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COMP9321

Data Services Engineering

Term 3, 2019

Week 7 Lecture 2

Decision Tree

Supervised Learning

Background

- Decision trees have a long history in machine learning
- The first popular algorithm dates back to 1979
- Very popular in many real world problems
 - Intuitive to understand
 - Easy to build

Motivation

- How do people make decisions?
 - Consider a variety of factors
 - Follow a logical path of checks
- An Example
 - Should I eat at this restaurant?
 - If there is no wait
 - Yes
 - If there is short wait and I am hungry
 - Yes
 - Else
 - No

Advantages

Handling of categorical variables

Handling of missing values and unknown labels

Detection of nonlinear relationships

Visualization and interpretation in decision trees

When to consider Decision Trees

Instances describable by attribute-value pairs

Target function is discrete valued

Disjunctive hypothesis may be required

Possibly noisy training data

Missing attribute values

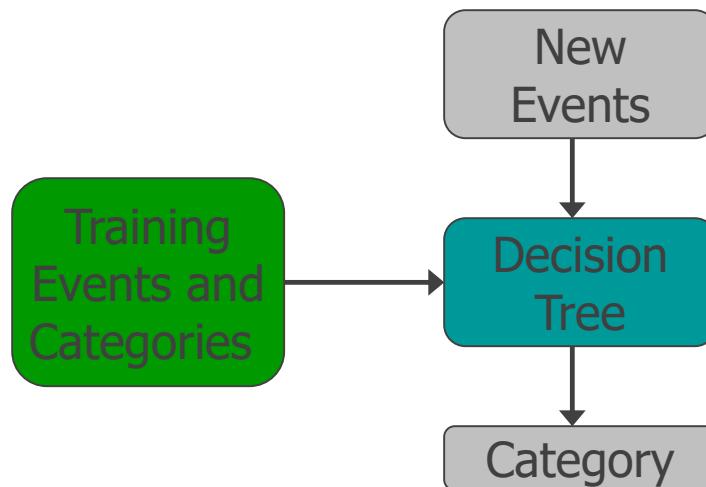
Examples:

- Medical diagnosis
- Credit risk analysis
- Object classification for robot manipulator (Tan 1993)

Introduction

Use a decision tree to predict categories for new events.

Use training data to build the decision tree.



Introduction

A decision tree has 2 kinds of nodes

1. Each leaf node has a class label, determined by majority vote of training examples reaching that leaf.
2. Each internal node is a question on features. It branches out according to the answers.

Decision Tree for Play Tennis

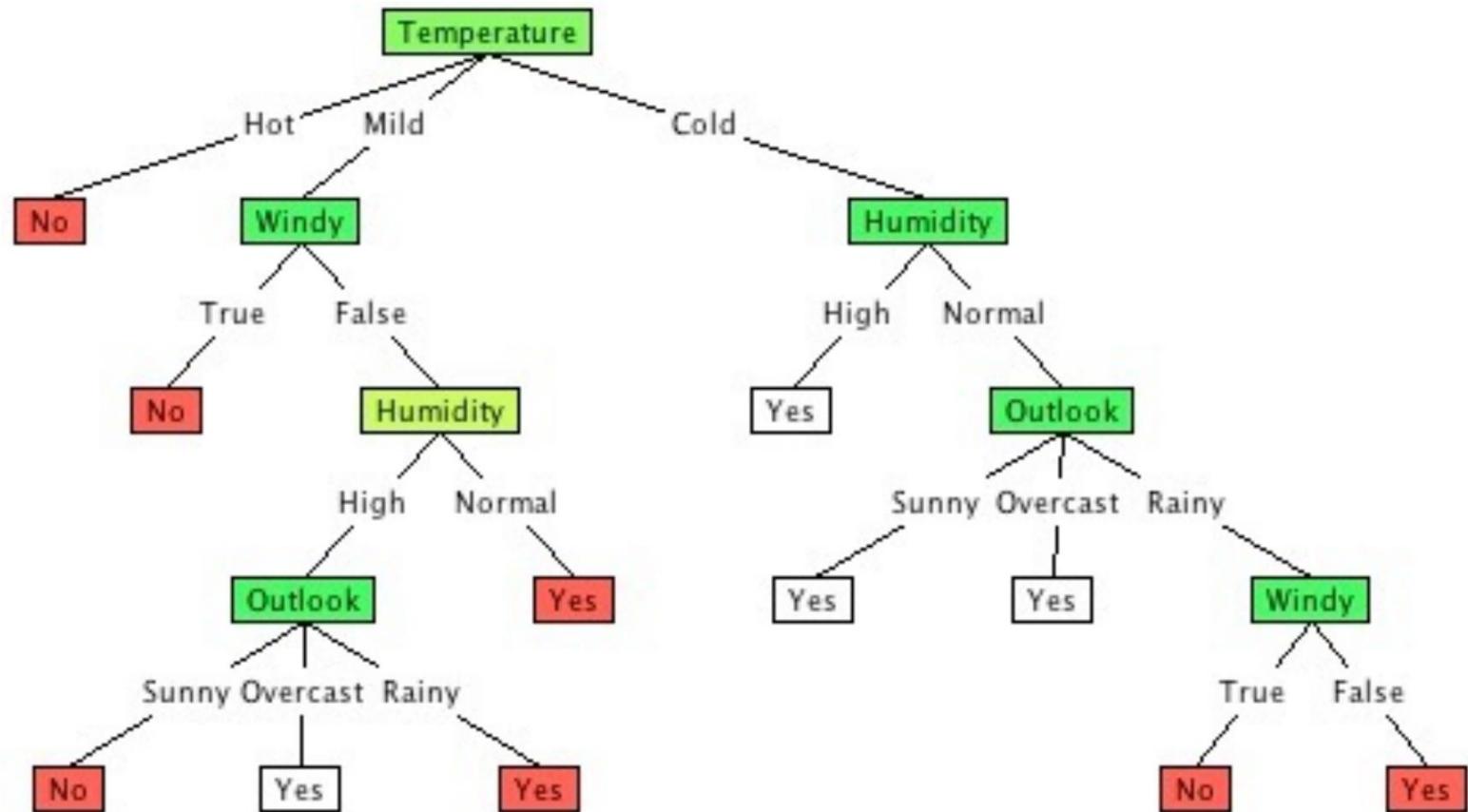
Play Tennis	Outlook	Temperature	Humidity	Windy
No	Sunny	Hot	High	No
No	Sunny	Hot	High	Yes
Yes	Overcast	Hot	High	No
Yes	Rainy	Mild	High	No
Yes	Rainy	Cold	Normal	No

