The Importance of Input-Output Network Structures in the U.S. Economy

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1 Introduction

In order to produce final consumption goods, industries (or firms) need to purchase a lot of commodities used as inputs in production process from other sectors (or firms). For instance, the production of cellphones needs not only electrical components, plastics, screens, but also transportation and financial services[1], etc. Therefore, Cellphone manufacturing industry trade with those industries as their common customer. In addition, the suppliers of the Cellphone manufacturing industry might also purchase intermediate inputs from a wider range of other sectors. The interconnected trade relationships across industries in the economy is seen as a production network or an input-output network . Intuitively, any shocks affecting the production of intermediate-good suppliers would ultimately impact the production of cellphones over linkages connecting one another. Then two questions might be raised: (i) Will the impact of microeconomic shocks, referring to sectoral-specific productivity (or TFP) shocks, be transmitted into macroeconomic fluctuations, or fade away eventually; (ii) What is the role of production network structures[2] in shaping the aggregate effect of sectoral productivity shocks. The first question has been resolved by (Acemoglu, Carvalho, Ozdaglar, & Tahbaz-Salehi, 2012) that at industry level, the impact of idiosyncratic sectoral shocks can be propagated through the input-output network, and finally transferred into significant aggregate fluctuations. Also, (Gabaix, 2011) and (V. Carvalho & Gabaix, 2013) point out that the "fundamental volatility" derived only by firm level idiosyncratic shocks can track the volatility of U.S. GDP growth quiet well before and after the Great Moderation, indicating aggregate volatility being primitively the results of micro shocks.

The main purpose of this paper is to evaluate the role of heterogeneous network structures in deciding aggregate fluctuations in the U.S. economy. I start by providing a set of empirical facts on the changing nature of the U.S. production network structure over time, as well as the relationship between input-output network structure and aggregate fluctuations. In particular, I use the Katz-Bonacich centrality, a measure of the relative importance of one industry as an input-supplier in the economy, to capture the heterogeneity in production network structure. On one hand, I observe a small number of industries, such as the Finance & Insurance or Professional Services, are connecting with more of other sectors and have become central suppliers in the U.S. economy from 1970 to 2017; while some other industries, like the Paper Product, are more isolated in the input-output network. On the other hand, this paper also presents a significant relationship between network heterogeneity and macroeconomic fluctuations. Specifically, in a panel of 46 industries for the period 1970-2017, I document three facts: (i) Industries with a wider connection to the rest of the economy tend to produce more outputs; and (ii) The economy where most industries are connected by a few central nodes or sectors has a relatively low aggre-

[1] In the example, the intermediate goods used in the production of cellphones are produced by the following industries: Electrical equipment, appliances, and components, Plastics and Rubber products, Transportation and Warehousing, and Finance and Insurance.

[2] The U.S. production network structure refers to the way that industries (upon different classifications) in the U.S. economy are connected or trade with one another.

gate real GDP growth, even controlling for other macroeconomic features; (iii) Real GDP growth is less volatile in an economy where industries centrality levels are more dispersive.

From a theoretical standpoint, I employ a static variant of multi-sector real business cycle model developed by (Baqaee & Farhi, 2019) to quantitatively assess the role of heterogeneous network structures in shaping macroeconomic impact of idiosyncratic TFP shocks. I evaluate the impact with a nonlinear characterization, since (Baqaee & Farhi, 2019) argue that some key features, such as production network details, has been ignored when only using a first-order approximation. Specifically, I generalize their findings to gauge the changes in aggregate fluctuations due to network heterogeneity. For example, if the U.S. economy experienced the same negative shocks as the recent financial crisis, but twenty years earlier with a less interconnected production network structure, whether or not a similar aggregate output collapse would reappear.

My main findings in this paper (so far) are as follows. First, the model suggests the input-output network structure has a crucial role in determining the aggregate impact of sector-specific TFP shocks, but the impact has to be evaluated under a nonlinear approximation. The welfare loss due to sector-specific TFP shocks is significantly greater in an economy where most industries are connected by a few primary suppliers, such as the Finance & Insurance. Second, in accordance with one of the empirical findings, in an economy where industries have more decentralized centrality levels, the real GDP growth has a smaller volatility. Third, the position of one industry as an input supplier to the rest of the economy (measured as the Katz-Bonacich centrality of an industry) can determine the macroeconomic effect of TFP shocks to this specific sector. The distinctions in aggregate impact due to heterogeneous network structures can be identified through both linear and nonlinear approximation.

[**literature review**]

2 Features of the Input-Output structure in the U.S. economy

In this section, I present some distinct features of the U.S. input-output network structure, as well as the empirical relationship between the network structure and macroeconomic fluctuations. I start by describing the data.

2.1 Data

Firstly, in order to map the U.S. production network to data, I use the input-output tables from the Bureau of Economic Analysis (BEA), which capture the input flow of goods from one industry to the other industries in the U.S. economy. The industrial annual data in this paper is collected at the three-digit industry level from 1970 to 2017[3]. The industries are classified according to the North American Industry Classification System (NAICS). The BEA defined 46 industries in the year 1947; while in 1963 and 1997, it revised the data collection mechanism and redefined 65 and 71 industries, respectively. In order to make the number of industries consistent in my sample period, I aggregate several industries back into the original 46-industry definition.[4]

Secondly, I combine the BEA's Make and Use tables[5] in each year to derive the Commodity-by-Commodity Direct Requirements (CCDR) tables from 1970 to 2017. The Make table documents the value (in producer's price) of each commodity produced in each industry; while the Use table reports the consumption of commodities by each industry or the final user. The Direct Requirements table is organized in the following way: each nonzero (i, j) entry captures an edge from the supplying industry i to its customer j in the network, implying the value of spending on good i per dollar of the production of good j. In each table, I normalize columns to sum to one, therefore, each element in jth column indicates j's input purchases from a supplying industry i as a fraction of its total sales.

Finally, the variables chosen for the estimation are sectoral centrality growth, sectoral real output growth and two corresponding cross-sectional volatilities of growth. The centrality of an industry indicates its relative importance as an input supplier in the production network, and more details will be described in subsection 2.4. I measure 46 sectors' real outputs from 1970 to 2017 using the chain-type quantity indexes for gross output by industry[6] from the BEA. In addition, two volatilities of growth are unweighed cross-sectional standard deviations.

2.2 The small world of input flows: Distance and Diameter

According to (V. M. Carvalho, 2014), the small-world network is a type of network in which most nodes are not neighbors of one another, but where most nodes can be reached from every other by a small number of hops or steps, resulting a short average distance and a small diameter within the network. In addition, in a small-world network, the typical distance l between two randomly chosen nodes grows proportionally to the logarithm of the number of nodes n. Intuitively, a small-world production network might lead to faster propagation of an idiosyncratic shock from one sector to the entire economy.

Formally, define the diameter of a network, d, as the maximum length of all ordered

[3] The BEA collects the two-digit and three-digit industry level data at an annual frequency from 1974 to 2017, and the five-digit industry level data in a five-year interval from 1972 to 2012.

[4] The BEA increases the number of industries in 1963 and 1997 by disaggregating existing industries.

[5] The Make and Use tables used in this paper are before redefinitions of secondary products. A redefinition is a transfer of a secondary product from the industry that produced it to the industry in which it is primary (Horowitz, Planting, et al., 2006). For example, the output and associated inputs for restaurants located in hotels are moved from the hotels and lodging places industry to the eating and drinking places industry.

[6] The gross output by industry data is adjusted using year 2002's prices and is available at https://apps.bea.gov/iTable/.

entries (i, j) of the shortest path from i to j. The average distance, l, represents the average length of shortest path for all entries (i, j). Table 1 lists the diameters and average distances of the U.S. input-output networks from 1970 to 2017.

When the U.S. economy is categorised into 46 industries, it has a diameter of 4 and an average distance about 2. In other words, it takes about two steps on average for one industry to reach another one within the network. The short average distance and a small diameter jointly indicate that industries with indirect demand-supply relationships are more likely to be connected through a few central nodes or sectors, which is consistent with (V. M. Carvalho, 2013) findings. For example, suppose industry A and B don't trade with each other, thus there is no direct connection between them. However, if both industries trade with a common industry C, two industries then will build indirect relationship with a relative small distance of two. Therefore, negative shocks, say shutdown or default, to a primary supplier in such an economy, the negative impact will be propagated very quickly to other industries, then generate aggregate fluctuations. In addition, since both diameters and average distances are consistent across years, in this context, the U.S. network structures have barely changed.

Table 1: Diameters and average distances of the U.S. IO networks from 1970 to 2017.

