

PLEASE HAND IN

UNIVERSITY OF TORONTO  
Faculty of Arts and Science  
AUGUST 2020 EXAMINATIONS  
CSC 418 H1F

PLEASE HAND IN

Duration—2 hours + 15 minutes to upload answers

No Aids Allowed

Student Number: \_\_\_\_\_

UTORid: \_\_\_\_\_

Last (Family) Name(s): \_\_\_\_\_

First (Given) Name(s): \_\_\_\_\_

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*Do **not** turn this page until you have received the signal to start.  
In the meantime, please read the instructions below carefully.*

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This final examination consists of 9 questions on 9 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the examination is complete and fill in the identification section above.*

Answer each question on a separate sheet of paper or in any digital form you want (e.g. physical paper, iPad note taking app, etc.). Please *indicate clearly which question your work belongs to*. You will have 10 minutes (after the 50 minutes allotted for the exam) to organize and upload your answers to MarkUs. You can take photos of your answers on your phone, upload your answers as a PDF, PNGs, etc.

As a student, you help create a fair and inclusive writing environment. If you possess an unauthorized aid during an exam, you may be charged with an academic offence.

MARKING GUIDE

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TOTAL: \_\_\_\_\_/80

**Question 1.** Raster Images [6 MARKS]**Part (a)** Bayer Filter [2 MARKS]

i- [1 MARK]

we use a linear interpolation-based demosaic method.

ii- [1 MARK]

where there are some big change in color.

**Part (b)** Image Manipulation [4 MARKS]

```

for (int i = 0; i < rgb.size(); i++) {
    rgb[i * 3 + 1] = 0.2 * rgb[i * 3 + 1];
    rgb[i * 3 + 2] = 0;
}

```

**Question 2.** Ray Casting [6 MARKS]**Part (a)** Planes [3 MARKS]

i- [1 MARK]

say the two points  $p_0, p_1$ , normal  $n$   
 if  $\tilde{n} \cdot (p_0 - p_1) = 0$ , it's on the plane, otherwise no.

ii- [2 MARKS]

$$\tilde{n} \cdot (1, 1, 1)$$

**Part (b)** Ray-Sphere Intersection [2 MARKS]

i- [1 MARK]

quadratic.

ii- [1 MARK]

say  $ax^2 + bx + c = 0$ .

$$\sqrt{b^2 - 4ac} < 0.$$

**Part (c) Complexity [1 MARK]**

i- [1 MARK]

$$O(mn)$$

**Question 3. Ray Tracing [6 MARKS]****Part (a) Occlusions [3 MARKS]**

i- [1 MARK]

compute the value of  $t$  of hitting that object.  
if the resulted  $t$  is negative, ignore it.

ii- [2 MARKS]

compare the value of  $t$ , the smaller one take into account.

**Part (b) Blinn-Phong [3 MARKS]**

The specular term of Blinn-Phong shading is below, with the specular exponent  $\alpha$ :

$$\underbrace{\mathbf{k}_s}_{\text{specular color}} * \underbrace{\mathbf{I}_s}_{\text{light color}} \max((\hat{\mathbf{h}} \cdot \hat{\mathbf{n}}), 0)^\alpha \quad (1)$$

i- [1 MARK]

$$\frac{e-p + l-p}{\|e-p + l-p\|}$$

ii- [2 MARKS]

the material will look less glossy to reflect light

**Question 4. Bounding Volume Hierarchies [6 MARKS]****Part (a) Bounding Volumes [4 MARKS]**

We covered 4 types of bounding volumes: sphere, AABB, OBB, and convex hull.

i- [2 MARKS]

OBB: because its oriented, it has direction.

ii- [2 MARKS]

convex hull: the total space used is the least.  
so spent least time.

**Part (b) AABBs [2 MARKS]**

Consider a set of  $n$  objects in space.

i- [1 MARK]

$$O(n)$$

, ii- [1 MARK]

Ocn)

**Question 5. Meshes** [6 MARKS]**Part (a) Normals** [3 MARKS]

i- [1 MARK]

It's a common way, we can use other method.

ii- [2 MARKS] Match each method to its best suited example.

per-face normals ○ ————— ○ a cube  
 per-vertex normals ○ ————— ○ a cylinder  
 per-corner normals ○ ————— ○ a sphere

**Part (b) Subdivision** [3 MARKS]Consider a *polygonal* mesh with  $v$  vertices,  $e$  edges and  $f$  faces.

i- [2 MARKS] True or false? Prove if true, or provide counter example if false.