If temperature is not hot → Play

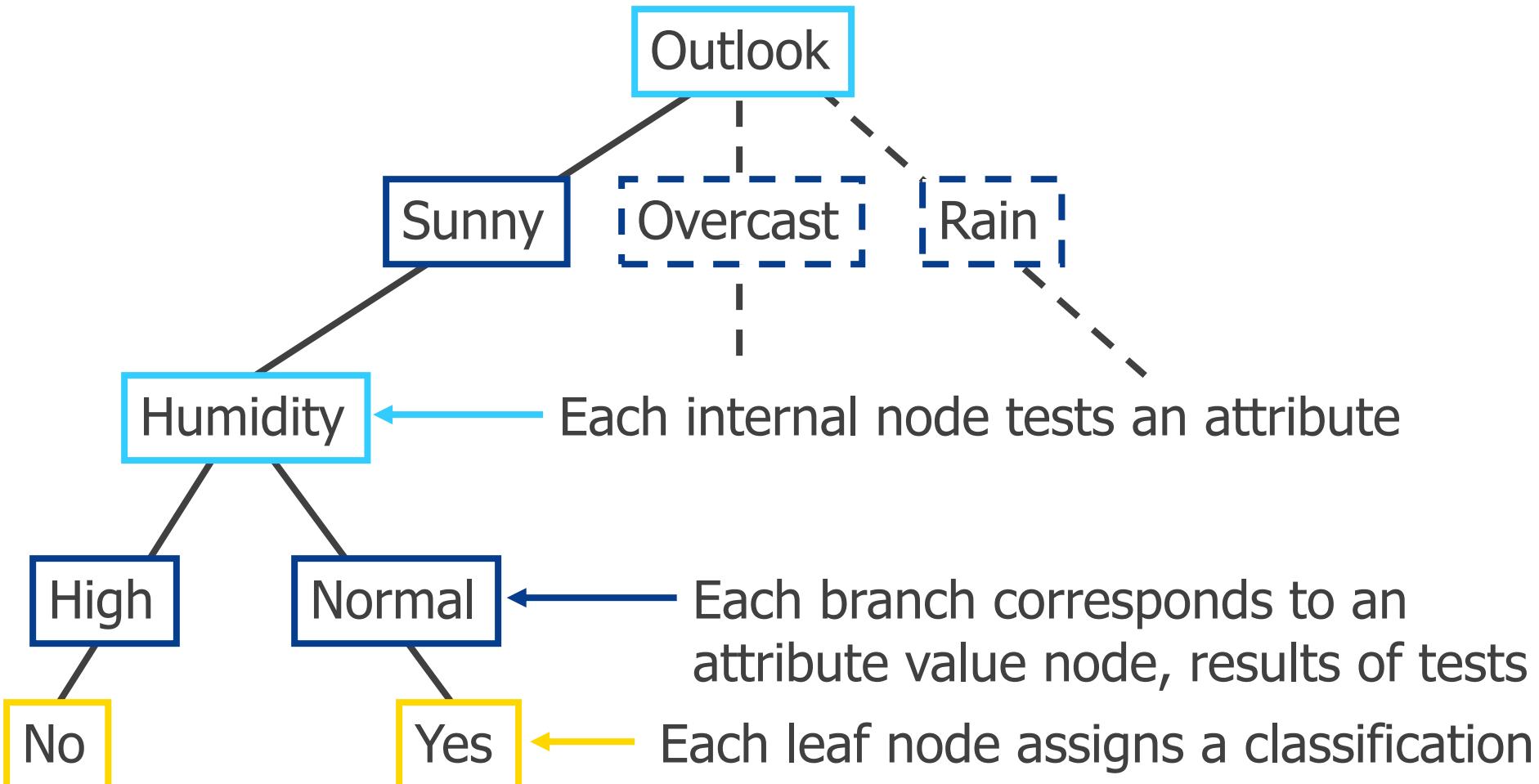
If outlook is overcast → Play

Otherwise → Don't play tennis

Decision Tree for Play Tennis



Decision Tree for PlayTennis - Structure



What Makes a Good Tree

Small tree:

- Occam's razor: a guideline to help us choose. Simpler is better
- Avoids over-fitting



What Makes a Good Tree

- Occam's razor: a guideline to help us choose. Simpler is better



What Makes a Good Tree

- Occam's razor: a guideline to help us choose. Simpler is better

Prank explanation requires:

1. Human (not observed)
2. Ability to enter house
(unknown)
3. Motive to play prank
(unknown)
4. Leaving no other trace
(observed)

Licking explanation requires:

1. Cat (observed)
2. Licking (observed)

What Makes a Good Tree

Small tree:

- Occam's razor: a guideline to help us choose. Simpler is better
- Avoids over-fitting

A decision tree may be human readable, but not use human logic!

