerge. I extend the baseline model to the case where the leader may not observe a private signal (the Dye friction). Since the disclosure is about endogenous actions, the uninformed type has the option to disclose the production plan as well. I show that this friction allows the leader firm to save the signaling cost by mimicking the uninformed type. This paper offers a theory of endogenous disclosure cost.

Keywords: Voluntary disclosure, Leader-follower game, Signaling, Oligopoly

JEL Classification: D43, D83, L13, M41

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1 Introduction

In a strategic situation, being better informed does not necessarily mean that one is better off. One prominent example of this is the first-mover disadvantage in leader-follower games with strategic substitutes (Gal-Or, 1987). If both parties are symmetrically informed, then the leader is in general better off than the follower due to the commitment power endowed by moving first (first-mover advantage). If the leader is better informed, however, the leader's action signals the private information of the leader. Understanding this signaling value of the action, the leader has the incentive to distort his action to change the follower's belief in a way that is beneficial to the leader. In a fully separating equilibrium, the private information is perfectly recovered by the follower, but the leader is trapped into distorting his action and incurring an endogenous signaling cost. Under some conditions, this signaling cost is so large that the leader's benefit from the commitment power by moving first is completely offset. That is, the leader is worse off than the follower, precisely because he owns valuable information and moves first (first-mover disadvantage).

However, the leader can avoid this "signaling curse" by concealing his action. Of course, this comes at a cost: by concealing the action, the leader cannot enjoy the first-mover advantage. Moreover, concealing the action does not completely hinder the follower's inference about the leader's private information. The follower makes an inference about the leader's private information from the fact that the action is concealed. Thus, one might suspect that the unraveling result applies, and the leader discloses all information in equilibrium, rendering the voluntary disclosure moot (Milgrom, 1981). But, in light of the endogenous signaling cost that ensues after disclosing the action, it is not clear if the leader discloses all information. Does the unraveling result apply? If not, what is the optimal disclosure strategy of the leader? What is the equilibrium action when there is an option to conceal the action from the follower? Can the leader restore the first-mover advantage through selective disclosure of the production plan?

To answer these questions, I consider a leader-follower game in a product market where firms compete in quantities. The firms compete in imperfectly differentiated goods. The demand system is linear. The leader firm privately observes a possibly imperfect signal about the demand intercept. Upon observing the private signal, the leader commits to a production plan.[1] The leader then decides whether to disclose the production plan.[2] After

[1] Alternatively, I could assume that the leader actually produces the quantity at this stage. As long as the leader commits to it, assuming that the leader actually produces and that the leader makes a production plan are the same.

[2] The disclosure of the production plan is assumed to be truthful as in the voluntary disclosure literature.

this voluntary disclosure decision, the follower chooses an output level, and then the market clears.

To illustrate the economic forces in the model, I start with the case of mandatory disclosure. This corresponds to the model by Gal-Or (1987). On the one hand, the leader can commit to producing more than the Cournot quantity to take a larger market share than the follower. I call this the "Stackelberg effect." On the other hand, since the leader wants to signal that demand is low, the leader has an incentive to distort production downward. This "signaling effect" counteracts the "Stackelberg effect." Under the set up of my model, I show that the leader is always worse off than the follower.[3] That is, the endogenous signaling cost–contraction of the production–outweighs the first-mover advantage, and the leader ends up producing less than the follower.

In the main model, the leader is always informed and has the option to disclose or withhold the production plan. The voluntary disclosure decision is made to maximize the cash flow from the product market. I show that the unraveling result does not apply. The equilibrium disclosure strategy is a threshold one, and the leader discloses quantity only when the demand signal is low (lower-tail disclosure). Intuitively, when the signal is low, the leader wants the follower to know this information; otherwise, the follower would overproduce. Alternatively, when the demand signal is high, if the leader decides to disclose the production plan, the leader has to engage in a significant amount of distortion of production (signaling) in an attempt to convince the follower that demand is not high, only to ultimately be unsuccessful (in a fully separating equilibrium). I show that there is a unique disclosure threshold under a production equilibrium that is linear and fully separating.

Does voluntary disclosure help the leader partially restore the first-mover advantage? I show that when signal realization is high enough, the leader earns more than the follower and the symmetric Cournot profit. Moreover, the expected payoff of the leader before observing the signal realization can also be higher than the follower and more than the symmetric Cournot profit when the market is competitive enough. Thus, both ex-ante and ex-post, the leader can mitigate the first-mover disadvantage by strategically releasing production information.

The unique disclosure threshold exhibits intuitive comparative statics. As the competitiveness of the market, measured by the degree of the homogeneity of the products, increases,

However, in the current setting, the leader is free to choose any (nonnegative) production quantity.

[3] Unless otherwise mentioned, I consider a fully separating equilibrium. In general, there exists a pooling equilibrium, but it should be supported by an "unreasonable" off-path belief (Gal-Or, 1987). Indeed, one can show that such an equilibrium does not survive equilibrium refinements.

the threshold decreases, i.e., the leader discloses less often. This is consistent with the empirical papers that document a negative association between competition and disclosure (Li, 2010; Huang, Jennings and Yu, 2017; Li, Lin and Zhang, 2018; Glaeser, 2018). The mechanism is that the competitiveness of the market increases the endogenous signaling cost. Moreover, as the precision of the leader's signal decreases, the threshold increases, i.e., the leader discloses more often. This feature is shared in standard voluntary disclosure models (Verrecchia, 1990). In addition, the analysis offers some implications for empirical research that examines managers' voluntary disclosure of good vs. bad news (Skinner, 1994; Verrecchia and Weber, 2006; Kothari, Shu and Wysocki, 2009). If one views disclosure cost through the lens of product market competition, then the leader firm selectively discloses bad news.

I also analyze the case where the leader firm has a short-term incentive to maximize stock price.[4] This captures the idea that cash flows may be realized with some delay, and that the manager of the firm is myopic, possibly because the stock price is tied to the manager's compensations or the manager has to sell the stock for a liquidity reason (Stein, 1989). The stock market does not observe the signal realization, so the stock price after nondisclosure is constant with respect to the signal realization. This is in contrast to the expected cash flow, which changes with the signal realization even after nondisclosure. I characterize all disclosure equilibria and show that only lower-tailed disclosure or interval disclosure is possible, depending on the weights on the disclosure objectives. I show that there is an interval disclosure equilibrium, where the leader firm discloses only when the signal is "modest." Importantly, upper-tail disclosure cannot be an equilibrium even when the leader firm cares almost exclusively about stock price. This is because the leader can mimic the production of the high-signal type. This mimicry is costly in terms of cash flow perspective, but a myopic firm ignores this cost.

Finally, I extend the baseline model to incorporate friction in information arrival (Dye, 1985; Jung and Kwon, 1988). The leader observes the private demand signal with some probability. It is reasonable to think that an industry leader may not always possess superior information compared to followers. A novel feature of my model is that the disclosure is not about private signals but about real actions. Therefore, even an uninformed agent can disclose. This new feature allows an informed agent to be pooled with the uninformed one even if the informed decides to disclose. Specifically, the uninformed leader can commit to a production plan using the common prior assessment of the demand signal. I identify two types of equilibria, depending on whether the uninformed leader discloses the quantity.

[4]See Glaeser, Michels and Verrecchia (2020) for evidence that (the investor's perception of) the manager's horizon shapes the disclosure behavior and affects real decisions.

If the uninformed leader conceals the quantity, then upon disclosure the follower knows that the leader is informed, so the subgame ensuing the disclosure stage is the same as in the main model. Upon nondisclosure, the follower updates beliefs, taking into account the possibilities that the leader is informed or uninformed. Thus, the disclosure equilibrium is still a threshold one, but now the threshold is lower (less disclosure), just like in standard disclosure models with the Dye friction.

Another type of equilibria is one in which the uninformed leader discloses the quantity. In this equilibrium, I show that a fully separating equilibrium no longer exists. The reason is, if there was a fully separating equilibrium, then there is a zero probability that the informed leader's disclosed quantity coincides with the uninformed leader's disclosed quantity. Thus, the follower believes that the leader is uninformed with probability one upon observing the quantity that the uninformed discloses in a candidate equilibrium. But then, some types of the informed leader benefit from mimicking the uninformed leader's production.

To capture this mimicking incentive of the informed leader, I consider a semi-separating equilibrium. The informed leader can save the signaling cost by producing and disclosing the quantity that the uninformed would produce and disclose. The uninformed leader devises a production plan understanding this mimicking incentive of the informed leader. The disclosure equilibrium continues to be a lower-tail disclosure. I show that the disclosure region can expand compared to the perfect information endowment case. In particular, for some parameter values, there exists a full disclosure equilibrium. Intuitively, the leader has two options to avoid the signaling distortion: withholding the production plan and mimicking the uninformed type. The latter additional device used to avoid the signaling distortion makes disclosure more profitable compared to nondisclosure for the informed leader.

The critical assumption in the model is that the leader firm, upon being informed, commits to a production plan and has the option to disclose it. The commitment assumption, which is shared in the standard Stackelberg model, may not be perfectly true in the real world. For example, a firm may have the freedom to adjust its production plan. Even after an initial round of production is completed, a firm may expand production or decide not to sell some of the inventory. Regarding this commitment assumption and the announcement of production plans, Doyle and Snyder (1999) present important evidence.[5] They find that in the U.S. automobile industry, firms frequently announce their production plans, and announcements of increased production leads to increase in production among competitors. They interpret the empirical pattern as consistent with the idea that the production plan

[5]See also Caruana and Einav (2008) for the role of production schedule as a commitment device.

acts as a signal for a common demand intercept. Even in a situation where a firm does not directly commits to and disclose a production plan, the expansion of capacity–for example through building new factory–could serve as a credible signal of a high production level (Milgrom and Roberts, 1992).[6]

In some industries, firms directly disclose their production forecast as part of investor relations, even though it is not required by law. For example, Toyota has been consistently disclosing planned vehicle production quantities in the next fiscal year in its financial statements. BMW, in its 2021 annual report, disclosed expected deliveries of electrified vehicles, but in 2022 they stopped disclosing this information. The world's largest gold mining corporation, Newmont, regularly releases its gold production forecast and explains if it delivered it's past forecasts. These examples suggest that the disclosure of production data has a voluntary and strategic nature. Thus, analyzing voluntary disclosure of production activities in product-market competition is important to have a better understanding of what drives firms' disclosure behavior.

The rest of the paper is organized as follows. In the following subsection, I discuss the related literature. In Section 2, I describe the main model. In Section 3, I analyze a couple of benchmark cases. In Section 4, I analyze the main model. In Section 5, I consider short-term disclosure incentives. In Section 6, I discuss welfare results. In Section 7, I extend the model to incorporate the Dye friction. Section 8 concludes the paper. All proofs are relegated to Appendix.

1.1 Related Literature

There is a vast theoretical literature on disclosure and competition (Gal-Or, 1985; Li, 1985; Gal-Or, 1986; Ziv, 1993; Darrough, 1993; Darrough and Stoughton, 1990; Wagenhofer, 1990; Sankar, 1995; Raith, 1996; Clinch and Verrecchia, 1997; Pae, 2002; Cheynel and Ziv, 2021). My paper is concerned with ex-post voluntary disclosure and endogenous proprietary costs in a product market. It is well known that, without any friction, the manager discloses all verifiable information (Viscusi, 1978; Grossman and Hart, 1980; Grossman, 1981; Milgrom, 1981; Milgrom and Roberts, 1986). This is called the unraveling/full disclosure result. Verrecchia (1983) introduced an exogenous disclosure cost to generate partial disclosure. Although he writes that such proprietary costs could arise from a competitor's use of informa-

[6]For example, in January 2023, Tesla, a market leader in the electric vehicle industry, announced that it is building a $3.6 billion factory in Nevada (WSJ, 2023). Bloomfield and Tuijn (2019) provide evidence that the announcement of capacity expansion has an information content.

tion, it is not clear if a model that explicitly models competition generates partial disclosure. Indeed, Cheynel and Ziv (2021) find that in a Cournot competition model, there are multiple disclosure equilibria. In particular, they find that there is always a full-disclosure equilibrium, even with the competitive disadvantage of the disclosure. Similarly, considering an endogenous entry model, Wagenhofer (1990) shows that a full-disclosure equilibrium always exists. Therefore, these models are not sufficient to provide a micro foundation of proprietary costs. In contrast, in my base model, the disclosure equilibrium is unique,[7] and full disclosure is impossible. Darrough and Stoughton (1990) consider a similar model to Wagenhofer (1990) and obtain multiple disclosure equilibria. As with my model, Sankar (1995) also considers an ex-post voluntary disclosure with the Dye friction in a product market. In a setting where a firm directly discloses the signal, the paper shows that the firm selectively discloses bad news. See also Ackert, Church and Sankar (2000) for experimental evidence that shows firms disclose bad news and withhold good news in the setting of Sankar (1995).

This paper is closely related to Gal-Or (1987), who studies signaling equilibria in leader-follower games where the leader is endowed with private information. In her model, the privately informed leader does not have the option to withhold the quantity information. Thus, given that the leader discloses the production plan, my model reduces to the one in Gal-Or (1987). She considers the case where the follower also observes a demand signal, which can be correlated with the leader's signal. She shows that if the correlation is not too high, the leader is worse off than the follower in a fully separating equilibrium. Privately informed leaders are also analyzed in Shinkai (2000); Nakamura (2015); Cumbul (2021). My paper extends this line of research by introducing the possibility of strategic disclosure.

The main model is also closely related to a note by Mailath (1993). In Mailath (1993), a privately informed leader has the option to delay production, which plays a similar role as voluntary disclosure in my model. He analyzes the three-type case and focuses on the issue of equilibrium selection.

In my model, voluntary disclosure has endogenous costs, as the leader firm has to distort its production to signal that the demand is low. Beyer and Guttman (2012) also analyze a model where firms engage in costly manipulation given disclosure.[8] They introduce voluntary disclosure to the model of Myers and Majluf (1984) and show that firms have an incentive to overinvest given disclosure. In contrast to their model, in my model, there is a strategic interaction in the real decisions that firms make. Thus, there is an endogenous

[7] To be precise, the uniqueness of the disclosure equilibrium is obtained given that linear fully separating signaling production equilibrium is played upon disclosure.

[8] See also Einhorn and Ziv (2012) for a related model of biased disclosure.

benefit through the first-mover advantage, in addition to the signaling cost.

Chen and Göx (2022) is also related to my paper as it considers disclosure decisions in leader-follower games (see also Corona and Nan, 2013). In their paper, the disclosure is about private signals. Moreover, when a firm does not disclose its private signal, it is not allowed to signal the information through real actions. Thus, the trade-off associated with signaling and disclosure–the central concern of my paper–is absent. Interestingly, they show that under the Dye friction the manager discloses *more*, a result that I also obtain, but for a somewhat different reason. In their paper, it is because after nondisclosure the manager has to distort the production to make it consistent with no information endowment, which they call the endogenous consistency cost. In my paper, it is because the informed firm can be pooled with the uninformed through production.

The paper also contributes to the literature on earnings management by providing a signaling perspective of real earnings management (Roychowdhury, 2006; Dechow, Ge and Schrand, 2010). The leader firm distorts production to signal that the private demand signal is low. The leader does so in an attempt to increase the expected cash flow (earnings). Seen this way, the analysis provides insight into how real earnings management interacts with voluntary disclosure. If the cost of signaling through the contraction of production is too high, then the leader firm prefers to stay silent.

Finally, the analysis of the Dye friction contributes to the literature that examines the interaction between real decisions and disclosure (Ben-Porath, Dekel and Lipman, 2018; Guttman and Meng, 2021). In my model, the uninformed leader can also disclose the production plan, because disclosure is about real actions and not about signals. When the uninformed type discloses, some informed types mimic the uninformed type by adjusting production. To the best of my knowledge, this type of strategic interaction between informed and uninformed types is absent in prior research that studies the Dye friction.

2 Model

I consider a duopolistic market for differentiated goods. There are two firms, firm 1, a Stackelberg leader, and firm 2, a follower. The goods are imperfect substitutes. The inverse demand for firm i's good is given by a linear form:

$$P_i(a, q_i, q_j) = a - q_i - tq_j, \qquad i, j \in \{1, 2\}, i \neq j.$$

The parameter $t \in (0,1)$ measures the substitutability of the two goods, and a high t means high substitutability and competitiveness.[9] The demand intercept, a, is a random variable, which follows a uniform distribution over $\mathscr{S} := [2\underline{a}, 2\overline{a}]$, where $0 \leq \underline{a} < \overline{a}$.[10] I further assume that $\underline{a} \geq \overline{a}/3$ to avoid negative prices.[11] The distribution of a is commonly known. The leader privately observes a noisy signal about demand.[12] I denote such a signal by s_1. I assume that the signal s_1 is a "truth-noise" one: it is a mixture of a degenerate distribution on a with probability $\rho \in (0,1]$ and and a uniform distribution on $\mathscr{S}$ with probability $1-\rho$ (Kanodia, Singh and Spero, 2005; Guttman and Marinovic, 2018). Thus, the posterior expectation of a conditional on s_1 is given by $\mathbb{E}[a \mid s_1] = \rho s_1 + (1-\rho)\mathbb{E}[a]$. The parameter ρ captures the precision of the signal, where higher ρ corresponds to a more precise signal. Note that $\rho = 1$ means the signal is perfect, and $\rho = 0$ means s_1 is pure noise. The follower does not observe any private signal.

The marginal cost of production is constant and normalized to zero, so the demand intercept should be understood as the difference between the stochastic demand and a deterministic marginal cost. The realized profit is denoted by $\pi_i := P_i q_i$.

Upon observing the private signal s_1, the leader makes a production plan and commits to the production quantity $q_1(s_1)$.[13] The leader can voluntarily disclose q_1 to the follower. I require that the disclosure of production plan q_1 be truthful, as in the voluntary disclosure literature and a standard Stackelberg model. If the quantity q_1 is disclosed, the timing of the game is the same as the standard Stackelberg model. The follower chooses a production level given that the leader produces q_1. If instead the production plan is not disclosed, the follower does not observe the leader's production plan when it makes a production decision, so the game is essentially one of Cournot competition. Unlike the standard Cournot model, the leader has better information, and the follower makes an inference about the private signal of the leader from the leader choosing to conceal the production plan.

In summary, the timeline is as follows. The leader privately observes s_1. The leader commits to a production quantity q_1. The leader decides whether to disclose the quantity q_1.

[9] I exclude $t = 0$, which corresponds to a monopoly case. The perfectly-homogeneous goods case $t = 1$ is also excluded, because a fully separating equilibrium does not exist.

[10] See Appendix C for the discussion of the demand distribution.

[11] The follower firm may overestimate the demand signal compared to the realization, in which case the follower may overproduce to the point where the prices of the goods are negative. If the lower bound of the signal is high enough, then this problem does not arise. Lemma 1 formally shows this point.

[12] The noise in the signal is introduced to obtain comparative statistics with respect the precision of the signal. Partial disclosure is obtained even if there is no noise in the signal.

[13] When there is no risk of confusion, I abuse notation and use q_1 to denote a function of s_1 and a specific value (for a realized signal).

Upon observing the disclosure or nondisclosure, the follower decides a production quantity q_2. The market clears and cash flows realize.

