Assignment 1-2

Part A:

Bradley Schmidt (T00711584)

$$L(s) = \frac{(s^2+3s+10)(s+5)}{(s+3)(s+4)(s^2+2s+100)} = \frac{A}{s+3} + \frac{B}{s+4} + \frac{Cs+D}{s^2+2s+100}$$

$$(s^2+3s+10)(s+5) = A(s+4)(s^2+2s+100) + B(s+3)(s^2+2s+100) + (Cs+D)(s+3)(s+4)$$

$$(s^3+8s^2+25s+50) = A(s+4)(s^2+2s+100) + B(s+3)(s^2+2s+100) + (Cs+D)(s+3)(s+4)$$

RHS:

$$A(s^3+6s^2+108s+400) + B(s^3+5s^2+106s+300) + (Cs+D)(s^2+7s+12)$$

$$As^3+6As^2+108As+400A+Bs^3+5Bs^2+106Bs+300B) + Cs^3+7Cs^2+12Cs+Ds^2+7Ds+12D$$

$$s^3(A+B+C) + s^2(6A+5B+7C+D) + s(108A+106s+12C+7D) + (400A+300B+12D)$$

Equations:

$$1 = A+B+C$$

$$8 = 6A+5B+7C+D$$

$$25 = 108A+106B+12C+7D$$

$$50 = 400A+300B+12D$$

Solve for variables

$$\begin{bmatrix} 1 & 1A & 1B & 1C & 0D \\ 8 & 6A & 5B & 7C & 1D \\ 25 & 108A & 106B & 12C & 7D \\ 50 & 400A & 300B & 0C & 12D \end{bmatrix}$$

$$A = \frac{20}{103}, B = \frac{-7}{54}, C = \frac{5203}{5562}, D = \frac{2600}{2781}$$

$$L(s) = \frac{20}{103(s+3)} - \frac{7}{54(s+4)} + (Cs+D)\frac{1}{(s+1)^2+99}$$

$$Cs+D = C(s+1) - C + D$$

$$Cs+D = C(s+1) - \frac{1}{1852}$$

$$L(s) = \frac{20}{103(s+3)} - \frac{7}{54(s+4)} + \left(\frac{5203}{5562}\frac{s+1}{(s+1)^2+99} - \frac{1}{1852}\frac{1}{(s+1)^2+99} \right)$$

Un-laplace now:

$$F(t) = \frac{20}{103}e^{-3t} - \frac{7}{54}e^{-4t} + \frac{5203}{5562}e^{-t}\cos(\sqrt{99}t) - \frac{1}{1852}\frac{1}{\sqrt{99}}e^{-t}\sin(\sqrt{99}t)$$

Part B:

Bradley Schmidt (T00711584)

$$L(s) = \frac{s^3 + 4s^2 + 2s + 6}{(s+8)(s^2+8s+3)(s^2+5s+7)}$$

$$= \frac{A}{s+8} + \frac{Bs+C}{s^2+8s+3} + \frac{Ds+E}{s^2+5s+7}$$

$$= A(s^2+8s+3)(s^2+5s+7) + (Bs+C)(s+8)(s^2+5s+7) + (Ds+E)((s+8)(s^2+8s+3)$$

$$= A(s^4+13s^3+40s^2+71s+21) + (Bs+C)(s^3+13s^2+47s+56) + (Ds+E)(s^3+16s^2+67s+24)$$

$$= s^4(A+B+D) + s^3(13A+13B+C+16D+E) + s^2(50A+47B+13C+67D+16E) + s(71A+56B+47C+$$

Equations:

$$A = -\frac{266}{93}$$

$$0 = A + B + D$$

$$1 = 13A + 13B + C + 16D + E$$

$$4 = 50A + 47B + 13C + 67D + 16E$$

$$2 = 71A + 56B + 47C + 24D + 67E$$

$$6 = 21A + 56C + 24E$$

$$\begin{bmatrix} -\frac{266}{93} & A & 0B & 0C & 0D & 0E \\ 0 & A & B & C & 0D & 0E \\ 1 & 13A & 13B & C & 16D & 16E \\ 4 & 50A & 47B & 13C & 67D & 16E \\ 2 & 71A & 56B & 47C & 24D & 67E \\ 6 & 21A & 0B & 56C & 0D & 24E \end{bmatrix}$$

$$A = -\frac{266}{93}, B = \frac{1199}{417}, C = \frac{2091}{1729}, D = -\frac{170}{19083}, E = -\frac{40}{211}$$

$$= \frac{-266}{93}\frac{1}{s+8} + \left(\frac{1199}{417}s + \frac{2091}{1729}\right)\frac{1}{(s+4)^2-13} + \left(\frac{-170}{19083}s - \frac{40}{211}\right)\frac{1}{(s+2.5)^2+0.75}$$

$$= \frac{-266}{93}\frac{1}{s+8} + \left(Bs+C\right)\frac{1}{(s+4)^2-13} + \left(Ds-E\right)\frac{1}{(s+2.5)^2+0.75}$$

First Term:

$$-\frac{266}{93}e^{-8t}$$

Second Term:

$$\left(Bs+C\right)\frac{1}{(s+4)^2-13}$$

$$\frac{1199}{417}\frac{(s+4)}{(s+4)^2-13}+\frac{2091}{1729}\frac{1}{(s+4)^2-13}$$

$$Bs+C=B(s+4)-4B+C=\frac{1199}{417}(s+4)-\frac{1029}{100}$$

$$\frac{1199}{417}\cosh(\sqrt{13}t)+\frac{2091}{1729}\frac{\sinh(\sqrt{13}t)}{\sqrt{13}}$$

Third Term:

$$\left(Ds-E\right)\frac{1}{(s+2.5)^2+0.75}$$

$$Ds+E=D(s+2.5)-2.5D+E=$$

$$\left(-\frac{170}{19083}\right)\frac{s+2.5}{(s+2.5)^2+0.75}+\left(-\frac{40}{211}\right)\frac{1}{(s+2.5)^2+0.75}$$

$$-\frac{170}{19083}\cosh(\sqrt{0.75}t)-\frac{40}{211}\frac{\sinh(\sqrt{0.75}t)}{\sqrt{(0.75)}}$$

Final Answer:

$$f(t)=-\frac{266}{93}e^{-8t}+\frac{1199}{417}\cosh(\sqrt{13}t)+\frac{2091}{1729}\frac{\sinh(\sqrt{13}t)}{\sqrt{13}}-\frac{170}{19083}\cosh(\sqrt{0.75}t)-\frac{40}{211}\frac{\sinh(\sqrt{0.75}t)}{\sqrt{0.75}}$$