Assignment 4

1.

(a)

$$T(s) = \frac{6}{s+7}$$

$$t(t) = 6e^{7t}$$

(b)

$$T(s) = \frac{8}{(s+8)(s+9)} = \frac{8}{s^2+17s+72}$$

poles at s = -8, -9

$$s^2 + 17s + 72 = s^2 + 2\zeta\omega_n + \omega_n^2$$

$$2\zeta\omega_n = 17 \text{ and } \omega_n^2 = 72$$

$$\boxed{\omega_n = 8.485 \text{ and } \zeta = 1.002}$$

since the damping ratio is slightly greater than 1, the system is slightly overdamped.

(c)

$$T(s) = \frac{11(s+7)}{(s+10)(s+20)} = \frac{11s+77}{s^2+30s+200}$$

Poles at s = -10, -20
Zeros at s = -7

$$s^2 + 30s + 200 = s^2 + 2\zeta\omega_n s + \omega_n$$

$$2\zeta\omega_n = 30 \text{ and } \omega_n^2 = 200$$

$$\omega_n = 14.142 \text{ and } \zeta = 1.061$$

Since the damping ratio is slightly greater than 1, the system is slightly overdamped.

(d)

$$T(s) = \frac{21}{s^2 + 6s + 144}$$

Poles at s = (-3 + 11.619j), (-3 - 11.619j)

$$s^2 + 6s + 144 = s^2 + 2\zeta\omega_n s + \omega_n$$

$$2\zeta\omega_n = 6 \text{ and } \omega_n^2 = 144$$

$$\omega_n = 12 \text{ and } \zeta = 0.25$$

Since the damping ratio is less than 1, the system is underdamped.

(e)

$$T(s) = \frac{s + 2}{s^2 + 0s + 9}$$

Poles at s = 3j, -3j

$$s^2 + 0s + 9 = s^2 + 2\zeta\omega_n s + \omega_n$$

$$2\zeta\omega_n s = 0 \text{ and } \omega_n^2 = 9$$

$$\omega_n = 3 \text{ and } \zeta = 0$$

Since the damping frequency is less than 1, is is underdamped. Since it is 0, the damping frequency is equal to the natural frequency.

(f)

$$T(s) = \frac{s + 5}{(s + 10)^2} = \frac{s + 5}{s^2 + 20s + 100}$$

Poles at s = -10, -10

$$s^2 + 20s + 100 = s^2 + 2\zeta\omega_n s + \omega_n$$

$$2\zeta\omega_n = 20 \text{ and } \omega_n^2 = 100$$

$$\omega_n = 10 \text{ and } \zeta = 1$$

Since the damping ratio is equal to 1, it is critically damped.

2.

$$\zeta = 0.5, \omega_n = 100\frac{rad}{s}, dcgain = 1$$

$$T(s) = \frac{100^2}{s^2 + (2)(0.5)(100)s + 100^2}$$

$$= \frac{10000}{s^2 + 100s + 10000}$$

$$\text{Step input in laplace domain, } X(s) = \frac{1}{s}$$

$$\frac{Y(s)}{X(s)} = T(s) = \frac{10000}{s^2 + 100s + 10000}$$

$$Y(s) = X(s)T(s) = \frac{10000}{s(s^2 + 100s + 10000)}$$

$$Y(s) = \frac{10000}{s(s^2 + 100s + 10000)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 100 + 10000}$$

$$10000 = A(s^2 + 100s + 10000) + s(Bs + C)$$

$$10000 = As^2 + 100As + 10000A + Bs^2 + Cs$$

$$10000 = s^2(A + B) + s(100A + C) + 10000A$$

$$A + B = 0 \Rightarrow B = -A$$

$$100A + C = 0 \Rightarrow C = -100A$$

$$10000A = 10000 \Rightarrow A = 1$$

$$\boxed{A = 1, B = -1, C = -100}$$

$$Y(s) = \frac{1}{s} - \frac{s + 100}{s^2 + 100 + 10000}$$

$$Y(s) = \frac{1}{s} - \frac{s + 100}{(s + 50)^2 + 7500}$$

$$\text{Let u} = s + 50$$

$$Y(s) = \frac{1}{s} - \frac{u + 50}{u^2 + 7500}$$

$$Y(s) = \frac{1}{s} - \frac{u}{u^2 + 7500} - \frac{50}{u^2 + 7500}$$

$$Y(s) = \frac{1}{s} - \frac{u}{u^2 + 7500} - \frac{50}{\sqrt{7500}} \frac{\sqrt{7500}}{u^2 + 7500}$$

$$\boxed{y(t) = u(t) - e^{-50t}\left(\cos(\sqrt{7500}t) - \frac{50}{\sqrt{7500}}\sin(7500)\right)}$$

3.