To define equilibrium, let $d : s_1 \mapsto \{0,1\}$ be the disclosure decision of the leader, where $d(s_1) = 1$ means the leader discloses q_1 for signal realization s_1, and $d(s_1) = 0$ means nondisclosure. I focus on pure strategies for production and disclosure decisions.[14] The leader's production decision q_1 is a function of the private signal s_1. Let $q_2^D(q_1)$ be the production strategy of the follower upon disclosure and q_2^N be the production strategy after nondisclosure. The solution concept I employ is weak Perfect Bayesian Equilibrium, which is defined as follows.

Definition 1. The equilibrium is a disclosure strategy and a production strategy $\{d^*, q_1^*, (q_2^D)^*, (q_2^N)^*\}$, together with a belief-updating rule, satisfying the following conditions:

1. Optimal production by the follower

$$\forall q_1^*, \quad (q_2^D)^* \in \arg\max_{q_2} \mathbb{E}\left[P_2(a, q_2, q_1^*)q_2 \mid q_1^*\right]$$

$$(q_2^N)^* \in \arg\max_{q_2} \mathbb{E}\left[P_2(a, q_2, q_1^*(s_1))q_2 \mid d^*(s_1) = 0\right]$$

2. Optimal disclosure and production by the leader

$$\forall s_1, \quad (d^*, q_1^*) \in \arg\max_{d, q_1} \mathbb{E}\left[dP_1(a, q_1, (q_2^D)^*)q_1 + (1-d)P_1(a, q_1, (q_2^N)^*)q_1 \mid s_1\right]$$

3. Beliefs are updated according to Bayes law on the equilibrium path. Let $\hat{q}_1$ and $\hat{d}$ be a follower's conjecture about the leader's strategy. After the leader's disclosure decision, the follower updates belief to

$$\mathbb{E}[a \mid \hat{d}(s_1) = 1, \hat{q}_1(s_1) = q_1] \tag{1}$$

if $q_1 \in \{q_1^*(s_1) \mid d(s_1) = 1\}$ is disclosed and to

$$\mathbb{E}[a \mid \hat{d}(s_1) = 0]$$

if the production plan is not disclosed. In equilibrium, the conjecture is correct: $\hat{q}_1 = q_1^*$

[14] Given that q_1 is disclosed, the leader cannot mix in a fully separating equilibrium. The follower does not randomize for any production and disclosure decisions of the leader, because the follower's profit function is concave.

and $\hat{d} = d^*$.

4. Off the equilibrium path, the belief is any probability measure on $\mathbb{R}_{++}$.

Given that q_1 is disclosed, it acts as a signal of the private demand information s_1 to the follower. As in standard signaling models, upon disclosure, there are many production equilibria. I focus on the most informative equilibrium among the possible equilibria. Therefore, when a fully separating equilibrium exists upon disclosure, which is indeed the case in the main model, I assume that the separating equilibrium is being played. As a result, the expression (1) reduces to $\mathbb{E}[a \mid s_1 = (q_1^*)^{-1}(q_1)]$.

Note that in Condition 4, I take the state space to be the entire positive part of the real line, instead of $[2\underline{a}, 2\overline{a}]$. This is to ensure the existence of linear fully separating equilibrium. Recall that in weak Perfect Bayesian equilibrium, the belief in a zero-probability history is *completely arbitrary* (Fudenberg and Tirole, 1991). In Appendix C, I introduce a payoff-irrelevant auxiliary state variable, so that the Bayes rule is applied for any production in the mandatory disclosure benchmark. That is, there is a restriction on zero-probability events as well. In the same appendix, I also explain in detail the distribution assumption on demand.[15]

I assume that the signal s_1 cannot be credibly disclosed. This assumption is rather natural given that it is hard to provide evidence that the industry demand is high/low. If the signal is verifiable and voluntary disclosed, then the unraveling prevails given that there is no exogenous or endogenous disclosure cost.

3 Benchmark: Mandatory Disclosure

Before I analyze the main model, I consider the case where the leader firm is forced to disclose the production plan, q_1. Although the setup of is slightly different, the analysis is essentially the same as in Gal-Or (1987). Thus, I only highlight main economic forces. A detailed discussion of this benchmark can be found in Appendix B.

Let $q_1 = Q_L(s_1)$ and $q_2 = Q_F(q_1)$ be the strategies of the leader and the follower. Since disclosure is mandatory, I omit the superscript D from the follower's production decision. As in Gal-Or (1987), there is a linear equilibrium when appropriate off-path beliefs are assigned. Thus, I let $Q_L(s_1) = A\mathbb{E}[a \mid s_1]$ and $Q_F(q_1) = Bq_1$, where A and B are unknown

[15] If I use a distribution that has full (essential) support on $\mathbb{R}_{++}$, then Condition 4 is unnecessary in the solution concept. In Appendix D, I show that the results in Sections 3 and 4 remain qualitatively the same when I use an exponential distribution. The analyses in Sections 5 and 6 would also hold under an exponential distribution, but the uniform distribution significantly simplifies the computation.

coefficients.[16] Notice that $A > 0$ in a fully separating equilibrium (see Lemma B.1). Upon seeing the leader's production decision, the follower's belief updating is given by

$$\mathbb{E}\left[a \mid s_1 = Q_L^{-1}(q_1)\right] = A^{-1}\mathbb{E}[a \mid s_1].$$

The first-order condition gives the follower's best response:

$$Q_F(q_1) = q_2(q_1) = \frac{1}{2}\left[\underbrace{A^{-1}q_1}_{\text{Signaling effeect}} - \underbrace{tq_1}_{\text{Stackelberg effect}}\right]. \tag{2}$$

This expression illustrates the signaling effect of the quantity. The last term in the bracket of (2) is a standard Stackelberg effect, which appears in the complete/symmetric information Stackelberg model. The more the leader produces, the less market share is left, so the follower curtails production. The first term, on the other hand, is unique to the setting where the leader possesses private demand information. Since $A > 0$, the follower produces more as the leader produces more. This is because the leader produces more if the demand is high, so the follower infers that the leader is expanding its output because the demand is high.

Given the best response of the follower, the leader solves

$$\max_{q_1} \mathbb{E}[q_1 P_1(a, q_1, Q_F(q_1)) \mid s_1].$$

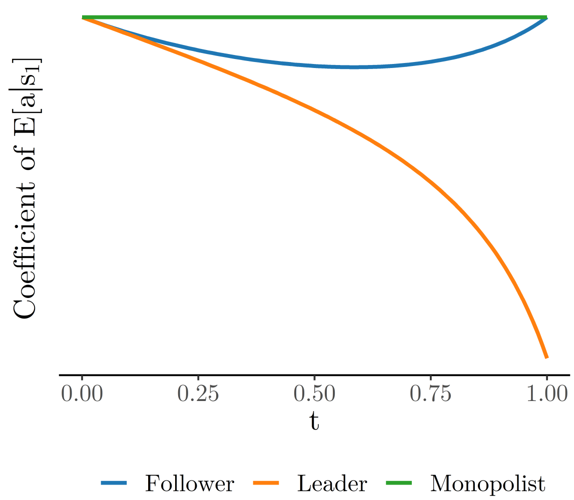
Here, the leader faces two offsetting forces on the follower's production $Q_F(q_1)$ when choosing q_1. If the leader increases production by one more unit, then the leader produces less from the Stackelberg effect but produces more from the signaling effect.

The leader's first-order condition, together with the follower's optimality condition (2), yield a system of equations for the unknown coefficients (A, B). Solving this, I obtain the equilibrium production. Due to the signaling effect, it can be shown that *the leader produces and earns less than the follower for any parameter values*. Thus, the first-mover disadvantage outweighs first-mover advantage in the current setting. See Figure 1 for the coefficients on $\mathbb{E}[a \mid s_1]$ in the production function of each firm.

This benchmark shows that, without the option to withhold the production quantity, the more informed leader is worse off compared to the less informed follower. In the main analy-

[16]The follower's production is linear in the leader's production function regardless of the structure of signaling equilibrium. It is without loss to omit constant terms from the strategies.

Figure 1: Quantities in the Mandatory Disclosure Regime



Note: The figure shows the coefficients on $\mathbb{E}[a \mid s_1]$ in each firm's production for each $t \in (0,1)$ in the mandatory regime.

sis below, I explore the possibility for the leader to escape from the first-mover disadvantage by strategically disclosing the production plan.

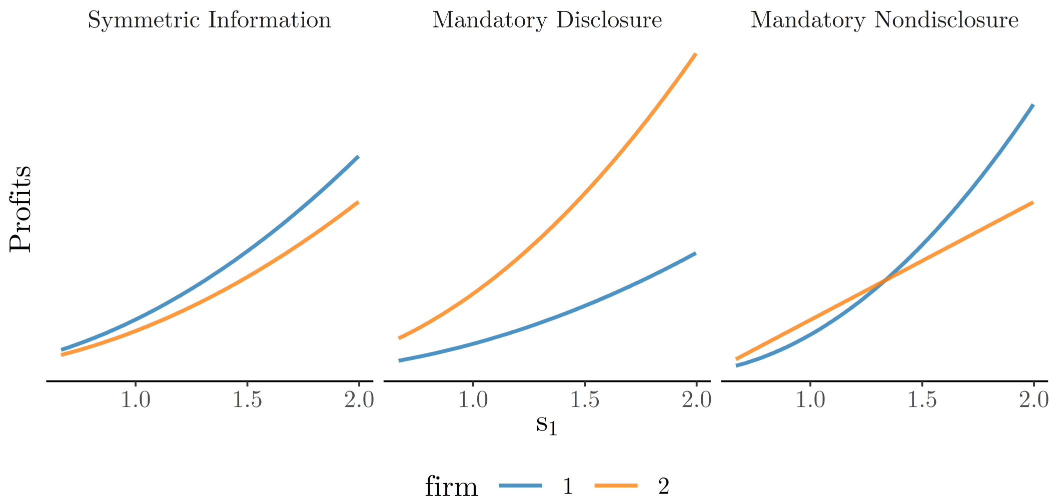
Summary In Figure 2, I plot the equilibrium profits for each signal realization of the mandatory disclosure benchmark (the middle panel). For comparison, I also show the case of symmetric information (the left panel) and mandatory nondisclosure (the right panel). Under symmetric information, the leader can enjoy the first-mover advantage without incurring any signaling cost, so the leader is always better off than the follower. Under mandatory disclosure, the signaling cost is so high that the leader always earns less than the follower. Under mandatory nondisclosure, the follower's production is based on the prior expectation of the demand, so the leader firm is better off than the follower only when the signal realization is higher than the average.

4 Main Analysis

4.1 Equilibrium

In this section, I derive the equilibrium disclosure strategy and production strategies. I start with the production strategies, fixing the disclosure decision. The production equilibrium given the leader disclosing the quantity is the one described in the mandatory disclosure

Figure 2: Profits in Benchmarks



Note: The figure shows the profits of the leader (firm 1) and the follower (firm 2) as a function of signal realizations in each benchmark case. The parameter is set as follows: $t = 0.8, \underline{a} = 1/3, \overline{a} = 1$.

benchmark. In particular, in Appendix B, I show that the leader's profit given disclosure, π_1^D, is given by $\pi_1^D = (1+t\psi)^2(q_1^D)^2$, where $q_1^D = \mathbb{E}[a \mid s_1]/(2(1+t\psi))$ is the leader's production and $\psi := \frac{2-t}{2(1-t)}$.

I now derive the production equilibrium given nondisclosure. This case reduces to a game of simultaneous moves (Cournot) where the leader possesses private information about demand. The leader solves $\max_{q_1} q_1(\mathbb{E}[a \mid s_1] - q_1 - tq_2)$, while the follower solves $\max_{q_2} q_2(\mathbb{E}[a \mid d(s_1) = 0] - \mathbb{E}[q_1(s_1)] - tq_2)$. The (Bayesian-Nash) equilibrium of this game is

$$(q_1^N, q_2^N) = \left(\frac{1}{2}\mathbb{E}[a \mid s_1] - \frac{t}{2(2+t)}\mathbb{E}[a \mid d(s_1) = 0], \quad \frac{1}{2+t}\mathbb{E}[a \mid d(s_1) = 0]\right). \tag{3}$$

The leader's production given nondisclosure, denoted by q_1^N, has $\max\{\cdot, 0\}$ because the production quantity cannot be negative, which I call the non-negativity constraint. When the signal realization is close to the lower bound and the leader withholds for sure, or more generally when the nondisclosure set is large, the follower may overproduce and flood the market up until the point the prices of the goods become zero. However, given the assumption that $\underline{a} \geq \overline{a}/3$, the non-negativity constraint never binds, stated for a general nondisclosure set.

Lemma 1. *Suppose that $\underline{a} \geq \overline{a}/3$. Then, regardless of the nondisclosure set $\{s_1 \mid d(s_1) = 0\}$, the nonnegativity constraint never binds for any disclosure strategy.*

Given this lemma, I omit $\max\{\cdot,0\}$ from the production from now on.

Next, I derive the disclosure equilibrium. I argue that the unique disclosure strategy is lower-tailed. To see this, suppose that the signal realization is increased by a unit. This increase is good news to the leader, so it increases the production level regardless of disclosure behavior. However, given disclosure, an increase in production will be countered by an increase in the follower's production, while the follower's production does not change given nondisclosure. Hence, the leader's expected profit increases in the signal realization by less given disclosure than given nondisclosure. Thus, the leader discloses the production plan only for low signal realizations. Hence, the unique disclosure strategy is a threshold one, and it takes a lower-tail disclosure form,

$$\{s_1 \mid d(s_1) = 1\} = \{s_1 \mid s_1 \leq \tau\},$$

where τ is the disclosure threshold. When the demand signal is low, the leader wishes to let the follower know this fact to avoid overproduction by the follower.

If $\pi_1^D(\tau) \geq (\leq) \pi_1^N(\tau)$ holds for all $\tau \in [2\underline{a}, 2\overline{a}]$, then full disclosure (nondisclosure) is the equilibrium. Otherwise, partial disclosure emerges in equilibrium, and the equilibrium threshold τ^* is determined by the following indifference condition of the marginal type:

$$\underbrace{\frac{1}{2}\mathbb{E}[a \mid \tau] - \frac{t}{2(2+t)}\mathbb{E}[a \mid s_1 > \tau]}_{\text{Nondisclosure}} = \underbrace{\frac{1}{2\sqrt{1+t\psi}}\mathbb{E}[a \mid \tau]}_{\text{Disclosure}}. \tag{4}$$

In order for the cutoff strategy to be an equilibrium, one needs to be careful about the deviation incentive of nondisclosure types. A type that does not disclose in a equilibrium has on-path deviation, where the type discloses and produces a quantity that is in the range of productions disclosed in equilibrium, and off-path deviation, where the type discloses and produces a quantity that is not disclosed and produced in equilibrium by any type. The on-path deviation is not profitable by construction. To see there is no off-path deviation, one needs to specify off-equilibrium beliefs. Clearly, if the follower assigns sufficiently "bad" belief for an off-path deviation, then it deters any off-path deviation.[17] As the following proposition states, there is a specification of off-path beliefs such that the disclosure equilibrium identified below survives the D1 criterion, a standard signaling refinement in the literature (Cho and Kreps, 1987; Banks and Sobel, 1987).

[17]To be precise, by the unboundedness of the follower's belief, the follower can assign probability one to an arbitrarily high s_1.

Proposition 1.

(i) The optimal disclosure strategy is unique and is an lower-tail disclosure, where the threshold τ^* is uniquely determined by the indifference condition (4).

(ii) Full disclosure is impossible for any t and ρ.

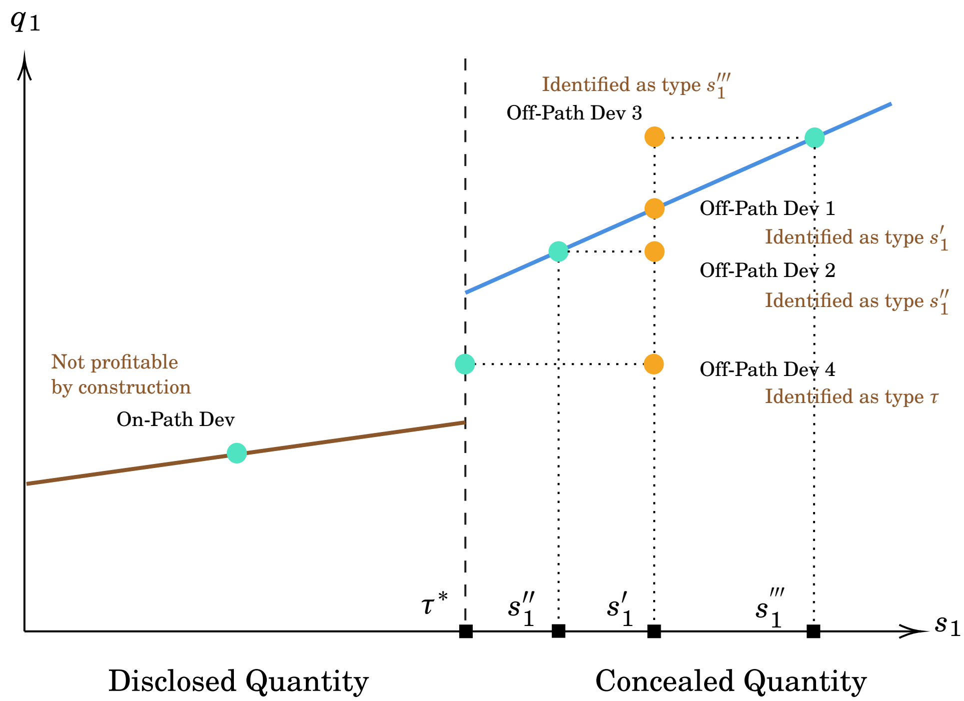
(iii) There are off-path beliefs such that the disclosure equilibrium survives the D1 criterion.

Part (i) of Proposition 1 explicitly characterizes a unique disclosure threshold in case it is interior. Notice that the uniqueness of the disclosure strategy is within a linear fully separating equilibrium. Since there may be nonlinear fully separating equilibria, an equilibrium in the entire game is not unique in general. That said, the trade-off associated with disclosure–the Stackelberg effect and the signaling effect–is present for any fully separating equilibrium. The explicit form of the unique threshold τ^* is given in the Appendix.

Part (ii) of Proposition 1 states that the unraveling result does not hold in this model. The reason is that, for high-signal types, the cost of contracting quantity to signal that the demand is low is too high, so these types would rather be pooled together with higher signal types. This result provides a micro-foundation for the "proprietary cost" of Verrecchia (1983). In my model, the proprietary cost is endogenous, and it is the cost of signaling through the production quantity. Cheynel and Ziv (2021) also endogenize a proprietary cost in the Cournot (simultaneous-move) competition setting where disclosure is about the demand signal. However, in their model, when the manager only cares about the cash flow, the unraveling result holds, so the paper does not provide a satisfactory explanation for the proprietary costs if one views the manager to be long-term oriented. Moreover, part (ii) of Proposition 1 highlights that the difference in what to disclose–signal or action as a function of signal–leads to different implications.