Year	l	d	Year	l	d	Year	l	d
1970	1.9245	4	1986	1.9269	4	2002	1.9586	4
1971	2.0108	4	1987	1.9653	4	2003	1.9667	4
1972	1.9643	4	1988	1.9638	4	2004	2.0063	4
1973	1.9724	4	1989	1.9563	4	2005	2.0034	4
1974	1.9945	5	1990	1.9533	4	2006	2.0000	5
1975	1.9574	4	1991	1.9484	4	2007	1.9411	4
1976	1.9259	4	1992	1.9837	4	2008	1.9463	4
1977	2.0006	4	1993	1.9863	4	2009	1.9164	4
1978	1.9716	4	1994	1.9273	4	2010	1.9473	4
1979	1.9317	4	1995	1.9173	4	2011	1.9731	5
1980	1.9148	4	1996	1.9430	5	2012	1.9670	4
1981	1.8981	4	1997	1.9204	4	2013	1.9766	4
1982	1.9240	4	1998	1.9261	4	2014	1.9550	4
1983	1.9376	4	1999	1.9795	4	2015	2.0138	5
1984	1.9311	4	2000	1.9653	5	2016	1.9877	4
1985	1.9149	4	2001	1.9704	4	2017	2.0511	5

2.3 A measure of network interconnection: Network Density

Network density measures industrial interconnection in a network, that is, the portion of all potential linkages among industries that are actually connected. The value of network density ranges from 0 to 1. The lower limit corresponds to the network without any inter-industrial connections; while the upper limit represents the network with all possible sectoral relationships. Industries in a denser production network tend to trade with a wider range of other industries. As a result, the adverse effect of negative shocks to a sector in such a network is more likely to be spread to other sectors. One potential drawback of the network density measure is that it only captures the number of linkages among industries in an economy, but is not able to identify which industries are actually connected. For instance, even though some industries have been changing their trading partners over time, the network density might still be the same.

As defined by (Foerster & Choi, 2017), an economy with N industries has a network density of L/N^2, where N^2 is the number of all possible links, and L is the actual interconnection. As mentioned earlier, I use the 46-industry classification throughout the sample period. In addition, I assume a link exists between industry i and j if j's intermediate goods purchases from i is at least one percent of its total input expenditures. On average, the network density from 1970 to 2017 is about 0.239, implying 506 out of 2,116 possible linkages between industries each year. Also, the network density has a standard deviation of 0.0086 over time. This 18-link deviation reveals that the variation of the degree of network interconnection is relatively small.

Figure 1: Network Density in the U.S. Economy from 1970 to 2017.

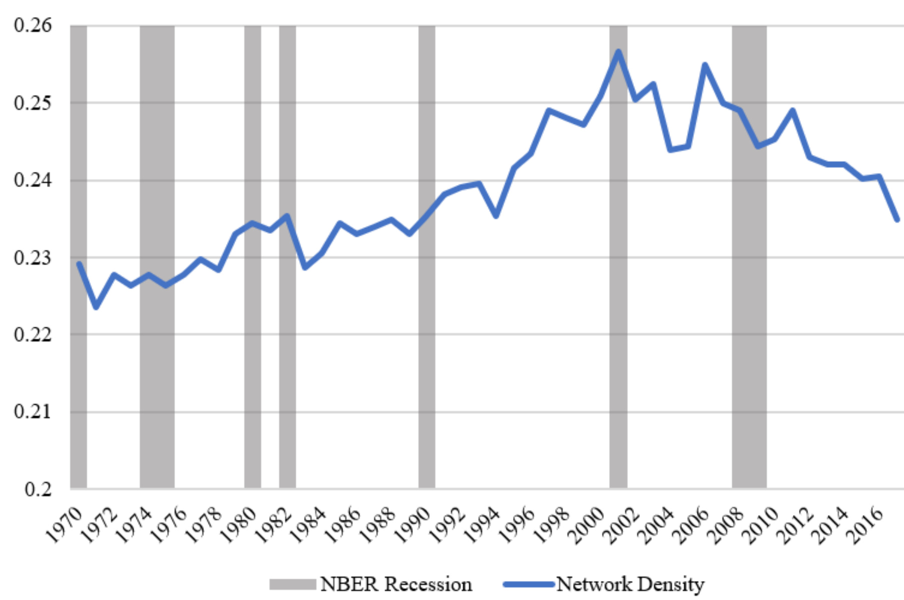


Figure 1 plots the network densities in the U.S. economy from 1970 to 2017. As seen in the figure, the U.S. network densities vary over time, but are always below 0.3. This number implies the actual connections across industries are less than 30% of all possible relationships. To some extent, the U.S. input-output network might not be very efficient in transmitting the impact of shocks, as it has to be propagated from sector to sector. Specifically, the network density exhibits an increasing trend until early 2000, indicating industries tend to form new trade relationship with more other industries over time. However, the trend has started to decline in 2001. One possible explanation might lie in the rapid technological improvement since late 1990s, industries have the opportunities to alter their intermediate input bundles in order to produce more efficiently. On one hand, supplying industries with sophisticated techniques tend to attract more customers. On the other hand, demanding sectors might break down relationships with previous low productivity suppliers and switch to better ones. Instead of trading with a wide range of partners, industries have a tendency to purchase inputs from a few major supplying industries, and that might lead to a less interconnected network structure in the U.S. economy. Also seen in the figure, there are a few large drops around the year 1970, 1982, 2001 and 2008, which corresponds to historical economic recessions. A drop in network density suggests a temporary loss in the interconnectedness among industries. It might be the case that during economic downturns, financial contraction leads sectors to cut input expenditures from existing partners, or even end previous trade relationship and switch to buy cheaper substitutes.

2.4 A measure of the relative importance of an industry: Katz-Bonacich Centrality

Centrality is one way to to identify the "key" industries in the economy, the industries with maximum interconnection with other industries in a production network.

While among several different measures, in this paper, I use the Katz-Bonacich (Katz for short) measure of centrality. It captures the relative importance of each industry in an input-output network.[7] Industries are considered to be more central if their neighbors are themselves well-connected industries. Intuitively, the higher the centrality is, the more important an industry becomes as a supplier. For instance, an industry with a centrality value of 0.2 has twice as much influence as a 0.1 centrality value industry. More importantly, this measure captures both direct and higher-order indirect connections of one industry with other industries. According to (V. M. Carvalho, 2014), the Katz centrality of industry j is proportional to a weighted sum of its neighbors' centralities, which is given by

$$c_j = \lambda \sum_{i=1}^{N} w_{ij} c_i + \eta$$

where $N = 46$ is the total number of industries in my sample. w_{ij} is the (i, j) element of a $N \times N$ input-output matrix $\mathbf{W}$, denoting a directed link from supplying industry i to demanding industry j. Specifically, $\mathbf{W}$ is the matrix representation of the U.S. production network. Besides, $\lambda > 0$ is an attenuation factor, and η is the identical baseline centrality level across industries. Since the Katz centrality captures both direct and undirected connections from industry i to others in the network, the

[7] According to the Perron-Frobenius theorem, the key of the eigenvector centrality measure is to look for the one with the largest nonnegative eigenvalue.

longer walks (more than an one-edge distance from i) will be penalized through the attenuation factor (Zhan, Gurung, & Parsa, 2017). In addition, the centrality value of each individual sector ranges from 0 to 1; while in any given year, the centralities of all sectors are normalized to one.

Rewrite the previous equation into a matrix form,

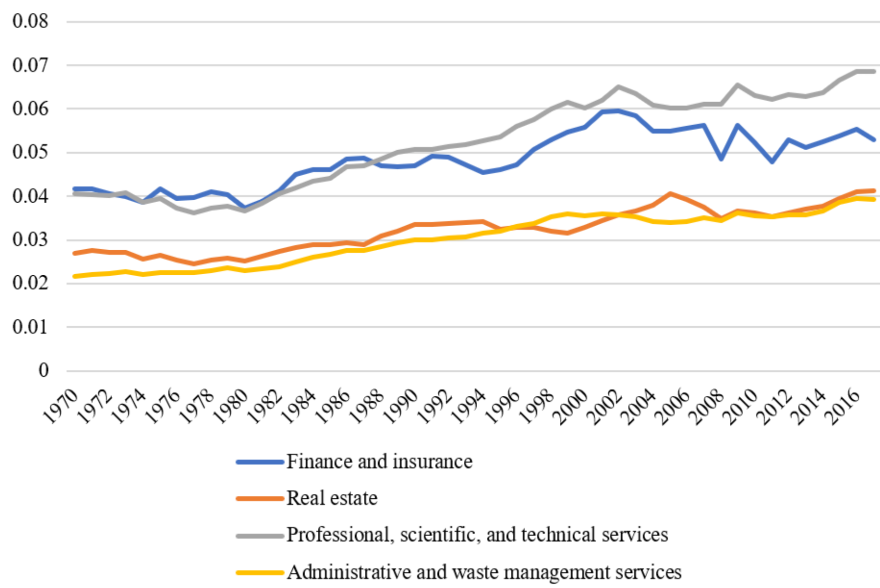
$$\mathbf{c} = \lambda \mathbf{W}'\mathbf{c} + \eta \mathbf{1}$$

therefore,

$$\mathbf{c} = \eta[\mathbf{I} - \lambda \mathbf{W}']^{-1}\mathbf{1}$$

where $\mathbf{c}$ and $\mathbf{1}$ are $N \times 1$ vectors, $\mathbf{I}$ is the identity matrix. Following (V. M. Carvalho, 2014), I set the attenuate factor $\lambda = 0.5$, and $\eta = (1 - \lambda)/N$.

Figure 2: Selected Industries that Increased in Centrality.