How do we build small trees that accurately capture data?

Learning an optimal decision tree is computationally intractable

Greedy Algorithm

We can get good trees by simple greedy algorithms

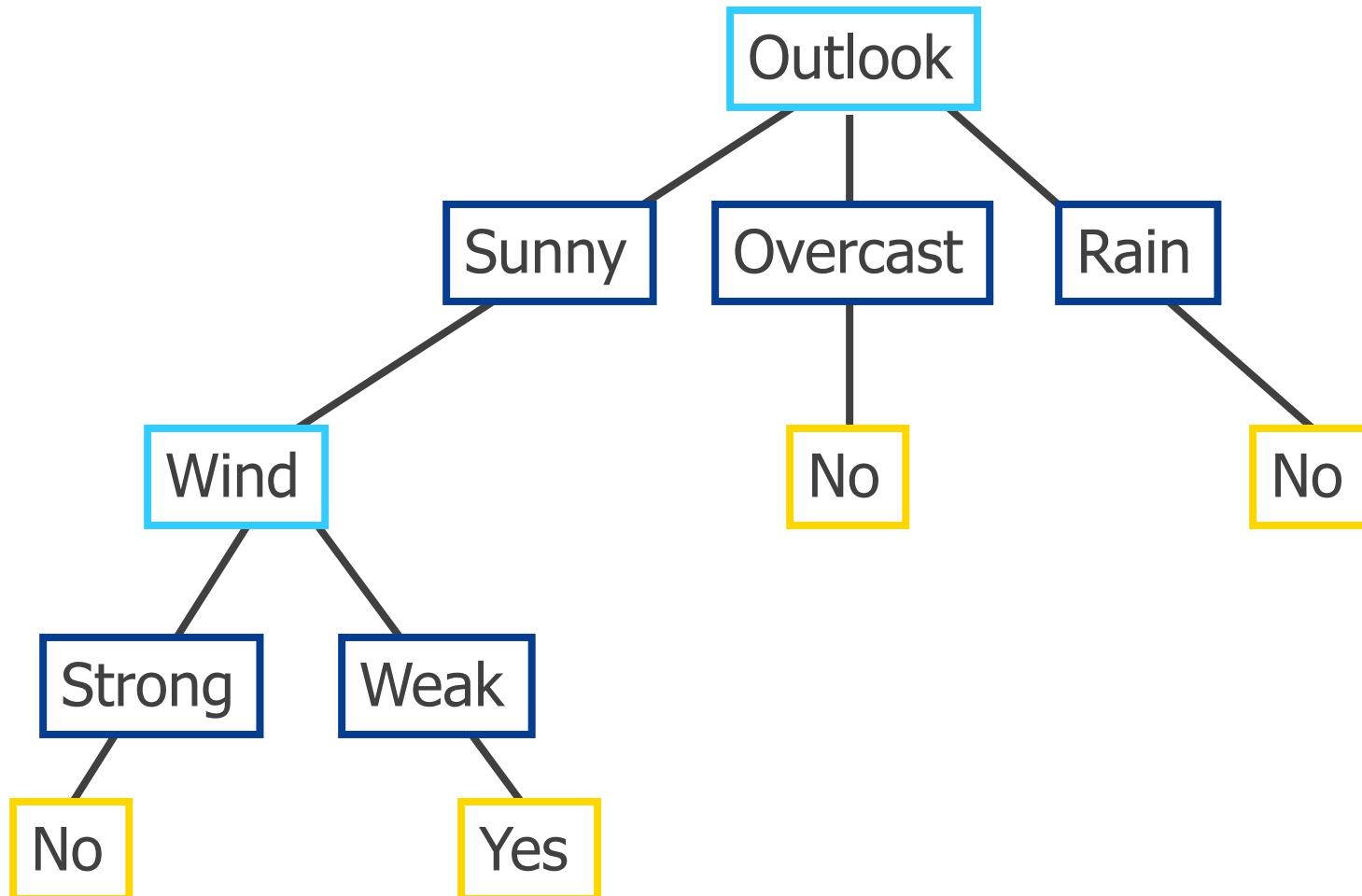
Adjustments are usually to fix greedy selection problems

Recursive:

1. Select the “best” variable, and generate child nodes: One for each possible value;
2. Partition samples using the possible values, and assign these subsets of samples to the child nodes;
3. Repeat for each child node until all samples associated with a node that are either all positive or all negative.