Part (iii) Proposition 1 states that the equilibrium survives the D1 criterion. I explain this point using Figure 3. For types that are supposed to withhold the production plan, there are possibly two types of off-path deviations. One type of deviation is for the leader to produce and disclose the quantity in $\{q_1^*(s_1) \mid s_1 \geq \tau^*\}$, i.e., the quantity that some type would produce and conceal in the proposed equilibrium. In the figure, this corresponds to off-path deviations 1 and 2. After the deviation to this quantity, the D1 criterion restricts the off-path belief to the type that would produce that quantity in equilibrium. For example, in the figure, if type s_1' produces and discloses the quantity denoted by off-path deviation 1, which is the quantity that the type produces and conceals in equilibrium, then the follower believes that the discloser is type s_1'. Similarly, the deviation to the production and disclosure of off-path deviation 2 (3) results in being identified as type s_1'' (s_1'''). Since type s_1' prefers to be pooled with other nondisclosure types, the deviation is unprofitable.[18] Alternatively, the leader can produce and disclose some quantity in the discontinuous region of the production schedule. In the figure, this corresponds to off-path deviations 4. The D1 criterion restricts the off-path belief to the marginal type τ^*. This again renders the deviation unprofitable.

Figure 3: Off-Path Beliefs under the D1 Criterion



Note: The red line shows that production schedule given disclosure, and the blue line shows the production schedule given nondisclosure. The dotted line denotes the disclosure threshold, τ.

In summary, the equilibrium disclosure strategy is given by $d^*(s_1) = 1 \iff s_1 \le \tau^*$ and the production strategies are

$$q_1^*(s_1) = \begin{cases} \frac{1}{2(1+t\psi)}\mathbb{E}[a \mid s_1] & \text{if } s_1 \le \tau^* \\ \frac{1}{2}\mathbb{E}[a \mid s_1] - \frac{t}{2(2+t)}\mathbb{E}[a \mid s_1 > \tau^*] & \text{if } s_1 > \tau^* \end{cases}$$
$$(q_2^D)^* = \frac{\psi}{2(1+t\psi)}\mathbb{E}[a \mid s_1], \qquad (q_2^N)^* = \frac{1}{2+t}\mathbb{E}[a \mid s_1 > \tau^*]. \tag{5}$$

The equilibrium profits can be easily computed from (5). Figure 4 shows the equilibrium

[18]More precisely, being pooled means that the follower believes that the type (signal) is $\mathbb{E}[a \mid a > \tau] = (\tau + 2\overline{a})/2$. Thus, the type $s_1 = \mathbb{E}[a \mid a > \tau]$ is indifferent between disclosing and withholding for the belief restricted by D1. I assume that this type withholds.

profits as a function of s_1, where the dotted vertical line indicates the disclosure threshold, τ^*. Since type-τ^* leader is indifferent between producing the disclosure quantity and nondisclosure quantity, the leader's profit is continuous. On the other hand, the follower is not made indifferent at the threshold, so the profit function exhibits a discontinuous jump at τ^*. As analyzed in Section 3, if the leader does not have the option to strategically conceal the quantity, then the leader always earns less profit compared to the follower. That is, the signaling cost (the first-mover disadvantage) always outweighs the first-mover advantage. If the leader is allowed to conceal the quantity, the leader optimally does so for high realizations of s_1, and the leader can partially restore the first-mover advantage.

Notice that the leader does not always earn a higher profit than the follower when the leader conceals the quantity. This is because the nondisclosure types are pooled together, and because the follower's production is determined by the average signal over the nondisclosure types. Thus, for low types in the nondisclosure region (i.e., types such that $\mathbb{E}[a \mid s_1] \in [\tau^*, \mathbb{E}[a \mid s_1 > \tau^*]]$), the follower overproduces in relative terms. However, in equilibrium, these low types in the nondisclosure types are better off than disclosing and contracting the quantity. Alternatively, high types in the nondisclosure region (i.e., types such that $\mathbb{E}[a \mid s_1] > \mathbb{E}[a \mid s_1 > \tau^*]$) earns more than the follower. Moreover, one can show that the leader earns more than the symmetric Cournot profit if and only if $\mathbb{E}[a \mid s_1] > \mathbb{E}[a \mid s_1 > \tau^*]$.[19] In these senses, the ex-post first-mover advantage is partially restored for types $s_1 > \mathbb{E}[a \mid s_1 > \tau^*]$.

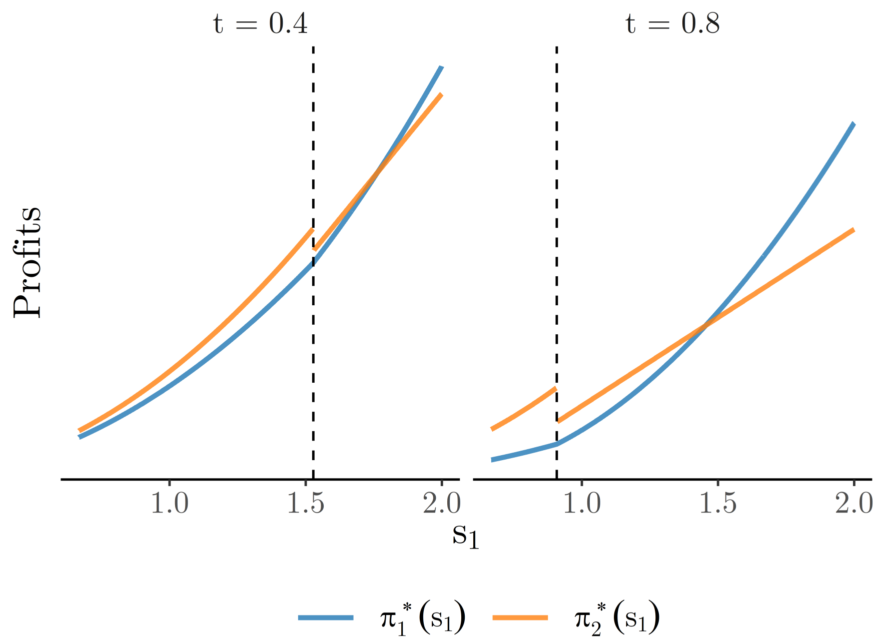
4.2 Comparative Statics

The natural question is how the disclosure behavior changes as the parameters of the model change. The relationship between product-market competition and disclosure has received considerable attention in accounting research (Li, 2010; Huang, Jennings and Yu, 2017; Li, Lin and Zhang, 2018; Glaeser, 2018). Since the disclosure equilibrium (within the class of linear signaling equilibria) is unique and it is characterized by a threshold, I can conduct comparative statics analysis. The following two corollaries show how the disclosure threshold changes when the competitiveness of the market, t, and the precision of the signal, ρ, changes.

Corollary 1. *The leader discloses less if the competitiveness of the market increases. That is, the disclosure threshold τ^*, if it is interior, is monotonically decreasing in t.*

[19] The symmetric Cournot profit is defined as the expected profit of each firm when they both observe the signal and move simultaneously. It is given by $\frac{1}{(2+t)^2}\mathbb{E}[a \mid s_1]^2$. Using this expression, one can compare the ex-post Cournot profit and the nondisclosure profit to obtain the result.

Figure 4: Equilibrium Profit and Disclosure Threshold



Note: The figure shows the profits of the leader (firm 1) and the follower (firm 2) as a function of signal realizations s_1 in equilibrium. The dashed vertical line shows the equilibrium disclosure threshold. The parameter is set as follows: $\underline{a} = 1/3, \overline{a} = 1$.

The left and the right panel in Figure 4 illustrates the change in the disclosure threshold as t increases. The corollary, which can be derived by differentiating τ^* with respect to t, shows that the leader discloses less when the market becomes more competitive. This is consistent with the proprietary cost hypothesis, where more competition implies less disclosure. In my model, as the two goods become more similar, the leader's profit depends more on the follower's production, so the leader's signaling incentive in the production stage given disclosure becomes stronger. Thus, if t becomes high, the leader would rather choose not to disclose the quantity. Moreover, as the competition parameter t goes to the the boundaries, the threshold also goes to boundaries. That is, $\lim_{t\to 0} \tau^* = 2\overline{a}$ and $\lim_{t\to 1} \tau^* = 2\underline{a}$ for any ρ.

The effect of the signal precision ρ on disclosure is as follows:

Corollary 2. *The leader discloses more if ρ increases. That is, the disclosure threshold τ^*, if it is interior, is monotonically increasing in ρ.*

In the model, the disclosure profit (quantity) and nondisclosure profit (quantity) both change with respect to ρ. Moreover, they may be increasing or decreasing with respect to ρ. However, it is always the case that the nondisclosure profit of the marginal type increases (decreases) in ρ at the rate lower (higher) than the disclosure profit does. Therefore, as the signal becomes more precise, the marginal type prefers to disclose more information. Intuitively, when the leader firm is known to have more precise information, the follower revises the belief in a more unfavorable way to the leader upon nondisclosure, so the leader firm is compelled to disclose more.[20]

4.3 Ex-ante Welfare: Voluntary vs Mandatory Disclosure

Compared to the mandatory disclosure case, the leader is unambiguously better off with the voluntary disclosure option, since the leader can save the signaling cost. On the flip side, the follower prefers the mandatory disclosure regime. However, the net effect is not obvious. How does the aggregate industry profit change as the voluntary disclosure option is introduced? Consumers are also affected by the disclosure regime through the firms' production. Are consumers better off under voluntary disclosure compared to mandatory disclosure? In this section, I look at the ex-ante welfare consequences of voluntary disclosure.

Assuming quasilinear quadratic utility on the part of consumers, consumer surplus (CS), total producer surplus or aggregate profits (PS), and total surplus (TS) are defined as follows (Choné and Linnemer, 2020):

$$CS(s_1) = \frac{1}{2}(q_1^2 + 2q_1q_2t + q_2^2)$$
$$PS(s_1) = \mathbb{E}[a \mid s_1](q_1 + q_2) - (q_1^2 + 2q_1q_2t + q_2^2)$$
$$TS(s_1) = \mathbb{E}[a \mid s_1](q_1 + q_2) - \frac{1}{2}(q_1^2 + 2q_1q_2t + q_2^2).$$

Ex-ante welfare is given by taking the expectation of these expressions over s_1.

Given that the production plan is disclosed, the total production of the mandatory disclosure regime Q^M and the voluntary disclosure regime Q^V are the same. Specifically, from (13),

$$Q^M(s_1) = Q^V(s_1) = \frac{1+\psi}{2(1+t\psi)}\mathbb{E}[a \mid s_1]$$

Given that the production plan is concealed, from (3) the total quantity in voluntary disclosure regime is given by

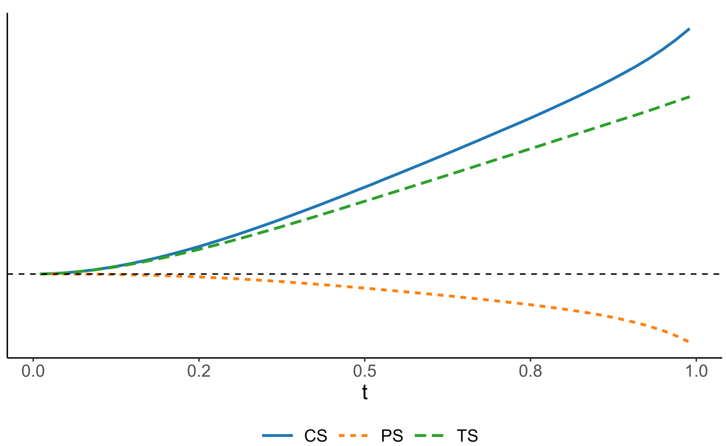
$$Q^V(s_1) = \frac{1}{2}\mathbb{E}[a \mid s_1] + \frac{2-t}{2(2+t)}\mathbb{E}[a \mid d(s_1) = 0].$$

[20]This mechanism holds for standard disclosure models such as Dye (1985) and Jung and Kwon (1988).

After some algebra, it turns out that, on average, the total quantity given nondisclosure is higher in the voluntary disclosure regime than in the mandatory disclosure regime. In a duopolistic market with perfect information, in general, firms produce less than the socially optimal level, and an increase in total production improves consumer welfare. Therefore, it can be conjectured that voluntary disclosure improves consumer surplus while it harms producer surplus. Moreover, it is expected that the total surplus is improved as well, because the total quantity Q^M is smaller than the socially optimal level. However, analytically comparing welfare is difficult for the following reasons: (i) since the disclosure threshold does not have an explicit form in general or is complicated, it is hard to compute the nondisclosure quantity; (ii) since CS, PS, and TS are nonlinear in s_1, it is hard to compute the expectation. Therefore, I present a numerical simulation result.

Figure 5 shows the difference in welfare between the voluntary regime and the mandatory regime. The horizontal dashed line denotes the zero line. For any competitiveness parameter t, the consumer surplus improves while the producer surplus deteriorates due to voluntary disclosure. The net effect for the economy as a whole is always positive: voluntary disclosure is welfare-enhancing. Hence, the numerical analysis suggests that the conjecture from the analysis of total quantity applies.

Figure 5: Welfare Difference Between the Voluntary Regime and the Mandatory Regime



Note: The figure shows the difference in welfare between the voluntary regime and the mandatory regime. The line $\mathrm{VD}_{\mathrm{CF}}$ shows the welfare difference when the disclosure objective is cash flow. The dashed line indicates the zero line. The parameters are set as $\underline{a} = 1/3, \overline{a} = 1, \rho = 1$.

5 Myopic Disclosure Incentive

In the main model, I assumed that firms maximize profits. If the manager of the firm is long-term-oriented, profit/cash flow maximization makes sense. However, managers of public firms are subject to short-term stock market incentives. In this section, I modify the model so that firms make disclosure decisions to maximize both stock price (market expectation of the cash flow) and cash flow. In particular, following the convention of the literature (Stein, 1989), I specify the leader firm's payoff, denoted by U_1, as a convex combination of cash flow and stock price:

$$U_1 = (1-\alpha)\pi_1 + \alpha MP_1, \qquad \alpha \in [0,1). \tag{6}$$

The parameter α captures the degree of the manager's myopia.[21] When α is higher, the leader care's more about the short-term stock price. The timeline of the model is now as follows. Firm 1 (leader) privately learns the signal and commits to a production plan. Then, firm 1 makes a disclosure decision, i.e., whether to disclose the committed production. After observing the disclosure decision, firm 1 (and 2) is priced by the stock market, where the price is the expectation of the profits given the available information. Firm 2 then produces a quantity.[22] The market clears, and the profits realize.

I focus on fully separating equilibrium as in the previous section. Therefore, upon disclosure of the quantity, the profit π_1^D is known to the market. I let $MP_1(d)$ be the market price of the leader firm, so $MP_1(d=1) = \pi_1^D$. Upon nondisclosure, the leader's profit is $\pi_1^N = (q_1^N)^2$ as before. However, since q_1^N depends on s_1, which is not observable to the market, the stock price takes expectation over signal realization under which the leader withholds the quantity: $MP_1(d=0) = \mathbb{E}[\pi_1^N \mid d(s_1) = 0]$. This highlights the difference in disclosure incentives from the long-term manager case. In particular, upon nondisclosure, although the follower does not observe the signal and takes an expectation of the signal realizations over the nondisclosure region, the leader's expected cash flow depends on signal realizations, as it can tailor production according to the private signal. Indeed, I have shown that, as the signal realization increases, nondisclosure becomes more attractive. On the other hand, the market price takes an expectation over the signal realizations over nondisclosure regions as the follower does, so the nondisclosure price is constant with respect to signal realizations. Hence, as the signal realization increases, disclosure becomes relatively attractive

[21] I exclude the case of pure myopia, $\alpha = 1$. If the leader maximizes only stock price in the production stage, then the leader is indifferent for any production levels given nondisclosure, because the market does not observe the quantity. In particular, any stock price can be supported in equilibrium.

[22] The order of the stock market's pricing and firm 2's production is interchangeable.

compared to nondisclosure. Depending on the myopia parameter α, these countervailing forces determine the disclosure equilibrium. Since both the expected payoff given disclosure and nondisclosure changes with respect to the signal in a nonlinear way, the disclosure equilibrium could potentially be complicated. The following proposition establishes that any disclosure equilibrium takes a simple form: lower-tail disclosure or interval disclosure.

Lemma 2. *Suppose that the leader firm makes a disclosure decision to maximize (6). Any disclosure equilibrium takes the following form:* $\{s_1 \mid d(s_1) = 0\} = \{s_1 \mid s_1 \leq \tau_1 \text{ or } s_1 \geq \tau_2\}$ *for some* $2\underline{a} \leq \tau_1 \leq \tau_2 < 2\overline{a}$.

Notice that $\tau_2 = 2\overline{a}$ corresponds to upper-tail disclosure and $\tau_1 = 2\underline{a}$ to lower-tail disclosure. Thus, Lemma 2 says that disclosure equilibrium is either lower-tail disclosure ($\tau_1 = 2\underline{a}$ and $\tau_2 < 2\overline{a}$) or interval disclosure ($\tau_1, \tau_2 \in (2\underline{a}, 2\overline{a})$). Interestingly, upper tail disclosure ($\tau_1 > 2\underline{a}$ and $\tau_2 = 2\overline{a}$) is impossible even if the leader firm puts a lot of weight on the stock price (i.e., α is high). To see this, suppose that α is close to one, so that the leader firm cares almost exclusively about stock price. Then, the nondisclosure payoff is almost constant, as the stock price given nondisclosure is constant. Since the expected payoff given disclosure is increasing in the signal realization, the disclosure equilibrium should be upper-tail disclosure. But then, any types who are in the nondisclosure region has have an incentive to mimic the types who disclose in equilibrium, because the stock price is weakly increasing in types. Doing so hurts the cash flow as the leader firm is overproducing compared to the signal realization. However, since the leader firm does not care about the cash flow consequences, this cost of deviation does not affect the deviation incentive.

Another implication of Lemma 2 is that interval nondisclosure is impossible. To gain some intuition behind this result, recall that the expected payoff given nondisclosure is constant with respect to the signal realization when the leader cares only about stock price, while it is increasing when the leader maximizes cash flow. Thus, for any $\alpha \in [0,1]$, the nondisclosure stock price is higher than the nondisclosure cash flow when the signal realization is low. Now, suppose on the contrary that interval nondisclosure is possible, and let $\tau_1 < \tau_2$ be the marginal types. In the neighborhood of τ_1, the marginal type's disclosure strategy can be seen as lower-tail disclosure. However, from the previous observation that the nondisclosure stock price dominates when signal realization is low, the lower-tail disclosure incentive dominates in the neighborhood of τ_1. Thus, it is a contradiction.

Having characterized possible disclosure equilibrium in Lemma 2, I now turn to the existence and properties of equilibria.

Proposition 2. *Suppose that the manager maximizes (6). (i) The equilibrium exists for α low enough. (ii) If disclosure equilibrium exists, full disclosure is impossible for any α, t, and ρ. (iii) An interval disclosure equilibrium exists.*

As the above discussion about the impossibility of upper-tail disclosure implies, the equilibrium may not exist for some value of α. Part (i) of the proposition states that as long as α is not high enough, the equilibrium exists. Moreover, the no-full disclosure result of Proposition 1 extends to the case of $\alpha > 0$ (Part (ii)). It is possible to construct a proper interval disclosure equilibrium, where the leader firm discloses the production plan only when the signal realization is "modest."