Using the above algorithm, I then calculate the centralities of 46 industries in the U.S. economy from 1970 to 2017, respectively. Figure 2 plots four selected industries that become more central, thus more important in the network over time. They are "Finance and Insurance", "Real Estate", "Professional, scientific, and technical services", and "Administrative and waste management services". Firstly, two industries, "Professional, scientific, and technical services" and "Administrative and waste management services", exhibit more than 50% increase in their centrality levels. It means these two industries have become more important suppliers in the input-output network. One possible explanation for the centrality jumps is that rather than hiring workers by themselves, sectors tend to outsource some particular job types, such as accounting and cleaning, to specialized servicing companies (Yuskavage, Strassner, & Medeiros, 2008). As a result, those service-related industries offer functional services to more other industries over time, thus have their centralities risen. Imagine the same idiosyncratic shock hitting the Professional Service industry but in different years, say 1970 and 2010. We might expect greater impact on the aggregate economy in the later year, since there were more other industries connected to the Professional Service in 2010 than that in 1970. Secondly, although the centralities of "Finance and insurance" and "Real estate" industries are different in magnitudes, they present similar periodic patterns. For example, the centrality levels of both industries were roughly the same from the late 1980s to early 1990s, respectively, and then started to rise. The increases in the importance of the Finance and the Real Estate sectors were coincident with the U.S. real estate boom starting around the mid-1990s. However, the centralities began to decline in 2007, when the recent financial crisis happened. A large number of subprime mortgage defaults triggered the collapse in the financial system. As the value of mortgage backed securities plummeted, the market for these securities evaporated and banks who were heavily invested in these assets began to experience a liquidity crisis (Zandi, 2009). The liquidity shortage forced banks to increase lending standards or even stop offering funds to other sectors. Consequently, industries lose their connections with financial institutions, thus the centrality of the Finance industry has declined.

Figure 3: Selected Industries that Decreased in Centrality.

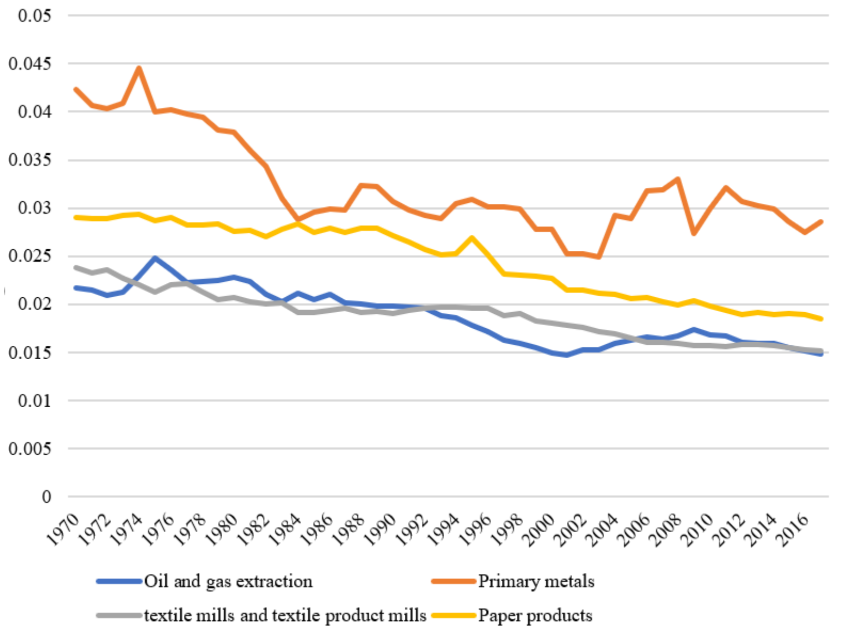
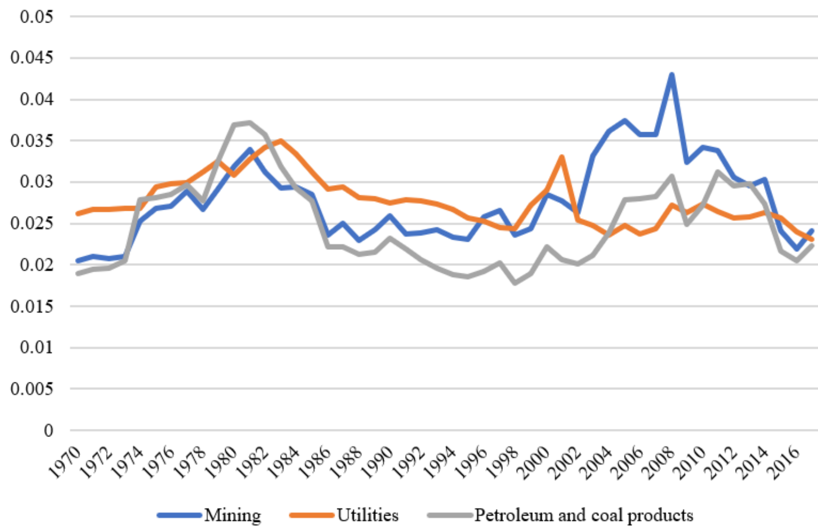


Figure 3 presents four selected industries that have decreased in centrality over time. In other words, the selected sectors turn out to be less important supplying products to the rest of the economy. Among all 46 U.S. industries, those with declined centralities mostly fall into the Manufacturing category. That is to say, the Manufacturing industry has been gradually losing its central position in the production network. For example, starting from the 1980s, more stringent environmental regulations have increased the costs of production in the Primary Metal industry in the U.S., as old equipment needed to be replaced by new environment-friendly ones (Andres, Ortiz, Viguri, & Irabien, 1995). Along with rising foreign competitions, domestic primary metal firms lose their market shares over time (Peden et al., 1998). Another possible explanation states that the industries used to buy raw ingredients from wide-ranging industries to produce on their own, now choose to outsource those particular products or (business service) activities to specialized sectors, and thus lose connections with previous suppliers (Yuskavage et al., 2008). In this sense, we should expect to observe declines in centralities of those industries over time.

Figure 4: Selected Industries that Fluctuate in Centrality.



In Figure 4, three selected industries share similarities in the centrality level at the beginning and the end of the sample period, and also in the spikes emerged in the early 1980s and late 2000s. Moreover, the centrality cycle has a countercyclical pattern, but with a lower frequency than the business cycle.

2.5 Summary Statistics of Centrality Across Years

In this section, I use two statistics, the skewness and kurtosis of industrial centrality distribution, as the determinants of network structures[8]. Learning how the U.S. production networks have changed over time is crucial to the study of aggregate fluctuations. Intuitively, (idiosyncratic) shocks will be propagated differently through different network structures, and generate inconsistent aggregate impacts.

I calculate the skewness and kurtosis of industrial centrality distribution in each year, and depict them in Figure 5 and 6, respectively. Skewness measures the asymmetry of a real-valued random variable about its mean in a probability distribution; while kurtosis is the measure of the "tailedness" of a probability distribution. The increasing trend of skewness in Figure 5 indicates a gradual right-skewed centrality distribution over time. In other words, the "body" of the distribution has been shifting to the left, which makes its tail heavier. Also in Figure 6, as the value of kur-

[8]The ways industries are connected with one another in the input-output production network.

Figure 5: Centrality Skewness from 1970 to 2017.

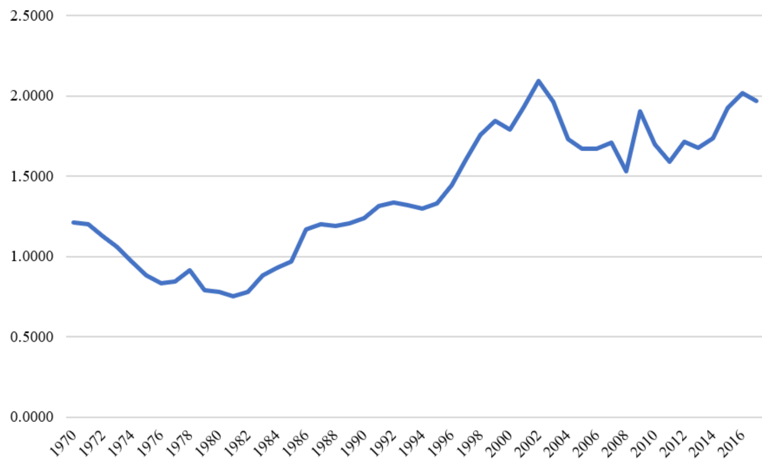
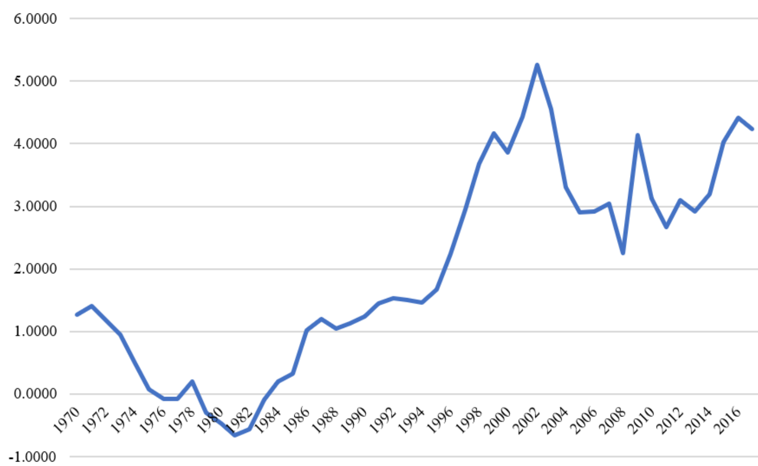


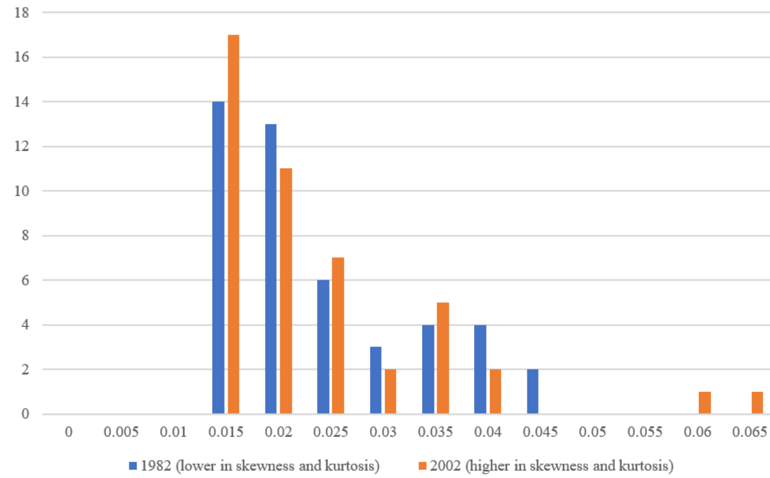
Figure 6: Centrality Kurtosis from 1970 to 2017.



tosis getting higher, the “body” of probability distribution would become narrower or sharper, which implies a thicker right tail.