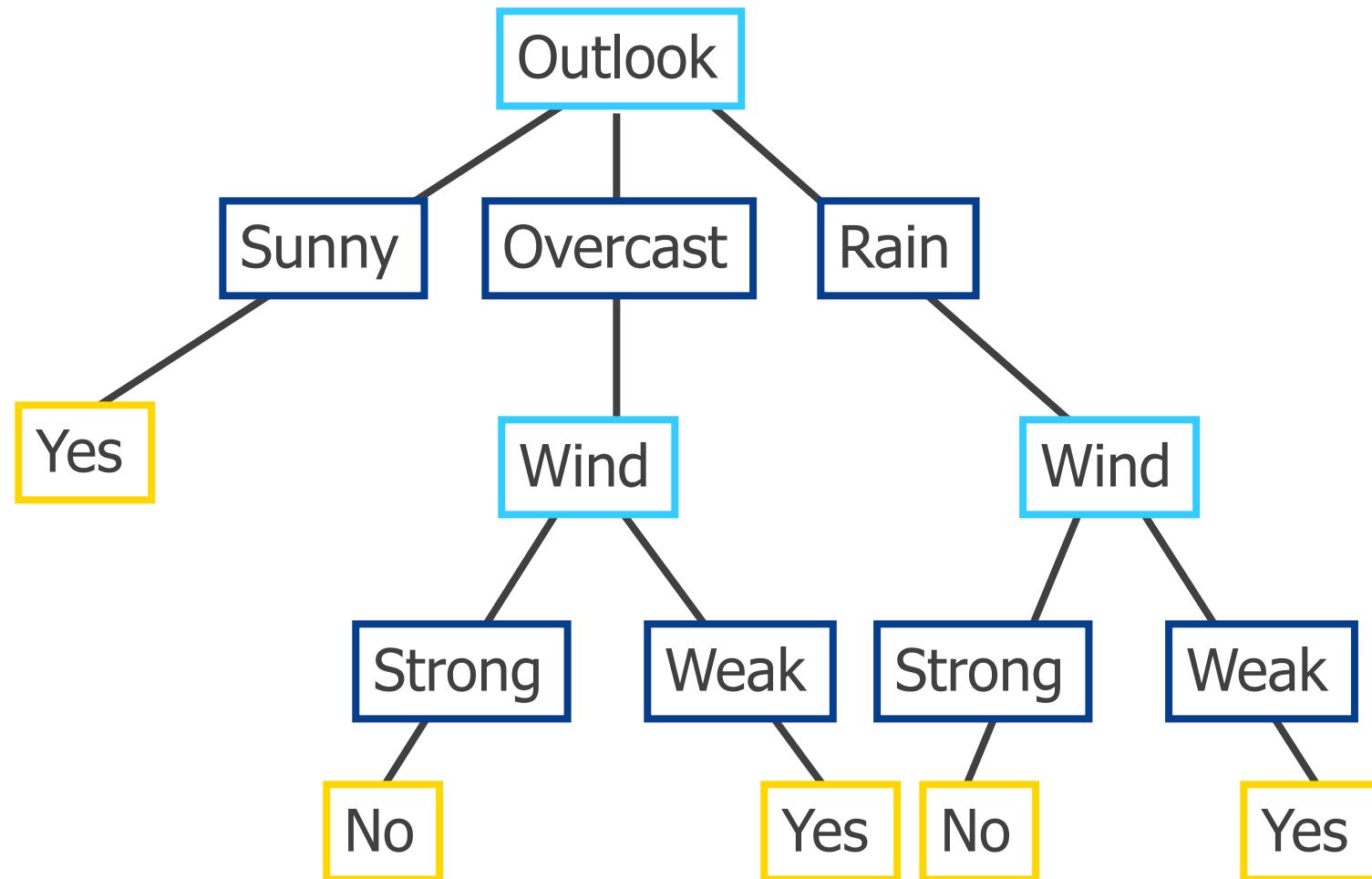
Decision Tree for Conjunction

Outlook=Sunny \wedge Wind=Weak



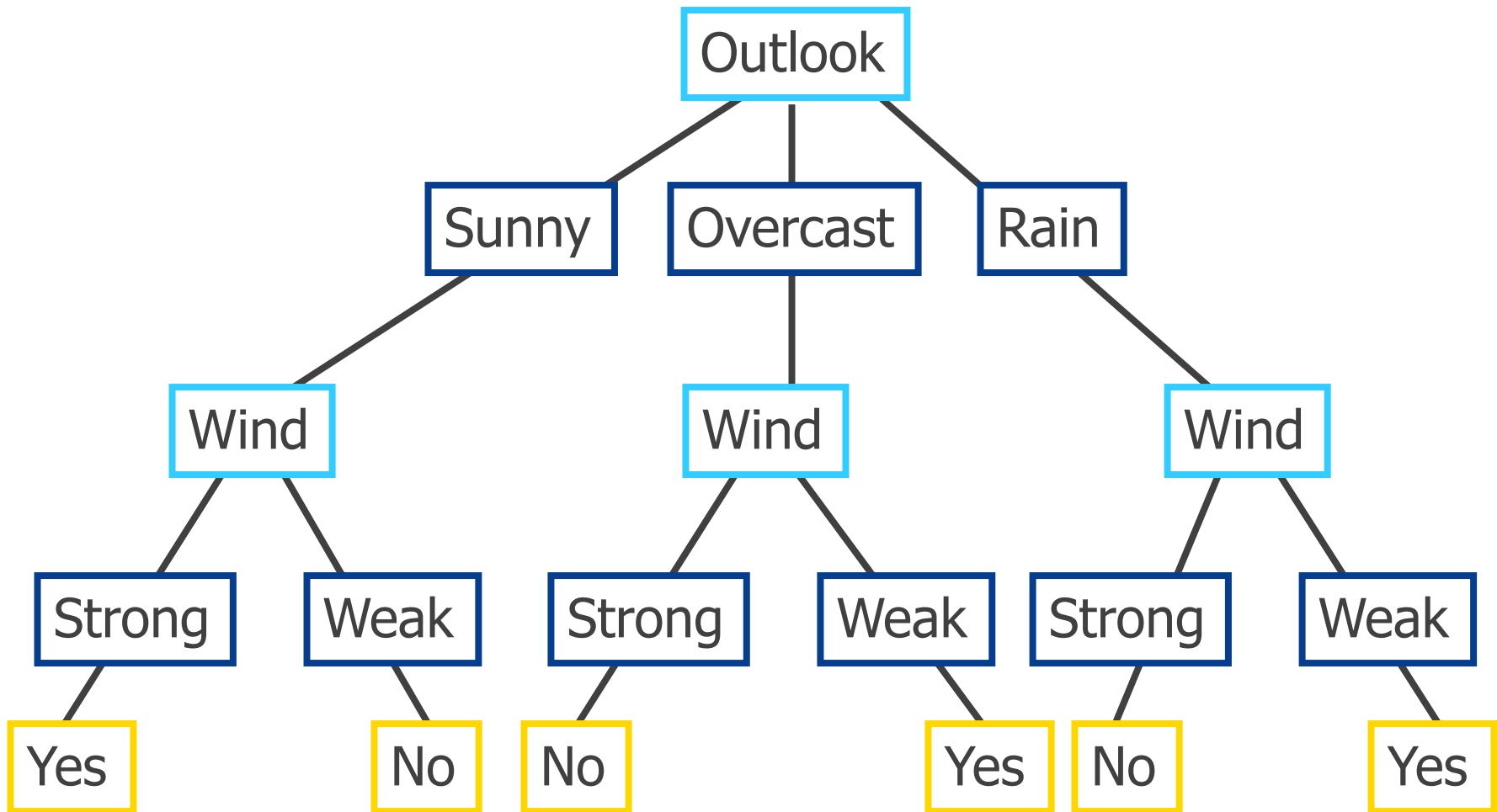
Decision Tree for Disjunction

$\text{Outlook} = \text{Sunny} \vee \text{Wind} = \text{Weak}$



Decision Tree for XOR

Outlook=Sunny XOR Wind=Weak



Top-Down Induction of Decision Trees ID3

1. $A \leftarrow$ the “best” decision attribute for next *node*
2. Assign A as decision attribute for *node*
3. For each value of A create new descendant
4. Sort training examples to leaf node according to the attribute value of the branch
5. If all training examples are perfectly classified (same value of target attribute) stop, else iterate over new leaf nodes.