Figure 6 illustrates an example of interval disclosure.[23] When the signal realization is low ($s_1 \leq \tau_1$) or high ($s_1 \geq \tau_2$), the leader firm conceals the production plan. Thus, upon nondisclosure, the follower is not sure if the demand signal is low or high. Each marginal type is indifferent between disclosing and not disclosing. When the production plan is disclosed ($s_1 \in [\tau_1, \tau_2]$), the leader is worse off than the follower by the signaling distortion. Thus, as the disclosure interval, which is determined in equilibrium, gets smaller, the leader benefits more from voluntary disclosure. The nondisclosing types should be prohibited from mimicking the disclosing types. When α is not high enough, such deviation is not profitable due to the cash flow consequences. Under the setting of Figure 6, this incentive constraint can be checked manually.

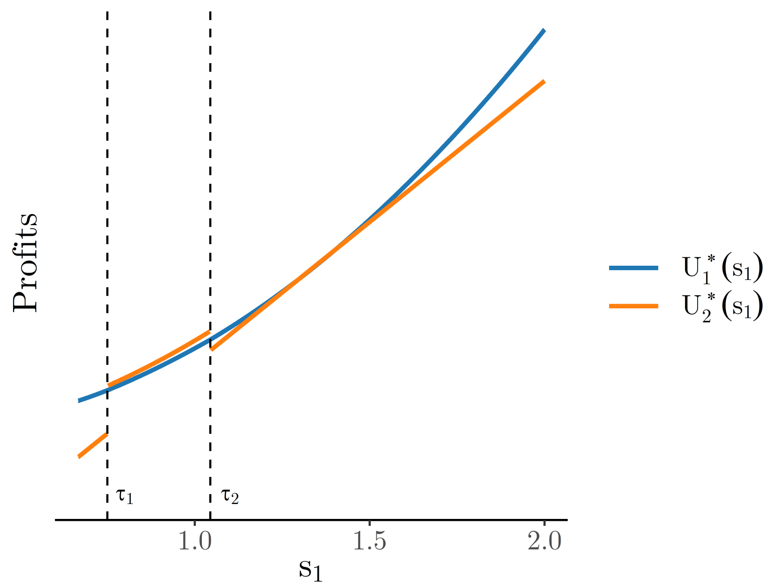
6 Uncertain Information Endowment

In this section, I extend the main model to allow the possibility that the leader firm does not receive information (Dye, 1985; Jung and Kwon, 1988). I simplify the model by assuming that the leader observes a perfect signal, i.e., $\rho = 1$. Specifically, the leader now observes the private signal $s_1 = a$ with probability $p \in (0,1)$ and observes nothing, denoted by $s_1 = \varnothing$, with probability $1-p$. The remaining structure of the game is the same as before. Importantly, the leader can disclose the production quantity q_1 even when the leader observes a null signal $s_1 = \varnothing$, because the leader commits to a production decision regardless of the information endowment.

This twist from the standard Dye-Jung-Kwon model, i.e., the disclosure by the *uninformed* type, introduces an interesting economic force in the model. There are two types of

[23]Since the follower's expected cash flow given nondisclosure is the same as the stock price, the specification of the follower's payoff does not matter as long as the production decision is made to maximize expected cash flow.

Figure 6: Equilibrium Profit and Disclosure Threshold: Dual Objective



Note: The figure shows the payoff of the leader (firm 1) and the profits of the follower (firm 2) as a function of the signal realization s_1 in equilibrium. The dashed vertical line shows the equilibrium disclosure thresholds. The leader firm discloses q_1 if $s_1 \in [\tau_1, \tau_2]$. The parameter is set as follows: $\underline{a} = 1/3, \overline{a} = 1, \alpha = 0.07, t = 0.3$.

equilibria, depending on whether the uninformed type discloses the quantity. Since the uninformed leader does not observe the private signal about demand, the uninformed leader's disclosure choice does not depend on the signal realization. If the uninformed leader does not disclose the production plan, then upon disclosure, the follower knows that the quantity is from the informed and that it contains information about the demand, so the production stage is the same as the main analysis. However, upon nondisclosure of the quantity, the follower does not know if the nondisclosure is coming from the informed or uninformed, so the follower's inference depends on the prior probability that the information arrives at the leader. I show that the the effect of the information friction on disclosure equilibrium is the same as in the standard Dye-Jung-Kwon model.

Alternatively, if the uninformed discloses its production plan, then the information friction gives different implications. The follower, upon observing the disclosed quantity, considers the possibility that the leader may be uninformed. The production schedule by the uninformed does not contain any value as a signal of the leader's private demand information. The informed leader could possibly mimic the uninformed type's production to signal that the quantity has no information value in an attempt to avoid signaling distortion. Indeed, I will show that there is no fully separating equilibrium. I analyze semi-separating equilibria, where the informed leader mimics the uninformed leader's production for some signal realizations to avoid the first-mover disadvantage but still enjoys the first-mover advantage. This new device used to restore the first-mover advantage–producing the quantity that the uninformed would produce–increases the value of disclosure. Hence, it is possible that, under some parameter values, the information friction *expands* the disclosure region.

6.1 The Uninformed Leader Conceals Its Production

Suppose that there is an equilibrium in which the uninformed leader conceals the production plan. In this case, any disclosure about quantity comes from the informed leader for sure, so the equilibrium given disclosure is the same as the main model. In particular, a separating production equilibrium exists in which disclosed types are separated. If the informed instead withholds the quantity, then the follower is not sure if the concealment is from the informed leader's strategic decision or the leader being uninformed. The informed leader follows a lower-tail threshold disclosure strategy by the same reasoning as in the main model, so upon nondisclosure, the follower's posterior expectation of the demand is

$$\frac{p\mathbb{P}(a \geq \tau)}{p\mathbb{P}(a \geq \tau)+(1-p)}\mathbb{E}[a \mid a > \tau] + \frac{(1-p)}{p\mathbb{P}(a \geq \tau)+(1-p)}\mathbb{E}[a]. \tag{7}$$

The uninformed leader's quantity is determined by solving $\max_{q_1} \mathbb{E}[q_1 P_1(a, q_1, q_2)]$.

The disclosure threshold is determined by the following indifference condition of the marginal type:

$$\underbrace{\frac{1}{2}\mathbb{E}[a \mid \tau] - \frac{t}{2(2+t)}\left[\frac{p\mathbb{P}(a \geq \tau)}{p\mathbb{P}(a \geq \tau)+(1-p)}\mathbb{E}[a \mid a > \tau] + \frac{(1-p)}{p\mathbb{P}(a \geq \tau)+(1-p)}\mathbb{E}[a]\right]}_{=\text{Nondisclosure}} = \underbrace{\frac{1}{2\sqrt{1+t\psi}}\mathbb{E}[a \mid \tau]}_{=\text{Disclosure}}. \tag{8}$$

For this type of equilibrium to exist, the uninformed leader's incentive compatibility must be satisfied. That is, if the uninformed leader has the incentive to deviate and disclose the quantity, it is no longer an equilibrium. There are two types of deviations. The uninformed can mimic the informed by committing to q_1 that is in the range of the informed leader's equilibrium disclosed production plan (on-path deviation). In this case, the follower thinks that the disclosure is from the informed and backs out the signal according to the fully separating equilibrium. Alternatively, the uninformed can disclose a quantity that the informed leader would not disclose (off-path deviation). In this case, the follower can assign any off-path belief.[24] I show that if the market is competitive enough, the equilibrium in which the uninformed conceals information exists.

Proposition 3. *Let τ^* be the solution to (8). There exists a cutoff t^* such that for $t \geq t^*$, the disclosure threshold τ^*, the uninformed leader's production $\frac{1}{2}\mathbb{E}[a] - \frac{t}{2(2+t)}\mathbb{E}[a \mid \text{Nondisclosure}]$, and the informed leader's and the follower's production plan identified in (5), where the nondisclosure belief is modified to (7), constitute an equilibrium in which the uninformed leader conceals the production plan. In particular, the uninformed does not have an incentive to deviate and disclose its production plan.*

Intuitively, for an on-path deviation, the uninformed leader mimics the informed leader by choosing the quantity that the informed leader would produce and disclose for some signal $s_1 \leq \tau^*$. Therefore, choosing the optimal deviation from the informed leader's production schedule is equivalent to choosing the signal realization $s_1 \leq \tau^*$. Since the uninformed leader does not observe the signal, it has to guess the private signal. When the best guess $\mathbb{E}[a]$ is available ($\mathbb{E}[a] \leq \tau^*$), this mimicry is profitable. Otherwise, the gain from deviation is nonpositive.

Explicitly solving (8) is infeasible, but I have the following comparative statics.

Corollary 3. *The disclosure threshold τ^* is increasing in p. That is, the leader discloses more as p increases.*

This result is in line with the original Dye model. When the leader becomes more likely to be uninformed, the follower thinks that nondisclosure is more likely to be coming from the uninformed, and thus the marginal type now finds it more attractive to be pooled with the uninformed. Note that in the extreme case where $p \to 1$, the condition (8) reduces to (4).

Not surprisingly, the comparative statics with respect to t continues to hold. That is, as the market becomes more competitive, the leader firm discloses less.

Corollary 4. *The disclosure threshold τ^* is increasing in p. That is, the leader discloses more as p increases.*

6.2 The Uninformed Leader Discloses Its Production

Next, I consider the case in which the uninformed leader discloses its production plan.

[24] As in Proposition 1, there are off-path beliefs that survive the D1 criterion.

Full Separation is Impossible

I first show that there is no linear fully separating equilibrium.

Lemma 3. *Suppose that the uninformed discloses its production plan. Then, there is no linear fully separating equilibrium.*

Intuitively, if there is a fully separating equilibrium, since the signal space is a continuum, the probability that the informed produces and discloses the quantity that the uninformed produces and discloses is zero. Therefore, upon seeing the production level that the uninformed would commit to, the follower assigns probability one to the event that the leader is uninformed. For this uninformed's production level, the follower believes that there is no signaling value, so the production levels are in Stackelberg equilibrium. If the informed leader produces this uninformed's quantity, then the follower evaluates that the demand is $\mathbb{E}[a]$. When the leader observes a signal around $\mathbb{E}[a]$, the leader can save the cost of signaling by being pooled with the uninformed, while the mimicked production level is close to the optimal production under perfect information. Thus, there is an interval of informed types who wish to deviate to the uninformed type's production level.

The logic above applies to any fully separating equilibria, which is not necessarily linear. Thus, one can predict that Lemma 3 is true across *any* fully separating equilibria. However, it is hard to formally show the result, as I do not have a closed-form solution for the equilibrium and it is difficult to compute the deviation profit.

Semi-Separating Equilibria: Production Equilibrium

In the proof of Lemma 3, I show that there is an interval of informed types who deviates to the uninformed production. Following this observation, I look for semi-separating equilibria of the following form. The informed leader's strategy, denoted by $q_{1,info}$, is given by

$$q^D_{1,info}(s_1) = \begin{cases} q_{1,uninfo} & \text{if } s_1 \in \mathscr{S}_{pool} = [\underline{s}, \overline{s}] \\ q_{1,sep} = Q_L(s_1) & \text{if } s_1 \in \mathscr{S}_{sep} = \mathscr{S} \setminus [\underline{s}, \overline{s}]. \end{cases} \tag{9}$$

if it is disclosed and $q^N_{1,info}(s_1)$ if it is concealed. The uninformed leader produces $q_{1,uninfo}$. That is, the informed leader types in $\mathscr{S}_{pool}$ produce the quantity that the uninformed leader produce, and other types produces linear separating equilibrium levels. It turns out that under a mild condition this is the only type of linear semi-separating equilibrium.[25]

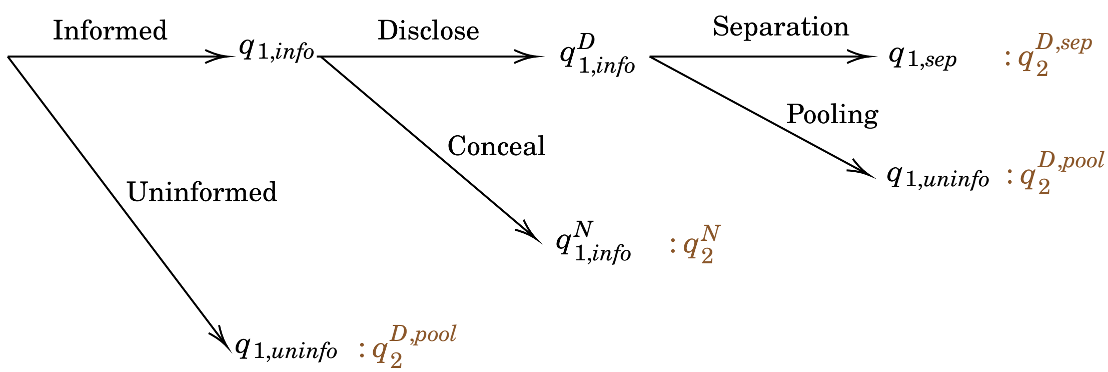
Lemma 4. *Suppose that $\mathscr{S}_{pool}$ is a union of (nondegenerate) intervals. Any semi-separating equilibrium takes the form (4).*

The follower's strategy when q_1 is disclosed thus can be written as follows:

$$q_2^D(q_1) = \begin{cases} q_2^{D,sep} = Q_{F,sep}(q_1) & \text{if } q_1 \in Q_L(\mathscr{S}_{sep}) \\ q_2^{D,pool} = Q_{F,pool}(q_1) & \text{if } q_1 = q_{1,uninfo}. \end{cases}$$

I use q_2^N to denote the follower's production when the leader conceals the production. The equilibrium is now defined by $\{d, q_{1,info}^D, q_{1,info}^N, q_{1,uninfo}, q_2^D, q_2^N\}$ such that the optimality and belief consistency analogous to Definition 1 hold. Notice that the uninformed production $q_{2,uninfo}$ is determined in equilibrium given that certain types of the informed wish to mimic the uninformed. Figure 7 summarizes the notation in the production stage.

Figure 7: Production Stage



Note: The figure shows the notation for the production stage.

To derive the production equilibrium, fix a pooling interval $\mathscr{S}_{pool}$. The equilibrium pooling interval is later uniquely identified as a fixed point. Consider the follower's reaction to the disclosure of $q_{1,sep}$. The follower knows that the leader is informed, and the production equilibrium is exactly the same as the main model. The follower uncovers the private signal in equilibrium from the disclosure. Next, suppose that the leader discloses $q_1 = q_{1,uninfo}$. The follower infers that the leader is either informed or uninformed and uses the Bayes rule to

[25] If $\mathscr{S}_{pool}$ contains a countable points $\{s^i\}_{i=1}^{\infty}$ where each point is not a limit point of $\mathscr{S}_{pool}$, then it does not affect belief updating, as $\mathscr{S}_{pool} = \mathscr{S}_{pool} \setminus \{s^i\}_{i=1}^{\infty}$ almost everywhere. However, the statement cannot be strengthened using an equivalent relation defined by almost everywhere.

update beliefs. In particular, the posterior expectation of the demand is given by

$$\mathbb{E}[a \mid q_{1,uninfo}] = \frac{p\mathbb{P}(s_1 \in \mathscr{S}_{pool})}{p\mathbb{P}(s_1 \in \mathscr{S}_{pool}) + (1-p)}\mathbb{E}[a \mid s_1 \in \mathscr{S}_{pool}] + \frac{1-p}{p\mathbb{P}(s_1 \in \mathscr{S}_{pool}) + (1-p)}\mathbb{E}[a]. \tag{10}$$

The above expression of $\mathbb{E}[a \mid q_{1,uninfo}]$ does not directly depend on s_1, because $q_{1,uninfo}$ is the pooling level, which does not vary with the signal realizations s_1. Finally, suppose that q_1 is not disclosed. Since the uninformed leader discloses, the follower knows that the leader is informed. The posterior expectation is determined by the nondisclosure set.

Given the best responses of the follower, consider the leader's optimal production. The uninformed leader solves

$$\max_{q_{1,uninfo}} q_{1,uninfo}(\mathbb{E}[a] - q_{1,uninfo} - tQ_{F,pool}(q_{1,uninfo})). \tag{11}$$

Here, because of the pooling by the informed types, the follower reacts to $q_{1,uninfo}$ with $Q_{F,pool}(q_{1,uninfo})$, which in turn is affected by the pooling interval. The uninformed leader chooses the quantity understanding this reaction by the follower. This strategic interaction between the informed and the uninformed is the new economic force that arises in the equilibrium where the uninformed discloses its quantity. The informed leader has a choice of choosing a pooling level or separation level. If it decides to choose the pooling level, it must produce the level identified in Equation (11). If it decides to choose a separation level, then the leader solves the same problem as the case in which there is no Dye friction.

The above discussion describes how the production equilibrium is determined for a fixed pooling interval. Thus, I am left to determine the pooling interval $[\underline{s}, \overline{s}]$ to characterize production equilibrium. An equilibrium interval $[\underline{s}^*, \overline{s}^*]$ must satisfy the following condition:

$$\pi_{1,pool}(s_1; \underline{s}^*, \overline{s}^*) \geq \pi_{1,sep}(s_1) \iff s_1 \in \mathscr{S}^*_{pool} = [\underline{s}^*, \overline{s}^*], \tag{12}$$

where $\pi_{1,sep}$ ($\pi_{1,pool}$) is the leader's profit when the leader chooses the separation (pooling) level of production.[26] Condition (12) requires that an equilibrium pooling interval is a fixed point. For given $\underline{s}$ and $\overline{s}$, the production equilibrium is pinned down, and one can compute the interval $[\underline{s}^\circ, \overline{s}^\circ]$ such that $\pi_{1,pool} \geq \pi_{1,sep}$. For $\underline{s}$ and $\overline{s}$ to be an equilibrium, it must be $\underline{s} = \underline{s}^\circ$ and $\overline{s} = \overline{s}^\circ$. The following result shows that this fixed point uniquely exists. Therefore, the equilibrium production and profit given disclosure of the form (9) is uniquely determined.

[26]Notice that $\pi_{1,pool}$ is quadratic and $\pi_{1,sep}$ is linear in s_1, so $\pi_{1,pool} - \pi_{1,sep}$ has at most two roots. Since $\pi_{1,sep}$ is convex, $\{s_1 \mid \pi_{1,pool} - \pi_{1,sep} \geq 0\}$ is an interval.

Lemma 5. *Suppose that q_1 is disclosed. Then, at the production stage, a unique semi-separating equilibrium of the form (9) exists.*

To understand the uniqueness of the pooling interval, consider the equilibrium interval $[\underline{s}^*, \overline{s}^*]$. Suppose that the interval is shifted slightly to the left, i.e,. $[\underline{s}^* - \varepsilon, \overline{s}^* - \varepsilon]$ for some $\varepsilon > 0$. This affects the production equilibrium through the posterior belief (10). In particular, in the uniform case, the probability that the signal is in intervals of the same lengths is the same, so $\mathbb{E}[a \mid q_{1,uninfo}]$ decreases.[27] Moreover, the pooling quantity, $q_{1,uninfo}$, is determined as a function of the posterior expectation $\mathbb{E}[a \mid q_{1,uninfo}]$ through the follower's reaction in the uninformed leader's optimal condition. Since the uninformed leader produces more as the follower's posterior belief given its production decreases, i.e., $q_{1,uninfo}$ increases as $\mathbb{E}[a \mid q_{1,uninfo}]$ decreases, $\pi_{1,pool}(s_1; \underline{s}^* - \varepsilon, \overline{s}^* - \varepsilon) > \pi_{1,pool}(s_1; \underline{s}^*, \overline{s}^*)$ for all s_1. But then, $\pi_{1,pool}(s_1; \underline{s}^* - \varepsilon, \overline{s}^* - \varepsilon) \geq \pi_{1,sep}(s_1)$ holds on a interval $[\underline{s}^\circ, \overline{s}^\circ] \supsetneq [\underline{s}^*, \overline{s}^*]$, as $\pi_{1,sep}(s_1)$ does not depend on the pooling interval. Hence, $[\underline{s}^* - \varepsilon, \overline{s}^* - \varepsilon]$ cannot be an equilibrium. By similar arguments, one can see that shifting, expanding, or shrinking the equilibrium interval $[\underline{s}^*, \overline{s}^*]$ cannot be supported as an equilibrium.