Moreover, in Figure 7, I plot two frequency distributions of industrial centrality, in the year 1982 and 2002 respectively, to illustrate the way two statistics altering the network structures. The blue bars show the frequency distribution in 1982 with relatively low skewness and kurtosis; while the orange bars represent year 2002, which has higher skewness and kurtosis. On one hand, there are more industries in 2002 with centralities smaller than 0.025, comparing with 1982. It implies those less important industries in 1982 have maintained low connections with other industries in the latter year. On the other hand, two industries, “Finance and Insurance” and “Professional, scientific, and technical services” in particular, increased their centralities sharply from about 0.4 to 0.6. An increase in centrality level suggests a more central position as an input-supplier in the network. Imagine an industry hit by a negative productivity shocks in a network where it is more intensively connected to the rest of economy like the year 2002. The shocks will generate a greater decline in

Figure 7: Frequency Distribution of Centrality in 1982 and 2002.



GDP than it would in 1982.

3 Aggregate Fluctuations and the Network Structure

In the previous section, I have shown that the U.S. input-output network structure has been changing over time due to changes in industrial interconnections and in the roles of industries as input-suppliers in the production network. In this section, I explore the empirical relationship between the changing network structure and aggregate fluctuations[9].

3.1 Sectoral Real Output and the Network Structure

First, I estimate the correlation between sectoral real output and its centrality from 1970 to 2017 by running the following OLS regression:

$$log(real\ output_{iT}) = \beta_1 * log(centrality_{iT}) + \mathbf{X'_{iT}}\gamma_1 + e_{iT} \tag{1}$$

where $real\ output_{iT}$ is the quantity indexes of gross output in sector i at time T and $centrality_{iT}$ represents the centrality score of industry i in period T. I also choose log intermediate sales to gross output ratio (Miranda-Pinto, 2019) as the (only) control variable in vector $\mathbf{X_{iT}}$.

Fact 1 *There is a positive relationship between sectoral real output and its centrality.*

Table 2 presents the results of pooled OLS and fixed effects regression. Based on fixed effect results, there is a significant positive correlation between sector's centrality

[9] Aggregate fluctuations in this section refer to changes in real outputs, in real GDP growth and in growth volatility.

and its real output. A one percent increase in centrality level leads to a 0.99 percent rise in real output. Intuitively, an industry with a higher centrality value means it supplies more products to its customers, thus produces more outputs (greater real outputs). The result is also robust by controlling for the sectoral intermediate sales share. In addition, I estimate the same relationship but using sectoral real value added as the independent variable instead, the results are very similar as shown in table 2 and robust as well.

Table 2: Real Output and the Centrality, 1970-2017.

Variables	(1) log $r.\ output_{iT}$	(2) log $r.\ output_{iT}$	(3) log $r.\ output_{iT}$	(4) log $r.\ output_{iT}$
log $centrality_{iT}$	-0.154***	-0.147***	0.991***	0.880***
	(0.03)	(0.028)	(0.092)	(0.090)
log $int.\ ratio_{iT}$		0.628***		0.683***
		(0.035)		(0.068)
Sector Fixed Effects	No	No	Yes	Yes
R-squared	0.012	0.140	0.506	0.528
Observations	2208	2208	2208	2208

[1] Note: Column (1) and (2) are the coefficients of a pooled OLS regression; while column (3) and (4) present the results of fixed effects. There is one observation in each year from 1970 to 2017, bringing the number of observations to 48 for every sector, and 2028 (48×46) in total.
[2] $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$, and robust standard errors are in the parentheses.

Second, I regress sectoral real output on its centrality score by each sector, and lay out all coefficients in Table 3. Thirty-one out of forty-six industries in the U.S. production network have shown strong correlations between sectoral real output and its centrality. That is to say, the changes in industries centrality do affect their outputs. For example, when the centrality level of the Finance & Insurance industry rises by one percent, its real output increases by 1.1 percent. These results are also robust when (i) I choose log intermediate sales to gross output ratio (Miranda-Pinto, 2019) as the control variable; (ii) I use sectoral real value added as the dependent variable.

Table 3: Correlation between Real Output and Centrality by Industry.

Industry	Coefficient	Industry	Coefficient	Industry	Coefficient
Farms	-0.537***	Motor vehicles, bodies and	2.175***	Transportation and	-0.373***
	(0.139)	trailers, and parts	(0.515)	Warehousing	(0.127)
Forestry, fishing, and	1.25***	Other transportation	0.761***	Information	0.281**
related activities	(0.329)	equipment	(0.246)		(0.137)
Oil and gas extraction	-0.361***	Furniture and related	1.533	Finance and Insurance	1.105***
	(0.123)	products	(1.262)		(0.154)
Mining, except oil	-0.398**	Miscellaneous	0.443	Real estate	0.24*
and gas	(0.168)	manufacturing	(0.778)		(0.139)
Support activities for	8.214***	Food and beverage and	-0.672***	Rental and leasing	1.546***
mining	(0.962)	tobacco products	(0.084)	services	(0.144)
Utilities	0.43**	Textile mills and textile	2.824***	Professional, scientific,	1.172***
	(0.189)	product mills	(0.781)	and technical services	(0.242)
Construction	-0.164	Apparel and leather and	-1.89	Management of companies	-0.165
	(0.317)	allied products	(1.512)	and enterprises	(0.199)
Wood products	2.896***	Paper products	1.825***	Administrative and	1.985***
	(0.346)		(0.195)	waste management services	(0.125)
Nonmetallic mineral	2.537***	Printing and related	3.192***	Educational services	-0.281
products	(0.792)	support activities	(0.173)		(0.247)
Primary metals	0.817***	Petroleum and coal	0.216***	Health care and	2.795***
	(0.129)	products	(0.036)	social services	(0.411)
Fabricated	-0.054	Chemical products	0.016	Arts, entertainment,	-1.879*
metal products	(0.287)		(0.287)	and recreation	(1.055)
Machinery	0.836	Plastics and rubber	3.18***	Accommodation	2.21***
	(0.568)	products	(0.346)		(0.632)
Computer and	1.72***	Wholesale trade	-0.121	Food services and	0.37**
electronic products	(0.362)		(0.302)	drinking places	(0.184)
Electrical equipment,	0.516	Retail trade	0.305	Other services,	1.195***
and components	(0.845)		(0.257)	except government	(0.09)

[1] Note: This table presents coefficients of OLS regression in 42 sectors, using log sectoral real output as the dependent variable and log sectoral centrality as the only independent variable. There are 48 observations in each regression. Government-related sectors are excluded from the sample due to collinearity.
[2] $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$, and robust standard errors are in the parentheses.

3.2 Aggregate Real GDP Growth, Volatility and the Network Structure

In the previous section, regression results imply a positive correlation between industry centrality level and its real gross outputs. In this section, I confirm that the centrality level also significantly correlated with the aggregate GDP growth and the growth volatility.

To begin with, I consider the following OLS regression:

$$RGDP\ growth_T = \beta_2 * Std.centrality_T + \bar{\mathbf{X}}'_{\mathbf{T}}\gamma_2 + \bar{e}_T \tag{2}$$

where $RGDP\ growth_T$ represents the U.S. real GDP growth at time T, and the independent variable $Std.centrality_T$ is the cross-sectional standard deviation of centrality in period T. The purpose to choose $Std.centrality_T$ as the most interested independent variable is that it can be considered as a critical indicator capturing network structure heterogeneity. I also introduce other determinants of real GDP growth as controls from previous literature, such as the ratio of sales in the service sector[10] over GDP (Moro, 2012), and the intermediate sales to GDP ratio (Miranda-Pinto, 2019).

Table 4: Real GDP Growth and the Std. of Centrality, 1970-2017.

Variables	(1) $RGDP\ growth_T$	(2) $RGDP\ growth_T$	(3) $RGDP\ growth_T$
$Std.centrality_T$	-1.522***	-1.885***	-1.976***
	(0.425)	(0.486)	(0.483)
$Serv.\ ratio_T$		-0.377	-0.811**
		(0.268)	(0.411)
$Int.\ ratio_T$			-0.144
			(0.153)
Observations	48	48	48

[1] Note: This table presents the coefficients of OLS regression, using real GDP growth as the dependent variable.
[2] $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$, and robust standard errors are in the parentheses.