Variable Selection

The best variable for partition

- The most informative variable
- Select the variable that is most informative about the labels

The quantification of information

Founded by Claude Shannon

Basic concepts

$$\text{Entropy: } H(X) = - \sum_x \mathbb{P}(X = x) \log \mathbb{P}(X = x)$$

$$\text{Conditional Entropy: } H(Y|X) = \sum_x \mathbb{P}(X = x) H(Y|X = x)$$

$$\text{Information Gain: } \text{IG}(Y|X) = H(Y) - H(Y|X)$$

Select the variable with the highest information gain

Entropy

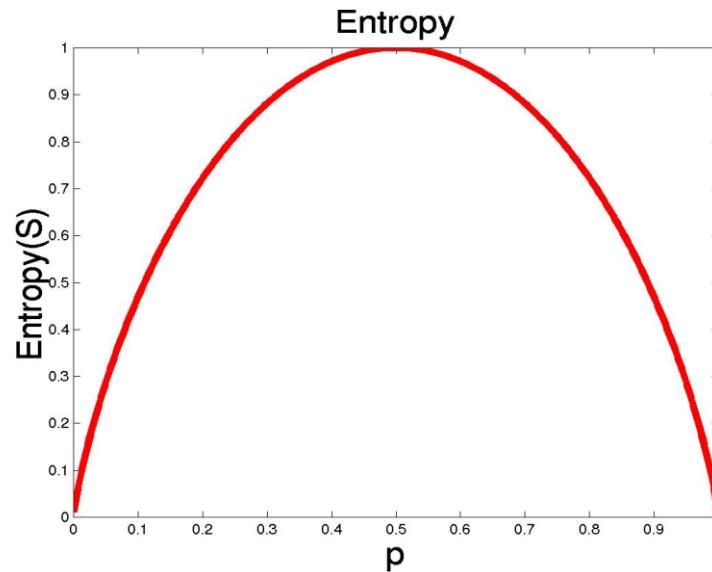
S is a sample of training examples

p_+ is the proportion of positive examples

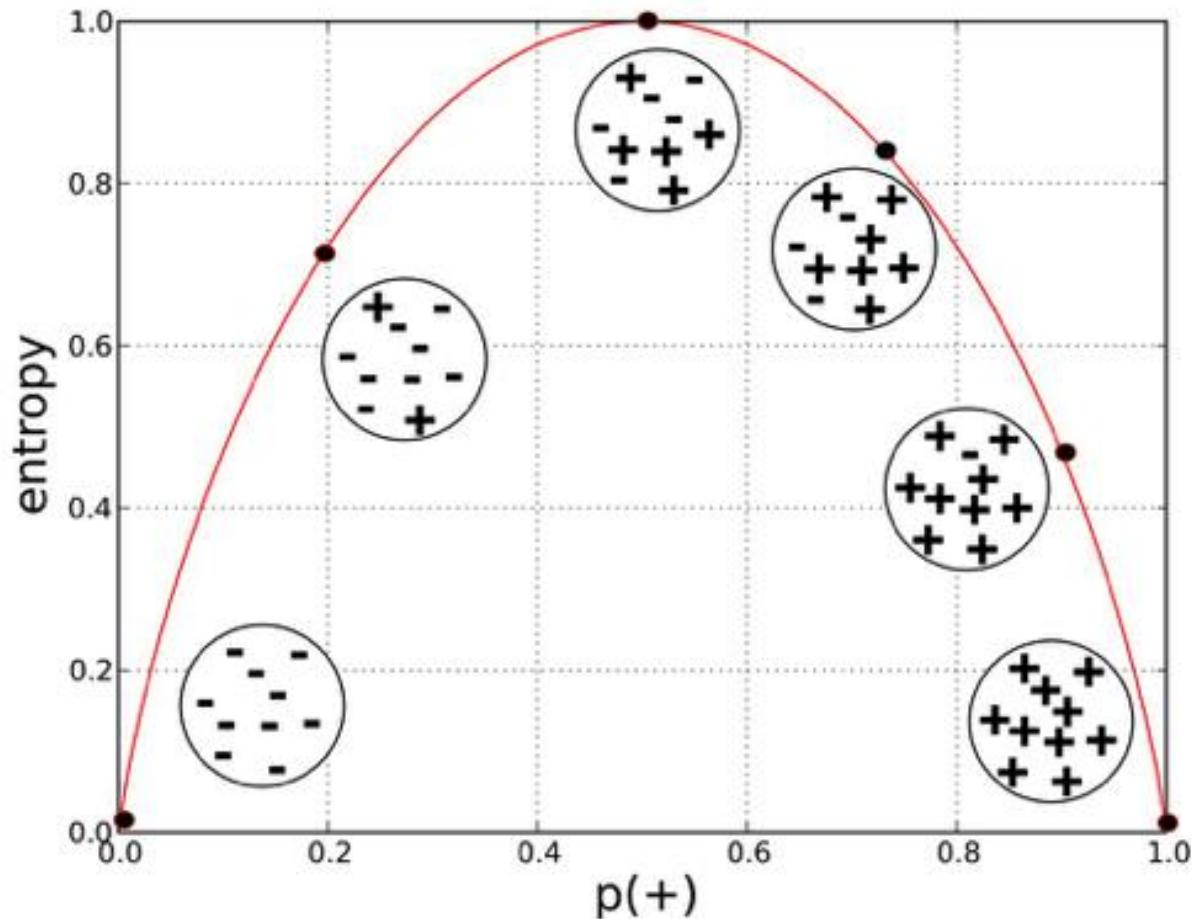
p_- is the proportion of negative examples