Semi-Separating Equilibria: Disclosure Equilibrium

To verify if a disclosure strategy is indeed equilibrium, one needs to check the uninformed leader's IC. Namely, the uninformed leader should be deterred from deviating to nondisclosure. Moreover, the informed leader's IC can also be a problem, because the informed leader's pooling profit depends not only on its type but also on other types in the pooling interval. In particular, if the pooling interval $[\underline{s}^*, \overline{s}^*]$ overlaps with the nondisclosure set, then types in the overlapping set prefers to deviate from the pooling level. The following lemma gives a partial characterization of the uninformed leader's IC.[28]

Lemma 6. *Suppose that $\mathbb{E}[a] \in [\underline{s}^*, \overline{s}^*]$. Then, the uninformed leader does not have an incentive to deviate to nondisclosure if and only if the informed type $s_1 = \mathbb{E}[a]$ prefers to disclose the production plan. Suppose that $\mathbb{E}[a] \notin [\underline{s}^*, \overline{s}^*]$. Then, it is necessary for the IC to hold that the informed type $s_1 = \mathbb{E}[a]$ prefers to disclose the production plan.*

Since the uninformed leader does not receive any signal, the evaluation of the demand is $\mathbb{E}[a]$. Thus, the uninformed can be regarded as the informed type $s_1 = \mathbb{E}[a]$. If $\mathbb{E}[a]$ is in

[27] When the density of a is not constant, then the change in $\mathbb{E}[a \mid q_{1,uninfo}]$ depends on the shape of the distribution, but it does change except for knife-edge cases.

[28] This characterization is only partial, because the disclosure incentive of type $s_1 = \mathbb{E}[a]$ gives only a necessary condition for the IC when $\mathbb{E}[a] \notin [\underline{s}^*, \overline{s}^*]$.

the pooling interval, then type $\mathbb{E}[a]$ produces the pooling level, so the informed type $\mathbb{E}[a]$ has exactly the same incentive as the uninformed. Thus, Lemma 6 states that the uninformed's IC is satisfied if and only if the informed leader does not deviate to nondisclosure. However, if $\mathbb{E}[a]$ is in the pooling interval, then the informed type $\mathbb{E}[a]$ produces a separating level, so the uninformed and the type-$\mathbb{E}[a]$ informed have different incentives. Thus, Lemma 6 gives only a partial characterization of the IC.

Next, I characterize all disclosure equilibria. Define the disclosure profit by $\pi_1^D := \max\{\pi_{1,pool}, \pi_{1,sep}\}$ and nondisclosure profit by π_1^N. To identify disclosure equilibrium, I need to compare the behavior of π_1^D and π_1^N with respect to the signal realizations. Although it is no longer feasible to write down the rate at which the expected profits increase with respect to the signal, I can characterize disclosure equilibrium using available information about expected profits. Specifically, $\pi_{1,sep}$ and π_1^N are both quadratic in s_1, and from Proposition 1, they intersect at most once. Moreover, $\pi_{1,pool}$ is linear in s_1. Using these properties together with the uninformed and informed leader's IC described above, I obtain the following result.[29]

Proposition 4. *A disclosure equilibrium, if it exists, takes the lower-tail disclosure form. That is, the informed leader discloses the production plan if and only if $s_1 \le \tau$ for some $\tau \in [2\underline{a}, 2\overline{a})$.*

Since the nondisclosure profit is quadratic and a part of the disclosure profit is linear, it may be surprising that only lower tail disclosure is possible. To understand the result, note that all the pooling types should disclose the production plan. Thus, the nondisclosure profit and the disclosure profit cannot intersect at the interior of the pooling interval. The separating part of the disclosure profit is the same as in the model without the Dye friction, so the disclosure equilibrium takes the same form as the model with perfect separation. Note that, unlike the model without the Dye friction, an equilibrium may not exist for some parameter values, because of the additional incentive constraints for the pooling types.

Compared to the model without the Dye friction, the disclosure profit is $\pi_1^D = \max\{\pi_{1,pool}, \pi_{1,sep}\}$ instead of $\pi_{1,sep}$. Since $\max\{\pi_{1,pool}, \pi_{1,sep}\} \ge \pi_{1,sep}$, for the informed leader, the disclosure is more profitable. Moreover, since the uninformed type discloses the production plan, the follower's posterior belief upon nondisclosure does not depend on p. Hence, the leader discloses more compare to the no-friction case ($p = 1$). This observation is summarized in the following proposition, which is the main result of the extended model.

[29] Assigning sufficiently bad off-path beliefs deters off-path deviations.

Proposition 5. *Suppose $p \in (0,1)$ and that lower-tail disclosure equilibrium exists. Then, the disclosure region weakly expands compared to the baseline model, which corresponds to $p = 1$.*

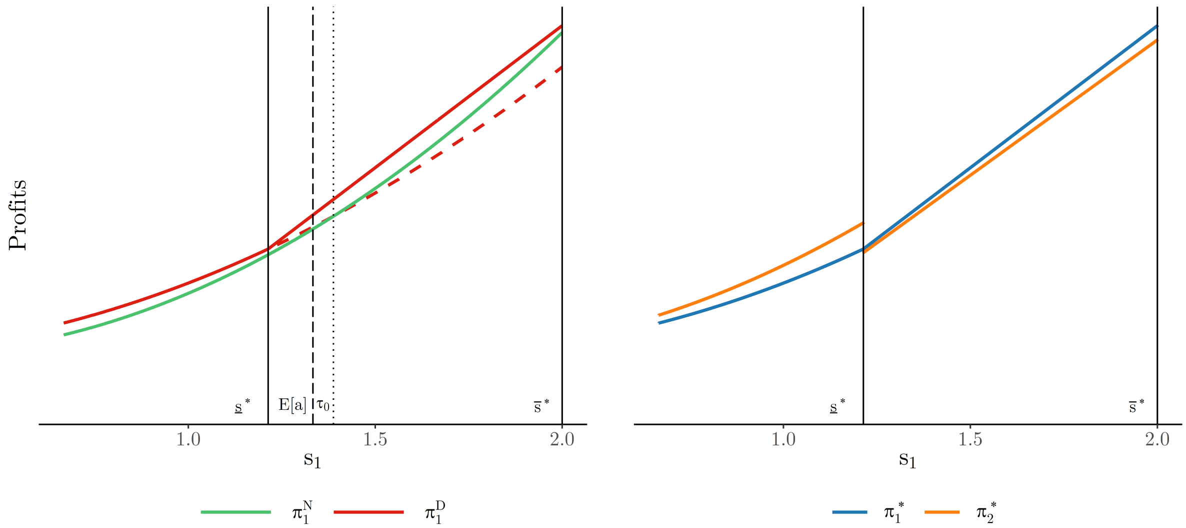
Figure 8 illustrates this proposition. The left panel shows the leader's indifference condition, assuming that the disclosure equilibrium is a lower-tail disclosure, i.e., $\{d(s_1) = 1\} = \{s_1 \le \tau\}$. It plots the leader's disclosure profit $\pi_1^D(s_1)$ and nondisclosure profit $\pi_1^N(s_1, \tau = s_1)$. The equilibrium pooling interval is $[\underline{s}^*, \overline{s}^*] = [1.21, 2\overline{a}]$. The leader's disclosure profit bifurcates at $\underline{s}^*$, and the dotted part of π_1^D shows the disclosure profit given that there is no Dye friction (i.e., full separation). Since the dotted part of π_1^D and π_1^N intersects at $\tau_0 \in (2\underline{a}, 2\overline{a})$, this is the disclosure equilibrium in the model without Dye friction. On the other hand, the solid part of π_1^D is always greater than π_1^N, indicating that with the information friction the leader always prefers to disclose the production plan. This is in contrast with the model without the information friction, where full disclosure is impossible. Since all pooling types disclose the production plan, the informed leader's IC is satisfied. Since the informed type $s_1 = \mathbb{E}[a]$ is in the pooling interval and prefers to disclose the production plan, by Lemma 6, the uninformed leader's IC is satisfied.

The right panel shows the equilibrium profits given that equilibrium disclosure threshold $\tau^* = 2\overline{a}$. Given that the production plan is disclosed and is a separation level, the equilibrium is the same as the mandatory disclosure model, so the follower earns profit higher than the leader (first-mover disadvantage). However, if the disclosed production plan is the pooling level, then the leader earns profits higher than the follower. This is because the leader can exploit the first-mover advantage and save the signaling cost at the same time by disclosing the production level but being pooled with the uninformed.

6.3 Discussion

I have identified two types of equilibria, depending on the unformed leader's disclosure strategy. Which equilibrium would likely prevail? One way to select an equilibrium is to apply a refinement. In equilibrium where the uninformed discloses the quantity, given disclosure, the quantity is pooled for some signal types. Therefore, there is a discontinuous jump in the quantity when the signal moves from the separating types to pooling types. One can show that off-path beliefs that deter a deviation does not survives the D1 criterion. Thus, from the equilibrium refinement perspective, the equilibrium where the uninformed withholds quantity should prevail.

Figure 8: Full Disclosure Equilibrium



Note: The left panel shows the indifference condition of the informed leader. That is, it plots the leader's disclosure profit $\pi_1^D(s_1)$ and the leader's nondisclosure profit given that the threshold is s_1, $\pi_1^N(s_1, \tau = s_1)$, assuming that the disclosure set takes the form $\{s_1 \leq \tau\}$. The right panel shows the equilibrium profits of the leader and the follower. In both panels, the solid vertical line indicates the boundaries of the pooling interval: $[\underline{s}^*, \overline{s}^*] = [1.21, 2]$. In the left panel, the dotted vertical line indicates the disclosure threshold if there is no Dye friction ($p = 1$). The dashed line indicates $\mathbb{E}[a]$. The parameters are set as $a_L = 1/3, a_H = 1, t = 0.5, p = 0.3$.

Another intuitive equilibrium selection is to compute the uninformed leader's expected profit. If the uninformed earns on average higher expected profit in one type of equilibrium than the other, one could expect that the uninformed prefers to follow the disclosure strategy in a higher-profit equilibrium. Unfortunately, analytically comparing the expected profit of the uninformed leader is not feasible, as I do not have an explicit form for the pooling interval. Numerical analysis suggests that the uninformed earns higher expected profit in equilibrium where the uninformed discloses quantity. This is intuitive, because the uninformed can enjoy the first-mover advantage without much of the signaling cost due to the pooling. Therefore, when both types of equilibrium exist, the uninformed prefers the equilibrium where it discloses the production plan. However, one could argue that the semi-separating equilibrium seems more "complicated" than the other equilibrium, because it involves a pooling interval, which is determined as a fixed point. From this viewpoint, the simple equilibrium where the uninformed conceals the production is selected.

7 Conclusion

In this paper, I examine the role of voluntary disclosure in a leader-follower game. In an oligopolistic market, if a better-informed leader firm is forced to disclose its production plan, then the leader is worse off than the follower due to signaling costs. When the leader has the option to withhold the production plan, the leader does so when the private demand information is sufficiently bad. As the market becomes more competitive, the leader firm discloses less often. When the leader's disclosure decision is based both on stock price and cash flow, an interval disclosure can emerge. When the leader may not always receive private information, the leader can mimic the uninformed type to avoid the signaling cost.

My study provides a theory of endogenous disclosure cost. The comparative statics I obtain for the main model are largely consistent with the extant empirical findings. Moreover, my analysis provides some new empirical predictions. For example, the leader firm's disclosure behavior changes depending on the leader firm's disclosure objective. The analysis of the Dye friction reveals that when the leader firm does not always possess information, empirical predictions are nontrivial.

References

Ackert, Lucy F., Bryan K. Church, and Mandira Roy Sankar. 2000. "Voluntary Disclosure under Imperfect Competition: Experimental Evidence." *International Journal of Industrial Organization*, 18(1): 81–105.

Banks, Jeffrey S., and Joel Sobel. 1987. "Equilibrium Selection in Signaling Games." *Econometrica*, 647–661.

Ben-Porath, Elchanan, Eddie Dekel, and Barton L. Lipman. 2018. "Disclosure and Choice." *The Review of Economic Studies*, 85(3): 1471–1501.

Beyer, Anne, and Ilan Guttman. 2012. "Voluntary Disclosure, Manipulation, and Real Effects: VOLUNTARY DISCLOSURE, MANIPULATION, AND REAL EFFECTS." *Journal of Accounting Research*, 50(5): 1141–1177.

Bloomfield, Matthew J., and Marcel Tuijn. 2019. "Do Firms Strategically Announce Capacity Expansions in Response to Heightened Entry Threats?"

Caruana, Guillermo, and Liran Einav. 2008. "Production Targets." *The RAND Journal of Economics*, 39(4): 990–1017.

Chen, Hui, and Robert F. Göx. 2022. "Voluntary Disclosure in Leader-Follower Games." *Available at SSRN 4268220*.

Cheynel, Edwige, and Amir Ziv. 2021. "On Market Concentration and Disclosure." *Journal of Financial Reporting*, 6(2): 1–18.

Cho, In-Koo, and David M. Kreps. 1987. "Signaling Games and Stable Equilibria." *The Quarterly Journal of Economics*, 102(2): 179–221.

Choné, Philippe, and Laurent Linnemer. 2020. "Linear Demand Systems for Differentiated Goods: Overview and User's Guide." *International Journal of Industrial Organization*, 73: 102663.

Clinch, Greg, and Robert E. Verrecchia. 1997. "Competitive Disadvantage and Discretionary Disclosure in Industries." *Australian Journal of Management*, 22(2): 125–137.

Corona, Carlos, and Lin Nan. 2013. "Preannouncing Competitive Decisions in Oligopoly Markets." *Journal of Accounting and Economics*, 56(1): 73–90.

Cumbul, Eray. 2021. "Stackelberg versus Cournot Oligopoly with Private Information." *International Journal of Industrial Organization*, 74: 102674.

Darrough, Masako N. 1993. "Disclosure Policy and Competition: Cournot vs. Bertrand." *The Accounting Review*, 534–561.

Darrough, Masako N., and Neal M. Stoughton. 1990. "Financial Disclosure Policy in an Entry Game." *Journal of Accounting and Economics*, 12(1-3): 219–243.

Dechow, Patricia, Weili Ge, and Catherine Schrand. 2010. "Understanding Earnings Quality: A Review of the Proxies, Their Determinants and Their Consequences." *Journal of Accounting and Economics*, 50(2-3): 344–401.

Doyle, Maura P., and Christopher M. Snyder. 1999. "Information Sharing and Competition in the Motor Vehicle Industry." *Journal of Political Economy*, 107(6): 1326–1364.

Dye, Ronald A. 1985. "Disclosure of Nonproprietary Information." *Journal of Accounting Research*, 123–145.

Einhorn, Eti, and Amir Ziv. 2012. "Biased Voluntary Disclosure." *Review of Accounting Studies*, 17(2): 420–442.

Fudenberg, Drew, and Jean Tirole. 1991. "Perfect Bayesian Equilibrium and Sequential Equilibrium." *Journal of Economic Theory*, 53(2): 236–260.

Gal-Or, Esther. 1985. "Information Sharing in Oligopoly." *Econometrica*, 329–343.

Gal-Or, Esther. 1986. "Information Transmission—Cournot and Bertrand Equilibria." *The Review of Economic Studies*, 53(1): 85–92.

Gal-Or, Esther. 1987. "First Mover Disadvantages with Private Information." *The Review of Economic Studies*, 54(2): 279–292.

Glaeser, Stephen. 2018. "The Effects of Proprietary Information on Corporate Disclosure and Transparency: Evidence from Trade Secrets." *Journal of Accounting and Economics*, 66(1): 163–193.

Glaeser, Stephen, Jeremy Michels, and Robert E. Verrecchia. 2020. "Discretionary Disclosure and Manager Horizon: Evidence from Patenting." *Review of Accounting Studies*, 25(2): 597–635.

Grossman, Sanford J. 1981. "The Informational Role of Warranties and Private Disclosure about Product Quality." *The Journal of Law and Economics*, 24(3): 461–483.

Grossman, Sanford J., and Oliver D. Hart. 1980. "Disclosure Laws and Takeover Bids." *The Journal of Finance*, 35(2): 323–334.

Guttman, Ilan, and Iván Marinovic. 2018. "Debt Contracts in the Presence of Performance Manipulation." *Review of Accounting Studies*, 23: 1005–1041.

Guttman, Ilan, and Xiaojing Meng. 2021. "The Effect of Voluntary Disclosure on Investment Inefficiency." *The Accounting Review*, 96(1): 199–223.

Huang, Ying, Ross Jennings, and Yong Yu. 2017. "Product Market Competition and Managerial Disclosure of Earnings Forecasts: Evidence from Import Tariff Rate Reductions." *The Accounting Review*, 92(3): 185–207.

Jung, Woon-Oh, and Young K. Kwon. 1988. "Disclosure When the Market Is Unsure of Information Endowment of Managers." *Journal of Accounting research*, 146–153.

Kanodia, Chandra, Rajdeep Singh, and Andrew E. Spero. 2005. "Imprecision in Accounting Measurement: Can It Be Value Enhancing?" *Journal of Accounting Research*, 43(3): 487–519.

Kothari, Sabino P., Susan Shu, and Peter D. Wysocki. 2009. "Do Managers Withhold Bad News?" *Journal of Accounting research*, 47(1): 241–276.

Li, Lode. 1985. "Cournot Oligopoly with Information Sharing." *The RAND Journal of Economics*, 521–536.

Li, Xi. 2010. "The Impacts of Product Market Competition on the Quantity and Quality of Voluntary Disclosures." *Review of Accounting Studies*, 15(3): 663–711.

Li, Yinghua, Yupeng Lin, and Liandong Zhang. 2018. "Trade Secrets Law and Corporate Disclosure: Causal Evidence on the Proprietary Cost Hypothesis." *Journal of Accounting Research*, 56(1): 265–308.

Mailath, George J. 1987. "Incentive Compatibility in Signaling Games with a Continuum of Types." *Econometrica*, 1349–1365.

Mailath, George J. 1993. "Endogenous Sequencing of Firm Decisions." *Journal of Economic Theory*, 59(1): 169–182.

Milgrom, Paul, and John Roberts. 1986. "Relying on the Information of Interested Parties." *The RAND Journal of Economics*, 18–32.

Milgrom, Paul, and John Roberts. 1992. *Economics, Organization and Management.* Prentice-Hall International.

Milgrom, Paul R. 1981. "Good News and Bad News: Representation Theorems and Applications." *The Bell Journal of Economics*, 380–391.