Fact 2 *Aggregate real GDP growth and the standard deviation of cross-sectional centrality are negatively correlated.*

The regression coefficients are exhibited in Table 4, which suggest that an increase in the standard deviation of centrality corresponds to declines in aggregate GDP

[10] 21 out of 46 industries are selected to be the members of the broad service sector, including Utility, 16 private service producing industries and 4 government-related industries.

growth. A greater centrality variation implies that industries in a production network have more dispersive centrality levels, such as the network structure in the latter years of my sample period. Also, the empirical facts have shown earlier that service-related sectors have become more important as input suppliers in the U.S. economy. Along with the findings of (Moro, 2015) that service sectors usually have relatively lower TFP growth in the economy, it is reasonable to expect a declining aggregate GDP growth over time. As seen in the table, my findings are also robust when adding some controls.

Next, I estimate the correlation between centrality variation and the volatility of real GDP growth using the equation below.

$$Growth\ volatility_T = \beta_3 * Std.centrality_T + \tilde{\mathbf{X}}'_{\mathbf{T}}\gamma_3 + \tilde{e}_T \tag{3}$$

where $Growth\ volatility_T$ denotes the standard deviation of U.S. real GDP growth at time T, and the independent variable $Std.centrality_T$ is still the cross-sectional standard deviation of centrality in period T. Again, the variable is chosen as an index of network structure heterogeneity. I follow (Cecchetti, Flores-Lagunes, & Krause, 2006) methodology to estimate volatility of real GDP growth. Specifically, I regress the first difference of log real GDP on its first lags from 1970 to 2017, and obtain a series of estimated residuals, say $\hat{\varepsilon}_t$. As argued by (McConnell & Perez-Quiros, 2000), if residual ε_t follows a normal distribution, then the transformed residual, $\sqrt{\frac{\pi}{2}}\ |\hat{\varepsilon}_t|$, are unbiased estimator of the standard deviation of ε_t. I use the unbiased estimator as real GDP growth volatility, which is also the dependent variable in the regression. I also include some other control variables in the regression.

Table 5: Real GDP Growth Volatility and the Centrality Variation, 1970-2017.

Variables	(1) $Growth\ Volatility_T$	(2) $Growth\ Volatility_T$	(3) $Growth\ Volatility_T$
$Std.centrality_T$	-0.439***	-0.652***	-0.725***
	(0.137)	(0.155)	(0.172)
$Serv.\ ratio_T$		-0.222**	-0.231**
		(0.010)	(0.096)
$Std.cent \times Serv.\ ratio_T$			0.125
			(0.175)
Observations	48	48	48
R-squared	0.136	0.247	0.254

[1] Note: This table presents the coefficients of OLS regression, using the standard deviation of real GDP growth as the dependent variable.
[2] $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$, and robust standard errors are in the parentheses.

Fact 3 *The relationship between real GDP growth volatility and the centrality variation is negative.*

Table 5 presents the results of estimating equation (3). There is a strong negative relationship between the cross-sectional standard deviation of centrality and real GDP growth volatility. A one percent increase in centrality variation is associated with a 0.44% decline in aggregate volatility. The centrality variation alone accounts for about 14% ($R^2 = 0.136$) of the variation in growth volatility.

The result is also robust when adding control variables such as the ratio of service sales over GDP (Moro, 2012), and the intersection between centrality volatility and the service sales share. The service share in the economy mitigates the negative correlation between centrality and growth volatilities. As shown in column (3), the coefficient of the intersection term is (insignificantly) positive, which means that in a high-service-share input-output network, industries' centrality variation has a weaker (less negative) impact on growth volatility. In addition, centrality volatility and the service share together explain aroung 25% of the observed variation in growth volatility.

Overall, the innovative empirical facts shown in Section 3.1 and 3.2 contribute to understanding the relationship between the network structure[11] and macroeconomic fluctuations.

4 The Benchmark Nested-CES Network

Following (Atalay, 2017) and (Baqaee & Farhi, 2019), a nested constant elasticity of substitution (CES) economy with intersectoral linkages is displayed in this section. Throughout paragraphs, variables with over-lines are normalizing constants equal to their steady state values. Since I focus on percentage changes in GDP, the normalizing constants are irrelevant.

Firms

The benchmark economy consists of N competitive industries. Each industry $i \in \{1, 2, ..., N\}$ produces a distinct good with a single factor of production (labor), intermediate inputs and one CES nest. The production function is given by

$$\frac{y_i}{\bar{y}_i} = A_i \left[a_i \left(\frac{L_i}{\bar{L}_i} \right)^{\frac{\theta_i - 1}{\theta_i}} + (1 - a_i) \left(\frac{X_i}{\bar{X}_i} \right)^{\frac{\theta_i - 1}{\theta_i}} \right]^{\frac{\theta_i}{\theta_i - 1}} \tag{4}$$

where L_i is the amount of labor in production, and X_i is the intermediate bundle used by industry i. A_i is the factor-neutral technology level, and parameter a_i reflects the long-run average usage of labor in i. The elasticity of substitution parameter θ_i measures how easily factors of production are substituted.

The intermediate bundle X_i used by industry i is produced through a combination

[11] In particular, I use the characteristics of the Katz-Bonacich centrality measure as the key determinants of the U.S. network structure heterogeneity.

of intermediate goods purchased from other sectors:

$$\frac{X_i}{\bar{X}_i} = \left(\sum_{j=1}^{N} \gamma_{ij} \left(\frac{x_{ij}}{\bar{x}_{ij}} \right)^{\frac{\varepsilon_i - 1}{\varepsilon_i}} \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}} \tag{5}$$

where x_{ij} is the quantity of inputs purchased by i from its supplier j. $\gamma_{ij} \geq 0$ is designated as the share of good j in the total intermediate input use in sector i, which captures the importance of j's goods in the production of industry i's intermediate bundle. Also, I assume intermediate good bundle is produced with constant return to scale technology, $\sum_{j=1}^{N} \gamma_{ij} = 1$. In the empirical study, γ_{ij} corresponds to the (j, i) element in the input-output matrix $\mathbf{W}$, measuring the expenditures on input j per dollar of production of good i. The elasticity of substitution parameter ε_i captures the substitutability across intermediate goods demanded by sector i.

Following (Baqaee & Farhi, 2019), I allow two types of labor[12], the specific l_{is_i} and the general labor l_{ig}, in production function. In particular, the specific labor can only be used by sector i, since it cannot be reallocated anywhere else; while the general labor can be freely used by all industries without transaction costs. Consider two polar cases (i) when $\theta_i = 1$, only industry-specific labor exsits, but it cannot be moved around; (ii) when $\theta_i = 0$, all labor can be flexibly reallocated across sectors. In a short time horizon, as labor is hard to adjust, we might expect a lower degree of labor reallocation than in a longer horizon.

The total labor supplied in industry i is organized as

$$\frac{L_i}{\bar{L}_i} = \left(\frac{l_{is_i}}{\bar{l}_{is_i}}\right)^{\beta_i} \left(\frac{l_{ig}}{\bar{l}_{ig}}\right)^{1-\beta_i} \tag{6}$$

where β_i is the portion of specific labor in total labor supplied in sector i. In addition, two types of labor are in fixed supplies, such that $\bar{l}_{s_i} = \bar{l}_{is_i}$ and $\bar{l}_g = \sum_{i=1}^{N} \bar{l}_{ig}$.

Household's Preferences

There is one representative household, who has the preference over leisure and N different consumption goods. The utility function is as follows

$$U(C, L) = C - \frac{\varepsilon_{LS}}{\varepsilon_{LS} + 1} L^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} \tag{7}$$

subject to budget constraint

$$wL + \sum_{i=1}^{N} \pi_i = P_c C$$

and aggregate consumption function

$$\frac{C}{\bar{C}} = \left(\sum_{i=1}^{N} b_i \left(\frac{c_i}{\bar{c}_i}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

[12](Baqaee & Farhi, 2019) argues that the degree of factor (labor) reallocation can affect aggregate impact of sectoral TFP shocks in a nonlinear approximation

where C represents household's aggregate consumption bundle over all final goods c_i, and P_c is the associated ideal price index, which is assumed to be a numeraire. The household also supplies labor L in total, and receives a profit of π_i from sector i ($\pi_i = 0$ for each i in equilibrium). The Frish elasticity of labor supply $\varepsilon_{LS} > 0$ describes the sensitivity of household's desired labor supply to a given wage w. In addition, b_i illustrates the importance of good i in his/her preference which satisfies $\sum_{i=1}^{N} b_i = 1$. The elasticity of substitution σ describes how easily different consumption goods are substituted.

Competitive Equilibrium

A decentralized competitive equilibrium of this CES-nested economy is a set of prices $\{w, p_i, p_i^X\}_{i=1}^N$, an allocation $\{C, L, c_i\}_{i=1}^N$ for the representative household, and an allocation $\{L_i, y_i, \{x_{ij}\}_{j=1}^N\}_{i=1}^N$ for firms (industries) given a vector of sectoral productivity shocks $\{A_i\}_{i=1}^N$, such that

- Firms (Industries) maximize profits;
- The representative household maximizes utility as in equation (7) subject to constraints;
- Goods and labor markets clear

$$y_i = c_i + \sum_{j=1}^{N} x_{ji},$$

$$L = \sum_{i=1}^{N} L_i.$$

4.1 The Role of Network Structure in Shaping Second-Order Aggregate Impact

In this section, I will study model implications for the relationship between the network structure and the aggregate impact of sector-specific productivity shocks. The impact is estimated up to a second-order approximation where the network structure plays in crucial role.