False.

ii- [1 MARK]

 $4k \cdot f$ **Question 6. Shader Pipeline** [15 MARKS]**Part (a) Projection** [2 MARKS]

i- [1 MARK]

for example, for art purpose.

ii- [1 MARK] A model matrix transforms object space into world space.**Part (b) Bump and Displacement Mapping** [4 MARKS]

i- [1 MARK]

ii- [1 MARK]

iii- [2 MARKS]

$$\bar{n} = \frac{\partial p}{\partial \gamma} \times \frac{\partial p}{\partial \beta}.$$

**Part (c)** Perlin Noise [3 MARKS]

The Perlin noise algorithm starts by assigning random unit gradient vectors to the nodes of a grid. Say we have a 2D grid where grid points are at integer values.

i- [1 MARK]

$$v = g \cdot (p - c)$$

ii- [1 MARK]

iii- [1 MARK]

increase order of the chosen function ~~or~~ used in smooth step.

**Part (d)** Shader [6 MARKS]

i- [1 MARK] Circle one.

mesh vertex positions

camera transformation

high-resolution color

ii- [1 MARK] Circle one.

- a) vertex shader
- b) tessellation control shader
- c) tessellation eval shader
- d) none (rasterization does shading automatically)
- ☒ e) fragment shader

iii- [2 MARKS] Prove or counterexample.

counter example.

$$\begin{matrix} \nearrow & (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0) \\ \searrow & (1, 0, 0) \end{matrix} \times = (0, 0, -\frac{\sqrt{2}}{2})$$

$|(0, 0, -\frac{\sqrt{2}}{2})| \neq 1.$

iv- [2 MARKS] Demonstrate how or give counterexample?

**Question 7.** Kinematics [15 MARKS]**Part (a)** Linear Blend Skinning [3 MARKS]

i- [2 MARKS]

no. for a counter example.  $R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  $R_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and  $0.5 \cdot R_1 + 0.5 \cdot R_2$  results in a matrix  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  which projects to z-axis.

ii- [1 MARK]

0

**Part (b)** Catmull-Rom Spline [3 MARKS]

Consider keyframes stored as parameter-timestamp pairs:

$$(\mathbf{p}_1, t_1 = 0.0), (\mathbf{p}_2, t_2 = 1.0), (\mathbf{p}_3, t_3 = 1.5), (\mathbf{p}_4, t_4 = 2.25), (\mathbf{p}_5, t_5 = 3)$$

i- [1 MARK]

$$\mathbf{p}_1$$

ii- [2 MARKS]

$$\frac{\mathbf{p}_4 - \mathbf{p}_2}{\|\mathbf{p}_4 - \mathbf{p}_2\|}$$

**Part (c)** Inverse Kinematics [8 MARKS]

i- [1 MARK]

$$E(x) =$$

ii- [7 MARKS]

```

a ← initial joint angles
E_cur = E( X(a) )
for outer = 1...100
  J = d X(a) / d a
  g = d E(a) / d a
  step_dir = - J * g
  lambda = lambda_max
  for inner = 1...10
    a_next = a + lambda * step_dir
    E_next = E( X(a_next) )
    if E_cur < E_next
      lambda = lambda/2
    else
      break
  a = a_next
E_cur = E_next

```

**Part (d)** Line Search [1 MARK]

make sure that energy  $E$  decreased after gradient descent by thinking of  $a$  as a line and scale it from large to small scalar.

**Question 8.** Mass-Spring Systems [15 MARKS]**Part (a)** Sparse Matrices [5 MARKS]

i- [1 MARK]

 $O(nk)$ 

ii- [1 MARK]

 $O(n)$ 

iii- [1 MARK] For every double precision floating point entry in a dense matrix, we need 8 bytes. For every entry in a sparse matrix we need 16 bytes.

iv- [1 MARK] For every integer entry in a dense matrix, we need 4 bytes. For every entry in a sparse matrix we need 12 bytes.

v- [1 MARK]

over 33.3%

**Part (b)** Physical simulation [9 MARKS]

i- [2 MARKS]

$$\mathbf{f} \in \mathbb{R}^3$$

$$m \in \mathbb{R}^1$$

$$\mathbf{a} \in \mathbb{R}^3$$

ii- [1 MARK]

$$V(\mathbf{x}_i, \mathbf{x}_j) = \left( \|\mathbf{x}_i - \mathbf{x}_j\| - r_{ij} \right)^2 \cdot \frac{1}{2} k$$

iii- [1 MARK]

$$\mathbf{f}_{ij} = \sqrt{V} \cdot 2k \cdot$$



iv- [1 MARK]

when the length of spring equal to the rest length of spring.

v- [2 MARKS]

$$a_t = \frac{x_{t+\Delta t} - 2x_t + x_{t-\Delta t}}{\Delta t^2}$$

vi- [2 MARKS]

$$f_{\text{internal}} =$$

**Part (c)** Time Integration [1 MARK]

i- [1 MARK]

**Question 9.** Computational Fabrication [5 MARKS]**Part (a)** Manufacturing [5 MARKS]

i- [2 MARKS]

additive manufacturing. because objects are constructed by depositing material in layers.

ii- [2 MARKS]

subtractive manufacturing. because objects are constructed by cutting away from a solid block of materials.

iii- [1 MARK]

for additive manufacturing,  
The more complex the object is, the time and expense it spend are much less than subtractive methods.