Myers, Stewart C., and Nicholas S. Majluf. 1984. "Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have." *Journal of financial economics*, 13(2): 187–221.

Nakamura, Tomoya. 2015. "One-Leader and Multiple-Follower Stackelberg Games with Private Information." *Economics Letters*, 127: 27–30.

Pae, Suil. 2002. "Optimal Disclosure Policy in Oligopoly Markets." *Journal of Accounting Research*, 40(3): 901–932.

Raith, Michael. 1996. "A General Model of Information Sharing in Oligopoly." *Journal of Economic Theory*, 71(1): 260–288.

Roychowdhury, Sugata. 2006. "Earnings Management through Real Activities Manipulation." *Journal of Accounting and Economics*, 42(3): 335–370.

Sankar, Mandira Roy. 1995. "Disclosure of Predecision Information in a Duopoly." *Contemporary Accounting Research*, 11(2): 829–859.

Shinkai, Tetsuya. 2000. "Second Mover Disadvantages in a Three-Player Stackelberg Game with Private Information." *Journal of Economic Theory*, 90(2): 293–304.

Skinner, Douglas J. 1994. "Why Firms Voluntarily Disclose Bad News." *Journal of accounting research*, 32(1): 38–60.

Stein, Jeremy C. 1989. "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior." *The Quarterly Journal of Economics*, 104(4): 655–669.

Verrecchia, Robert E. 1983. "Discretionary Disclosure." *Journal of Accounting and Economics*, 5: 179–194.

Verrecchia, Robert E. 1990. "Information Quality and Discretionary Disclosure." *Journal of Accounting and Economics*, 12(4): 365–380.

Verrecchia, Robert E., and Joseph Weber. 2006. "Redacted Disclosure." *Journal of Accounting Research*, 44(4): 791–814.

Viscusi, W. Kip. 1978. "A Note on" Lemons" Markets with Quality Certification." *The Bell Journal of Economics*, 277–279.

Wagenhofer, Alfred. 1990. "Voluntary Disclosure with a Strategic Opponent." *Journal of Accounting and Economics*, 12(4): 341–363.

WSJ. 2023. "Tesla Plans $3.6 Billion Factory Expansion in Nevada." *Wall Street Journal*.

Ziv, Amir. 1993. "Information Sharing in Oligopoly: The Truth-Telling Problem." *The RAND Journal of Economics*, 455–465.

Appendix

A Proofs

In this appendix, I use notation ND to denote the event of nondisclosure. When there is no Dye friction, $ND = \{d(s_1) = 0\}$. When there is the Dye friction, $ND = \{\text{the leader is uninformed}\} \cup \{\text{the leader is informed and } s_1 \in \{d(s_1) = 0\}\}$. When the disclosure strategy is a τ-threshold strategy, sometimes I use $ND(\tau)$ to emphasize the dependence of ND on τ.

Proof of Lemma 1

Note that $\frac{1}{2}\mathbb{E}[a \mid s_1] - \frac{t}{2(2+t)}\mathbb{E}[a \mid d(s_1) = 0] \geq \frac{1}{2}\mathbb{E}[a \mid 2\underline{a}] - \frac{t}{2(2+t)}\mathbb{E}[a \mid s_1 = 2\overline{a}]$. Given $\underline{a} \geq \overline{a}/3$, one can show that the right hand side is nonnegative for any t and ρ.

Proof of Proposition 1

(i) Since the leader's disclosure and nondisclosure profits are both squares of production quantity (with additional coefficients for π_1^D), to derive the disclosure strategy, it suffices to compare the term inside of the square. The square root of nondisclosure profit increases with respect to $\mathbb{E}[a \mid s_1]$ at the rate of 1/2, while the square root of disclosure profit increases with respect to $\mathbb{E}[a \mid s_1]$ at the rate of $(2\sqrt{1+t\psi})^{-1} < 1/2$ for all s_1. Hence, the unique disclosure strategy is lower-tail disclosure. Define

$$f(\tau; t, \rho) = \underbrace{\frac{1}{2\sqrt{1+t\psi}}\mathbb{E}[a \mid \tau]}_{\text{Disclosure}} - \underbrace{\left\{\frac{1}{2}\mathbb{E}[a \mid \tau] - \frac{t}{2(2+t)}\mathbb{E}[a \mid a > \tau]\right\}}_{\text{Nondisclosure}}.$$

Note that $f(\tau^*, t, \rho) = 0$ at some point $\tau^* \in (2\underline{a}, 2\overline{a})$ means partial disclosure, while $f(\tau, t, \rho) \geq (\leq) 0, \forall \tau$ means full (non)disclosure. The unique solution to $f(\tau^*; t, \rho) = 0$ is given by the expression. The solution is

$$\tau^* = \frac{\left(\frac{t}{2+t} - \zeta\right)(1-\rho)\mathbb{E}[a] + \frac{t}{2+t}\rho\overline{a}}{\left[\zeta - \frac{t}{2(2+t)}\right]\rho}, \qquad \zeta = 1 - \frac{1}{\sqrt{1+t\psi}}.$$

(ii) Note that

$$f(2\overline{a}; t, \rho) = \frac{(2\sqrt{2} + \sqrt{2}t - 2Z)(\mathbb{E}[a] + \rho(\overline{a} - \underline{a}))}{2(2+t)\sqrt{Z}}, \quad Z = 1 + t + \frac{1}{1-t}.$$

Note that $Z > 2$ and Z is increasing in t and that $2\sqrt{2}+\sqrt{2}t-2Z$ is monotonically decreasing in t and at $\lim_{t\to 0} 0$. Thus, $2\sqrt{2}+\sqrt{2}t-2Z<0$ for all $t\in(0,1)$. Since $\mathbb{E}[a]+\rho(\overline{a}-\underline{a})>0$ for all $\rho\in(0,1]$, it follows that$f(2\overline{a},t,\rho)<0$ for all t and ρ. That is, the highest type strictly prefers to conceal the production plan for any parameter values. By the continuity of $f(\cdot;t,\rho)$, there is a type $s_N < 2\underline{a}$, which depends on t and ρ, such that types in $(s_N, 2\overline{a}]$ conceal the production plan.

(iii) Without loss, I assume that the leader's equilibrium production schedule is right-continuous. Moreover, for this proof, I use the demand structure in Appendix 1, so that the equilibrium production schedule is strictly increasing everywhere on $\mathbb{R}_{++}$.The only relevant type of off-path deviation is such that a nondisclosure type $s_1 \in ND = \{s_1 \mid s_1 \geq \tau^*\}$ produces and discloses[30]

$$q_1^o \notin \{q_1(s_1) \mid s_1 < \tau^*\} \iff q_1^o \geq \lim_{s_1\to(\tau^*)^-} q_1(s_1)$$

I use superscript N to denote that the production level is concealed in equilibrium. Take any $q_1^o > q_1^N(\tau^*)$. Thus, there is a single type that corresponds to q_1^o. Let such type be $s_1' = \{s_1' \mid q_1^N(s_1') = q_1^o\}$. Define $D_{s_1}(q_1^o) = \{q_2^o \mid \pi_1^* < \pi_1(q_1^o, q_2^o, s_1)\}$ and $D_{s_1}(q_1^o) = \{q_2^o \mid \pi_1^* = \pi_1(q_1^o, q_2^o, s_1)\}$, where $\pi_1(q_1^o, q_2^o, s_1) = q_1^o(\mathbb{E}[a \mid s_1] - q_1^o - tq_2^o)$. Fixing the equilibrium nondisclosure quantity of the follower, the s_1-type has to incur a cost by adjusting to the non-optimal quantity $q_1^N(s_1')$:

$$\pi_1(q_1^N(s_1'), q_2^N, s_1') > \pi_1(q_1^N(s_1'), q_2^N, s_1).$$

Thus, for the deviation q_1^o, the follower's production should decrease by larger amount for type s_1 than for type s_1', i.e., $D_{s_1}(q_1^o) \subsetneq D_{s_1'}(q_1^o) \cup D^0_{s_1'}(q_1^o)$. Thus, by D1, upon observing q_1^o, the follower assigns probability one to type s_1'. Type s_1 chooses to be pooled with other nondisclosure types in equilibrium, so being identified as s_1 for sure is strictly dominated for $s_1 \neq \mathbb{E}[s_1 \mid s_1 \geq \tau^*]$ and indifferent for $s_1 = \mathbb{E}[s_1 \mid s_1 \geq \tau^*]$. By the similar argument, for a off-path deviation $q_1^o \in [\lim_{s_1\to(\tau^*)^-} q_1(s_1), q_1^N(\tau^*))$, the deviation is most likely from type τ^* in the D1 sense. Thus, types in $s_1 \in ND$ have no incentive to choose such production levels and disclose them.

[30]An off-path deviation to $q_1^o < q_1(2\underline{a})$ is clearly not profitable as it is dominated by on-path deviations.

Proof of Corollary 1

Using the inequality $(1+t)+1/(1-t)\geq 2$,

$$\begin{aligned}\frac{\partial}{\partial t}\left[\zeta-\frac{t}{2(2+t)}\right]&=-\frac{1}{(2-t)^2}+\frac{(2-(2-t)t)\sqrt{2(1+t+1/(1-t))}}{2(2-t^2)^2}\\&\geq-\frac{1}{(2-t)^2}+\frac{2-(2-t)t}{(2-t^2)^2}\\&=\frac{t^2(6+(2-t)t)}{(2+t)^2(2-t^2)^2}>0\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial}{\partial t}\left[\frac{t}{2+t}-\zeta\right]&=\frac{2}{(2+t)^2}-\frac{(2-(2-t)t)\sqrt{1+t+1/(1-t})}{\sqrt{2}(2-t^2)^2}\\&\leq\frac{2}{(2+t)^2}-\frac{(2-(2-t)t)}{(2-t^2)^2}\\&=-\frac{t^2(6+(2-t)t)}{(2+t)^2(2-t^2)^2}<0.\end{aligned}$$

Thus, τ^* is decreasing in t.

Proof of Corollary 2

Let $z=1+t+1/(1-t)\geq 2$.

$$\frac{\partial\tau^*}{\partial\rho}=\frac{2\mathbb{E}[a](2\sqrt{z}-2\sqrt{2}-\sqrt{2}t)}{(4(\sqrt{z}-\sqrt{2})+t(\sqrt{z}-2\sqrt{2}))\rho^2}>0,$$

where the inequality follows from the fact that both the denominator and numerator is positive.

Proof of Lemma 2

Given that both disclosure payoff and nondisclosure payoff are quadratic in signal, the disclosure equilibrium is either one of the following: lower-tail disclosure, upper tail disclosure, interval disclosure, and interval nondisclosure. The discussion of the main text can be easily formalized to show that upper-tail disclosure is impossible. Thus, it suffices to show that interval nondisclosure is impossible. The payoff upon disclosure, $U_1(d=1)=\pi_1^d$, increases with respect to $\mathbb{E}[a \mid s_1]$ at the rate $\frac{1}{2(1+t\psi)}\mathbb{E}[a \mid s_1]$, while payoff upon nondisclosure increases with respect to $\mathbb{E}[a \mid s_1]$ at the rate

$$(1-\alpha)\left[\frac{1}{2}\mathbb{E}[a \mid s_1] - \frac{t}{2(2+t)}\mathbb{E}[a \mid ND]\right].$$

When $\alpha = 0$, $U_1(d=0)$ increases at the rate $\frac{1}{2}\mathbb{E}[a \mid s_1] > \frac{1}{2(1+t\psi)}\mathbb{E}[a \mid s_1]$. In this case, from Proposition 1, $U_1(\alpha=0, d=0, s_1=2\overline{a}) > U_1(\alpha=0, d=1, s_1=2\overline{a})$, When $\alpha = 1$,[31] $U_1(d=0)$ is constant. In this case,

$$\begin{aligned}
&U_1(\alpha=1, d=0, s_1=2\underline{a}) - U_1(\alpha=1, d=1, s_1=2\underline{a}) \\
&\qquad = \left[\frac{1}{4}\mathrm{Var}(\mathbb{E}[a \mid s_1] \mid ND) + \frac{1}{(2+t)^2}\mathbb{E}[a \mid ND]^2\right] - \frac{1}{4(1+t\psi)}\mathbb{E}[a \mid s_1 = 2\underline{a}]^2 \\
&\qquad \geq \frac{1}{(2+t)^2}\mathbb{E}[a \mid 2\underline{a}]^2 - \frac{1}{4(1+t\psi)}\mathbb{E}[a \mid s_1 = 2\underline{a}]^2 \\
&\qquad = \frac{t^2(1+t)}{2(2+t)^2(2-t^2)} > 0,
\end{aligned}$$

so $U_1(\alpha=1, d=0, s_1=2\underline{a}) > U_1(\alpha=1, d=1, s_1=2\underline{a})$.

Suppose toward contradiction that interval nondisclosure equilibrium exists. First, suppose that $U_1(\alpha=0, d=0, s_1=2\underline{a}) < U_1(\alpha=0, d=1, s_1=2\underline{a})$ and $U_1(\alpha=1, d=0, s_1=2\overline{a}) < U_1(\alpha=1, d=1, s_1=2\overline{a})$. It is necessary for interval nondisclosure that$U_1(\alpha, d=0, s_1=2\underline{a}) \leq U_1(\alpha, d=1, s_1=2\underline{a})$. For the disclosure and nondisclosure payoffs to intersect twice, it is necessary that the slope of the disclosure payoff at $s_1 = 2\underline{a}$ ($s_1 = 2\overline{a}$) is smaller (larger) than the nondisclosure payoff:

$$\frac{1}{2(1+t\psi)}\mathbb{E}[a \mid 2\underline{a}] < (1-\alpha)\left[\frac{1}{2}\mathbb{E}[a \mid 2\underline{a}] - \frac{t}{2(2+t)}\mathbb{E}[a \mid ND]\right]$$

$$\frac{1}{2(1+t\psi)}\mathbb{E}[a \mid 2\overline{a}] > (1-\alpha)\left[\frac{1}{2}\mathbb{E}[a \mid 2\overline{a}] - \frac{t}{2(2+t)}\mathbb{E}[a \mid ND]\right]$$

which implies

$$\frac{1}{2}\left(\frac{1}{(1+t\psi)(1-\alpha)} - 1\right)\mathbb{E}[a \mid 2\overline{a}] < \frac{t}{2(2+t)}\mathbb{E}[a \mid ND] < \frac{1}{2}\left(\frac{1}{(1+t\psi)(1-\alpha)} - 1\right)\mathbb{E}[a \mid 2\underline{a}]$$

Furthermore, the slope of the disclosure payoff has to increase faster than the nondisclosure

[31] To be precise, $\alpha < 1$, so this should be understood as α is arbitrarily close to 1.

price:

$$\frac{1}{2(1+t\psi)} > \frac{1}{2}(1-\alpha).$$

This contradicts with the above inequality.

Second, suppose that $U_1(\alpha=0,d=0,s_1=2\underline{a}) < U_1(\alpha=0,d=1,s_1=2\underline{a})$ and $U_1(\alpha=1,d=0,s_1=2\overline{a}) > U_1(\alpha=1,d=1,s_1=2\overline{a})$. For interval nondisclosure, it is necessary that $U_1(\alpha,d=0,s_1=2\underline{a}) \leq U_1(\alpha,d=1,s_1=2\underline{a})$. But then, by the same argument as above, it can be shown that the slope condition is not satisfied. Third, suppose that $U_1(\alpha=0,d=0,s_1=2\underline{a}) > U_1(\alpha=0,d=1,s_1=2\underline{a})$ and $U_1(\alpha=1,d=0,s_1=2\overline{a}) < U_1(\alpha=1,d=1,s_1=2\overline{a})$. In this case, there is no α such that $U_1(\alpha,d=0,s_1=2\underline{a}) \leq U_1(\alpha,d=1,s_1=2\underline{a})$, so interval nondisclosure is impossible. Finally, suppose that $U_1(\alpha=0,d=0,s_1=2\underline{a}) > U_1(\alpha=0,d=1,s_1=2\underline{a})$ and $U_1(\alpha=1,d=0,s_1=2\overline{a}) > U_1(\alpha=1,d=1,s_1=2\overline{a})$. Then, it is clear that for any α, the nondisclosure payoff does not cross with the disclosure payoff.

Proof of Proposition 2

(i) Under the conjecture of a τ-threshold lower-tail disclosure, the indifference condition is given by $U_1(\alpha,d=1,s_1=\tau) = U_1(\alpha,d=1,s_1=\tau)$. For $\alpha=0$, there is a unique solution by Proposition 1. Since U_1 is continuous in α, for a small level of α, an indifference type still exists.

(ii) Suppose that the equilibrium is lower-tail disclosure ($\tau_1 = 2\underline{a}$). Then

$$U_1(\alpha,d=1,\tau=2\overline{a}) - U_1(\alpha,d=0,\tau=2\overline{a}) = -\frac{t^2(1+t)(\mathbb{E}[a]+\rho(\overline{a}-\underline{a}))}{2(2+t^2)(2-t^2)} < 0.$$

Thus, the lowest type strictly prefers to conceal the production plan. Suppose that the equilibrium is interval disclosure ($\tau_1 > 2\underline{a}, \tau_2 < 2\overline{a}$). By definition, full disclosure is impossible.

(iii) See the construction in Figure 6.

Proof of Proposition 3

It suffices to check the uninformed's IC. The off-path deviation is prevented by assigning the worst off-path belief for such deviation. Suppose that the uninformed deviates to produce and disclose quantity that some informed type s_1 would produce and disclose in equilibrium. This means that the uninformed leader produces $\hat{q}_1 = \frac{1}{2(1+t\psi)}s$, where $s \leq \tau^*$. The uninformed's expected profit by producing $\hat{q}_1$ is

$$\frac{1}{2(1+t\psi)}s\left[\mathbb{E}[a]-\frac{1}{2(1+t\psi)}s-t\frac{\psi}{2(1+t\psi)}s\right]=\frac{1}{2(1+t\psi)}s\left[\mathbb{E}[a]-\frac{1}{2}s\right].$$

Thus, the uninformed leader optimally chooses $s=\min\{\tau^*,\mathbb{E}[a]\}$. I make use of the following lemma.