I start by presenting the definition of the input-output covariance operator, proposed by (Baqaee & Farhi, 2019).

Definition 1 *The input-output covariance operator is defined as*

$$cov_{\Gamma^k}(\Psi_i, \Psi_j) = \sum_{r=1}^{N} \gamma_{kr}\Psi_{ri}\Psi_{rj} - \left(\sum_{r=1}^{N} \gamma_{kr}\Psi_{ri}\right)\left(\sum_{r=1}^{N} \gamma_{kr}\Psi_{rj}\right). \tag{8}$$

The operator illustrates the covariance between ith and jth columns of the Leontief inverse using the kth row of an input-output matrix as the distribution. The economy's Leontief inverse is defined as $\Psi = (I - \Gamma)^{-1}$, where Γ is the model-implied input-output matrix. Intuitively, Ψ_{ij} captures industry i's total (directed and indirected) reliance on j as its input-supplier. Moreover, the element γ_{ij} is specified as the steady state value of $\frac{p_j x_{ij}}{p_i y_i}$.

Proposition 1 *In any efficient economy, the impact of an idiosyncratic TFP shock to industry i on the aggregate economy is i's Domar weight up to a first-order approximation.*

$$\frac{d\ logY}{d\ logA_i} = \lambda_i \tag{9}$$

where $\lambda_i = p_i y_i / \sum_{j=1}^{N} p_j c_j$ *is i's total sales over GDP ratio, which is defined as the Domar weight of industry i.*

(Hulten, 1978) states that as long as an economy is efficient, Domar weight is a sufficient statistics in determining the impact of industry-specific TFP shocks on aggregate GDP. In other words, other microeconomic details, such as the inter-industrial linkages, etc., are completely irrelevant. However, it seems implausible when you consider the following example. In 2010, Construction and Finance industries had very similar gross sales, so does the Domar weights, which was about 7 percent. If Hulten's Theorem holds true, productivity shocks should generate analogous fluctuations on GDP. But it is more persuasive, as the Finance sector supplies to more sectors than Construction, adverse productivity shocks to Finance will be more damaging to the whole economy. This example tells us that industries interconnection might play an important role in modelling the macroeconomic impact of productivity shocks, which has been ignored by a first-order approximation.

Proposition 2 *In a nested-CES economy with only one factor of production, the second-order aggregate impact of idiosyncratic shocks is*

$$\frac{d^2\ logY}{d\ logA_j\ d\ logA_i} = \frac{d\ \lambda_i}{d\ logA_j} = \sum_{k=1}^{N} (\varepsilon_k - 1)\lambda_k cov_{\Gamma^k}(\Psi_i, \Psi_j). \tag{10}$$

Proposition 2 specifies three key microeconomic determinants in analysing the aggregate impact on GDP up to a second-order approximation. They are the elasticity of substitution across intermediates used by industry k, ε_k, Domar weight of k, λ_k, and the input-output covariance operator defined earlier.

As shown in equation (10), the second-order impact can also be expressed by the change in i's Domar weight in response to a productivity shock to industry j. Intuitively, the change in i's sales depends on how much i's direct and indirect customers are exposed to j's productivity shocks simultaneously. Suppose there are negative TFP shocks $d\ logA_j < 0$ hitting industry j, which leads to an increase in good j's price. Now considering any given producer k, if $\varepsilon_k > 1$, the producer will substitute away from the intermediates that are (directly or indirectly) affected by negative shocks. In other words, industry k will decrease its demanding expenditures on good j-related inputs[13] and switch to cheaper substitutes. Meanwhile, if those affected

[13]The total reliance on good j by all industries in the network is captured by Ψ_j, the jth column of Leontief inverse.

inputs are also (directly or indirectly) related to industry i, the demand of i's output will be affected as well, which is measured by Ψ_i.

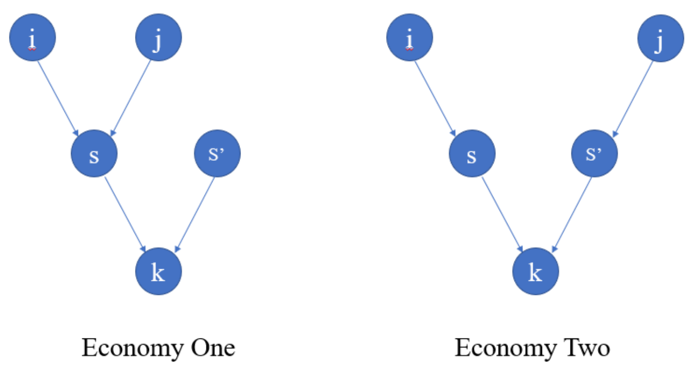
Again, the covariance operator is calculated with the input-output matrix Γ, which is also used in the Katz-Bonacich centrality measure. Therefore, the network structure (inter-sectoral linkages) does matter in shaping the second-order macroeconomic fluctuation.

4.2 The Role of Elasticity of Substitution in Deciding Macroeconomic Fluctuations

To begin with, I explain how the elasticity of substitution and input-output network structure are correlated. In general, negative productivity shocks to industry i can be propagated through the production network in two different channels. On one hand, it leads to an increase in good i's price. Consequently, all sectors using good i as intermediate inputs in production, either directly or indirectly, will be affected negatively. On the other hand, shocks to i might also cause reallocations in the demand for good j as intermediates, depending on the complementarity across two sectors.

To elaborate the second channel, let's consider two simple economies in Figure 8. In each economy, producer (or industry) k has two (potential) indirect suppliers i and j ($j \neq i$). Then I present how the productivity shocks to sector j affect i's sales differently, given k's complementarity across inputs in production.

Figure 8: Two Simple Economies



In economy one, sector i and j share a common customer s, which is also the direct supplier of producer k. When sector j hit by negative productivity shocks, the price of j's output rises. Now suppose k has a structural elasticity of substitution across intermediate inputs being greater than one, $\varepsilon_k > 1$. Therefore, k will substitute away from good s, that is affected negatively by shocks, to a new product s'. This reallocation leads to a decline in the demand for good i as well, since i and j overlap in the production chain. However, if good s and s' are complements to producer k, the need for i might stay the same or decline with much less amount than the substitutes case.

In the second economy, industry i and j connect with k through two separate supply chains. i and j are also direct suppliers of s and s', respectively. Again, negative TFP shocks to j make j-related goods more expensive. When $\varepsilon_k > 1$, k will purchase good s instead of s', as they are gross substitutes. As a result, the demand for good i will increase. But when $\varepsilon_k < 1$, i's sales will probably drop.

This example illustrates that given different complementarity values, the reallocation of the demand for inputs in production (by sector k) changes correspondingly subject to productivity shocks. Differences in reallocation pattern lead to various input-output network structures. Thus, the correlation between elasticities of substitution across intermediates and the network structure can be confirmed. Recall that inter-sectoral linkages[14] is one of the key features of the second-order aggregate impact. Therefore, elasticity of substitution does play a non-trivial role in deciding macroeconomic fluctuations.

5 Quantitative Application

In this section, I develop some quantitative applications to assess the role of U.S. input-output structures[15] in shaping aggregate impact of idiosyncratic TFP shocks. I consider both the nonlinear impact evaluated using the benchmark CES-nested model specified in the previous section, and the linear effect via a (close to) Cobb-Douglas calibration. First, I calibrate the multi-sector business cycle model with sectoral TFP shocks using the U.S. input-output tables. In particular, based on the estimates of previous literature, I choose two structural elasticities of substitution in firm's production. Second, in order to evaluate the role of network structures in determining macroeconomic impact, I compare model-simulated GDP growth given different network structures. In addition, I compare the outcomes between the benchmark nonlinear model and a Cobb-Douglas model up to the first-order approximation. The results emphasize the practical importance of nonlinearities. Finally, I study the aggregate impact of the recent financial crisis in both linear and nonlinear frameworks.

5.1 Calibration Targets

5.1.1 Household's Preferences

1. b_i is the demand parameter reflecting the importance of consumption good i in household's preference. Thus I match b_i with industry i's final consumption to its total output ratio. The consumption expenditures are taken from the BEA's input-output tables, as sales to the following industry codes: F010 (personal consumption expenditures), F02R (residential private fixed investment), and F040 (exports). With the aim of improving numerical performance of steady state and simulation, I bound the preference weights, b_i, at 0.006.

[14] The network structure again is defined as the way of industries being connected or trading with one another, which is specified by the input-output matrix.

[15] The change of the U.S. input-output structures refers to the change in industries' positions as direct and indirect suppliers to one another.

2. According to (Atalay, 2017) and (Herrendorf, Rogerson, & Valentinyi, 2013), I set the elasticity of substitution in consumption across industries slightly less than one, which is $\sigma = 0.9$ in the benchmark case. Intuitively, a higher σ implies that households respond to an increase in relative price of one consumption good (produced in the corresponding industry) by substituting away from it.

3. Following (V. Carvalho & Gabaix, 2013), I interpret the Frisch elasticity of labor supply broadly, including changes in hours worked per worker, changes in employment, as well as changes in effort. Therefore, I choose $\varepsilon_{LS} = 2$ for this elasticity.