Entropy measures the impurity of S

- $\text{Entropy}(S) = -p_+ \log_2 p_+ - p_- \log_2 p_-$



Entropy

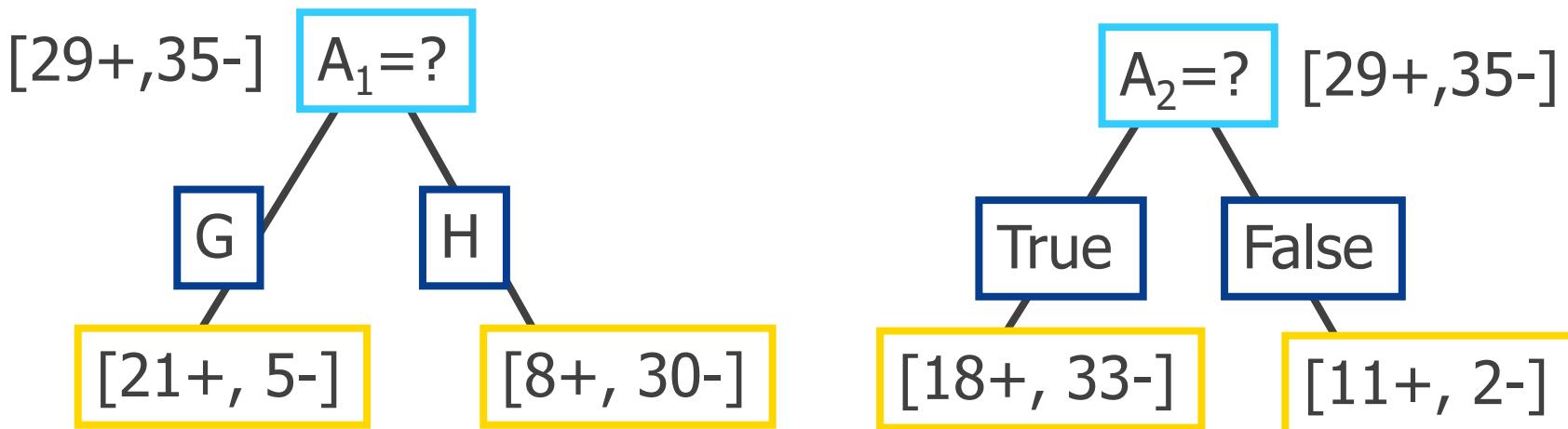


Information Gain (S=E)

$\text{Gain}(S, A)$: expected reduction in entropy due to sorting S on attribute A

$$\begin{aligned}\text{Entropy}([29+, 35-]) &= -\frac{29}{64} \log_2 \frac{29}{64} - \frac{35}{64} \log_2 \frac{35}{64} \\ &= 0.99\end{aligned}$$

$$\text{Gain}(S, A) \equiv \text{Entropy}(S) - \sum_{v \in D_A} \frac{|S_v|}{|S|} \text{Entropy}(S_v)$$



Training Examples

Day	Outlook	Temp.	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Weak	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cold	Normal	Weak	Yes
D10	Rain	Mild	Normal	Strong	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Selecting the Next Attribute

The information gain values for the 4 attributes are:

- $\text{Gain}(S, \text{Outlook}) = 0.247$
- $\text{Gain}(S, \text{Humidity}) = 0.151$
- $\text{Gain}(S, \text{Wind}) = 0.048$
- $\text{Gain}(S, \text{Temperature}) = 0.029$

where S denotes the collection of training examples

Occam's Razor

"If two theories explain the facts equally well, then the simpler theory is to be preferred"