Lemma A.1. *The posterior belief after nondisclosure, $\mathbb{E}[a\mid ND(\tau)]$, is increasing at $\tau=\tau^*$.*

Proof. Suppose on the contrary that $\mathbb{E}[a\mid ND(\tau)]$ is decreasing in τ at $\tau=\tau^*$ Let $\pi^D(s_1;t)=\left[\frac{1}{2\sqrt{1+t\psi}}\mathbb{E}[a\mid s_1]\right]^2$ and $\pi^N(s_1,\tau;t)=\left[\frac{1}{2}\mathbb{E}[a\mid s_1]-\frac{t}{2(2+t)}\mathbb{E}[a\mid ND(\tau)]\right]^2$. From the equilibrium condition, $\pi^D(\tau^*)=\pi^N(\tau^*,\tau^*)$. Since π^N is increasing in τ, for $\tau'<\tau^*$, I have

$$\pi^D(\tau^*)-\pi^N(\tau^*,\tau')>\pi^D(\tau^*)-\pi^N(\tau^*,\tau^*)=0.$$

Thus, it must be $\tau'>\tau^*$, a contradiction. □

Suppose that $\tau^*\le\mathbb{E}[a]$. Since the expected profit given nondisclosure increases at a higher rate than the expected profit given disclosure and they coincide at τ^*, I obtain

$$\begin{aligned}\left[\frac{1}{2}\mathbb{E}[a]-\frac{t}{2(2+t)}\mathbb{E}[a\mid ND(\tau^*)]\right]^2&\ge\left[\frac{1}{2}\mathbb{E}[a]-\frac{t}{2(2+t)}\mathbb{E}[a\mid ND(\mathbb{E}[a])]\right]^2\\&\ge\frac{1}{4(1+t\psi)}\mathbb{E}[a]^2\\&\ge\frac{1}{2(1+t\psi)}\tau^*\left[\mathbb{E}[a]-\frac{1}{2}\tau^*\right],\end{aligned}$$

where the last inequality is from $\tau^*\le\mathbb{E}[a]$. Since the first expression is the equilibrium profit of the uninformed, this shows that the uninformed does not have a deviation incentive. Suppose instead that $\mathbb{E}[a]<\tau^*$. For the same reason, I obtain

$$\left[\frac{1}{2}\mathbb{E}[a]-\frac{t}{2(2+t)}\mathbb{E}[a\mid ND(\tau^*)]\right]\le\left[\frac{1}{2}\mathbb{E}[a]-\frac{t}{2(2+t)}\mathbb{E}[a\mid ND(\mathbb{E}[a])]\right]^2<\frac{1}{4(1+t\psi)}\mathbb{E}[a]^2,$$

which shows that the deviation is profitable.

Hence, the equilibrium exists if and only if $\tau^*\le\mathbb{E}[a]$. Since τ^* is monotonically decreasing in t, $\lim_{t\to0}\tau^*>\mathbb{E}[a]$, and $\lim_{t\to1}\tau^*=2\underline{a}$, there exists a cutoff t^* such that $\tau^*\le\mathbb{E}[a]$ if and only if $t\ge t^*$.

Proof of Corollary 3

The derivative of the LHS of (8) with respect to p is

$$\frac{(\overline{a}-\underline{a})(2\overline{a}-s_1)(s_1-2\underline{a})t}{2(2\overline{a}-2\underline{a}(1-p)-ps_1)^2(2+t)}<0.$$

Since the RHS is constant with respect to p and the nondisclosure price increases in s_1 at a higher rate than the disclosure price, the solution to (8) is increasing in p.

Proof of Corollary 4

Note that the indifference conditions (4) and (8) are the same except for the nondisclosure belief $\mathbb{E}[a\,|\,ND]$. Since $\mathbb{E}[a\,|\,ND]$ does not depend on t in either case, Corollary (1) implies the desired result.

Proof of Lemma 3

Suppose toward a contradiction that there exists an equilibrium in which the informed leader plays linear fully separating equilibrium. Since the probability that the uninformed's production and the informed production coincides is zero, the follower knows if a disclosed quantity is from the informed or uninformed. Therefore, for the informed leader, the equilibrium is exactly the same as in the main model without the Dye friction. The uninformed leader optimally discloses the quantity, as disclosure induces the standard Stackelberg game. In particular, the uninformed leader earns $\frac{(2-t)^2}{8(2-t^2)}\mathbb{E}[a]^2$.

I argue that the type $s_1=\mathbb{E}[a]$ always have an incentive to deviate. Suppose that $\tau^*\leq\mathbb{E}[a]$. The informed leader's expected profit is $\frac{1}{4(1+t\psi)}\mathbb{E}[a]^2$. Since $\frac{(2-t)^2}{8(2-t^2)}>\frac{1}{4(1+t\psi)}$, the informed leader deviates. Suppose that $\mathbb{E}[a]\leq\tau^*$. The informed leader's expected profit is

$$\left[\frac{1}{2}\mathbb{E}[a]-\frac{t}{2(2+t)}\frac{2\overline{a}+\tau}{2}\right]^2\leq\left[\frac{1}{2}\mathbb{E}[a]-\frac{t}{2(2+t)}\mathbb{E}[a]\right]^2=\frac{1}{(2+t)^2}\mathbb{E}[a]^2<\frac{(2-t)^2}{8(2-t^2)}\mathbb{E}[a]^2,$$

so the deviation is again profitabl(6)e. This shows that there is an open interval of types containing $\mathbb{E}[a]$ such that those types deviate to the uninformed production. Hence, there is no linear fully separating equilibrium.

Proof of Lemma 4

It suffices to show that $\mathscr{S}_{pool}$ is a connected interval in $\mathscr{S}$. Suppose toward a contradiction that $\mathscr{S}_{pool}$ is not a connected interval. It is without loss to assume that $\mathscr{S}_{pool}$ consists of two disjoint intervals. I write $\mathscr{S}_{pool} = [\underline{s}_1, \overline{s}^1] \cup [\underline{s}_2, \overline{s}^2]$, where $\underline{a} \leq \underline{s}_1 < \overline{s}^1 < \underline{s}_2 < \overline{s}^2 \leq \overline{a}$. Both intervals cannot intersect with the nondisclosure set $\{s_1 \mid d(s_1) = 0\}$, because if the production is withheld the informed leader tailors production to the private signal.

If type s_1 produces $q_{1,uninfo}$, the expected profit is $q_{1,uninfo}(\mathbb{E}[a \mid s_1] - q_{1,uninfo} - tq_2^{D,pool})$. Note that this is linear in s_1. The expected profit by producing $q_{1,sep}$ is $\frac{1}{4(1+t\psi)}\mathbb{E}[a \mid s_1]^2$, which is convex in s_1. The informed leader's IC requires that, for all $s_1 \in \mathscr{S}_{pool}$,

$$q_{1,uninfo}(\mathbb{E}[a \mid s_1] - q_{1,uninfo} - tq_2^{D,pool}) \geq \frac{1}{4(1+t\psi)}\mathbb{E}[a \mid s_1]^2.$$

The range of s_1 that satisfies this inequality takes the form $[\underline{s}, \overline{s}]$, so this is a contradiction.

Proof of Lemma 5

First, I derive the production equilibrium for a fixed pooling interval. Let $Q_L(s_1) = A_0 + A_1 s_1$. The follower's production q_2^D is necessarily linear in q_1, so I let $Q_{F,sep}(q_1) = B_0 + B_1 q_1$ and $Q_{F,pool}(q_1) = C_0 + C_1 q_1$. Therefore, the production equilibrium is pinned down by $\{(A_0, A_1), (B_0, B_1), (C_0, C_1), q_{1,uninfo}, q_{1,info}^N, q_2^N\}$. Given that the leader chooses the separating level of production, the equilibrium is the same as in Section 4.1. Thus, $\{(A_0, A_1), (B_0, B_1)\}$ is already derived.[32]

Suppose instead that $q_1 = q_{1,uninfo}$. Then, the follower infers that the leader is either informed or uninformed and uses the Bayes rule to update belief. Thus,

$$\mathbb{E}[a \mid q_{1,uninfo}] = \frac{p\mathbb{P}(s_1 \in \mathscr{S}_{pool})}{p\mathbb{P}(s_1 \in \mathscr{S}_{pool}) + (1-p)}\mathbb{E}[a \mid s_1 \in \mathscr{S}_{pool}] + \frac{1-p}{p\mathbb{P}(s_1 \in \mathscr{S}_{pool}) + (1-p)}\mathbb{E}[a].$$

The follower's optimality condition gives

$$C_0 = \frac{1}{2}\mathbb{E}[a \mid q_{1,uninfo}], \qquad C_1 = -\frac{t}{2}.$$

Notice that $\mathbb{E}[a \mid q_{1,uninfo}]$ is determined by the pooling interval $[\underline{s}, \overline{s}]$. Suppose that q_1 is not disclosed. As in Section 4.1, $(q_{1,info}^N, q_2^N)$ is determined as a Nash equilibrium.

[32] In particular, $A_0 = B_0 = 0$, $A_1 = \frac{1}{2(1+t\psi)}$, and $B_1 = \psi$.

Finally, The uninformed leader solves

$$\max_{q_{1,uninfo}} q_{1,uninfo}(\mathbb{E}[a] - q_{1,uninfo} - tQ_{F,pool}(q_{1,uninfo})).$$

The FOC gives

$$q_{1,uninfo} = \frac{\mathbb{E}[a] - tC_0}{2(1+tC_1)} = \frac{\mathbb{E}[a] - \frac{t}{2}\mathbb{E}[a \mid q_{1,uninfo}],}{2(1-\frac{t^2}{2})}.$$

Hence, I have derived the production equilibrium $\{(A_0, A_1), (B_0, B_1), (C_0, C_1), q_{1,uninfo}, q^N_{1,info}, q^N_2\}$. Accordingly, I have obtained the profit functions $\pi_{1,pool}$ and $\pi_{1,sep}$. The leader's off-path deviation is deterred by assigning sufficiently bad belief upon such deviation.

Let $X = \{(\underline{s}, \overline{s}) \mid 2\underline{a} \le \underline{s} \le \overline{s} \le 2\overline{a}\} \subset \mathbb{R}^2$ be the upper-triangle in $\mathscr{S}^2$. As noted in the main text, $\{s_1 \mid \pi_{1,pool}(s_1) \ge \pi_{1,sep}(s_1)\}$ is an interval if it is not empty. Therefore, $\{s_1 \mid \pi_{1,pool}(s_1) \ge \pi_{1,sep}(s_1)\}$ can be identified as $(\underline{s}^\circ, \overline{s}^\circ) \in X \subset \mathbb{R}^2$, where $\{s_1 \mid \pi_{1,pool}(s_1) \ge \pi_{1,sep}(s_1)\} = [\underline{s}^\circ, \overline{s}^\circ]$. Define a map $S_p : X \to X$ by

$$S_p(\underline{s}, \overline{s}) = \begin{cases} (\underline{s}^\circ, \overline{s}^\circ) & \{s_1 \mid \pi_{1,pool}(s_1) \ge \pi_{1,sep}(s_1)\} \neq \varnothing \\ (v, v) & \text{otherwise}, \end{cases}$$

where v is the y-coordinate of the vertex of the parabola $\pi_{1,pool}(s_1) - \pi_{1,sep}(s_1)$. When the informed leader strictly prefers to be pooled with the uninformed, the map S_p returns (v, v), which means the informed leader is indifferent between pooling and separation. This seemingly inconsistent property of S_p can be reconciled by noting that $\{v\} \in \mathscr{S}$ is measure zero, so the strategy at $\{v\}$ does not matter.[33] Since $\pi_{1,pool}(s_1) - \pi_{1,sep}(s_1)$ is continuous in $(\underline{s}, \overline{s})$, the map S_p is also continuous. Since X is nonempty, compact, and convex, by Brouwer's fixed point theorem, there is a fixed point $(\underline{s}^*, \overline{s}^*)$ such that $S_p(\underline{s}^*, \overline{s}^*) = (\underline{s}^*, \overline{s}^*)$.

Proof of Lemma 6

Suppose that the uninformed leader deviates to nondisclosure. Since the follower's posterior upon nondisclosure is

$$\mathbb{E}[a \mid ND] = \mathbb{E}[a \mid d(s_1) = 1],$$

[33] Another way to reconcile is to assume that if the leader chooses a pooling level if indifferent.

the optimal deviation is $q_1' = \frac{1}{2}\mathbb{E}[a] - \frac{t}{2(2+t)}\mathbb{E}[a \mid d(s_1)=0] = q^N_{1,info}(\mathbb{E}[a])$. Thus, the uninformed leader's IC is given by

$$q_{1,uninfo}(\mathbb{E}[a] - q_{1,uninfo} - tq_2^{D,pool}) \geq q^N_{1,info}(\mathbb{E}[a])(\mathbb{E}[a] - q^N_{1,info}(\mathbb{E}[a]) - q_2^N).$$

This is equivalent to the condition that the expected profit of the informed type $s_1 = \mathbb{E}[a]$ when it chooses a pooling level is equal to or greater than the expected profit when it chooses nondisclosure: $\pi_{1,pool}(\mathbb{E}[a]) \geq \pi_1^N(\mathbb{E}[a])$, where π_1^N is the leader's nondisclosure profit. Let $\pi_1^D(s_1) = \max\{\pi_{1,sep}(s_1), \pi_{1,pool}(s_1)\}$ be the disclosure profit of the informed leader.

Suppose that $\mathbb{E}[a] \in [\underline{s}^*, \overline{s}^*]$. Observe that

$$s_1 = \mathbb{E}[a] \text{ discloses } q_{1,info} \iff \pi_1^D(\mathbb{E}[a]) = \pi_{1,pool}(\mathbb{E}[a]) \geq \pi_1^N(\mathbb{E}[a]),$$

$$s_1 = \mathbb{E}[a] \text{ withholds } q_{1,info} \iff \pi_1^N(\mathbb{E}[a]) > \pi_1^D(\mathbb{E}[a]) = \pi_{1,pool}(\mathbb{E}[a]).$$

Hence, the uninformed leader's IC is characterized by the disclosure incentive of the informed leader with type $s_1 = \mathbb{E}[a]$.

Suppose instead that $\mathbb{E}[a] \notin [\underline{s}^*, \overline{s}^*]$. If $s_1 = \mathbb{E}[a]$ withholds, then

$$s_1 = \mathbb{E}[a] \text{ withholds } q_{1,info} \iff \pi_1^N(\mathbb{E}[a]) > \pi_1^D(\mathbb{E}[a]) = \pi_{1,sep}(\mathbb{E}[a]) \geq \pi_{1,pool}(\mathbb{E}[a]),$$

so the IC does not hold. Therefore, it is necessary that the informed type $s_1 = \mathbb{E}[a]$ does discloses the production plan.

Proof of Proposition 4

To prove Proposition 4, I use the following lemma.

Lemma A.2. *Let $[\underline{s}^*, \overline{s}^*]$ be the equilibrium pooling interval. Then, in any equilibrium, all types in $(\underline{s}^*, \overline{s}^*)$ discloses the production plan, i.e., $(\underline{s}^*, \overline{s}^*) \subset \{d(s_1) = 1\}$.*

Proof. Suppose toward a contradiction there is a type $s_0 \in (\underline{s}^*, \overline{s}^*)$ that withholds the production plan in equilibrium. This type is supposed to produce the pooling level and conceals the production plan. However, since the production is not disclosed, the follower produces q_2^N. The best response to this follower's production level is the nondisclosure level of production, $q^N_{1,info}$. The deviation to $q^N_{1,info}$ is not observable to the follower, so the belief is not affected. Thus, type $s_1 = s_0 + \varepsilon$ deviates from $q_{1,uninfo}$ to $q^N_{1,info}$. □

Using Lemma A.2, I prove that the disclosure equilibrium is a lower-tail disclosure.

Case 1. $\pi_{1,sep}(2\overline{a}) < \pi_1^N(2\overline{a}) \le \pi_{1,pool}(2\overline{a})$. Then, by Lemma A.2, only upper-tail disclosure is possible. However, given that $\{d(s_1) = 1\} = \{s_1 \ge \tau\}$, one can show that $\pi_{1,pool}(2\overline{a}) < \pi_1^N(2\overline{a})$, a contradiction.

Case 2. $\pi_{1,sep}(2\overline{a}) \le \pi_{1,pool} < \pi_1^N(2\overline{a})$. Then, by Lemma A.2, only possible case is $\pi_{1,sep}$ and π_1^N intersect at a point in $[\underline{s}^*, \overline{s}^*]$. By the assumption, $\pi_1^D > \pi_1^N$ for all s_1, i.e., full disclosure is the only possibility. This can be expressed as a lower-tail disclosure, where the threshold is $2\overline{a}$.

Case 3. $\pi_{1,pool}(2\overline{a}) < \pi_1^N(2\overline{a}) \le \pi_{1,sep}(2\overline{a})$. Then, by Lemma A.2, only upper-tail disclosure is possible, where $\pi_{1,sep}$ and π_1^N intersect at a point greater than $\overline{s}^*$. But then, as in Case 1, this is a contradiction.

Case 4. $\pi_{1,pool}(2\overline{a}) \le \pi_{1,sep}(2\overline{a}) < \pi_1^N(2\overline{a})$. Then, by Lemma A.2, only possible case is $\pi_{1,sep}$ and π_1^N intersect at a point a greater than $\overline{s}^*$. Therefore, it is a lower-tail disclosure, and the disclosure is always partial.

B Analysis of Mandatory Disclosure Benchmark

The analysis of the benchmark case where the leader is forced to disclose production quantity is essentially the same as Gal-Or (1987). However, my setting is slight different from hers in terms of the demand system and distributional assumptions. Therefore, for completeness, I detail the derivation of the equilibrium in this section.

First, a linear fully separating equilibrium continues to exist under mild assumptions.

Lemma B.1. *In a fully separating equilibrium, Q_L is strictly increasing. If Q_L is continuous and differentiable, then there is a linear equilibrium, where both Q_L and Q_F are linear.*

Proof. That Q_L is strictly increasing can be shown in a similar manner as in Gal-Or (1987, Lemma 2). The leader's incentive compatibility constraint implies the following first-order differential equation:

$$Q_L'(s_1) = \frac{t}{2} \frac{Q_L(s_1)}{\left(1 - \frac{t}{2}\right)\mathbb{E}[a \mid s_1] - \left(2 - t^2\right) Q_L(s_1)}.$$

It is straightforward to verify that a linear function Q_L can satisfy this differential equation. If Q_L is linear, then by the first-order condition of the follower's production, Q_F is linear as well. Let the off-path belief of the follower be such that the leader is as if following the linear strategy for all $s_1 \in \mathbb{R}_{++}$. Then, the structure of the game is exactly the same as Gal-Or (1987), and the linear strategy is indeed an equilibrium. $\square$

Remark 1. Unlike standard signaling models with an exogenous smooth cost function (e.g. Mailath, 1987), the signaling function Q_L is not necessarily continuous and differentiable. To see this, note that, if Q_L is discontinuous, then the leader's production discontinuously changes around some signal realizations. But, this discontinuous change in the leader's production induces a discontinuous change in the follower's production. Since the cost of signaling is determined by the follower's production, the leader with signal realization around the discontinuous point does not necessarily find it profitable to deviate. That is, since the signaling cost is endogenous, the signaling function can be discontinuous.[34] A similar argument applies to differentiability. It is the convention of the literature to assume linear perfectly separating equilibrium (Shinkai, 2000; Cumbul, 2021).

Remark 2. Gal-Or (1987) derives essentially the same differential equation and claims that the only solution is linear, but this is not necessarily true. By Picard-Lindelöf theorem, there are infinitely many solutions for each choice of initial conditions for the differential equation at a point in $(0,\infty)$.[35]

I solve by backward induction. The follower solves

$$\max_{q_2} \mathbb{E}[q_2 P_2(a, q_2, q_1) \mid q_1 = Q_L(s_1)].$$

The best response of the follower is obtained in the main text. Given the best response of the follower, the leader solves

$$\max_{q_1} \mathbb{E}[q_1 P_1(a, q_1, Q_F(q_1)) \mid s_1].$$

The leader's first order condition, together with the follower's optimality condition, gives a system of equations for the unknown coefficients (A, B):

$$\begin{cases} Q_F(q_1) = \frac{1}{2}(A^{-1} q_1 - t q_1), \\ Q_L(s_1) = \frac{1}{2(1+tB)} \mathbb{E}[a \mid s_1]. \end{cases}$$

[34] When the cost function is smooth, a local deviation around a point where a signaling function is discontinuous gives a discontinuous benefit at a second-order cost. See Mailath (1987).