5.1.2 Firm's Production

Since the estimation of elasticities of substitution in a more disaggregated level is not applicable (Baqaee & Farhi, 2019), in the paper I assume two structural elasticities in firm's production functions. Therefore, the i subscript in the elasticity parameters will be dropped.

1. According to the estimation of (Atalay, 2017), the structural elasticity of substitution across factors of production should be between 0.4 and 0.8. Thus I choose $\theta = 0.5$ for the benchmark model.

2. The parameter $(1 - a_i)$ reflects long-run average usage of intermediate inputs in industry i. Therefore, I match $(1 - a_i)$ with industry i's intermediate input cost over gross output ratio; thus a_i is left to match with the value added to total output ratio in each industry.

3. Based on (Atalay, 2017) estimates, one cannot reject zero or slightly positive values for the structural elasticity of substitution across intermediate goods. I set $\varepsilon = 0.001$ in the benchmark case to be consistent with his estimation.

4. The parameter γ_{ij} in the intermediate goods production function matches with the (j, i) element of the input-output matrix $\mathbf{W}$, which reflects the importance of industry j in the production of industry i's intermediate bundle. The empirical counterpart of $\mathbf{W}$ is the Commodity-by-Commodity Direct Requirements table derived from BEA's Make and Use tables.

5.1.3 Sectoral TFP Shocks

To construct sectoral productivity shock series at an annual frequency from 1970 to 2014, I combine the 46 industries[16] US KLEMS annual input-output data organized by (D. W. Jorgenson, Ho, & Samuels, 2017), with the BEA's Use tables. I use the methodology developed by (D. Jorgenson, Gollop, & Fraumeni, 2016), which is also used by (V. Carvalho & Gabaix, 2013).

[16] The original data contains 65 industries based on the NASIC classification, so I aggregate some sectors to have 46 industries in total.

Following (Baqaee & Farhi, 2019), I specify sectoral productivity shocks to be log-normally distributed in the way that $logA_i \sim N(-\Sigma_{ii}/2,\ \Sigma_{ii})$, where Σ_{ii} is the sample variance of log TFP growth in industry i. In this paper, I work with uncorrelated sectoral shocks. (The correlated case has not been tested yet.)

5.2 Primary Results

5.2.1 The Aggregate Effect of Sectoral TFP Shocks

Table 6 displays the mean, standard deviation and skewness of log GDP for different specifications based on the production network structure in 1982 and 2002, respectively. Two years are chosen as representatives of the most differentiated input-output network (in terms of centrality features) of the U.S. economy. In particular, year 2002 has the highest skewness and kurtosis of network centrality distribution in the sample period; while year 1982 is the reverse. The table also shows the standard deviation of real GDP growth from 1970 to 2014.

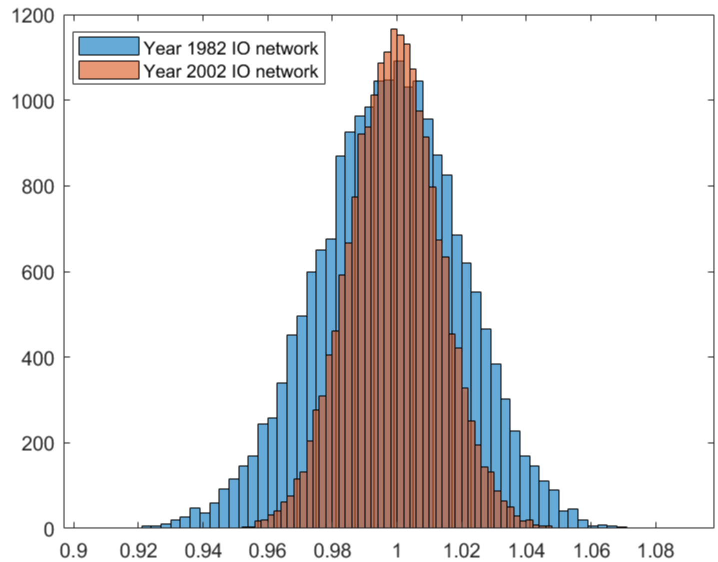
Table 6: Simulated Moments in 1982 and 2002.

$(\sigma, \theta, \varepsilon)$	Mean ($\times$100)	Standard Deviation ($\times$100)	Skewness
Data			
real GDP growth	-	2.2	-
Year 1982			
$(0.9, 0.5, 0.001)$	-0.22	2.2	-0.16
$(0.99, 0.99, 0.99)$	-0.12	2.2	-0.02
Year 2002			
$(0.9, 0.5, 0.001)$	-0.11	1.4	-0.04
$(0.99, 0.99, 0.99)$	-0.09	1.4	-0.01

[1] The sample moments shown in the table are model-simulated log GDP, and are calculated from 20,000 draws.
[2] For real GDP growth, I use the methodology of (D. Jorgenson et al., 2016).

Firstly, let's consider the benchmark calibration. As mentioned previously, I set three important structural elasticities as $(\sigma, \theta, \varepsilon) = (0.9, 0.5, 0.001)$ for the benchmark model. Again, they are time-invariant elasticity of substitution across consumption goods σ, between intermediate inputs and value added θ, and across intermediate goods ε. When the U.S. production network is organized in the manner of year 1982, the mean is -0.0022 log points. On one hand, it means the loss generated by empirical sector-specific productivity shocks is about 0.22% of output. On the other hand, the loss is due to nonlinearities in production. While within 2002's input-output network, the aggregate output only reduces by 0.11% subject to the same TFP shocks[17]. It is obvious with different production network structures, even identical sectoral TFP shocks might be propagated into significantly different aggregate impact. However, in a loglinear Cobb-Douglas characterization with $(\sigma, \theta, \varepsilon) = (0.99, 0.99, 0.99)$, the mean of log GDP are -0.12% and -0.09%, respectively. Even though two networks are distinctive in structures, the linear aggregate impact is not quiet different from each other, compared to the benchmark case. In other words, without nonlinearity, network structure plays minor role in determining macroeconomic impact of sectoral productivity shocks.

Figure 9: Model-Simulated log GDP.



Secondly, regardless of model specification, the standard deviations of log aggregate output[18] are 2.2 in year 1982, and 1.4 in 2002. This finding is consistent with Fact 3 that when industries in the network have more dispersive centrality scores (higher centrality variation), the real GDP growth is less volatile (a smaller standard deviation). Also, the standard deviation is constant across two specifications within the same network structure, respectively. With nonlinearity, second-order terms will amplify some shocks but mitigate other shocks, leaving a relatively stable variation. This finding is in accordance with (Baqaee & Farhi, 2019).

Finally, I plot two distributions of benchmark model-simulated log GDP based on different network structures in Figure 9. The standard deviation of model-simulated log aggregate output[19] in 1982 and 2002 is quiet different, which are 0.022 and 0.014,

[17]TFP shocks series are cross-sectional uncorrelated and drawn from a multivariate log-normal distribution with the same mean and variance.

[18]In the model, the stead state value of $Y/\bar{Y}$ equals one, which yields a log value of zero. Therefore, I calculate the standard deviation of log aggregate output as the simulated real GDP growth volatility.

[19]Model-simulated log GDPs are calculated from 20,000 draws.

respectively. As seen in the figure, a greater standard deviation along with a more negative skewness of log GDP indicates negative fluctuations are relatively more pronounced in year 1982, than in 2002. In addition, in the Cobb-Douglas model the skewness is nearly zero in both years which is intuitive as with linearity, on average negative shocks should be offset by positive ones.

5.2.2 The Effect of Financial Shocks

As documented in Section 2.4, Finance & Insurance industry has become a more important input-supplier in 2002 than the past years, as its centrality level increased substantially over time. Intuitively, the aggregate impact is expected to be stronger, if an sectoral productivity shock hits an industry that is more central in the production network. As the industry with higher centrality has wider direct and indirect connections in the input-output network, the effect is more likely to be spread to other sectors, then transformed into macroeconomic fluctuations. However, in a Cobb-Douglas world, the only determinant of macroeconomic fluctuation is each industry's Domar weight. In other words, as long as one sector maintains its sales to GDP ratio (Domar weight) over time, the aggregate impact will always be identical, regardless of changes in its centrality. Therefore, in this section, I want to examine quantitatively whether the effect of negative productivity shocks to the Finance industry in 2002 would be transferred into a greater GDP declines at the aggregate level compared to the early years.

To gauge distinctions in the macroeconomic effect of financial shocks[20] in various input-output networks, I first calibrate the size of productivity shocks using the changes in quantity of production[21] in the Finance & Insurance sector. In particular, I demean the log growth rate of gross output, and construct TFP shocks to be the cumulative change in demeaned growth rate from 2007 to 2009, which yields a one-time shock of -11.7%. For the benchmark calibration, financial shocks reduce GDP by 3.5% via the production network in 1982, while within 2002's input-output network, the declines in aggregate output is 5.4% ($> 3.5\%$). As the centrality of Finance & Insurance increasing from 0.4 in 1982 to 0.6 in 2002, GDP fluctuation rises significantly across two years[22] subject to the same TFP shocks. This finding is consistent with my expectation.

To provide supportive evidence to assess the relative importance of an industry in a production network, I select the Wood Products industry as an example. The Wood Products industry has very similar centralities in both 1982 and 2002, which is around 0.017. With the same negative shocks (e.g. a one-time negative TFP shock of -11.7%), the reduction in GDP is almost identical in two years, which is about 0.3%.