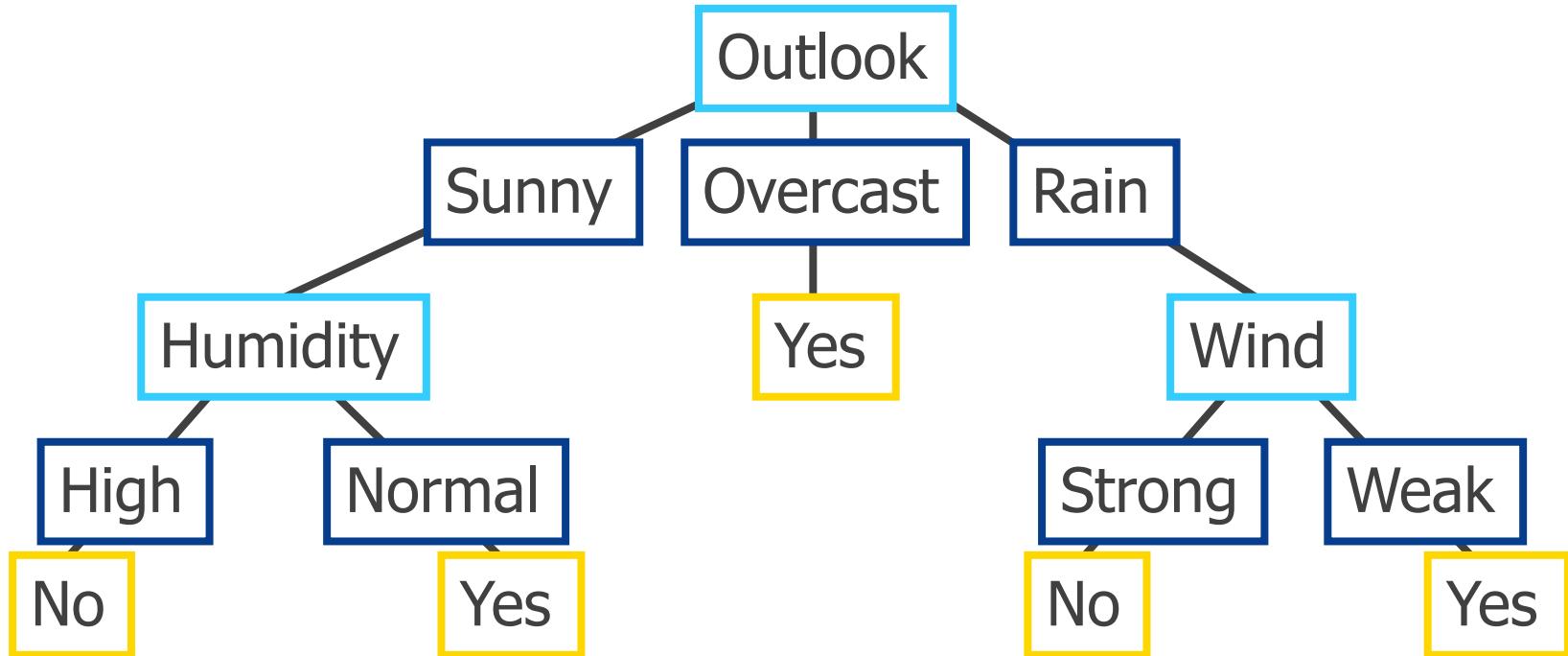
Arguments in favor:

- Fewer short hypotheses than long hypotheses
- A short hypothesis that fits the data is unlikely to be a coincidence
- A long hypothesis that fits the data might be a coincidence

Arguments opposed:

- There are many ways to define small sets of hypotheses

Converting a Tree to Rules



- R₁: If (Outlook=Sunny) \wedge (Humidity=High) Then PlayTennis>No
- R₂: If (Outlook=Sunny) \wedge (Humidity=Normal) Then PlayTennis=Yes
- R₃: If (Outlook=Overcast) Then PlayTennis=Yes
- R₄: If (Outlook=Rain) \wedge (Wind=Strong) Then PlayTennis>No
- R₅: If (Outlook=Rain) \wedge (Wind=Weak) Then PlayTennis=Yes

Continuous Valued Attributes

Create a discrete attribute to test continuous

Temperature = 24.50C

(Temperature > 20.00C) = {true, false}

Where to set the threshold?

Temperature	15 ^o C	18 ^o C	19 ^o C	22 ^o C	24 ^o C	27 ^o C
PlayTennis	No	No	Yes	Yes	Yes	No

Unknown Attribute Values

What if some examples have missing values of A?

Use training example anyway sort through tree

If node n tests A, assign most common value of A among other examples sorted to node n.

Assign most common value of A among other examples with same target value

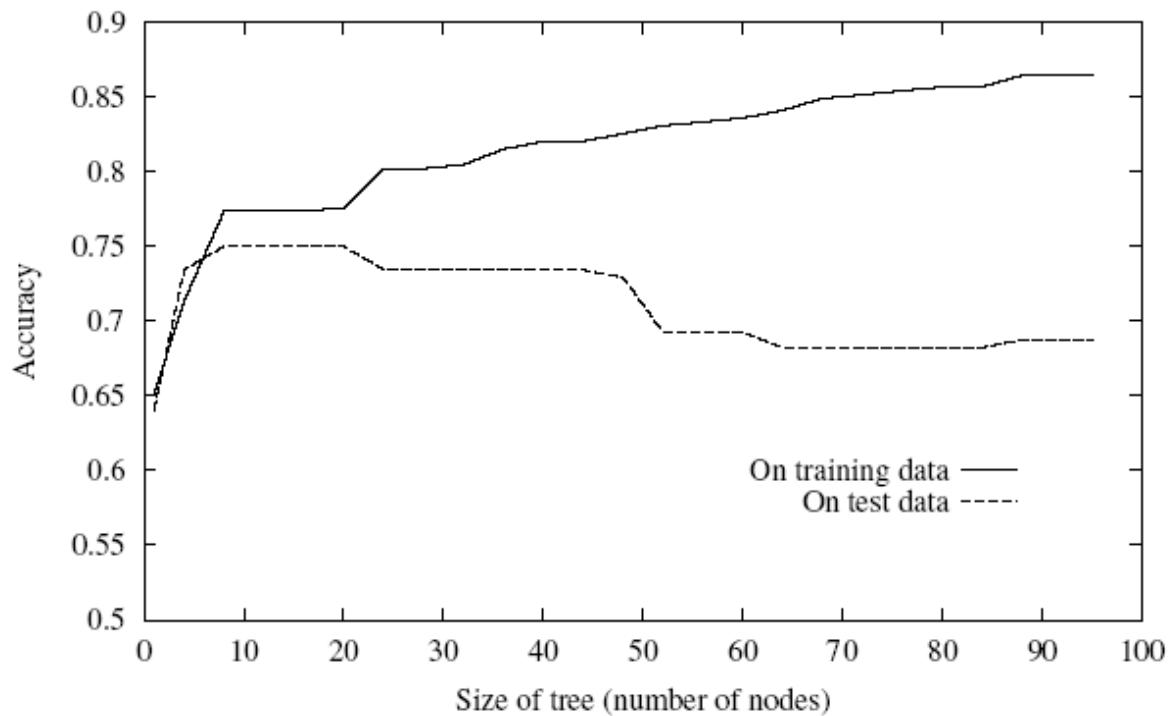
Assign probability π_i to each possible value v_i of A