(a)

$$T(s) = \frac{16}{s^2 + 3s + 16}$$

$$\omega_n = 4 \text{ and } \zeta = 0.375$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = 0.847$$

$$\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} * 100\% = 28.0\%$$

$$T_s = \frac{4}{\zeta\omega_n} = 2.666$$

$$T_r = \frac{\pi - \theta}{\sqrt{1 - \zeta^2}} = 0.536$$

(b)

$$T(s) = \frac{1.05 * 10^7}{s^2 + 1.6 * 10^3 s + 10^7}$$

$$\omega_n = 3162.278 \text{ and } \zeta = 0.253$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = 0.00103$$

$$\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} * 100\% = 44\%$$

$$T_s = \frac{4}{\zeta\omega_n} = 0.005$$

$$T_r = \frac{\pi - \theta}{\sqrt{1 - \zeta^2}} = 1.65$$

4.

(a)

$$\%OS = 13\% \text{ and } T_s = 0.8s$$

$$T_s = 0.8 = \frac{4}{\zeta\omega_n}$$

$$\boxed{\zeta\omega_n = 5}$$

$$OS = 0.13 = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$2.04 = \frac{\zeta\pi}{\sqrt{1 - \zeta^2}}$$

$$\boxed{0.649 = \frac{\zeta}{\sqrt{1-\zeta^2}}}$$

$$\zeta = 0.544 \text{ and } \omega_n = 9.19$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 10s + 84.46$$

Poles at s = (-5+7.711j), (-5-7.711j)

(b)

$$T_s = 6 \text{ and } T_p = 3$$

$$T_s = 6 = \frac{4}{\zeta\omega_n}$$

$$\zeta\omega_n = \frac{2}{3}$$

$$\boxed{\zeta - \frac{2}{3}\omega_n = 0}$$

$$T_p = 3 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$3\sqrt{1-\zeta^2} = \frac{\pi}{\omega_n}$$

$$9(1-\zeta^2) = \frac{\pi^2}{\omega_n^2}$$

$$9 - 9\zeta^2 = \frac{\pi^2}{\omega_n^2}$$

$$\zeta = 0.537 \text{ and } \omega_n = 1.241$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 1.33s + 1.54$$

$$\boxed{\text{Poles at } s = \left(-\frac{133}{200} + 1.048j\right), \left(-\frac{133}{200} - 1.048j\right)}$$

5.

$$T(s) = \frac{14.145}{(s^2 + 0.842s + 2.829)(s+5)} = \frac{14.145}{s^3 + 5.842s^2 + 7.052s + 14.145}$$

poles at s= -5, (-0.422-1.62j), (-0.411+1.62j)

$$\zeta\omega_n = 0.422$$

$$\omega_n\sqrt{1-\zeta^2} = 1.62$$

$$\omega_n = 1.682 \text{ and } \zeta = 0.25$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.93$$

$$\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} * 100\% = 44.4\%$$

$$T_s = \frac{4}{\zeta\omega_n} = 9.51$$

$$T_r = \frac{\pi-\theta}{\sqrt{1-\zeta^2}} = 1.88$$

$$K_p = \lim_{s\to 0}\frac{14.145}{s(s^2+5.842s+7.052)} = \propto$$

$$K_v = \lim_{s\to 0}\frac{14.145}{s^2+5.842s+7.052} = 2.01$$

$$K_a = \lim_{s\to 0}\frac{14.145s}{s^2+5.842s+7.052} = 0$$

$$e_{ss} = \frac{1}{1+K_p} = 0$$

$$e_{ss} = \frac{1}{K_v} = 0.498$$

$$e_{ss} = \frac{1}{K_a} = \propto$$