I also compare the results of the nonlinear benchmark model in the Finance example to a loglinear Cobb-Douglas model. When I set $(\sigma, \theta, \varepsilon) = (0.99, 0.99, 0.99)$, the

[20]The financial shocks refer to idiosyncratic TFP shocks to the Finance & Insurance sector.

[21]For the quantity of production, I use the chain-type quantity indexes for gross output by industry from the BEA.

[22]Overall, as the centrality of Finance increasing by 50%, the impact on aggregate GDP rises by about 54%.

losses in aggregate output, subject to the same financial shocks calculated previously, are 3.2% and 5.1% based on the 1982 and 2002 input-output network, respectively. This outcome only differs from the benchmark one in the impact levels, which tells us that even with a Cobb-Douglas calibration, network structure might still play a significant role in determining aggregate impact of productivity shocks.

6 Conclusions

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A Theoretical Framework

In this section, I start by presenting a non-parametric general equilibrium model of production network, which will be used to demonstrate the Hulten's theorem and the second-order approximation of the impact of an idiosyncratic shock on the aggregate economy. This static model is also used and analysed by Baqaee and Farhi (2019).

A.1 The Hulten Theorem

Consider a static economy consisting of N competitive industries, each industry $i \in \{1, 2, ..., N\}$ produces a distinct good using different factors of production and intermediate inputs. The production function is given by

$$y_i = A_i F_i(l_{i1}, ..., l_{iF}, x_{i1}, ..., x_{iN})$$

where A_i is the Hicks-neutral technology, l_{if} is the type$-f \in \{1, 2, ..., F\}$ factor used by industry i, and x_{ij} are the intermediate goods purchased by i from its directed supplier j. The profits π_i earned by industry i are

$$\pi_i = p_i y_i - \sum_{f=1}^{F} w_f l_{if} - \sum_{j=1}^{N} p_j x_{ij}$$

with p_i being good i's price, and w_f denoting the wage of type$-f$ factor.

The economy is also populated with one representative household. The aggregate final demand of the economy Y is the maximized final demand of all consumption goods

$$Y = \max_{\{c_1, ..., c_N\}} D(c_1, ..., c_N)$$

subject to a budget constraint

$$\sum_{i=1}^{N} p_i c_i = \sum_{f=1}^{F} w_f \bar{l}_f + \sum_{i=1}^{N} \pi_i$$

where c_i is the final consumption of good i, and each type of factor $\bar{l}_f$ offered by the household is in fixed supply.

In a competitive equilibrium, all prices are given. The markets for each good i and each type of factor f clear at

$$y_i = \sum_{j=1}^{N} x_{ji} + c_i \quad and \quad \bar{l}_f = \sum_{i=1}^{N} l_{if}$$

Theorem 1 (The Hulten's Theorem) *In any efficient economy, the impact of an idiosyncratic shock to industry i on the aggregate economy is i's Domar weight up to a first-order approximation.*

$$\frac{d\ logY}{d\ logA_i} = \lambda_i \tag{11}$$

where $\lambda_i = p_i y_i / \sum_{j=1}^{N} p_j c_j$ *is i's total sales over GDP ratio, which is defined as the Domar weight of industry i.*

According to Hulten's Theorem, when the economy is efficient, Domar weight is a sufficient statistics in determining the impact of an industry-specific TFP shock on GDP. In other words, other microeconomic details, such as the structure of a production network, household's perefernce over consumption goods, and the elasticities of substitution in production, are irrelevant. However, it seems implausible when you consider the following example. Construction and Finance industries have very similar gross sales, so does the Domar weights (about 7 percent in 2010). If Hulten's Theorem holds true, industry-specific productivity shocks should generate analogous macroeconomic impacts. But in reality, as the Finance sector supplies to a wider range of sectors than Construction, the adverse effects caused by negative productivity shocks to Finance will be more damaging. This example emphasizes that an industry's network interconnection does matter in translating the impact of idiosyncratic shocks to aggregate outputs, which is missing from the first-order approximation.

A.2 Second-Order Aggregate Impact of Idiosyncratic Shocks

As discussed above, the first order approximation cannot capture all characteristics of the aggregate impact of idiosyncratic shocks. I now provide the features of macroeconomic effects up to a second-order approximation, which is proposed by Baqaee and Farhi (2019). Two definitions, the GE elasticity of substitution and the input-output multiplier, need to be introduced at first.

Definition 2 *For any smooth function $f : \mathbb{R}^N \to \mathbb{R}$, the pseudo elasticity of substitution is*

$$\frac{1}{\rho_{ji}} \equiv \frac{d\ log(MRS_{ji})}{d\ logA_i} = \frac{d\ log(f_j/f_i)}{d\ logA_i}$$

where MRS_{ji} is the ratio of partial derivatives with respect to A_j and A_i, and $f_i = df/dA_i$.

When the definition is applied to the equilibrium aggregate final demand function, we call the general equilibrium pseudo elasticity of substitution the *general equilibrium (GE) elasticity of substitution* for short. The GE elasticity of substitution is essential because it captures the changes in relative sales shares of industry i and j in response to an industry-specific TFP shock to i. Recall the Hulten's Theorem $\frac{d\ logY}{d\ logA_i} = \lambda_i$, then

$$\frac{d\ log(\lambda_i/\lambda_j)}{d\ logA_i} = \frac{d\ log[(\frac{Y_iA_i}{Y})/(\frac{Y_jA_j}{Y})]}{d\ logA_i} = 1 + \frac{d\ log(Y_i/Y_j)}{d\ logA_i} = 1 - \frac{1}{\rho_{ji}}.$$

When $\rho_{ji} \in (0,1)$, industry j is defined as a *GE complement* for industry i; while i and j are *GE substitutes* if $\rho_{ji} \in (1, \infty)$. Besides, if f is a Cobb-Douglas function, two industries are neither GE substitutes nor GE complements. The equation above implies that an increase in i's productivity will reduce the relative sales shares if i and j are GE complements.

Definition 3 *The input-output multiplier is defined as*

$$\xi \equiv \sum_{i=1}^{N} \frac{d\ logY}{d\ logA_i} = \sum_{i=1}^{N} \lambda_i.$$

The input-output multiplier implies the gross changes in GDP in response to a one-percent technological change in each individual industry. When $\xi > 1$, the summation of all industries' Domar weights exceeds one, indicating that some industries are producing and supplying intermediate goods to other industries.

Along with two definitions above, we can characterize the second-order aggregate impact of microeconomic shocks.

Theorem 2 (Second-Order Aggregate Impact of Idiosyncratic Shocks) *The second-order aggregate impact of idiosyncratic shocks is given by*

$$\frac{d^2\ logY}{d\ logA_i^2} = \frac{d\ \lambda_i}{d\ logA_i} = \frac{\lambda_i}{\xi} \sum_{\substack{1\leqslant j\leqslant N \\ j\neq i}} \lambda_j \left(1 - \frac{1}{\rho_{ji}}\right) + \lambda_i \frac{d\ log\xi}{d\ logA_i}. \tag{12}$$

The second-order approximation of industry-specific shocks to i on GDP is the changes in i's Domar weight relative to log TFP changes. More precisely, it depends on industry i's Domar weight λ_i, the input-output multiplier ξ, and the GE elasticity of substitution ρ_{ji}. On one hand, when production and consumption functions are Cobb-Douglas, $\rho_{ji} = 1$ for each j, the first term on the RHS of equation (2) becomes zero. One the other hand, if the input-output multiplier is independent of TFP shocks, $d\ log\xi / d\ logA_i$ turns to zero. If two assumptions are satisfied simultaneously, second-order terms are irrelevant and the Hulten's theorem is globally accurate. However, as long as the economy deviates from what described above, the second-order impact is nonzero, and the first-order approximation is no longer reliable.

To expand aggregate output (GDP) with respect to an idiosyncratic shock to industry i up to second order, it yields

$$\begin{aligned} log\ Y &\approx log\ \bar{Y} + \frac{d\ logY}{d\ logA_i} logA_i + \frac{1}{2} \frac{d^2\ logY}{d\ logA_i^2} (logA_i)^2 \\ &\approx log\ \bar{Y} + \lambda_i logA_i + \frac{1}{2} \frac{\lambda_i}{\xi} \sum_{\substack{1 \leqslant j \leqslant N \\ j \neq i}} \lambda_j \left(1 - \frac{1}{\rho_{ji}}\right) (logA_i)^2 + \frac{1}{2} \lambda_i \frac{d\ log\xi}{d\ logA_i} (logA_i)^2 \end{aligned}$$

where $\bar{Y}$ is the steady state value of Y. Define the equilibrium aggregate output as a function of exogenous technology levels, which is $Y(A_1, ...A_N)$, then the steady state $\bar{Y}$ is evaluated at $(A_1, ...A_N) = (1, ..., 1)$. Holding the input-output multiplier constant, when goods are GE complements, the second-order term amplifies negative shocks, but mitigates positive shocks. Instead, if goods are GE substitute, the second-order term works oppositely. Similarly, if the input-output multiplier increases, the second-order approximation will magnify positive shocks and attenuate negative shocks. But if the multiplier is decreasing, the second-order approximation will do the reverse.

In summary, the presence of the nonlinearity (a second-order approximation) to some extent fixes the disparity between Hulton's theorem and the intuition regarding to the importance of actual network structures. The impact of idiosyncratic shocks on aggregate economy will be amplified by the production network itself through a second-order approximation.