[35] Note that the initial condition at zero does not pin down a solution, as the function that defines the ODE is not Lipshitz continuous at zero. The failure of Lipshitz continuity here is the reason why there are infinitely many solutions.

Solving this, I obtain the unknown coefficients:

$$A = \frac{1-t}{2-t^2}, \quad B = \frac{2-t}{2(1-t)}.$$

The equilibrium production can be written as

$$q_1^* = \frac{1}{2(1+t\psi)}\mathbb{E}[a \mid s_1], \quad q_2^* = \psi q_1^* = \frac{\psi}{2(1+t\psi)}\mathbb{E}[a \mid s_1], \tag{13}$$

where $\psi := B = \frac{2-t}{2(1-t)}$. The equilibrium profits are

$$(\pi_1^*, \pi_2^*) = \left(\frac{1}{4(1+t\psi)}\mathbb{E}[a \mid s_1]^2, \frac{(2-t)^2}{4(2-t^2)^2}\mathbb{E}[a \mid s_1]^2\right),$$

where the leader's profit can also be written as

$$\pi_1^* = (1+t\psi)^2(q_1^*)^2.$$

The parameter ψ is the multiplier on the leader's production by the follower. That is, the follower produces ψ times the leader's equilibrium production. The parameter ψ is determined by the competitiveness of the market, t. It has the property that $\lim_{t\to 0}\psi(t) = 1$, $\lim_{t\to 1}\psi(t) = \infty$, and monotonically increasing in t. In particular, since $\psi > 1$ for all $t \in (0,1)$, the follower produces *more* than the leader for any signal realization. As a consequence, the leader earns *less* profit compared to the follower, i.e., $\pi_1^*(s_1) \le \pi_2^*(s_2)$ for all s_1 and $t \in (0,1)$. This is the first-mover *disadvantage*. Since the quantity q_1 reveals the private demand information and the leader wants the follower to think that the demand is low in the hope of the follower produces less, the leader has an incentive to contract production. However, in equilibrium, the private information is perfectly recovered by the follower, so the leader is trapped into paying the cost of signaling only to reveal the private information.

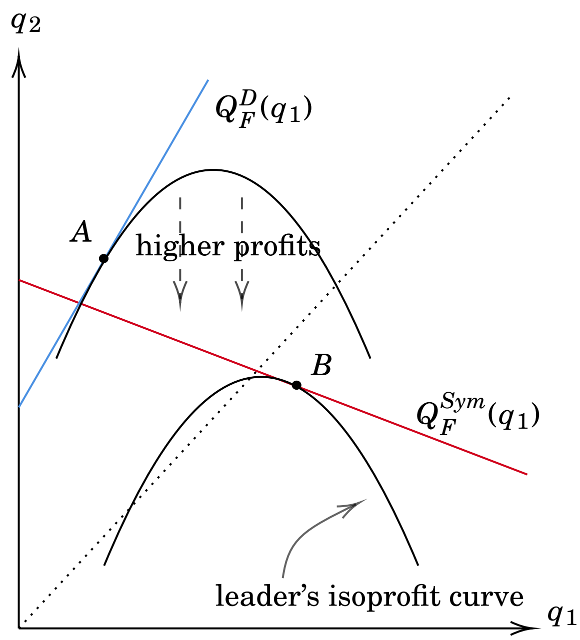
As t becomes large, the leader produces less in equilibrium, i.e., the coefficient $\frac{1}{2(1+t\psi)}$ in the leader's production function decreases in t. This means the leader's production reacts less to the signal, i.e., $\frac{\partial q_1^*}{\partial s_1}$ is decreasing in t. Since the follower wants to react to a unit change in s_1, the follower's response coefficient, ψ, becomes large as t becomes large. If t tends to one, the leader's production decreases to zero in the limit ($\lim_{t\to 1}\psi(t) = \infty$). The leader finds the signaling cost too high and stops producing. At $t = 1$, the follower becomes the monopolist and produces the monopoly quantity $\frac{1}{2}\mathbb{E}[a \mid s_1]$. However, this is a contradiction, as if $q_1^*(s_1) \equiv 0$, then the leader's production cannot reveal s_1, and the follower cannot tailor its production to s_1. This is why I exclude $t = 1$. In other words, the equilibrium is not (left-)continuous with respect to t at $t = 1$. Note that for all parameter values $t \in (0,1)$, both the leader's and the follower's production is less than the monopoly production. See Figure 1 for the coefficients on $\mathbb{E}[\alpha \mid s_1]$ in the production function of each firm.

Notice that the follower's response coefficient, ψ, does not depend on ρ. When ρ tends to zero, the leader reacts less to the signal, but what matters is the leader reacts the same way for a given posterior expectation $\mathbb{E}[\alpha \mid s_1]$. Thus, for a given production level q_1, regardless of the precision of the signal, as long as the signal is not a pure noise, the follower infers that the posterior expectation is $A^{-1}q_1$. Hence, the leader's signaling incentive is constant with respect to the precision of the signal. When ρ is zero, then the leader's action does not carry a signaling value, so the equilibrium above, which is derived by assuming that the leader reacts to the signal (i.e., $A > 0$), is no longer valid. In other words, the equilibrium above is not (right-)continuous with respect to ρ at $\rho = 0$. These somewhat counter-intuitive feature of the model is shared in the literature following Gal-Or (1987). As a practical matter, one can assume that ρ is high enough and the support is large enough.

The leader always produces and earns less than the follower, because the leader's quantity is the only information about the demand for the follower. As in Gal-Or (1987), suppose that the follower also observes a noisy signal about the demand. If the follower's signal is sufficiently correlated with the leader's signal, then the leader is limited in its ability to signal that the demand is low. If the follower observes a high signal, then the follower updates belief less when the leader decreases its production. In my model, the follower does not observe any signal, so the follower's belief is highly sensitive to the leader's decision.

To gain further insight into why the leader earns less than the follower for any signal realizations, consider the best response function of the follower. The leader can choose any point on the follower's best response curve by choosing appropriate production (and disclosure of it). In the symmetric information case, where the signal is observed by both firms, the follower's best response function's slope is negative ($-t/2$). In Figure 9, Q_F^{Sym} shows this best response function for some fixed realization of signal s_1. If this were the follower's BR function, the point B is the equilibrium, and the leader produces and earns more than the follower. On the other hand, in the asymmetric information case, from (2), the follower's best response function's slope is $\psi > 1$. This means that, if the leader increases a unit of production, then the follower also increases its production, and it does so at the rate higher than one. In the figure, the follower's BR function is shown as Q_F^D. Therefore, at any point of the follower's best response function, the leader's production is less than the follower's production. In the figure, the point A is the signaling equilibrium. Since the slope of the best response functions does not depend on the signal realizations, it follows that the leader is always worse off than the follower.[36]

Figure 9: Best Response Functions of the Follower



Note: The graph shows the best response function of the leader and the follower. The dotted line is the 45-degree line. The parabola shows the iso-profit curve of the leader.

C On Demand Distribution

C.1 Requirement for the Demand Distribution

What kind of requirements need to be imposed on the distribution of the demand? Instead of assuming that the demand a follows the uniform distribution, suppose that it has the distribution F. I continue to assume that the signal is given by the "truth-noise" structure. To avoid a negative demand intercept, F has to have support on a positive part of the real line. The truth-noise signal structure ensures that $\mathbb{E}[a \mid s_1]$ is linear in s_1, so, unlike Gal-Or (1987), I do not need to restrict F to make the posterior expectation linear in signal realizations. However, the truncated expectations, $\mathbb{E}[a \mid s_1 > \tau]$ and $\mathbb{E}[a \mid s_1 < \tau]$, are not

[36]To illustrate that the leader cannot achieve an outcome that is close to the Stackelberg outcome, assume that the leader produces as if it is a perfect information game, i.e., $q_1 = A^{\text{Stck}}\mathbb{E}[a \mid q_1]$, where $A^{\text{Stck}} = \frac{2-t}{2(2-t^2)}$. Then, one can show that $A^{-1} - t > 1$ for any $t \in (0,1)$. That is, the leader has an incentive to deviate to a production level that is *smaller* than the follower.

necessarily linear in a threshold τ for any F. Since these truncated expectations naturally arise in a disclosure game, it is desirable, though not necessary, that they are linear in thresholds. Finally, to ensure the existence of linear fully separating equilibrium, it must be the case that either F has full support on $\mathbb{R}_{++}$ or an appropriate specification of the off-path beliefs is necessary.

Unfortunately, if F has full support on $\mathbb{R}_{++}$, then it can be shown that the lower-truncated expectation $\mathbb{E}[a \mid s_1 < \tau]$ is not linear in τ for *any* distribution F. In contrast, there are distributions such that the upper-truncated expectation is linear in thresholds. One such example is an exponential distribution. This case is analyzed in Appendix D. If the support of F can be a subset of $\mathbb{R}_{++}$, then a uniform distribution has the property that both upper- and lower-truncated expectations are linear in thresholds. In the main text, In the main text, I employed weak Perfect Bayesian Equilibrium, which does not impose anything on off-path beliefs. In the next section, I explain that, by introducing a payoff-irrelevant state variable, the "weak" qualification from the PBE can be dropped.

C.2 Payoff-Irrelevant State

I introduce an auxiliary payoff-irrelevant state variable, which governs the demand distribution. Specifically, let $\omega \sim \mathscr{U}[0,1]$ be the auxiliary state variable. After ω realizes, the distribution of a is chosen as follows. If $\omega \in \mathbb{R} \cap [0,1] =: U$, then it is the uniform distribution $\mathscr{U}[2\underline{a}, 2\overline{a}]$. If $\omega \in \mathbb{Q} \cap [0,1] =: N$, then it is the gamma distribution $\Gamma(\underline{a}+\overline{a}, 1)$.[37] Note that $\mathbb{P}(\omega \in U) = 1$ and $\mathbb{P}(\omega \in N) = 0$. Thus, the support of a is $\mathbb{R}_{++}$, while the essential support of a is $[2\underline{a}, 2\overline{a}]$.[38] The signal is again the truth-noise structure. Hence, the timeline of the model is now as follows.

1. The auxiliary state variable ω realizes.

2. The distribution of a is chosen according to ω.

3. The demand intercept a realizes.

4. The leader privately observes the signal s_1, which is equal to a w.p. ρ and it is a pure noise with mean $\underline{a}+\overline{a}$.

[37] Any distribution with positive support does the job. Under the gamma distribution, all positive real numbers are supported. Whether the density has a support on $(0, 2\underline{a})$ is irrelevant, but one can shift $\Gamma(\underline{a}+\overline{a}, 1)$ by $2\underline{a}$ so that a has support on $[2\underline{a}, \infty)$.

[38] Recall that the essential support of a distribution is the smallest closed set such that the complement of the set has zero probability.

The rest of the timeline is the same as the main text.

The probability that $a \sim \mathscr{U}[2\underline{a}, 2\overline{a}]$ is one, so essentially the game is the one with bounded support. However, the leader forms a strategy for all $s_1 \in (0,\infty)$, as there is a state ω in which $s_1 > 2\overline{a}$ is possible. If the follower observes $s_1 \in [2\underline{a}, 2\overline{a}]$, the follower assigns probability one to the event that a follows the uniform distribution. If instead $s_1 > 2\overline{a}$, then the follower assigns probability one to the event that a follows the gamma distribution. That is, from an ex-ante perspective, the demand almost always follows the uniform distribution, but if the follower were to infer that $s_1 > 2\overline{a}$, then the follower believes that it is in a measure-zero state $\omega \in N$. In any case, the follower's posterior expectation is $\mathbb{E}[a \mid s_1] = \rho s_1 + (1-\rho)(\underline{a}+\overline{a})$ for all $s_1 \in (0,\infty)$. With this distribution assumption, the leader follows the linear strategy for all $s_1 \in [2\underline{a},\infty)$. In the mandatory benchmark, I do not specify off-path beliefs, because the Bayes law applies for every $q_1 \geq q_1^*(2\underline{a})$. Since the support of a is $[2\underline{a}, 2\overline{a}]$ with probability one, the description of the main text, where I restrict discussion to a and s_1 in $[2\underline{a}, 2\overline{a}]$ is without loss. For example, given that $\tau \in [2\underline{a}, 2\overline{a}]$, the upper-truncated mean is

$$\begin{aligned}\mathbb{E}[a \mid s_1 \geq \tau] &= \mathbb{P}(\omega \in N)\mathbb{E}[a \mid s_1 \geq \tau, N] + \mathbb{P}(\omega \in U)\mathbb{E}[a \mid s_1 \geq \tau, U] \\ &= 0 \times \mathbb{E}[a \mid s_1 \geq \tau, N] + 1 \times \frac{2\overline{a}+\tau}{2} = \frac{2\overline{a}+\tau}{2}.\end{aligned}$$

Remark 3. Alternatively, one could define equilibrium as the limit of games where the demand distribution is unboundedly supported. Call the original game Γ and define a sequence of games $\{\Gamma_n\}_{n\in\mathbb{N}}$ as follows. The demand parameter a follows a distribution whose density $f_n(a)$ is given by

$$f_n(a) = \begin{cases} \frac{1-e^{-2\underline{a}n}/n}{2(\overline{a}-\underline{a})} & \text{for } a \in [2\underline{a}, 2\overline{a}] \\ e^{-na} & \text{for } a > 2\overline{a}. \end{cases}$$

It is clear that $f_n \to f$ pointwise, where $f(a) = 1/(2(\overline{a}-\underline{a}))$ for $a \in [2\underline{a}, 2\overline{a}]$ and $f(a) = 0$ for $a > 2\overline{a}$. Since f_n is fully supported on $[2\overline{a},\infty)$, for each n, there is a diminishingly small probability that the demand realization is $a > 2\overline{a}$. As n becomes large, the small probability on the event $a > 2\overline{a}$ diminishes, and in the limit f_n converges to the density of $\mathscr{U}[2\underline{a}, 2\overline{a}]$. The signal for a continues to be the "truth-noise" structure, i.e., it is a mixture of a and u_n that has the density f_n. For each Γ_n, let E_n be the equilibrium where the leader plays linear fully separating equilibrium given disclosure.[39] Then, the linear fully separating equilibrium derived in the main text can be understood as the limit of E_n. By construction, $\mathbb{E}_n[a \mid s_1] \to \rho s_1 + (1-\rho)(\underline{a}+\overline{a})$, where $\mathbb{E}_n$ is the posterior expectation under the density f_n.

[39]The existence of linear fully separating equilibrium follows from the analysis of the main text.

For all n, the support of s_1 is unbounded above.

D Exponentially Distributed Demand

In the main text, I assumed that the demand intercept follows the uniform distribution. The use of the uniform distribution made it fairly easy to compute the truncated expectation. In this appendix, I assume that the demand intercept follows the exponential distribution with scale $\beta > 0$, $Exp(\beta^{-1})$. Since the support of a is unbounded above, there is no off-path action given disclosure. I show that the partial disclosure result Proposition 1 still holds. I also show that the comparative statics remain the same.

Specifically, let $a = \underline{a} + x$, where $\underline{a} > 0$ and $x \sim Exp(\beta^{-1})$. To simplify the analysis, suppose that the signal is the realization of the demand itself, so $s_1 = x$. The rest of the model is the same as the main text. Clearly, $\mathbb{E}[a \mid s_1] = \underline{a} + s_1$ and $\mathbb{E}[a] = \underline{a} + \beta$.

From the analysis of the main text, one can see that the production equilibrium is exactly the same, except that the values of $\mathbb{E}[a \mid s_1]$ and $\mathbb{E}[a \mid d(s_1) = 0]$ are now computed under the new distributional assumption. The argument for the lower-tail disclosure still applies. Thus, to derive the unique disclosure threshold, I solve Equation (4). To do so, define the function $f(\tau; t, \beta)$ as the difference of the adjusted disclosure quantity and nondisclosure quantity. (see the proof of Proposition 1). A useful feature of the exponential distribution is that the upper-truncated mean is linear in a truncation threshold. Under the current setting, $\mathbb{E}[a \mid s_1 > \tau] = \alpha + \beta + \tau$. Therefore,

$$f(\tau; t, \beta) = \underbrace{\frac{1}{2\sqrt{1+t\psi}}\mathbb{E}[a \mid \tau]}_{\text{Disclosure}} - \underbrace{\left(\frac{1}{2}\mathbb{E}[a \mid \tau] - \frac{t}{2(2+t)}\mathbb{E}[a \mid s_1 > \tau]\right)}_{\text{Nondisclosure}}$$
$$= -\left(\frac{2}{2+t} - \frac{1}{\sqrt{1+t\psi}}\right)(\tau - \underline{a}) + \frac{t}{2+t}\beta.$$

Since the coefficient on τ is negative for all $t \in (0,1)$, $f(\cdot; t)$ is monotonically decreasing. Hence, there is a unique solution. Solving $f(\tau; t) = 0$, I obtain the equilibrium disclosure threshold (if it is interior):

$$\tau^* = -\underline{a} + \frac{t\sqrt{1+t\psi}}{2\sqrt{1+t\psi} - (2+t)}\beta.$$

Since $\lim_{\tau\to\infty} f(\tau; t, \beta) = -\infty$, is is evident that full disclosure is impossible.

The nonnegativity constraint does not bind in equilibrium. To see this, note that

$$\mathbb{E}[a \mid s_1 = \tau^*] - \frac{t}{2+t}\mathbb{E}[a \mid s_1 > \tau^*] \geq 0 \iff \frac{2t\sqrt{1+t\psi}}{2\sqrt{1+t\psi}-(2+t)} \geq 0.$$

However, the left hand side only depends on t, and it is straightforward to show that it is positive for all $t \in (0,1)$. Since $\mathbb{E}[a \mid s_1] - \frac{t}{2+t}\mathbb{E}[a \mid s_1 > \tau^*] \geq \mathbb{E}[a \mid s_1 = \tau^*] - \frac{t}{2+t}\mathbb{E}[a \mid s_1 > \tau^*]$ for all $s_1 > \tau^*$, the nonnegativity constraint never binds in equilibrium.

After some algebra, one can verify that $\frac{\partial \tau^*}{\partial t} < 0$ for all parameter values. Hence, the leader firm discloses less as the market becomes more competitive. Additionally, I can consider the effect of the scale parameter β on the disclosure threshold. Since the coefficients on β in the expression for τ^* is positive, the disclosure threshold increases as the scale parameter increases. That is, the leader firm discloses more as the demand becomes higher on average. This is because, for a given type $s_1 = \tau$, as the demand becomes higher on average, the follower believes that the average demand of the nondisclosure types is higher. In other words, the follower's posterior $\mathbb{E}[a \mid s_1 > \tau]$ is increasing in β.

The other results–stock price maximization, dual objective, and the Dye friction–would also hold for this alternative signal structure. However, unlike the uniform distribution case, the lower-truncated mean is not linear in a truncation threshold for any distribution. Specifically, in the exponential case, $\mathbb{E}[a \mid s_1 < \tau] = \underline{a} + \beta - \frac{\tau}{e^{\beta^{-1}\tau}-1}$. Thus, the computation of equilibrium becomes more complicated, and one has to rely on numerical computation. For this reason, I prefer to use the uniform distribution.