- Assign fraction π_i of example to each descendant in tree

Classify new examples in the same fashion

Overfitting

One of the biggest problems with decision trees is Overfitting



Avoid Overfitting

stop growing when split not statistically significant
grow full tree, then post-prune

Select “best” tree:

measure performance over training data

measure performance over separate validation data set

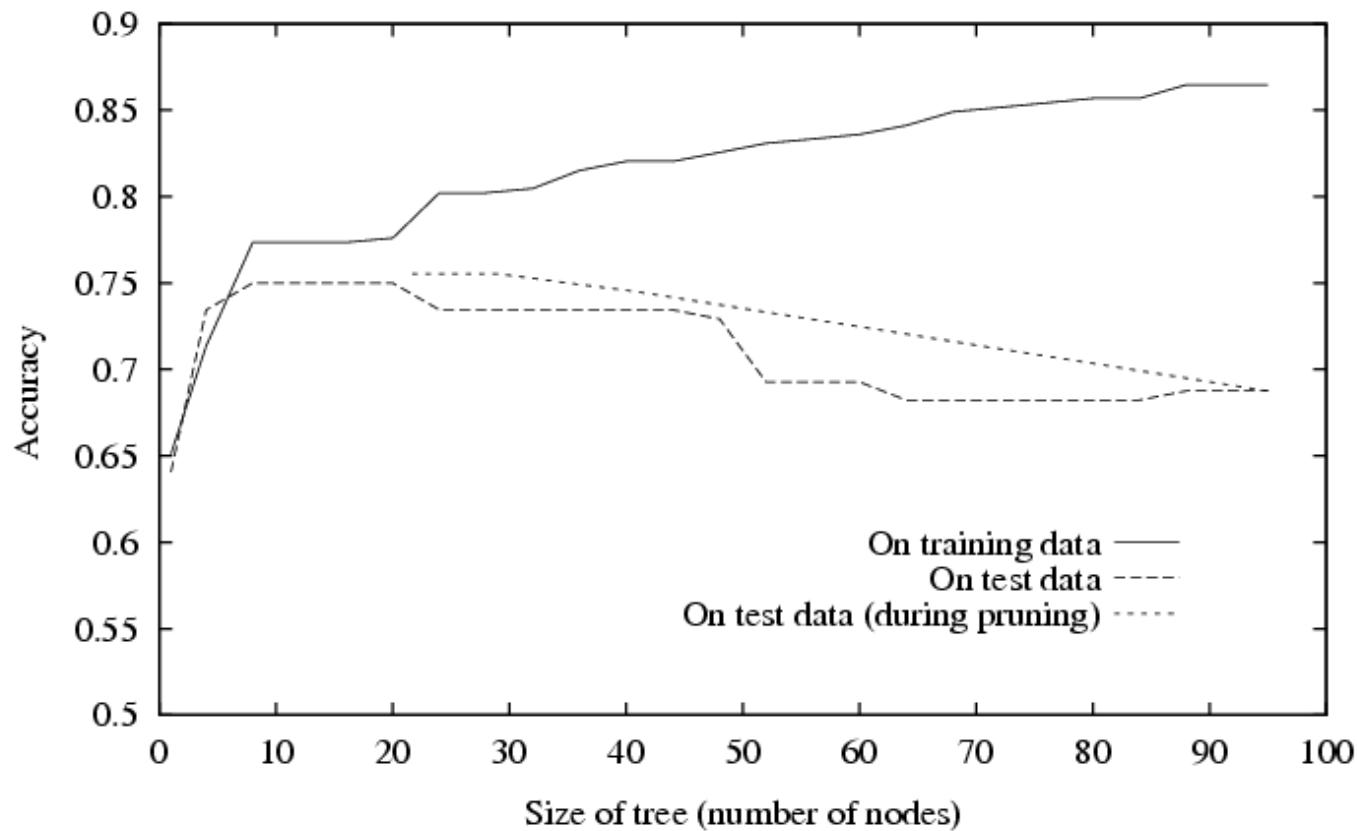
$\min(|\text{tree}| + |\text{misclassifications(tree)}|)$

Avoid Overfitting

Idea 1: Stop growing the tree when the error doesn't drop by more than a threshold with any new cut.

Idea 2: Prune a large tree from the leaves to the root. I
Weakest link pruning:

Effect of Reduced Error Pruning



Useful resources

- <https://dl.acm.org/citation.cfm?id=541177>
- <https://towardsdatascience.com/a-guide-to-decision-trees-for-machine-learning-and-data-science-fe2607241956>
- <https://towardsdatascience.com/entropy-how-decision-trees-make-decisions-2946b9c18c8>
- <https://victorzhou.com/blog/information-gain/>

Questions?